SUFFICIENT AND NECESSARY CONDITION FOR THE PERMANENCE OF PERIODIC PREDATOR-PREY SYSTEM

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We consider the permanence of a periodic predator-prey system, where the prey disperse in a two-patch environment. We assume the Volterra within-patch dynamics and provide a sufficient and necessary condition to guarantee the predator and prey species to be permanent by using the techniques of inequality analysis. Our work improves previous relevant results.

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1. Introduction. Dispersal predator-prey systems described by autonomous ordinary differential equations have long played an important role in population biology (see [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 20, 21, 22, 23, 24] and the references cited therein). Recently, Lou and Ma [15] studied the following predator-prey system in two-patch environment:

$$\begin{aligned}
\dot{x}_1 &= x_1(b_1 - a_1 x_1 - c_1 y) + D(x_2 - x_1), \\
\dot{x}_2 &= x_2(b_2 - a_2 x_2) + D(x_1 - x_2), \\
\dot{y} &= y(-d + c_2 x_1 - ly),
\end{aligned} \tag{1.1}$$

where $x_i(t)$ represents the prey population in the ith patch, $i = 1, 2$, at time $t \geq 0$, $y(t)$ stands for the predator population in patch 1 at time $t \geq 0$; coefficients a_i, b_i, c_i ($i = 1, 2$), d, l, and D are all positive constants. They proved that

$$-d + c_2 x_1^*(D) > 0 \tag{1.2}$$

is a necessary and sufficient condition of the strong persistence of system (1.1), where $(x_1^*(D), x_2^*(D))$ is the globally asymptotically stable equilibrium of the following system:

$$\begin{aligned}
\dot{x}_1 &= x_1(b_1 - a_1 x_1) + D(x_2 - x_1), \\
\dot{x}_2 &= x_2(b_2 - a_2 x_2) + D(x_1 - x_2).
\end{aligned} \tag{1.3}$$

Considering realistic models often requires the effects of the changing environment; we naturally expect that a similar condition should be selected for the permanence of the corresponding periodic predator-prey system,

$$
\begin{aligned}
\dot{x}_1 &= x_1[b_1(t) - a_1(t)x_1 - c_1(t)y] + D(t)(x_2 - x_1), \\
\dot{x}_2 &= x_2[b_2(t) - a_2(t)x_2] + D(t)(x_1 - x_2), \\
\dot{y} &= y[-d(t) + c_2(t)x_1 - l(t)y],
\end{aligned} \tag{1.4}
$$

under the assumptions that the functions $a_i(t)$, $b_i(t)$, $c_i(t)$ $(i = 1, 2)$, $D(t)$, $d(t)$, and $l(t)$ are all positive, ω-periodic, and continuous for $t \geq 0$.

Existing results on the permanence of system (1.4) have largely been restricted to some roughly sufficient conditions due to the increased complexity of global analysis for the nonautonomous systems (cf. Song and Chen [18]). The present paper provides a necessary and sufficient condition of the permanence of system (1.4) and removes some unnecessary conditions in [18].

The organization of this paper is as follows. In Section 2, we agree on some notations, give some definitions, and state three lemmas which will be essential to our proofs. In Section 3, by introducing the techniques found in [21], we obtain the necessary and sufficient condition which guarantees that system (1.4) is permanent.

2. Notations, definitions, and preliminaries. In this section, we introduce some definitions and notations and state some results which will be useful in subsequent sections. Let C denote the space of all bounded continuous functions $f : \mathbb{R} \to \mathbb{R}$, C_+^0 the set of nonnegative $f \in C$, and C_+ the set of all $f \in C$ such that f is bounded below by a positive constant. Given $f \in C$, we denote

$$
f^M = \sup_{t \geq 0} f(t), \qquad f^L = \inf_{t \geq 0} f(t), \tag{2.1}
$$

and define the lower average $A_L(f)$ and upper average $A_M(f)$ of f by

$$
\begin{aligned}
A_L(f) &= \lim_{r \to \infty} \inf_{t-s \geq r} (t-s)^{-1} \int_s^t f(\tau)d\tau, \\
A_M(f) &= \lim_{r \to \infty} \sup_{t-s \geq r} (t-s)^{-1} \int_s^t f(\tau)d\tau,
\end{aligned} \tag{2.2}
$$

respectively. If $f \in C$ is ω-periodic, we define the average $A_\omega(f)$ of f on the time interval $[0, \omega]$ by

$$
A_\omega(f) = \omega^{-1} \int_0^\omega f(t)dt. \tag{2.3}
$$

DEFINITION 2.1. The system of differential equations

$$
\dot{x} = F(t, x), \quad x \in \mathbb{R}^n, \tag{2.4}
$$

is said to be permanent if there exists a compact set K in the interior of $\mathbb{R}_+^n = \{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n : x_i \geq 0, i = 1, 2, \ldots, n\}$ such that all solutions starting in the interior of $\mathbb{R}_+^n$

ultimately enter and remain in K. The system is said to be strongly persistent if

$$\liminf_{t \to \infty} x_i(t) > 0, \quad i = 1, 2, \dots, n, \tag{2.5}$$

hold for all solutions $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$ starting in the interior of $\mathbb{R}^n_+$.

DEFINITION 2.2. The system of differential equations

$$\dot{x} = F(t, x), \quad (t, x) \in \mathbb{R} \times \mathbb{R}^n, \tag{2.6}$$

is said to be cooperative if the off-diagonal elements of $D_x F(t, x)$ are nonnegative and competitive if the off-diagonal elements are nonpositive, where $D_x F(t, x)$ is the $n \times n$ matrix derivative of F with respect to x.

LEMMA 2.3 [17]. *Let $x(t)$ and $y(t)$ be solution of*

$$\dot{x} = F(t, x), \qquad \dot{y} = G(t, y), \tag{2.7}$$

respectively, where both systems are assumed to have the uniqueness property for initial value problems. Assume both $x(t)$ and $y(t)$ belong to a domain $D \subset \mathbb{R}^n$ for $[t_0, t_1]$, in which one of the two systems is cooperative and

$$F(t, z) \leq G(t, z), \quad (t, z) \in [t_0, t_1] \times D. \tag{2.8}$$

If $x(t_0) \leq y(t_0)$, then $x(t_1) \leq y(t_1)$. If $F = G$ and $x(t_0) < y(t_0)$, then $x(t_1) < y(t_1)$.

To prove the permanence of the species in (1.4), we need the information on the periodic logistic models with and without dispersal.

LEMMA 2.4 [25]. *The problem*

$$\dot{x} = x[b(t) - a(t)x], \quad x \in C_+ \tag{2.9}$$

has exactly one canonical solution U if $a \in C_+$, $b \in C$, and $A_L(b) > 0$. Moreover, the following properties hold:
   (a) *U is ω-periodic (almost periodic) if a, b are ω-periodic (almost periodic);*
   (b) *U is constant if b/a is constant. In this case, $U = b/a$;*
   (c) *$u(t) - U(t) \to 0$ as $t \to \infty$, for any positive solution $u(t)$ of (2.9);*
   (d) *$(b/a)^L \leq U \leq (b/a)^M$.*

For the dispersal logistic equations

$$\begin{aligned}
\dot{x}_1 &= x_1[b_1(t) - a_1(t)x_1] + D(t)(x_2 - x_1), \\
\dot{x}_2 &= x_2[b_2(t) - a_2(t)x_2] + D(t)(x_1 - x_2),
\end{aligned} \tag{2.10}$$

we have the following result.

LEMMA 2.5 [16]. *Suppose that $b_i(t)$, $a_i(t)$ $(i = 1, 2)$, and $D(t)$ are all positive and ω-periodic functions; then (2.10) has a positive and ω-periodic solution $(x_1^*(t), x_2^*(t))$ which is globally asymptotically stable.*

3. Necessary and sufficient condition of permanence in (1.4)

THEOREM 3.1. *System (1.4) is permanent if and only if*

$$A_\omega \big[-d(t) + c_2(t)x_1^*(t) \big] > 0, \tag{3.1}$$

where $(x_1^*(t), x_2^*(t))$ is the globally asymptotically stable periodic solution of (2.10).

To prove this theorem, we need several propositions. In the rest of this paper, we denote by $(x_1(t), x_2(t), y(t))$ any solution of (1.4) with positive initial condition.

PROPOSITION 3.2. *There exist positive constants M_x and M_y such that*

$$\limsup_{t \to \infty} x_i(t) \le M_x, \quad \limsup_{t \to \infty} y(t) \le M_y, \quad i = 1, 2. \tag{3.2}$$

PROOF. Obviously, $\mathbb{R}_+^3$ is a positively invariant set of (1.4). Given any positive solution $(x_1(t), x_2(t), y(t))$ of (1.4), we have

$$\dot{x}_i \le x_i \big[b_i(t) - a_i(t)x_i \big] + D(t)(x_j - x_i), \quad i = 1, 2, \ j \ne i; \tag{3.3}$$

on the other hand, the auxiliary equations

$$\dot{u}_i = u_i \big[b_i(t) - a_i(t)u_i \big] + D(t)(u_j - u_i), \quad i = 1, 2, \ j \ne i, \tag{3.4}$$

have a unique globally asymptotically stable positive ω-periodic solution $(x_1^*(t), x_2^*(t))$. Let $(u_1(t), u_2(t))$ be the solution of (3.4) with $u_i(0) = x_i(0)$. By Lemma 2.3, we have

$$x_i(t) \le u_i(t), \quad i = 1, 2, \text{ for } t \ge 0. \tag{3.5}$$

Moreover, from the global stability of $(x_1^*(t), x_2^*(t))$, for every given $\varepsilon > 0$, there exists $T_0 > 0$ such that

$$u_i(t) < x_i^*(t) + \varepsilon \quad \text{for } t > T_0; \tag{3.6}$$

hence

$$x_i(t) < x_i^*(t) + \varepsilon, \quad i = 1, 2, \text{ for } t > T_0. \tag{3.7}$$

In addition, for $t \ge T_0$, we have

$$\dot{y} \le y \big[-d(t) + c_2(t)(x_1^*(t) + \varepsilon) - l(t)y \big]. \tag{3.8}$$

By (3.1), and Lemmas 2.3 and 2.4, there exists $T_1 > T_0$ such that

$$y(t) < y^*(t) + \varepsilon \quad \text{for } t > T_1, \tag{3.9}$$

where $y^*(t)$ is the positive and globally asymptotically stable ω-periodic solution of the auxiliary logistic equation

$$\dot{v} = v \big[-d(t) + c_2(t)(x_1^*(t) + \varepsilon) - l(t)v \big]. \tag{3.10}$$

Denote $M_x = \max_{0 \le t \le \omega}\{x_i^*(t) + \varepsilon : i = 1,2\}$ and $M_y = \max_{0 \le t \le \omega}\{y^*(t) + \varepsilon\}$; then (3.2) holds for system (1.4). □

PROPOSITION 3.3. *There exists a positive constant η_x such that*

$$\limsup_{t \to \infty} x_1(t) \ge \eta_x. \tag{3.11}$$

PROOF. Suppose that (3.11) is not true; then there is a sequence $\{z_m\} \subset \mathbb{R}_+^3$ such that

$$\limsup_{t \to \infty} x_1(t,z_m) < \frac{1}{m}, \quad m = 1,2,\ldots, \tag{3.12}$$

where $(x_1(t,z_m),x_2(t,z_m),y(t,z_m))$ is the solution of (1.4) with initial values $(x_1(0,z_m),x_2(0,z_m),y(0,z_m)) = z_m$. Choosing sufficiently small positive constants ε_x and ε_y such that $\varepsilon_x < 1$, $\varepsilon_y < 1$, and

$$A_\omega\big(-d(t) + c_2(t)\varepsilon_x\big) < 0, \tag{3.13}$$
$$A_\omega\big(\phi_\varepsilon(t)\big) > 0, \tag{3.14}$$

where $\phi_\varepsilon(t) = \min\{b_1(t) - c_1(t)\varepsilon_y \exp(\alpha\omega) - a_1(t)\varepsilon_x, b_2(t) - a_2(t)\varepsilon_x\}$, $\alpha = \max_{0 \le t \le \omega}\{|d(t)| + c_2(t) + l(t)\}$. By (3.12), for the given $\varepsilon_x > 0$, there exists a positive integer N_0 such that

$$\limsup_{t \to \infty} x_1(t,z_m) < \frac{1}{m} < \varepsilon_x \tag{3.15}$$

for $m > N_0$. In the rest of this proof, we always assume that $m > N_0$. The above inequality implies that there exists $\tau_1^{(m)} > 0$ such that

$$x_1(t,z_m) < \varepsilon_x \tag{3.16}$$

for $t \ge \tau_1^{(m)}$, and further

$$\dot{y}(t,z_m) \le y(t,z_m)\big[-d(t) + c_2(t)\varepsilon_x - l(t)y(t,z_m)\big] \tag{3.17}$$

for $t \ge \tau_1^{(m)}$. By (3.13), any solution $v(t)$ of the equation

$$\dot{v} = v\big[-d(t) + c_2(t)\varepsilon_x - l(t)v\big] \tag{3.18}$$

with positive initial condition satisfies

$$\lim_{t \to \infty} v(t) = 0. \tag{3.19}$$

By Lemma 2.3, we have

$$\lim_{t \to \infty} y(t,z_m) = 0. \tag{3.20}$$

Therefore, there is a $\tau_2^{(m)} > \tau_1^{(m)}$ such that

$$y(t,z_m) < \varepsilon_y \quad \text{for } t \geq \tau_2^{(m)}. \tag{3.21}$$

This leads to

$$\begin{aligned}
\dot{x}_1(t,z_m) &\geq x_1(t,z_m)[b_1(t) - c_1(t)\varepsilon_y - a_1(t)x_1(t,z_m)] \\
&\quad + D(t)(x_2(t,z_m) - x_1(t,z_m)), \\
\dot{x}_2(t,z_m) &= x_2(t,z_m)[b_2(t) - a_2(t)x_2(t,z_m)] + D(t)(x_1(t,z_m) - x_2(t,z_m))
\end{aligned} \tag{3.22}$$

for $t \geq \tau_2^{(m)}$. Let $(u_1(t),u_2(t))$ be any positive solution of the following auxiliary equations:

$$\begin{aligned}
\dot{u}_1 &= u_1[b_1(t) - a_1(t)u_1 - c_1(t)\varepsilon_y] + D(t)(u_2 - u_1), \\
\dot{u}_2 &= u_2[b_2(t) - a_2(t)u_2] + D(t)(u_1 - u_2).
\end{aligned} \tag{3.23}$$

By (3.14) and Lemma 2.5, (3.23) has a unique positive and ω-periodic solution $(u_1^*(t),u_2^*(t))$, which is globally asymptotically stable. So we have

$$x_i(t,z_m) > \frac{u_i^*(t)}{2}, \quad i = 1,2, \tag{3.24}$$

for sufficiently large $t > 0$ and $m > N_0$, which is a contradiction with (3.12). This completes the proof.   □

PROPOSITION 3.4. *There exists positive constants y_x such that*

$$\liminf_{t \to \infty} \rho_x(t) \geq y_x, \tag{3.25}$$

where $\rho_x(t) = x_1(t) + x_2(t)$.

PROOF. Suppose that (3.25) is not true; then there exists a sequence $\{z_m\} \subset \mathbb{R}_+^3$ such that

$$\liminf_{t \to \infty} \rho_x(t,z_m) < \frac{\eta_x}{2m^2}, \quad m = 1,2,\dots. \tag{3.26}$$

On the other hand, by Proposition 3.3, we have

$$\limsup_{t \to \infty} \rho_x(t,z_m) \geq \limsup_{t \to \infty} x_1(t,z_m) \geq \eta_x, \quad m = 1,2,\dots. \tag{3.27}$$

Hence there are two time sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$ satisfying the following conditions:

$$0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots, \quad s_q^{(m)} \longrightarrow \infty, \; t_q^{(m)} \longrightarrow \infty \; (q \longrightarrow \infty), \tag{3.28}$$

$$\begin{aligned}
\rho_x(s_q^{(m)},z_m) &= \frac{\eta_x}{m}, \qquad \rho_x(t_q^{(m)},z_m) = \frac{\eta_x}{m^2}, \\
\frac{\eta_x}{m^2} &< \rho_x(t,z_m) < \frac{\eta_x}{m}, \quad t \in (s_q^{(m)},t_q^{(m)}).
\end{aligned} \tag{3.29}$$

By Proposition 3.2, for a given integer $m > 0$, there is a $T_1^{(m)} > 0$ such that

$$x_i(t, z_m) \leq M_x, \quad y(t, z_m) \leq M_y, \quad \text{for } t \geq T_1^{(m)}, \ i = 1, 2. \tag{3.30}$$

Because of $s_q^{(m)} \to \infty$ as $q \to \infty$, there is a positive integer $K^{(m)}$ such that $s_q^{(m)} > T_1^{(m)}$ as $q \geq K^{(m)}$; hence

$$\begin{aligned}
\dot{x}_1(t, z_m) &\geq x_1(t, z_m)[b_1(t) - a_1(t)M_x - c_1(t)M_y] + D(t)(x_2(t, z_m) - x_1(t, z_m)), \\
\dot{x}_2(t, z_m) &\geq x_2(t, z_m)[b_2(t) - a_2(t)M_x] + D(t)(x_1(t, z_m) - x_2(t, z_m))
\end{aligned} \tag{3.31}$$

for $q \geq K^{(m)}$; so

$$\dot{\rho}_x(t, z_m) \geq \zeta(t)\rho_x(t, z_m) \tag{3.32}$$

for $q \geq K^{(m)}$, $t \in [s_q^{(m)}, t_q^{(m)}]$, where $\zeta(t) = \min\{b_1(t) - a_1(t)M_x - c_1(t)M_y, b_2(t) - a_2(t)M_x\}$. Integrating (3.32) from $s_q^{(m)}$ to $t_q^{(m)}$ yields

$$\rho_x(t_q^{(m)}, z_m) \geq \rho_x(s_q^{(m)}, z_m) \exp \int_{s_q^{(m)}}^{t_q^{(m)}} \zeta(t) dt \tag{3.33}$$

or

$$- \int_{s_q^{(m)}}^{t_q^{(m)}} \zeta(t) dt \geq \ln m \quad \text{for } q \geq K^{(m)}. \tag{3.34}$$

If $A_\omega(\zeta(t)) \geq 0$, this leads to a contradiction; otherwise, if $A_\omega(\zeta(t)) < 0$, we have

$$t_q^{(m)} - s_q^{(m)} \longrightarrow \infty \quad (m \longrightarrow \infty, \ q \geq K^{(m)}) \tag{3.35}$$

according to the boundedness of $\zeta(t)$. By (3.13) and (3.14), there are constants $P > 0$ and $N_0 > 0$ such that

$$\frac{\eta_x}{m} < \varepsilon_x, \qquad t_q^{(m)} - s_q^{(m)} > 2P, \tag{3.36}$$

$$M_y \exp \int_0^P [-d(t) + c_2(t)\varepsilon_x - l(t)\varepsilon_y] dt < \varepsilon_y, \qquad \int_0^a \phi_\varepsilon(t) dt > 0 \tag{3.37}$$

for $m \geq N_0$, $q \geq K^{(m)}$, and $a \geq P$. Inequality (3.36) implies

$$x_i(t, z_m) < \varepsilon_x, \quad i = 1, 2, \ t \in [s_q^{(m)}, t_q^{(m)}] \tag{3.38}$$

for $m \geq N_0$, $q \geq K^{(m)}$. For the positive ε_y satisfying (3.14) and (3.37), we have the following two circumstances:

(i) $y(t, z_m) \geq \varepsilon_y$ for all $t \in [s_q^{(m)}, s_q^{(m)} + P]$;

(ii) there exists $\tau_{q1}^{(m)} \in [s_q^{(m)}, s_q^{(m)} + P]$ such that $y(\tau_{q1}^{(m)}, z_m) < \varepsilon_y$.

If (i) holds, by (3.38) we have

$$
\begin{aligned}
\varepsilon_y &\le y\big(s_q^{(m)}+P,z_m\big) \\
&\le y\big(s_q^{(m)},z_m\big)\exp\int_{s_q^{(m)}}^{s_q^{(m)}+P}\big[-d(t)+c_2(t)\varepsilon_x-l(t)\varepsilon_y\big]dt \\
&\le M_y\exp\int_0^P\big[-d(t)+c_2(t)\varepsilon_x-l(t)\varepsilon_y\big]dt \\
&< \varepsilon_y,
\end{aligned}
\tag{3.39}
$$

which is a contradiction.

If (ii) holds, we now claim that

$$
y(t,z_m)\le\varepsilon_y\exp(\alpha\omega),\quad t\in(\tau_{q1}^{(m)},t_q^{(m)}].
\tag{3.40}
$$

Otherwise, there exists $\tau_{q2}^{(m)}\in(\tau_{q1}^{(m)},t_q^{(m)}]$ such that

$$
y(\tau_{q2}^{(m)},z_m)>\varepsilon_y\exp(\alpha\omega).
\tag{3.41}
$$

By the continuity of $y(t,z_m)$, there must exist $\tau_{q3}^{(m)}\in(\tau_{q1}^{(m)},\tau_{q2}^{(m)})$ such that

$$
\begin{aligned}
y(\tau_{q3}^{(m)},z_m)&=\varepsilon_y, \\
y(t,z_m)&>\varepsilon_y\quad\text{for }t\in(\tau_{q3}^{(m)},\tau_{q2}^{(m)}).
\end{aligned}
\tag{3.42}
$$

Denote by $P^{(m)}$ the nonnegative integer such that $\tau_{q2}^{(m)}\in(\tau_{q3}^{(m)}+P^{(m)}\omega,\tau_{q3}^{(m)}+(P^{(m)}+1)\omega]$. By (3.13), we obtain

$$
\begin{aligned}
\varepsilon_y\exp(\alpha\omega) &< y(\tau_{q2}^{(m)},z_m) \\
&< y(\tau_{q3}^{(m)},z_m)\exp\int_{\tau_{q3}^{(m)}}^{\tau_{q2}^{(m)}}\big[-d(t)+c_2(t)\varepsilon_x-l(t)\varepsilon_y\big]dt \\
&= \varepsilon_y\exp\left\{\int_{\tau_{q3}^{(m)}}^{\tau_{q3}^{(m)}+P^{(m)}\omega}+\int_{\tau_{q3}^{(m)}+P^{(m)}\omega}^{\tau_{q2}^{(m)}}\right\}\big[-d(t)+c_2(t)\varepsilon_x-l(t)\varepsilon_y\big]dt \\
&< \varepsilon_y\exp(\alpha\omega).
\end{aligned}
\tag{3.43}
$$

This contradiction establishes that (3.40) is true; particularly, (3.40) holds for $t\in[s_q^{(m)}+P,t_q^{(m)}]$. By (3.29) and (3.14), we have

$$
\begin{aligned}
\frac{\eta_x}{m^2} &= \rho_x(t_q^{(m)},z_m) \\
&\ge \rho_x\big(s_q^{(m)}+P,z_m\big)\exp\int_{s_q^{(m)}+P}^{t_q^{(m)}}\phi_\varepsilon(t)dt \\
&> \frac{\eta_x}{m^2},
\end{aligned}
\tag{3.44}
$$

which is also a contradiction. This completes the proof.   □

PROPOSITION 3.5. *There exist positive constants γ_{xi} $(i = 1, 2)$ such that*

$$\liminf_{t \to \infty} x_i(t) \geq \gamma_{xi} \quad (i = 1, 2). \tag{3.45}$$

PROOF. Inequality (3.25) implies that there exists $T_2 \geq T_1$ such that

$$\rho_x(t) = x_1(t) + x_2(t) \geq \gamma_x \quad \text{for } t \geq T_2. \tag{3.46}$$

Hence,

$$\begin{aligned}
\dot{x}_1 &= x_1[b_1(t) - 2D(t) - a_1(t)x_1 - c_1(t)y] + D(t)\rho_x(t) \\
&\geq -a_1^M x_1^2 + (b_1^L - 2D^M - c_1^M M_y)x_1 + D^L \gamma_x := F_1(x_1), \\
\dot{x}_2 &\geq -a_2^M x_2^2 + (b_2^L - 2D^M)x_2 + D^L \gamma_x := F_2(x_2)
\end{aligned} \tag{3.47}$$

for $t \geq T_2$. The algebraic equation $F_1(x_1) = 0$ gives us one positive root

$$\tilde{x}_1 = \frac{b_1^L - 2D^M - c_1^M M_y + \sqrt{(b_1^L - 2D^M - c_1^M M_y)^2 + 4D^L a_1^M \gamma_x}}{2a_1^M}. \tag{3.48}$$

Clearly, $F_1(x_1) > 0$ for every positive number x_1 $(0 < x_1 < \tilde{x}_1)$. Choose γ_{x1} $(0 < \gamma_{x1} < \tilde{x}_1)$, $\dot{x}_1|_{x_1 = \gamma_{x1}} \geq F_1(\gamma_{x1}) > 0$. If $x_1(T_2) \geq \gamma_{x1}$, then it also holds for $t \geq T_2$; if $x_1(T_2) < \gamma_{x1}$, then

$$\dot{x}_1(T_2) \geq \inf\{F_1(x_1) \mid 0 \leq x_1 < \gamma_{x1}\} > 0; \tag{3.49}$$

there exists $T_3(\geq T_2)$ such that $x_1(t) > \gamma_{x1}$ for $t \geq T_3$.

Similarly, there exists $\gamma_{x2} > 0$ and $T_4(\geq T_3)$ such that $x_2(t) > \gamma_{x2}$ for $t \geq T_4$. This completes the proof. $\square$

PROPOSITION 3.6. *Suppose that (3.1) holds. Then there exists a positive constant η_y such that*

$$\limsup_{t \to \infty} y(t) > \eta_y. \tag{3.50}$$

PROOF. By (3.1), we can choose constant $\varepsilon_0 > 0$ such that

$$A_\omega(\psi_{\varepsilon_0}(t)) > 0, \tag{3.51}$$

where

$$\psi_{\varepsilon_0}(t) = -d(t) + c_2(t)x_1^*(t) - c_2(t)\varepsilon_0 - l(t)\varepsilon_0. \tag{3.52}$$

Consider the following equations with parameter $\alpha(0 < \alpha < b_1^L/2c_1^M)$:

$$\begin{aligned}
\dot{x}_1 &= x_1[b_1(t) - 2\alpha c_1(t) - a_1(t)x_1] + D(t)(x_2 - x_1), \\
\dot{x}_2 &= x_2[b_2(t) - a_2(t)x_2] + D(t)(x_1 - x_2).
\end{aligned} \tag{3.53}$$

By Lemma 2.5, (3.53) has a unique positive ω-periodic solution $(x_{1\alpha}(t), x_{2\alpha}(t))$ which is globally asymptotically stable. Let $(\bar{x}_{1\alpha}(t), \bar{x}_{2\alpha}(t))$ be the solution of (3.53) with initial condition $\bar{x}_{i\alpha}(0) = x_i^*(0)$, $i = 1, 2$, where $(x_1^*(t), x_2^*(t))$ is the positive and ω-periodic solution of (2.10); then for the given ε_0, there exists $T_5 \geq T_4$ such that

$$\left| \bar{x}_{1\alpha}(t) - x_{1\alpha}(t) \right| < \frac{\varepsilon_0}{4} \quad \text{for } t \geq T_5. \tag{3.54}$$

By the continuity of solution to parameter α, we have $(\bar{x}_{1\alpha}(t), \bar{x}_{2\alpha}(t)) \to (x_1^*(t), x_2^*(t))$ uniformly in $[T_5, T_5 + \omega]$ as $\alpha \to 0$. Hence for $\varepsilon_0 > 0$, there exists positive $\alpha_0 = \alpha_0(\varepsilon_0) < b_1^L / 2c_1^M$ such that

$$\left| \bar{x}_{1\alpha}(t) - x_1^*(t) \right| < \frac{\varepsilon_0}{4} \quad \text{for } t \in [T_5, T_5 + \omega], \ 0 < \alpha < \alpha_0. \tag{3.55}$$

So we have

$$\left| x_{1\alpha}(t) - x_1^*(t) \right| \leq \left| \bar{x}_{1\alpha}(t) - x_{1\alpha}(t) \right| + \left| \bar{x}_{1\alpha}(t) - x_1^*(t) \right| < \frac{\varepsilon_0}{2} \tag{3.56}$$

for $t \in [T_5, T_5 + \omega]$. Since $x_{1\alpha}(t)$ and $x_1^*(t)$ are all ω-periodic, we have

$$\left| x_{1\alpha}(t) - x_1^*(t) \right| < \frac{\varepsilon_0}{2} \quad \text{for } t \geq 0, \ 0 < \alpha < \alpha_0. \tag{3.57}$$

Choosing constant α_1 ($0 < \alpha_1 < \alpha_0$, $2\alpha_1 < \varepsilon_0$), then

$$x_{1\alpha_1}(t) \geq x_1^*(t) - \frac{\varepsilon_0}{2}, \quad t \geq 0. \tag{3.58}$$

Suppose that the conclusion (3.50) is not true. Then there exists $Z \in \mathbb{R}_+^3$ such that for the positive solution $(x_1(t), x_2(t), y(t))$ of (1.4) with initial condition $(x_1(0), x_2(0), y(0)) = Z$, we have

$$\limsup_{t \to \infty} y(t) < \alpha_1. \tag{3.59}$$

So there exists $T_6 \geq T_5$ such that

$$y(t) < 2\alpha_1 \quad \text{for } t \geq T_6 \tag{3.60}$$

and hence,

$$\begin{aligned}
\dot{x}_1 &\geq x_1 \left[b_1(t) - 2\alpha_1 c_1(t) - a_1(t) x_1 \right] + D(t)(x_2 - x_1), \\
\dot{x}_2 &= x_2 \left[b_2(t) - a_2(t) x_2 \right] + D(t)(x_1 - x_2).
\end{aligned} \tag{3.61}$$

Let $(u_1(t), u_2(t))$ be the solution of (3.53) with $\alpha = \alpha_1$ and $u_i(T_6) = x_i(T_6)$, $i = 1, 2$. By Lemma 2.3, we know that

$$x_i(t) \geq u_i(t), \quad t \geq T_6, \ i = 1, 2. \tag{3.62}$$

By the globally asymptotically stability of $(x_{1\alpha_1}(t), x_{2\alpha_1}(t))$, for given $\varepsilon = \varepsilon_0/2$, there exists $T_7 \geq T_6$ such that

$$\left| u_1(t) - x_{1\alpha_1}(t) \right| < \frac{\varepsilon_0}{2} \quad \text{for } t \geq T_7. \tag{3.63}$$

So we have

$$x_1(t) \geq u_1(t) > x_{1\alpha_1}(t) - \frac{\varepsilon_0}{2}, \quad t \geq T_7, \tag{3.64}$$

and hence

$$x_1(t) > x_1^*(t) - \varepsilon_0, \quad t \geq T_7. \tag{3.65}$$

This implies

$$\dot{y}(t) \geq \psi_{\varepsilon_0}(t)y(t), \quad t \geq T_7. \tag{3.66}$$

Integrating the above inequality from T_7 to t yields

$$y(t) \geq y(T_7) \exp \int_{T_7}^t \psi_{\varepsilon_0}(t)dt. \tag{3.67}$$

By (3.51), we know that $y(t) \to \infty$ as $t \to \infty$, which is a contradiction. This completes the proof. □

PROPOSITION 3.7. *Under the assumption (3.1), there exists a positive constant y_y such that*

$$\liminf_{t \to \infty} y(t) \geq y_y. \tag{3.68}$$

PROOF. Otherwise, there must exist a sequence $\{z_m\} \subset \mathbb{R}_+^3$ such that

$$\liminf_{t \to \infty} y(t, z_m) < \frac{\eta_y}{(m+1)^2}, \quad m = 1, 2, \ldots. \tag{3.69}$$

But

$$\limsup_{t \to \infty} y(t, z_m) > \eta_y, \quad m = 1, 2, \ldots, \tag{3.70}$$

according to Proposition 3.6. Hence there are two time sequences $\{s_q^{(m)}\}$ and $\{t_q^{(m)}\}$ satisfying the following conditions:

$$\begin{aligned}
0 < s_1^{(m)} < t_1^{(m)} < s_2^{(m)} < t_2^{(m)} < \cdots < s_q^{(m)} < t_q^{(m)} < \cdots, \\
s_q^{(m)} \longrightarrow \infty, \quad t_q^{(m)} \longrightarrow \infty \text{ as } q \longrightarrow \infty, \\
y(s_q^{(m)}, z_m) = \frac{\eta_y}{m+1}, \qquad y(t_q^{(m)}, z_m) = \frac{\eta_y}{(m+1)^2}, \\
\frac{\eta_y}{(m+1)^2} < y(t, z_m) < \frac{\eta_y}{m+1}, \quad t \in (s_q^{(m)}, t_q^{(m)}).
\end{aligned} \tag{3.71}$$

By Proposition 3.2, for a given integer $m > 0$, there is a $T_1^{(m)} > 0$ such that

$$y(t, z_m) \leq M_y \quad \text{for } t \geq T_1^{(m)}. \tag{3.72}$$

Because $s_q^{(m)} \to \infty$ as $q \to \infty$, there is a positive integer $K^{(m)}$ such that $s_q^{(m)} > T_1^{(m)}$ as $q \geq K^{(m)}$; hence

$$\dot{y}(t, z_m) \geq y(t, z_m)\big[-d(t) - l(t)M_y\big] \tag{3.73}$$

for $q \geq K^{(m)}$, $t \in [s_q^{(m)}, t_q^{(m)}]$. Integrating the above inequality from $s_q^{(m)}$ to $t_q^{(m)}$, we get

$$y(t_q^{(m)}, z_m) \geq y(s_q^{(m)}, z_m) \exp \int_{s_q^{(m)}}^{t_q^{(m)}} \big[-d(t) - l(t)M_y\big]dt. \tag{3.74}$$

So we have

$$\int_{s_q^{(m)}}^{t_q^{(m)}} \big[d(t) + l(t)M_y\big]dt \geq \ln(m+1) \quad \text{for } q \geq K^{(m)}. \tag{3.75}$$

According to the boundedness of the function $d(t) + l(t)M_y$, we know that

$$t_q^{(m)} - s_q^{(m)} \longrightarrow \infty \quad \text{as } m \longrightarrow \infty, \, q \geq K^{(m)}. \tag{3.76}$$

By (3.51), there are constants $P > 0$ and an integer $N_0 > 0$ such that

$$\frac{\eta_y}{m+1} < \alpha_1 < \varepsilon_0, \qquad t_q^{(m)} - s_q^{(m)} > 2P, \tag{3.77}$$
$$\int_0^a \psi_{\varepsilon_0}(t)dt > 0$$

for $m \geq N_0$, $q \geq K^{(m)}$, and $a \geq P$. Further, we have

$$y(t, z_m) < \alpha_1, \quad t \in [s_q^{(m)}, t_q^{(m)}] \tag{3.78}$$

for $m \geq N_0$, $q \geq K^{(m)}$. In addition, for $t \in [s_q^{(m)}, t_q^{(m)}]$, we have

$$\begin{aligned}
\dot{x}_1(t, z_m) &\geq x_1(t, z_m)\big[b_1(t) - 2\alpha_1 c_1(t) - a_1(t)x_1(t, z_m)\big] \\
&\quad + D(t)(x_2(t, z_m) - x_1(t, z_m)), \\
\dot{x}_2(t, z_m) &= x_2(t, z_m)\big[b_2(t) - a_2(t)x_2(t, z_m)\big] + D(t)(x_1(t, z_m) - x_2(t, z_m)).
\end{aligned} \tag{3.79}$$

Let $(u_1(t), u_2(t))$ be the solution of (3.53) with $\alpha = \alpha_1$ and $u_i(s_q^{(m)}) = x_i(s_q^{(m)}, z_m)$. By Lemma 2.3, we have

$$x_i(t, z_m) \geq u_i(t), \quad t \in [s_q^{(m)}, t_q^{(m)}]. \tag{3.80}$$

Further, by Propositions 3.2, 3.5, and $s_q^{(m)} \to \infty$ as $q \to \infty$, we can choose $K_1^{(m)} > K^{(m)}$ such that

$$\gamma_{xi} \leq x_i(s_q^{(m)}, z_m) \leq M_x, \quad i = 1, 2, \tag{3.81}$$

for $q \geq K_1^{(m)}$. For $\alpha = \alpha_1$, (3.53) has a unique positive ω-periodic solution $(x_{1\alpha_1}(t), x_{2\alpha_1}(t))$ which is globally asymptotically stable. In addition, by the periodicity of (3.53),

the periodic solution $(x_{1\alpha_1}(t), x_{2\alpha_1}(t))$ is uniformly asymptotically stable with respect to the compact set $\Omega = \{(x_1, x_2) : \gamma_{xi} \le x_i \le M_x, i = 1, 2\}$. Hence, for the given ε_0 in Proposition 3.6, there exists $T_0(> P)$, which is independent of m and q, such that

$$u_1(t) \ge x_{1\alpha_1}(t) - \frac{\varepsilon_0}{2}, \quad t \ge T_0 + s_q^{(m)}. \tag{3.82}$$

Combining (3.58), we have

$$u_1(t) \ge x_1^*(t) - \varepsilon_0 \quad \text{for } t \ge T_0 + s_q^{(m)}. \tag{3.83}$$

From (3.76), there exists a positive integer $N_1 \ge N_0$ such that $t_q^{(m)} > s_q^{(m)} + 2T_0 > s_q^{(m)} + 2P$ for $m \ge N_1$ and $q \ge K_1^{(m)}$. So we have

$$x_1(t, z_m) \ge x_1^*(t) - \varepsilon_0, \quad t \in [s_q^{(m)} + T_0, t_q^{(m)}] \tag{3.84}$$

as $m \ge N_1$ and $q \ge K_1^{(m)}$. Hence,

$$\dot{y}(t, z_m) \ge \psi_{\varepsilon_0}(t) y(t, z_m) \tag{3.85}$$

for $t \in [s_q^{(m)} + T_0, t_q^{(m)}]$. Integrating the above inequality from $s_q^{(m)} + T_0$ to $t_q^{(m)}$ yields

$$y(t_q^{(m)}, z_m) \ge y(s_q^{(m)} + T_0, z_m) \exp \int_{s_q^{(m)} + T_0}^{t_q^{(m)}} \psi_{\varepsilon_0}(t) dt, \tag{3.86}$$

that is to say,

$$\frac{\eta_y}{(m+1)^2} \ge \frac{\eta_y}{(m+1)^2} \exp \int_{s_q^{(m)} + T_0}^{t_q^{(m)}} \psi_{\varepsilon_0}(t) dt > \frac{\eta_y}{(m+1)^2}, \tag{3.87}$$

which is a contradiction. This completes the proof. $\qquad\square$

Combining Propositions 3.2 to 3.6, we complete the proof of the sufficiency of Theorem 3.1.

To prove the necessity of Theorem 3.1, we will show that

$$\lim_{t \to \infty} y(t) = 0 \tag{3.88}$$

under the following condition:

$$A_\omega[-d(t) + c_2(t) x_1^*(t)] \le 0. \tag{3.89}$$

In fact, by (3.89), we know that for every given ε $(0 < \varepsilon < 1)$, there exists $\varepsilon_1 > 0$ and $\varepsilon_0 > 0$ such that

$$A_\omega[-d(t) + c_2(t)(x_1^*(t) + \varepsilon_1) - l(t)\varepsilon] \le \varepsilon_1 A_\omega(c_2(t)) - \varepsilon A_\omega(l(t)) \le -\varepsilon_0. \tag{3.90}$$

Since

$$\begin{aligned}
\dot{x}_1 &\le x_1[b_1(t) - a_1(t)x_1] + D(t)(x_2 - x_1), \\
\dot{x}_2 &= x_2[b_2(t) - a_2(t)x_2] + D(t)(x_1 - x_1),
\end{aligned} \tag{3.91}$$

we know that for the given ε_1, there exists $T^{(1)} > 0$ such that

$$x_1(t) \leq x_1^*(t) + \varepsilon_1 \quad \text{for } t \geq T^{(1)}. \tag{3.92}$$

By (3.90), we have

$$A_\omega\left[-d(t) + c_2(t)x_1(t) - l(t)\varepsilon\right] \leq -\varepsilon_0 \tag{3.93}$$

for $t \geq T^{(1)}$. Firstly, there must exist $T^{(2)}$ such that $y(T^{(2)}) < \varepsilon$. Otherwise, we have

$$\varepsilon \leq y(t) \leq y(T^{(1)}) \exp \int_{T^{(1)}}^t \left[-d(s) + c_2(s)x_1(s) - q(s)\varepsilon\right]ds \longrightarrow 0 \quad \text{as } t \longrightarrow \infty. \tag{3.94}$$

This implies $\varepsilon \leq 0$, which is a contradiction. Let $M(\varepsilon) = \max_{0 \leq t \leq \omega}\{d(t) + c_2(t)x_1(t) + l(t)\varepsilon\}$. By Proposition 3.2, we know that $x_1(t)$ is bounded. So $M(\varepsilon)$ is also bounded for $\varepsilon \in [0,1]$.

Secondly, we have

$$y(t) \leq \varepsilon \exp\left(M(\varepsilon)\omega\right) \quad \text{for } t \geq T^{(2)}. \tag{3.95}$$

Otherwise, there exists $T^{(3)} > T^{(2)}$ such that

$$y(T^{(3)}) > \varepsilon \exp\left(M(\varepsilon)\omega\right). \tag{3.96}$$

By the continuity of $y(t)$, there must exist $T^{(4)} \in (T^{(2)}, T^{(3)})$ such that $y(T^{(4)}) = \varepsilon$ and $y(t) > \varepsilon$ for $t \in (T^{(4)}, T^{(3)}]$. Let P_1 be the nonnegative integer such that $T^{(3)} \in (T^{(4)} + P_1\omega, T^{(4)} + (P_1 + 1)\omega]$. By (3.93), we have

$$
\begin{aligned}
\varepsilon \exp\left(M(\varepsilon)\omega\right) &< y(T^{(3)}) \\
&< y(T^{(4)}) \exp \int_{T^{(4)}}^{T^{(3)}} \left[-d(t) + c_2(t)x_1(t) - l(t)\varepsilon\right]dt \\
&= \varepsilon \exp\left\{\int_{T^{(4)}}^{T^{(4)} + P_1\omega} + \int_{T^{(4)} + P_1\omega}^{T^{(3)}}\right\}\left[-d(t) + c_2(t)x_1(t) - l(t)\varepsilon\right]dt \\
&< \varepsilon \exp\left(M(\varepsilon)\omega\right),
\end{aligned} \tag{3.97}
$$

which is a contradiction. This implies that (3.95) holds. Further by the arbitrariness of ε, we know that $y(t) \to 0$ as $t \to \infty$. This completes the proof.

Applying Theorem 3.1 to autonomous system (1.1) directly, we have the following corollary.

COROLLARY 3.8. *System (1.1) is permanent if and only if (1.2) holds.*

This corollary implies that the strong persistence is equivalent to the permanence for system (1.1), and hence improves the main result (cf. [15, Theorem 2]).

REMARK 3.9. In [18], Song and Chen obtained that if the following conditions

$$N_2^* := \frac{c_2^M}{l^L} \max\left\{ \frac{b_1^M}{a_1^L}, \frac{b_2^M}{a_2^L} \right\} < \frac{b_1^L}{c_1^M}, \tag{3.98}$$

$$m^* := \min\left\{ \frac{b_1^L - c_1^M N_2^*}{a_1^M}, \frac{b_2^L}{a_2^M} \right\} > \frac{d^M}{c_2^L}, \tag{3.99}$$

or

$$-d^M + c_2^L m^* > 0 \tag{3.100}$$

holds, then system (1.4) is permanent. According to [18, Theorem 1], we have $m^* \le x^*(t)$, and $A_\omega(-d(t) + c_2(t)x_1^*(t)) \ge -d^M + c_2^L m^*$. Hence (3.100) implies (3.1). Further, we give an example where condition (3.1) holds, but conditions (3.98) and (3.100) do not hold.

EXAMPLE 3.10. We consider the model

$$\dot{x}_1 = x_1\left[1 + \frac{1}{2}\sin t - x_1 - y \right] + \frac{1}{2}(x_2 - x_1),$$

$$\dot{x}_2 = x_2\left[1 - \frac{1}{2}\sin t - x_2 \right] + \frac{1}{2}(x_1 - x_2), \tag{3.101}$$

$$\dot{y} = y\left[-d_0 + \frac{1}{2}x_1 - y \right],$$

where d_0 is a positive number. By simple calculation, we have

$$N_2^* = \frac{3}{4}, \qquad m^* = -\frac{1}{4} < 0. \tag{3.102}$$

Hence (3.100) does not hold. We cannot get the permanence of (3.101) from the results of Song and Chen [18]. However, we can obtain its permanence according to our result. In fact, from Lemma 2.5, we know that the following system, without a predator,

$$\dot{x}_1 = x_1\left[1 + \frac{1}{2}\sin t - x_1 \right] + \frac{1}{2}(x_2 - x_1),$$

$$\dot{x}_2 = x_2\left[1 - \frac{1}{2}\sin t - x_2 \right] + \frac{1}{2}(x_1 - x_2) \tag{3.103}$$

has a positive periodic solution $(x_1^*(t), x_2^*(t))$ which is globally asymptotically stable. Denote $(2\pi)^{-1}\int_0^{2\pi} x_1^*(t)dt = l_0$. Then l_0 is positive and condition (3.1) holds for $d_0 < (1/2)l_0$. The permanence of (3.12) follows from Theorem 3.1, provided $d_0 < (1/2)l_0$.

REMARK 3.11. Xu, Chaplain, and Davidson studied a more general model than (1.4) (see [22]) and provided the existence, uniqueness, and global stability of periodic solutions of the more general periodic predator-prey system. Conditions for uniform persistence are also stated. We note that their condition (H5) in [22] does not hold for a weak

patchy environment (see [6]) in the sense that the intrinsic growth rate $b_i(t)$ may become negative on some time intervals. However, the discussion in this paper can be used to study the more reasonable weak patchy environment which is important for conservation of some endangered and rare species.

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