JORDAN AUTOMORPHISMS, JORDAN DERIVATIONS OF GENERALIZED TRIANGULAR MATRIX ALGEBRA

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We investigate Jordan automorphisms and Jordan derivations of a class of algebras called generalized triangular matrix algebras. We prove that any Jordan automorphism on such an algebra is either an automorphism or an antiautomorphism and any Jordan derivation on such an algebra is a derivation.

1. Introduction

Throughout this paper, let R be a 2-torsion-free commutative ring with identity 1. Consider an associative algebra A over R, then A can be viewed as a Jordan algebra with the usual product $x \circ y = (1/2)(xy + yx)$. An R-linear map $\delta : A \to A$ is called a derivation (resp., Jordan derivation) of A if

$$\delta(ab) = \delta(a)b + a\delta(b), \quad \forall a, b \in A \qquad (\text{resp., } \delta(a^2) = \delta(a)a + a\delta(a)\ \forall a \in A). \tag{1.1}$$

An R-linear map $\theta : A \to A$ is said to be a Jordan homomorphism of A if

$$\theta(a \circ b) = \theta(a) \circ \theta(b), \quad \forall a, b \in A, \qquad \text{or, equivalently, } \theta(a^2) = (\theta(a))^2, \quad \forall a \in A. \tag{1.2}$$

Derivations, Jordan derivations, as well as automorphisms and Jordan automorphisms of the algebra of triangular matrices and some class of their subalgebras have been the object of active research for a long time [1, 2, 5, 6, 9, 10].

A well-know result of Herstein [11] states that every Jordan isomorphism on a prime ring of characteristic different from 2 is either an isomorphism or an anti-isomorphism. We remark that the situation where the rings are semiprime rings does not hold. In the same time, he showed that every Jordan derivation on a prime ring of characteristic different from 2 is a derivation [12]. A brief proof of this result can be found in [4]. This result is extended by [3, 8] to the semiprime case.

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Let now $\mathcal{U}$ be the algebra of the form

$$\mathcal{U} = \begin{pmatrix} A & M \\ & B \end{pmatrix}, \tag{1.3}$$

where A and B are unital R-algebras and M is an (A,B)-bimodule. This algebra $\mathcal{U}$, endowed with the usual formal matrix addition and multiplication, will be called a generalized triangular matrix algebra. Many widely studied algebras, including upper-triangular matrix algebras, block-triangular matrix algebras, nest algebras, semi-nest algebras, and triangular Banach algebras, may be viewed as triangular algebras.

Khazal et al. [13] discuss the automorphism group of $\mathcal{U}$ so that A and B have only trivial idempotents. Cheung [7] gives sufficient conditions under which every Lie derivation is a sum of derivation on $\mathcal{U}$ and a mapping from $\mathcal{U}$ to its center.

In this paper, we consider linear operators on a class of algebras of the form $\mathcal{U}$; specifically, Jordan derivations and Jordan automorphisms. M is assumed to be faithful as a left A-module as well as a right B-module. We will prove that if both A and B have only trivial idempotents, any Jordan automorphism of the ring $\mathcal{U}$ is either an automorphism or an antiautomorphism, and we will prove that any Jordan derivation of such an algebra $\mathcal{U}$ is a derivation of $\mathcal{U}$.

2. The Jordan automorphism of generalized triangular matrix algebra

In this section, we suppose that $\mathcal{U}$ is the algebra of the form

$$\mathcal{U} = \begin{pmatrix} A & M \\ & B \end{pmatrix}, \tag{2.1}$$

where A and B are unital R-algebras and M is an (A,B)-bimodule, both A and B have only trivial idempotents.

This section is devoted to prove the following result.

Theorem 2.1. *If M is faithful as a left A-module as well as a right B-module and if both A and B have only trivial idempotents, then any Jordan automorphism θ of $\mathcal{U}$ is either an automorphism or an antiautomorphism.*

First, we start by recalling the next statement concerning the set of all idempotents of $\mathcal{U}$.

Lemma 2.2 [13]. *The idempotents of $\mathcal{U}$ are the elements of the forms $\begin{pmatrix} 0 & 0 \\ & 0 \end{pmatrix}$, $\begin{pmatrix} 1 & 0 \\ & 1 \end{pmatrix}$, $\begin{pmatrix} 1 & x \\ & 0 \end{pmatrix}$, and $\begin{pmatrix} 0 & x \\ & 1 \end{pmatrix}$, for any $x \in M$.*

We now introduce the notations $E_x = \begin{pmatrix} 1 & x \\ & 0 \end{pmatrix}$, $F_x = \begin{pmatrix} 0 & x \\ & 1 \end{pmatrix}$, and $X = \begin{pmatrix} 0 & x \\ & 0 \end{pmatrix}$, for some $x \in M$.

Then it is easy to check the following relations:

(i) $E_xE_y = E_y$, $F_xF_y = F_x$, $F_xE_y = 0$, $E_xF_y = \begin{pmatrix} 0 & x+y \\ & 0 \end{pmatrix}$;

(ii) $aE_0 = E_0 \circ (aE_0)$, $bF_0 = F_0 \circ (bF_0)$, $X = (2E_0) \circ X$, $X = (2F_0) \circ X$;

(iii) $2(aE_0) \circ X = \begin{pmatrix} 0 & ax \\ & 0 \end{pmatrix}$, $2(bF_0) \circ X = \begin{pmatrix} 0 & xb \\ & 0 \end{pmatrix}$.

On the other hand, if θ is a Jordan automorphism of $\mathcal{U}$, then either $\theta(E_0) = E_u$ or $\theta(E_0) = F_u$ for some $u \in M$, since E_0 is an idempotent.

The proof of Theorem 2.1 is an immediate consequence of the following two lemmas.

Lemma 2.3. *Assume that $\theta(E_0) = E_u$ for some $u \in M$, then θ is an automorphism of $\mathcal{U}$.*

Proof. Since $\theta(E_0) = E_u$, we have necessarily $\theta(F_0) = F_v$ for some $v \in M$. Indeed, if $\theta(F_0) = E_v$ for some $v \in M$, we obtain the contradiction $\theta(E_0 \circ F_0) \neq 0$. Now, by the relation $\theta(E_0 \circ F_0) = \theta(E_0) \circ \theta(F_0)$, we have $u + v = 0$, hence $\theta(F_0) = F_{-u}$. We observe that $W = \left(\begin{smallmatrix} 1 & -u \\ & 1 \end{smallmatrix}\right)$ is invertible of inverse $\left(\begin{smallmatrix} 1 & u \\ & 1 \end{smallmatrix}\right)$. So, one may consider the inner automorphism σ_W of $\mathcal{U}$ defined by $\sigma_W(Y) = WYW^{-1}$. It is not difficult to see that $\theta(E_0) = \sigma_W(E_0)$ and $\theta(F_0) = \sigma_W(F_0)$, which furnishes $\theta_1(E_0) = E_0$ and $\theta_1(F_0) = F_0$, where $\theta_1 = \sigma_{W^{-1}} \circ \theta$ is also a Jordan automorphism of $\mathcal{U}$.

By applying θ_1 to $aE_0 = E_0 \circ (aE_0)$ and $bF_0 = F_0 \circ (bF_0)$ for $a \in A$ and $b \in B$, we get that $\theta_1(aE_0) = \varphi_A(a)E_0$ and $\theta_1(bF_0) = \varphi_B(b)F_0$, where $\varphi_A : A \to A$ and $\varphi_B : B \to B$ are additive and bijective maps.

Now if we apply θ_1 to $(a^2E_0) = (aE_0)^2$ and $(b^2F_0) = (bF_0)^2$ for $a \in A$ and $b \in B$, we have $\varphi_A(a^2) = (\varphi_A(a))^2$ and $\varphi_B(b^2) = (\varphi_B(b))^2$, that is φ_A and φ_B are Jordan automorphisms of A and B, respectively.

Applying θ_1 to $X = 2E_0 \circ X$ yields $\theta_1(X) = \left(\begin{smallmatrix} 0 & f(x) \\ & 0 \end{smallmatrix}\right)$. It follows that

$$\theta_1 \begin{pmatrix} a & x \\ & b \end{pmatrix} = \begin{pmatrix} \varphi_A(a) & f(x) \\ & \varphi_B(b) \end{pmatrix}. \tag{2.2}$$

Applying again θ_1 to $\left(\begin{smallmatrix} 0 & ax \\ & 0 \end{smallmatrix}\right) = 2(aE_0) \circ X$ and $\left(\begin{smallmatrix} 0 & xb \\ & 0 \end{smallmatrix}\right) = 2(bF_0) \circ X$ for $a \in A$, for $x \in M$ and $b \in B$, we obtain $f(ax) = \varphi_A(a)f(x)$ and $f(xb) = f(x)\varphi_B(b)$.

Since $\theta = \sigma_W \circ \theta_1$, we deduce that

$$\theta \begin{pmatrix} a & x \\ & b \end{pmatrix} = \begin{pmatrix} \varphi_A(a) & f(x) + \varphi_A(a)u - u\varphi_B(b) \\ & \varphi_B(b) \end{pmatrix}, \tag{2.3}$$

where $\mathcal{U} \in M$, and $\varphi_A : A \to A$, $\varphi_B : B \to B$, $f : M \to M$ are maps satisfying that

(i) φ_A is a Jordan automorphism of A, $f(ax) = \varphi_A(a)f(x)$,

(ii) φ_B is a Jordan automorphism of B, $f(xb) = f(x)\varphi_B(b)$.

As a consequence, the following two identities are valid for all $a_1, a_2 \in A$ and $x \in M$:

$$f((a_1a_2)x) = \varphi_A(a_1a_2)f(x), \qquad f(a_1(a_2x)) = \varphi_A(a_1)f(a_2x) = \varphi_A(a_1)\varphi_A(a_2)f(x). \tag{2.4}$$

Thus,

$$\varphi_A(a_1a_2)f(x) = \varphi_A(a_1)\varphi_A(a_2)f(x). \tag{2.5}$$

Since M is faithful, we have $\varphi_A(a_1a_2) = \varphi_A(a_1)\varphi_A(a_2)$, which means that φ_A is an automorphism of A. Similarly, φ_B is an automorphism of B. Finally, in view of these arguments, one can easily check that θ is an automorphism of $\mathcal{U}$, which concludes the proof of the lemma. □

Lemma 2.4. *Assume that $\theta(E_0) = F_u$ for some $u \in M$, then θ is an antiautomorphism of $\mathcal{U}$.*

Proof. By hypothesis, we have $\theta(F_0) = F_v$ for some $v \in M$. Hence, it follows from the equality $\theta(E_0 \circ F_0) = \theta(E_0) \circ \theta(F_0)$ that $u + v = 0$, so $\theta(F_0) = E_{-u}$.

Applying θ to $aE_0 = E_0 \circ (aE_0)$ and $bF_0 = F_0 \circ (bF_0)$ for $a \in A$ and $b \in B$, we have

$$\theta(aE_0) = \begin{pmatrix} 0 & u\varphi_1(a) \\ & \varphi_1(a) \end{pmatrix}, \qquad \theta(bF_0) = \begin{pmatrix} \varphi_2(b) & -\varphi_2(b)u \\ & 0 \end{pmatrix}, \tag{2.6}$$

where $\varphi_1 : A \to B$ and $\varphi_2 : B \to A$ are obviously additive and bijective maps. In addition, by application of θ to $(a^2E_0) = (aE_0)^2$ and $(b^2F_0) = (bF_0)^2$ for $a \in A$ and $b \in B$, we can show by simple calculus that φ_1 and φ_2 are Jordan isomorphisms.

Applying now θ to $X = 2E_0 \circ X$ gives $\theta(X) = \begin{pmatrix} 0 & f(x) \\ & 0 \end{pmatrix}$.

Therefore,

$$\theta\begin{pmatrix} a & x \\ & b \end{pmatrix} = \begin{pmatrix} \varphi_2(b) & f(x) + u\varphi_1(a) - \varphi_2(b)u \\ & \varphi_1(a) \end{pmatrix}. \tag{2.7}$$

Applying θ to $2(aE_0) \circ X = \begin{pmatrix} 0 & ax \\ & 0 \end{pmatrix}$ and $2(bF_0) \circ X = \begin{pmatrix} 0 & xb \\ & 0 \end{pmatrix}$ for $a \in A$, $x \in M$, and $b \in M$, we have

$$f(ax) = f(x)\varphi_1(a), \qquad f(xb) = \varphi_2(b)f(x). \tag{2.8}$$

It follows that

$$\theta\begin{pmatrix} a & x \\ & b \end{pmatrix} = \begin{pmatrix} \varphi_2(b) & f(x) + u\varphi_1(a) - \varphi_2(b)u \\ & \varphi_1(a) \end{pmatrix}, \tag{2.9}$$

where $u \in M$ and $\varphi_2 : B \to A$, $\varphi_1 : A \to B$, $f : M \to M$ are maps satisfying that

(i) φ_A is an Jordan isomorphism on A into B and $f(ax) = f(x)\varphi_1(a)$,

(ii) φ_B is an Jordan isomorphism on B into A and $f(xb) = \varphi_2(b)f(x)$.

Hence, we have the following two identities:

$$f((a_1a_2)x) = f(x)\varphi_1(a_1a_2), \qquad f(a_1(a_2x)) = f(a_2x)\varphi_1(a_1) = f(x)\varphi_1(a_2)\varphi_1(a_1) \tag{2.10}$$

for any $a_1, a_2 \in A$ and $x \in M$. Consequently,

$$f(x)\varphi_1(a_1a_2) = f(x)\varphi_1(a_2)\varphi_1(a_1), \tag{2.11}$$

which shows that φ_1 is an anti-isomorphism from A onto B, since M is faithful. It is proved analogously that φ_2 is an anti- isomorphism from B onto A. Finally, the preceding arguments allows us to get by simple calculus that θ is an antiautomorphism of $\mathcal{U}$. This completes the proof of the lemma. □

3. The Jordan derivations of generalized triangular matrix algebra

In this section, we suppose that $\mathcal{U}$ is the algebra of the form

$$\mathcal{U}=\begin{pmatrix} A & M \\ & B \end{pmatrix}, \tag{3.1}$$

where A and B are unital R-algebras and M is an (A,B)-bimodule. The first principal result of this paper is the following.

THEOREM 3.1. *If M is faithful as a left A-module as well as a right B-module, then any Jordan derivation of $\mathcal{U}$ is an ordinary derivation.*

Before proving this theorem, we need to describe all Jordan derivations of $\mathcal{U}$.

LEMMA 3.2. *Every Jordan derivation ∂ of $\mathcal{U}$ is of the from*

$$\partial\begin{pmatrix} a & x \\ & b \end{pmatrix}=\begin{pmatrix} g_A(a) & au-ub+f(x) \\ & g_B(b) \end{pmatrix}, \tag{3.2}$$

where $u\in M$ and $g_A:A\to A$, $g_B:B\to B$, $f:M\to M$ are linear maps satisfying that
(i) *g_A is a Jordan derivation of A and $f(ax)=g_A(a)x+af(x)$,*
(ii) *g_B is a Jordan derivation of B and $f(xb)=xg_B(b)+f(x)b$.*

Proof. Write

$$\partial\begin{pmatrix} a & x \\ & b \end{pmatrix}=\begin{pmatrix} g_A(a)+h_B(b)+k_A(x) & f_A(a)+f_B(b)+f(x) \\ & h_A(a)+g_B(b)+k_B(x) \end{pmatrix}, \tag{3.3}$$

where $g_A:A\to A$, $h_B:B\to A$, $k_A:M\to A$, $f_A:A\to M$, $f_B:B\to M$, $f:M\to M$, $h_A:A\to B$, $g_B:B\to B$, and $k_B:M\to B$ are clearly linear maps.

Take $X=\begin{pmatrix} 1 & 0 \\ & 0 \end{pmatrix}$ and $Y=\begin{pmatrix} 0 & x \\ & 0 \end{pmatrix}$ in the equation

$$\partial(X\circ Y)=\partial(X)\circ Y+X\circ\partial(Y). \tag{3.4}$$

We have $\partial(X\circ Y)=(1/2)\begin{pmatrix} k_A(x) & f(x) \\ & k_B(x) \end{pmatrix}$, while

$$2\big(\partial(X)\circ Y+X\circ\partial(Y)\big)=\begin{pmatrix} 2k_A(x) & g_A(1)x+xh_A(1)+f(x) \\ & 0 \end{pmatrix}. \tag{3.5}$$

Hence, $k_A(x)=0$ and $k_B(x)=0$, which allow us to have

$$\partial\begin{pmatrix} a & x \\ & b \end{pmatrix}=\begin{pmatrix} g_A(a)+h_B(b) & f_A(a)+f_B(b)+f(x) \\ & h_A(a)+g_B(b) \end{pmatrix}. \tag{3.6}$$

Putting now $X=\begin{pmatrix} a & 0 \\ & 0 \end{pmatrix}$ and $Y=\begin{pmatrix} a' & 0 \\ & 0 \end{pmatrix}$, we obtain $\partial(X\circ Y)=\begin{pmatrix} g_A(a\circ a') & f_A(a\circ a') \\ & h_A(a\circ a') \end{pmatrix}$ and $\partial(X)\circ Y+X\circ\partial(Y)=\begin{pmatrix} g_A(a)\circ a'+a\circ g_A(a') & a'f_A(a)+af_A(a') \\ & 0 \end{pmatrix}$.

Then, $g_A(a\circ a')=g_A(a)\circ a'+a\circ g_A(a')$, that is, g_A is a Jordan derivation of A, and $h_A(a\circ a')=0$. Replacing a' by 1 in the last relation yields $h_A(a)=0$. Therefore,

$$\partial\begin{pmatrix} a & x \\ & b \end{pmatrix}=\begin{pmatrix} g_A(a)+h_B(b) & f_A(a)+f_B(b)+f(x) \\ & g_B(b) \end{pmatrix}. \tag{3.7}$$

Let now $X=\begin{pmatrix} 0 & 0 \\ & b \end{pmatrix}$ and $Y=\begin{pmatrix} 0 & 0 \\ & b' \end{pmatrix}$. Then $\partial(X\circ Y)=\begin{pmatrix} h_B(b\circ b') & f_B(b\circ b') \\ & g_B(b\circ b') \end{pmatrix}$ and

$$\partial(X)\circ Y+X\circ\partial(Y)=\begin{pmatrix} 0 & f_B(b)b'+f_B(b')b \\ & g_B(b)\circ b'+b\circ g_B(b') \end{pmatrix}, \tag{3.8}$$

showing that

$$g_B(b\circ b')=g_B(b)\circ b'+b\circ g_B(b'), \tag{3.9}$$

which means that g_B is a Jordan derivation of B, and $h_B(bb'+b'b)=0$. Substituting now $b'=1$ in the latter identity implies $h_B(b)=0$. Consequently, $\partial\begin{pmatrix} a & x \\ & b \end{pmatrix}=\begin{pmatrix} g_A(a) & f_A(a)+f_B(b)+f(x) \\ & g_B(b) \end{pmatrix}$.

We continue with the same method by taking $X=\begin{pmatrix} a & 0 \\ & b \end{pmatrix}$ and $Y=\begin{pmatrix} 1 & 0 \\ & 0 \end{pmatrix}$. It furnishes that $\partial(X\circ Y)=\begin{pmatrix} g_A(2a) & f_A(2a) \\ & 0 \end{pmatrix}$ and

$$\begin{aligned}\partial(X)\circ Y+X\circ\partial(Y)&=\begin{pmatrix} 2g_A(a)+g_A(1)a+ag_A(1) & f_A(a)+f_B(b)+f_A(1)b+af_A(1) \\ & 0 \end{pmatrix}\\ &=\begin{pmatrix} 2g_A(a) & f_A(a)+f_B(b)+f_A(1)b+af_A(1) \\ & 0 \end{pmatrix},\end{aligned} \tag{3.10}$$

since $g_A(1)=0$.

Hence $f_A(2a)=f_A(a)+f_B(b)+f_A(1)b+af_A(1)$, which implies clearly that $f_A(a)=af_A(1)$ and $f_B(b)+f_A(1)b=0$. So, $f_A(a)=au$ and $f_B(b)=-ub$, where $u=f_A(1)$.

It follows that

$$\partial\begin{pmatrix} a & x \\ & b \end{pmatrix}=\begin{pmatrix} g_A(a) & au-ub+f(x) \\ & g_B(b) \end{pmatrix}. \tag{3.11}$$

Now, if we take $X=\begin{pmatrix} a & 0 \\ & 0 \end{pmatrix}$ and $Y=\begin{pmatrix} 1 & x \\ & 0 \end{pmatrix}$, we find that

$$\begin{aligned}\partial(X\circ Y)&=\begin{pmatrix} g_A(2a) & 2au+f(ax) \\ & 0 \end{pmatrix},\\ \partial(X)\circ Y+X\circ\partial(Y)&=\begin{pmatrix} 2g_A(a) & g_A(a)x+2au+af(x) \\ & 0 \end{pmatrix},\end{aligned} \tag{3.12}$$

deducing the identity $f(ax)=g_A(a)x+af(x)$.

Finally, putting $X=\begin{pmatrix} 0 & 0 \\ & b \end{pmatrix}$ and $Y=\begin{pmatrix} 0 & x \\ & 0 \end{pmatrix}$ gives $\partial(XY+YX)=\begin{pmatrix} 0 & f(xb) \\ & 0 \end{pmatrix}$ and

$$\partial(X)\circ Y+X\circ\partial(Y)=\begin{pmatrix} 0 & xg_B(b)+f(x)b \\ & 0 \end{pmatrix}. \tag{3.13}$$

Hence, $f(xb) = xg_B(b) + f(x)b$, which ends the proof of the lemma. □

Now we are ready to establish our first principal theorem.

Proof of Theorem 3.1. Let ∂ be a Jordan derivation of $\mathcal{U}$, we have

$$\partial\begin{pmatrix} a & x \\ & b \end{pmatrix} = \begin{pmatrix} g_A(a) & au - ub + f(x) \\ & g_B(b) \end{pmatrix}, \tag{3.14}$$

where $u \in M$, $g_A : A \to A$, $g_B : B \to B$, and $f : M \to M$ are linear maps satisfying that

(i) g_A is a Jordan derivation of A, $f(ax) = g_A(a)x + af(x)$,

(ii) g_B is a Jordan derivation of B, $f(xb) = xg_B(b) + f(x)b$.

Hence, we have the following two identities:

$$\begin{gathered} f((aa')x) = g_A(aa')x + aa'f(x), \\ f(a(a'x)) = g_A(a)a'x + af(a'x) = g_A(a)a'x + ag_A(a')x + aa'f(x). \end{gathered} \tag{3.15}$$

As a consequence, we get that

$$g_A(aa')x = g_A(a)a'x + ag_A(a')x. \tag{3.16}$$

Since M is faithful, $g_A(aa') = g_A(a)a' + ag_A(a')$ and g_A is a derivation of A. A similar argument shows that g_B is a derivation of B.

Finally, one can now easily check that ∂ is a derivation of $\mathcal{U}$. Indeed, let $X = \begin{pmatrix} a & x \\ & b \end{pmatrix}$ and $Y = \begin{pmatrix} a' & x' \\ & b' \end{pmatrix}$ be arbitrary elements in $\mathcal{U}$. By straightforward computations, we have

$$\begin{gathered} \partial(XY) = \begin{pmatrix} g_A(aa') & aa'u - ubb' + f(ax' + xb') \\ & g_B(bb') \end{pmatrix}, \\ \partial(X)Y + X\partial(Y) = \begin{pmatrix} g_A(a)a' + ag_A(a') & aa'u - ubb' + g_A(a)x' + af(x') + xg_B(b') + f(x)b' \\ & g_B(b)b' + bg_B(b') \end{pmatrix}. \end{gathered} \tag{3.17}$$

This shows that $\partial(XY) = \partial(X)Y + X\partial(Y)$ and concludes the proof of the theorem. □

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