P-CLEAN RINGS

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In this paper we unify the structures of various clean rings by introducing the notion of P-clean rings. Some properties of P-clean rings are investigated, which generalize the known results on clean rings, semiclean rings, n-clean rings, and so forth. By the way, we answer a question of Xiao and Tong on n-clean rings in the negative.

1. Introduction

Throughout this paper R denotes an associative ring with identity and all modules are unitary. We use the symbol $U(R)$ to denote the group of units of R and $\mathrm{Id}(R)$ the set of idempotents of R, $U_n(R)$ the set of elements which are the sum of n units of R, $U_\Sigma(R)$ the set of elements each of which is the sum of finitely many units in R, $RE(R)$ ($URE(R)$) the set of regular (unit regular) elements of R, and $\mathrm{Peri}(R)$ the set of periodic elements of R. The Jacobson radical and the prime radical of R are denoted by $J(R)$ and $\mathrm{Nil}_*(R)$, respectively.

Following Han and Nicholson [4], an element x of a ring R is called clean if $x = e + u$ where $e \in \mathrm{Id}(R)$ and $u \in U(R)$. A ring R is clean if every element of R is clean. This notion was first introduced by Nicholson [5] as early as 1977 in his study of lifting idempotents and exchange rings. Since then, a great deal is known about clean rings and their generalizations (cf. [1–9]).

According to Ye [9], a ring R is called semiclean if every element of R has the form $x = f + u$, where $u \in U(R)$ and f is periodic, that is, $f^p = f^q$ for two different positive integers p and q . In [8], an element x of a ring R is called n-clean if $x = e + u_1 + \cdots + u_n$ where $e \in \mathrm{Id}(R)$, $u_i \in U(R)$, and n is a positive integer. The ring R is called n-clean if every element of R is n-clean for some fixed positive integer n. While R is called Σ-clean, if the n is a positive integer depending on x. Also Zhang and Tong in [10] defined R to be G-clean, if each $x \in R$ has the form $x = a + u$ where a is unit regular and $u \in U(R)$.

Motivated by the results of Han and Nicholson [4] on clean rings, Ye [9] on semiclean rings, Xiao and Tong [8] on n-clean rings and Σ-clean rings, and Zhang and Tong [10] on G-clean rings, in this paper we unify the structures of various clean rings by introducing

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the notion of P-clean rings and the common properties of those rings. By the way, we answer a question of Xiao and Tong [8] in the negative and extend some known results of [8, 9].

2. P-clean rings

We start this section by the following definitions.

For two subsets A and B of a ring R, the sum of A and B is defined as follows: $A + B = \{a + b \mid a \in A,\ b \in B\}$. The sum of more than two subsets of an R can be defined inductively.

Let P be a property which is meaningful for elements of a ring. For any ring R, let $P(R)$ be the subset $\{a \in R \mid a \text{ has property } P\}$ of R.

Definition 2.1. Property P will be called admissible if the following conditions are satisfied.

(1) For any ring homomorphism $\sigma: R \to S$, $\sigma(P(R)) \subseteq P(S)$.

(2) For any rings $R \subseteq S$, $P(R) \subseteq P(S)$.

(3) For any $e \in \mathrm{Id}(R)$, $P(eRe) + P((1-e)R(1-e)) \subseteq P(R)$.

For convenience, an element of $P(R)$ is called a P-element in R. In this paper P will always be an admissible property.

Proposition 2.2. (1) *If σ is a ring isomorphism from R onto S, then $\sigma(P(R)) = P(S)$.*

(2) *If $e_1, e_2, \ldots, e_n$ are orthogonal complete idempotents, that is, $e_ie_j = 0$ whenever $i \neq j$ and $e_i^2 = e_i$, and $e_1 + e_2 + \cdots + e_n = 1$, then $P(e_1Re_1) + P(e_2Re_2) + \cdots + P(e_nRe_n) \subseteq P(R)$.*

Proof. (1) By Definition 2.1, $\sigma(P(R)) \subseteq P(S)$, hence $\sigma^{-1}(\sigma(P(R))) \subseteq \sigma^{-1}(P(S)) \subseteq P(R)$. It follows that $P(R) \subseteq \sigma^{-1}(P(S)) \subseteq P(R)$, which gives $\sigma(P(R)) \subseteq P(S) \subseteq \sigma(P(R))$ and so $\sigma(P(R)) = P(S)$.

(2) We prove this by using induction on n. In fact, the case $n = 2$ is condition (3) of Definition 2.1. Assume (2) holds for $n-1$. Let $e_1 + e_2 + \cdots + e_{n-1} = f$. Then multiplied by e_i on the two sides of the above equation, we have $e_if = fe_i = e_i$, which gives $e_i = fe_if$ and so $e_i \in \mathrm{Id}(fRf)$. Note that fRf is a ring with identity f. It yields that $P(e_1Re_1) + P(e_2Re_2) + \cdots + P(e_{n-1}Re_{n-1}) \subseteq P(fRf)$ by inductive assumption. On the other hand, $f + e_n = 1$ implies $P(fRf) + P(e_nRe_n) \subseteq P(R)$ by Definition 2.1(3). Hence $P(e_1Re_1) + P(e_2Re_2) + \cdots + P(e_{n-1}Re_{n-1}) + P(e_nRe_n) \subseteq P(fRf) + P(e_nRe_n) \subseteq P(R)$. □

Definition 2.3. A ring R is called P-clean if every $x \in R$ has the form $x = p + u$, where $p \in P(R)$ and $u \in U(R)$.

Lemma 2.4. *Let R be a ring and $e \in \mathrm{Id}(R)$. Then the following hold.*

(1) *If $u \in U(eRe)$ and $v \in U((1-e)R(1-e))$, then $u + v \in U(R)$.*

(2) *If $e_1 \in \mathrm{Id}(eRe)$ and $e_2 \in \mathrm{Id}((1-e)R(1-e))$, then $e_1 + e_2 \in \mathrm{Id}(R)$.*

(3) *If $f \in \mathrm{Peri}(eRe)$ and $g \in \mathrm{Peri}((1-e)R(1-e))$, then $f + g \in \mathrm{Peri}(R)$.*

(4) *If $x \in RE(eRe)$ and $y \in RE((1-e)R(1-e))$, then $x + y \in RE(R)$.*

(5) *If $x \in URE(eRe)$ and $y \in URE((1-e)R(1-e))$, then $x + y \in URE(R)$.*

(6) *If $x \in U_n(eRe)$ and $y \in U_n((1-e)R(1-e))$, then $x + y \in U_n(R)$.*

(7) *If* $x \in \mathrm{Id}(eRe) + U_n(eRe)$ *and* $y \in \mathrm{Id}((1-e)R(1-e)) + U_n((1-e)R(1-e))$, *then* $x + y \in \mathrm{Id}(R) + U_n(R)$.

(8) *If* $x \in \mathrm{Id}(eRe) + U_\Sigma(eRe)$ *and* $y \in \mathrm{Id}((1-e)R(1-e)) + U_\Sigma((1-e)R(1-e))$, *then* $x + y \in \mathrm{Id}(R) + U_\Sigma(R)$.

Proof. We only prove (3) and (8), the others are very similar.

(3) Let $f \in \mathrm{Peri}(eRe)$ and $g \in \mathrm{Peri}((1-e)R(1-e))$. Then there exist positive integers $m > n$ and $p > q$ such that $f^m = f^n$ and $g^p = g^q$. By Ye [9, Lemma 5.2], $f^{n(m-n)}$ and $g^{q(p-q)}$ are both idempotents. Set $t = 2n(m-n)\ q(p-q)$. Then $f^{2t} = f^t$ and $g^{2t} = g^t$. Since $fg = gf = 0$, $(f+g)^{2t} = f^{2t} + g^{2t} = f^t + g^t = (f+g)^t$. Hence $f + g \subseteq \mathrm{Peri}(R)$.

(8) Assume $x \in \mathrm{Id}(eRe) + U_\Sigma(eRe)$ and $y \in \mathrm{Id}((1-e)R(1-e)) + U_\Sigma((1-e)R(1-e))$. Then $x = f + u_1 + \cdots + u_n$ and $y = g + v_1 + \cdots + v_m$ where $f \in \mathrm{Id}(eRe)$, $g \in \mathrm{Id}((1-e)R(1-e))$, $u_i \in U(eRe)$, and $v_j \in U((1-e)R(1-e))$. It is easy to show that an n-clean element is m-clean whenever $n \le m$, since for any $e \in \mathrm{Id}(R)$, $e = (1-e) + (2e-1)$ where $1 - e \in \mathrm{Id}(R)$ and $(2e-1)^2 = 1$. So without loss of generality, we can assume $n = m$. Using (1), $f + g \in \mathrm{Id}(R)$. And from (6), $u_i + v_i \in U(R)$, hence $x + y = (f+g) + (u_1 + v_1) + \cdots + (u_n + v_n) \in \mathrm{Id}(R) + U_\Sigma(R)$.

Using Lemma 2.4, it is easy to check that for any ring R, 0, R, $\mathrm{Id}(R)$, $\mathrm{Peri}(R)$, $U(R)$, $RE(R)$, $URE(R)$, $U_n(R)$, $\mathrm{Id}(R) + U_{n-1}(R)$ for $n \ge 2$, $\mathrm{Id}(R) + U_\Sigma(R)$ are all subsets of R defined by a suitable admissible property P. □

From the above arguments, the following proposition is immediate.

Proposition 2.5. *Let R be a ring. Then the following conclusions hold.*

(1) $\mathrm{Id}(R)$*-clean rings are precisely clean rings.*

(2) $Pri(R)$*-clean rings are precisely semiclean rings.*

(3) $U(R)$*-clean rings are precisely* $(S,2)$*-rings.*

(4) $Ure(R)$*-clean rings are precisely G-clean rings.*

(5) $\mathrm{Id}(R) + U_{n-1}(R)$*-clean rings are precisely n-clean rings when* $n \ge 2$.

(6) $\mathrm{Id}(R) + U_\Sigma(R)$*-clean rings are precisely* Σ*-clean rings.*

Note that here an $(S,2)$-ring is a ring in which every element can be expressed as a sum of two units of R. While in some literature it referred to a ring in which every element can be written as a sum of no more than two units.

Proposition 2.6. *Any homomorphic image of a P-clean ring is P-clean.*

Proof. Let R be a P-clean ring and let $f : R \to S$ be a ring surjective homomorphism. Then for any $y \in S$, there exists $x \in R$ such that $f(x) = y$. Since R is P-clean, $x = p + u$ with $p \in P(R)$ and $u \in U(R)$. Hence $f(x) = f(p) + f(u)$. Obviously $f(u) \in U(S)$ and $f(p) \in f(P) \subseteq P(S)$ by Definition 2.1, the proof is complete. □

Proposition 2.7. *A finite direct product* $R = \prod_{i=1}^n R_i$ *of rings* R_i *is P-clean if and only if each* R_i *is P-clean.*

Proof. If R is P-clean, then each R_i is P-clean by Proposition 2.6. Conversely, assume each R_i is P-clean, and $x = (x_i) \in R$. Then $x_i = p_i + u_i$ with $p_i \in P(R_i)$ and $u_i \in U(R_i)$

for each i. By Proposition 2.2, we can identify R_i with $(\dots,0,R_i,0,\dots)$ canonically. Let $e_i = (\dots,0,1,0,\dots)$. Then $(p_i) = (p_1,0,\dots,0) + (0,p_2,\dots,0) + \cdots + (0,0,\dots,p_n) \in P(e_1Re_1) + P(e_2Re_2) + \cdots + P(e_nPe_n) \subseteq P(R)$. Now $x = (x_i) = (p_i + u_i) = (p_i) + (u_i)$ with $(p_i) \in P(R)$ and $(u_i) \in U(R)$, so we are done. □

It should be noted that Proposition 2.7 is not true for an infinite direct product of rings R_i. For example, the ring $\mathbb{Z}$ of integers is a Σ-clean ring, but $R = \prod_{i=1}^{\infty} \mathbb{Z}$ is not Σ-clean since $(1,2,\dots,n,\dots)$ is obviously not Σ-clean.

Proposition 2.8. *The ring R is P-clean if and only if the ring $R[[x]]$ of formal power series over R is P-clean.*

Proof. If $R[[x]]$ is P-clean, then R is P-clean by Proposition 2.6. Now if R is P-clean, then for any $f(x) \in R[[x]]$, $f(x) = a_0 + a_1x + \cdots + a_nx^n + \cdots$. By assumption, $a_0 = p + u$ with $p \in P(R)$ and $u \in U(R)$. Hence $f(x) = p + u + a_1x + \cdots + a_nx^n + \cdots$ with $p \in P(R) \subseteq P(R[[x]])$ and $u + a_1x + \cdots + a_nx^n + \cdots \in U(R[[x]])$, as desired. □

The following corollary extends [8, Proposition 2.5] which states that for a commutative ring R, R is n-clean if and only if $R[[x]]$ is n-clean.

Corollary 2.9. *R is n-clean (Σ-clean) if and only if $R[[x]]$ is n-clean (Σ-clean).*

It has been proved by Han and Nicholson in [4] that if e is an idempotent in a ring R such that eRe and $(1-e)R(1-e)$ are both clean rings, then R is clean. Hence the ring of $n \times n$ matrices over R is clean. Similar results hold for semiclean rings, n-clean rings, and Σ-clean rings. We now extend these results to P-clean rings.

Lemma 2.10. *Let $e \in \mathrm{Id}(R)$ be such that eRe and $(1-e)R(1-e)$ are both P-clean rings. Then R is a P-clean ring.*

Proof. For convenience, write $\bar{r} = 1 - r$ for each $r \in R$. We use the Pierce decomposition of the ring R:

$$R = eRe + eR\bar{e} + \bar{e}Re + \bar{e}R\bar{e}. \tag{2.1}$$

Let $x = a + b + c + d$ where $a \in eRe$, $b \in eR\bar{e}$, $c \in \bar{e}Re$, and $d \in \bar{e}R\bar{e}$. By hypothesis, write $a = p + u$ where $p \in P(eRe)$ and $u \in U(eRe)$ with inverse u_1. Then $d - cu_1b \in \bar{e}R\bar{e}$, so write $d - cu_1b = q + v$ where $q \in P(\bar{e}R\bar{e})$ and $v \in U(\bar{e}R\bar{e})$ with inverse v_1. Hence $x = (p+q) + u + b + c + v + cu_1b$ and it suffices to show that $u + b + c + v + cu_1b$ is a unit in R. To this end compute

$$\begin{aligned}
&(u+b+c+v+cu_1b)(u_1+u_1bv_1cu_1-u_1bv_1-v_1cu_1+v_1)\\
&\quad= (e+bv_1cu_1-bv_1)+(-bv_1cu_1+bv_1)+(cu_1+cu_1bv_1cu_1-cu_1bv_1)\\
&\qquad+(-cu_1+1-e)+(-cu_1bv_1cu_1+cu_1bv_1)=1.
\end{aligned} \tag{2.2}$$

Similarly, $(u_1+u_1bv_1cu_1-u_1bv_1-v_1cu_1+v_1)(u+b+c+v+cu_1b)=1$.

Note that $p + q \in P(eRe) + P(\bar{e}R\bar{e}) \subseteq P(R)$ by Definition 2.1, the proof is complete. □

Using Lemma 2.10, an inductive argument gives immediately.

Theorem 2.11. *If $1 = e_1 + e_2 + \cdots + e_n$ in a ring R where e_i are orthogonal idempotents and each e_iRe_i is P-clean, then R is P-clean.*

The following two results are direct consequences of Theorem 2.11

Corollary 2.12. *If R is a P-clean ring, so also is the matrix ring $M_n(R)$.*

Corollary 2.13. *If $M = M_1 \oplus M_2 \oplus \cdots \oplus M_n$ are modules and end(M_i) is P-clean for each i, then end (M) is P-clean.*

Since any homomorphic image of a P-clean ring is again P-clean, with Theorem 2.11, this gives the following.

Corollary 2.14. *If A and B are rings and $V = {}_AV_B$ is a bimodule, the split-null extension R is P-clean if and only if both A and B are P-clean, where*

$$R = \begin{pmatrix} A & V \\ 0 & B \end{pmatrix}. \tag{2.3}$$

In particular, induction shows that for each $n \geq 1$, a ring R is P-clean if and only if the ring of all $n \times n$ upper triangular matrices over R is P-clean.

Let R be a ring and let I be an ideal of R. We say P-elements in R/I lift modulo I, if for any $p \in P(R/I)$ there exists $a \in P(R)$ such that $\pi(a) = p$ where π is the canonical ring homomorphism from R onto R/I.

We close this section with the following proposition whose proof is very easy.

Proposition 2.15. *Let R be a ring and let I be an ideal contained in $J(R)$. If R/I is a P-clean ring and P-elements lift modulo I, then R is a P-clean ring.*

3. Some remarks

It is known by [4, Proposition 6] that a ring R is clean if and only if R/I is clean for any ideal $I \subseteq J(R)$ and idempotents lift modulo I. Xiao and Tong [8] naturally claimed that they do not know whether for any n-clean ring R, idempotents of R/I lift modulo I where I is any ideal of R contained in $J(R)$. The following counterexample shows that the answer is negative.

Example 3.1. There is a 4-clean ring R in which idempotents of $R/J(R)$ cannot be lifted to R.

Proof. Let R be the subring of rational numbers Q given by $R = \{m/n \in Q \mid (m,n) = (n,6) = 1\}$. Then R has only two maximal ideals: $2R$ and $3R$, so $J(R) = 6R$. Denote the ring of integers modulo n by $\mathbb{Z}_n$, then $R/J(R) \cong \mathbb{Z}_2 \times \mathbb{Z}_3$, which has four idempotents. But R has only two idempotents. This shows that idempotents of $R/J(R)$ cannot be lifted to R modulo $J(R)$. But it can be shown that R is a 4-clean ring.

Clearly, $x = m/n \in U(R)$ if and only if $(m,6) = (n,6) = 1$. Now for any $x \in R$, x has the form $x = 3^p2^qm/n$ where $(m,6) = (n,6) = 1$. If $p,q \geq 1$, then $3^p2^qm = (3^p2^q - 1 + 1)m = (3^p2^q - 1)\, m + m \in U_2(R)$, so $x \in U_2(R)$. If $x = 3^pm/n$ with $p \geq 1$ and $(m,6) = (n,6) = 1$, then $3^pm = (3^p - 2 + 2)m = (3^p - 2)m + m + m$, which implies $x \in U_3(R)$. Similarly, in the case of $x = 2^q m/n$ where $(m,6) = (n,6) = 1$ and $q \geq 1$, then $2^q = 2^q - 3 + 3 = 2^q - 3 + 1 + 1 + 1$. It follows that $x \in U_4(R)$. Since any n-clean element must be m-clean for any $n \leq m$ (cf. the proof of Lemma 2.4(8)), R is a 4-clean ring from the above arguments. This ring R is clearly a k-clean for any $k \geq 4$ as the proof shows. □

The following two results are obtained by Xiao and Tong [8] for commutative rings, now we extend them to 2-primal rings (rings whose prime radical coincides with the set of nilpotent elements).

Proposition 3.2. *For any 2-primal ring R, the polynomial ring $R[x]$ is not Σ-clean.*

Proof. Assume the contrary, then $x = e(x) + u_1(x) + u_2(x) + \cdots + u_n(x)$ where $e(x) \in \mathrm{Id}(R[x])$, $u_i(x) \in U(R[x])$ for each i, and n is a positive integer. Let

$$\begin{aligned} e(x) &= e_0 + a_1 x + \cdots + a_m x^m, \\ u_1(x) &= u_{10} + u_{11} x + \cdots + u_{1m} x^m, \\ &\vdots \\ u_n(x) &= u_{n0} + u_{n1} x + \cdots + u_{nm} x^m. \end{aligned} \tag{3.1}$$

Since R is 2-primal, a polynomial over R is invertible if and only if its constant term is in $U(R)$ and the other coefficients are in $\mathrm{Nil}_*(R)$ by [3, Theorem 2.4], so $u_{ij} \in \mathrm{Nil}_*(R)$ for each $j \geq 1$. Hence $x = e(x) + u_1(x) + \cdots + u_n(x)$ gives $a_1 + u_{11} + \cdots + u_{n1} = 1$, so a_1 is a unit in R, and $a_2 + u_{12} + \cdots + u_{n2} = 0$ implies $a_2 \in \mathrm{Nil}_*(R)$. On the other hand, $e(x)^2 = e(x)$ implies $e_0^2 = e_0$, and $e(x)^2 = e_0 + (e_0 a_1 + a_1 e_0)\, x + (e_0 a_2 + a_1^2 + a_2 e_0)\, x^2 + \cdots + a_m^2 x^{2m}$. So $a_2 = e_0 a_2 + a_1^2 + a_2 e_0$ by comparing the coefficient of x^2 in $e(x)^2 = e(x)$. Note that the sum of a unit and a nilpotent element must be a unit and $e_0 a_2 + a_2 e_0 \in \mathrm{Nil}_*(R)$. It follows that $a_2 \in U(R)$. This is a contradiction, and the proof is complete. □

From Proposition 3.2, the following corollary is immediate.

Corollary 3.3. *For any 2-primal ring R, the polynomial ring $R[x]$ is not n-clean.*

We conclude this paper with the following proposition.

Proposition 3.4. *For any 2-primal ring R, the polynomial ring $R[x]$ is not semiclean.*

Proof. Assume the contrary, then $x = p(x) + u(x)$ where $p(x)$ is a periodic element and $u(x)$ is a unit. Let $p(x) = p_0 + p_1 x + \cdots + p_n x^n$ and $u(x) = u_0 + u_1 x + \cdots + u_n x^n$. Since R is 2-primal, $u_i \in \mathrm{Nil}_*(R)$ for each $i \geq 1$ by [3, Theorem 2.4]. By comparing the coefficient of $x = p(x) + u(x)$, we have $p_1 + u_1 = 1$, which implies p_1 is a unit in R, and $p_i + u_i = 0$ gives $p_i \in \mathrm{Nil}_*(R)$ for each $i \geq 2$. Clearly we can assume that $p(x)^s = p(x)^t$ for positive integers $s > t \geq 2$. Then a routine calculation shows that the coefficient of x^s in $p(x)^s$ is

$$\sum_{i_1 + i_2 + \cdots + i_s = s} p_{i_1} p_{i_2} \ldots p_{i_s} = p_1^s + a \quad \text{for some } a \in \mathrm{Nil}_*(R). \tag{3.2}$$

While the coefficient of x^s in $p(x)^t$ is

$$\sum_{j_1+j_2+\cdots+j_t=s} p_{j_1}p_{j_2}\dots p_{j_t}=b \quad \text{for some } b\in \mathrm{Nil}_*(R). \tag{3.3}$$

Comparing the coefficients of x^s on two sides of $p(x)^s = p(x)^t$, we have $p_1 \in \mathrm{Nil}_*(R)$, which is a contradiction. □

The above result is obtained by Ye [9] only for a commutative ring.

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