Hindawi Publishing Corporation
International Journal of Mathematics and Mathematical Sciences
Volume 2012, Article ID 528570, 10 pages
doi:10.1155/2012/528570

Research Article

Neighborhoods of Certain Multivalently Analytic Functions

Serap Bulut

Civil Aviation College, Kocaeli University, Arslanbey Campus, 41285 İzmit-Kocaeli, Turkey

Correspondence should be addressed to Serap Bulut, serap.bulut@kocaeli.edu.tr

Received 28 March 2012; Accepted 16 May 2012

Academic Editor: Hernando Quevedo

We introduce and investigate two new general subclasses of multivalently analytic functions of complex order by making use of the familiar convolution structure of analytic functions. Among the various results obtained here for each of these function classes, we derive the coefficient bounds, distortion inequalities, and other interesting properties and characteristics for functions belonging to the classes introduced here.

1. Introduction and Definitions

Let $\mathbb{R} = (-\infty, \infty)$ be the set of real numbers, and let $\mathbb{C}$ be the set of complex numbers,

$$\begin{aligned} \mathbb{N} &:= \{1, 2, 3, \ldots\} = \mathbb{N}_0 \setminus \{0\}, \\ \mathbb{N}^* &:= \mathbb{N} \setminus \{1\} = \{2, 3, 4, \ldots\}. \end{aligned} \tag{1.1}$$

Let $\mathcal{T}_p$ denote the class of functions of the form

$$f(z) = z^p - \sum_{j=k}^{\infty} a_j z^j \quad (p < k; a_j \geq 0 \ (j \geq k); k, p \in \mathbb{N}), \tag{1.2}$$

which are analytic and p-valent in the open unit disk

$$\mathbb{U} = \{z : z \in \mathbb{C}, |z| < 1\}. \tag{1.3}$$

Denote by $f * g$ the Hadamard product (or convolution) of the functions f and g, that is, if f is given by (1.2) and g is given by

$$g(z) = z^p + \sum_{j=k}^{\infty} b_j z^j \quad (p < k; b_j \geq 0 \ (j \geq k); k, p \in \mathbb{N}), \tag{1.4}$$

then

$$(f * g)(z) := z^p - \sum_{j=k}^{\infty} a_j b_j z^j =: (g * f)(z). \tag{1.5}$$

In [1], the author defined the following general class.

Definition 1.1. Let the function $f \in \mathcal{T}_p$. Then we say that f is in the class $\mathcal{S}_g(p, k, \lambda, \mu, b, \beta, m, n)$ if it satisfies the condition

$$\left| \frac{1}{b} \left(\frac{z^n (\mathcal{F}_{\lambda,\mu} * g)^{(m+n)}(z)}{(\mathcal{F}_{\lambda,\mu} * g)^{(m)}(z)} - (p - m)_n \right) \right| < \beta,$$
$$(m + n < p < k;\ p, n \in \mathbb{N};\ m \in \mathbb{N}_0;\ b \in \mathbb{C} \setminus \{0\};\ 0 \leq \mu \leq \lambda \leq 1;\ 0 < \beta \leq 1; z \in \mathbb{U}), \tag{1.6}$$

where

$$\mathcal{F}_{\lambda,\mu}(z) = \lambda\mu z^2 f''(z) + (\lambda - \mu) z f'(z) + (1 - \lambda + \mu) f(z), \tag{1.7}$$

g is given by (1.4), and $(\nu)_n$ denotes the falling factorial defined as follows:

$$\begin{aligned} (\nu)_0 &= 1 =: \binom{\nu}{0}, \\ (\nu)_n &= \nu(\nu - 1) \cdots (\nu - n + 1) =: n! \binom{\nu}{n} \quad (n \in \mathbb{N}). \end{aligned} \tag{1.8}$$

Various special cases of the class $\mathcal{S}_g(p, k, \lambda, \mu, b, \beta, m, n)$ were considered by many earlier researchers on this topic of geometric function theory. For example, $\mathcal{S}_g(p, k, \lambda, \mu, b, \beta, m, n)$ reduces to the function class

(i) $\mathcal{S}_g^n(p, \lambda, b, \beta)$ for $m = 0$, $n = 1$, and $\mu = 0$, studied by Mostafa and Aouf [2],

(ii) $\mathcal{S}_g(p, k, b, m, n)$ for $\lambda = \mu = 0$, and $\beta = 1$, studied by Srivastava et al. [3],

(iii) $\mathcal{S}_g(p, n, b, m)$ for $n = 1$, $\lambda = \mu = 0$, and $\beta = 1$, studied by Prajapat et al. [4],

(iv) $\mathcal{S}_{n,p}(g; \lambda, \mu, \alpha)$ for $m = 0$, $n = 1$, $\beta = 1$, and $b = p(1 - \alpha)$ $(0 \leq \alpha < 1)$, studied by Srivastava and Bulut [5],

(v) $\mathcal{TS}_g^*(p, m, \alpha)$ for $m = 0$, $n = 1$, $\lambda = \mu = 0$, $\beta = 1$, and $b = p(1 - \alpha)$ $(0 \leq \alpha < 1)$, studied by Ali et al. [6].

Definition 1.2. Let the function $f \in \mathcal{T}_p$. Then we say that f is in the class $\mathcal{K}_g(p,k,\lambda,\mu,b,\beta,m,n;q,u)$ if it satisfies the following nonhomogenous Cauchy-Euler differential equation (see, e.g., [7, page 1360, Equation (9)] and [5, page 6512, Equation (1.9)]):

$$z^q \frac{d^q w}{dz^q} + \binom{q}{1}(u+q-1)z^{q-1}\frac{d^{q-1}w}{dz^{q-1}} + \cdots + \binom{q}{q} w \prod_{\varepsilon=0}^{q-1}(u+\varepsilon) = h(z)\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p), \tag{1.9}$$

where

$$w = f(z) \in \mathcal{T}_p; \quad h \in \mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n); \quad q \in \mathbb{N}^*, \quad u \in (-p,\infty). \tag{1.10}$$

Setting $m = 0$, $n = 1$, $\mu = 0$, and $q = 2$ in Definition 1.2, we have the special class introduced by Mostafa and Aouf [2].

Following the works of Goodman [8] and Ruscheweyh [9] (see also [10, 11]), Altıntaş [12] defined the δ-neighborhood of a function $f \in \mathcal{T}(p)$ by

$$\mathcal{N}_k^\delta(f) = \left\{ h \in \mathcal{T}_p : h(z) = z^p - \sum_{j=k}^{\infty} c_j z^j, \sum_{j=k}^{\infty} j\left|a_j - c_j\right| \le \delta \right\}. \tag{1.11}$$

It follows from the definition (1.11) that if

$$e(z) = z^p \quad (p \in \mathbb{N}), \tag{1.12}$$

then

$$\mathcal{N}_k^\delta(e) = \left\{ h \in \mathcal{T}_p : h(z) = z^p - \sum_{j=k}^{\infty} c_j z^j, \sum_{j=k}^{\infty} j\left|c_j\right| \le \delta \right\}. \tag{1.13}$$

The main object of this paper is to investigate the various properties and characteristics of functions belonging to the above-defined classes

$$\mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n), \qquad \mathcal{K}_g(p,k,\lambda,\mu,b,\beta,m,n;q,u). \tag{1.14}$$

Apart from deriving coefficient bounds and distortion inequalities for each of these classes, we establish several inclusion relationships involving the δ-neighborhoods of functions belonging to the general classes which are introduced above.

2. Coefficient Bounds and Distortion Theorems

Lemma 2.1 (see [1]). *Let the function $f \in \mathcal{T}_p$ be given by* (1.2). *Then f is in the class $\mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n)$ if and only if*

$$\sum_{j=k}^{\infty}(j)_m\left[(j-m)_n-(p-m)_n+\beta|b|\right]\varphi(j)a_jb_j \le \beta|b|(p)_m\varphi(p),$$
$$\left(m+n<p<k;p,n\in\mathbb{N};m\in\mathbb{N}_0;b\in\mathbb{C}\setminus\{0\};0<\beta\le 1;z\in\mathbb{U}\right), \tag{2.1}$$

where

$$\varphi(s)=(s-1)(\lambda\mu s+\lambda-\mu)+1 \quad (0\le\mu\le\lambda\le 1). \tag{2.2}$$

Remark 2.2. If we set $m=0$, $n=1$, and $\mu=0$ in Lemma 2.1, then we have [2, Lemma 1].

Lemma 2.3 (See[1]). *Let the function $f \in \mathcal{T}_p$ given by* (1.2) *be in the class $\mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n)$. Then, for $b_j \ge b_k (j\ge k)$, one has*

$$\sum_{j=k}^{\infty}a_j \le \frac{\beta|b|(p)_m\varphi(p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\varphi(k)b_k}, \tag{2.3}$$

$$\sum_{j=k}^{\infty}ja_j \le \frac{(k-m)!\beta|b|(p)_m\varphi(p)}{(k-1)!\left[(k-m)_n-(p-m)_n+\beta|b|\right]\varphi(k)b_k} \quad (p>|b|), \tag{2.4}$$

where φ is defined by (2.2).

Remark 2.4. If we set $m=0$, $n=1$, and $\mu=0$ in Lemma 2.3, then we have [2, Lemma 2].

The distortion inequalities for functions in the class $\mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n)$ are given by the following Theorem 2.5.

Theorem 2.5. *Let a function $f \in \mathcal{T}_p$ be in the class $\mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n)$. Then*

$$|f(z)| \le |z|^p + \frac{\beta|b|(p)_m\varphi(p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\varphi(k)b_k}|z|^k, \tag{2.5}$$

$$|f(z)| \ge |z|^p - \frac{\beta|b|(p)_m\varphi(p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\varphi(k)b_k}|z|^k, \tag{2.6}$$

and in general

$$\left|f^{(r)}(z)\right| \leq (p)_r|z|^{p-r} + \frac{\beta|b|(p)_m(k)_r\varphi(p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\varphi(k)b_k}|z|^{k-r},$$
$$\left|f^{(r)}(z)\right| \geq (p)_r|z|^{p-r} - \frac{\beta|b|(p)_m(k)_r\varphi(p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\varphi(k)b_k}|z|^{k-r},$$
$$(p>r;\, r\in\mathbb{N}_0;\, z\in\mathbb{U}), \tag{2.7}$$

where φ *is defined by* (2.2).

Proof. Suppose that $f \in \mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n)$. We find from the inequality (2.3) that

$$\left|f(z)\right| \leq |z|^p + |z|^k\sum_{j=k}^{\infty}a_j \leq |z|^p + \frac{\beta|b|(p)_m\varphi(p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\varphi(k)b_k}|z|^k, \tag{2.8}$$

which is equivalent to (2.5) and

$$\left|f(z)\right| \geq |z|^p - |z|^k\sum_{j=k}^{\infty}a_j \geq |z|^p - \frac{\beta|b|(p)_m\varphi(p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\varphi(k)b_k}|z|^k, \tag{2.9}$$

which is precisely the assertion (2.6). □

If we set $m=0$, $n=1$, and $\mu=0$ in Theorem 2.5, then we get the following.

Corollary 2.6. *Let a function* $f \in \mathcal{T}_p$ *be in the class* $\mathcal{S}_g^n(p,\lambda,b,\beta)$*. Then*

$$\left|f(z)\right| \leq |z|^p + \frac{\beta|b|\left[1+\lambda(p-1)\right]}{\left[k-p+\beta|b|\right][1+\lambda(k-1)]b_k}|z|^k,$$
$$\left|f(z)\right| \geq |z|^p - \frac{\beta|b|\left[1+\lambda(p-1)\right]}{\left[k-p+\beta|b|\right][1+\lambda(k-1)]b_k}|z|^k, \tag{2.10}$$

and in general

$$\left|f^{(r)}(z)\right| \leq \frac{p!}{(p-r)!}|z|^{p-r} + \frac{k!\beta|b|\left[1+\lambda(p-1)\right]}{(k-r)!\left[k-p+\beta|b|\right][1+\lambda(k-1)]b_k}|z|^{k-r},$$
$$\left|f^{(r)}(z)\right| \geq \frac{p!}{(p-r)!}|z|^{p-r} - \frac{k!\beta|b|\left[1+\lambda(p-1)\right]}{(k-r)!\left[k-p+\beta|b|\right][1+\lambda(k-1)]b_k}|z|^{k-r},$$
$$(p>r;\, r\in\mathbb{N}_0;\, z\in\mathbb{U}), \tag{2.11}$$

where φ *is defined by* (2.2).

The distortion inequalities for functions in the class $\mathcal{K}_g(p,k,\lambda,\mu,b,\beta,m,n;q,u)$ are given by Theorem 2.7 below.

Theorem 2.7. *Let a function $f \in \mathcal{T}_p$ be in the class $\mathcal{K}_g(p,k,\lambda,\mu,b,\beta,m,n;q,u)$. Then*

$$\begin{aligned}
\left|f(z)\right| &\leq |z|^p \\
&\quad + \frac{\beta|b|(p)_m \psi(p) \prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\psi(k)(q-1)\prod_{\varepsilon=0}^{q-2}(u+\varepsilon+k)b_k}|z|^k,
\end{aligned} \tag{2.12}$$

$$\begin{aligned}
\left|f(z)\right| &\geq |z|^p \\
&\quad - \frac{\beta|b|(p)_m \psi(p) \prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\psi(k)(q-1)\prod_{\varepsilon=0}^{q-2}(u+\varepsilon+k)b_k}|z|^k,
\end{aligned} \tag{2.13}$$

and in general

$$\begin{aligned}
\left|f^{(r)}(z)\right| &\leq (p)_r|z|^{p-r} \\
&\quad + \frac{(k)_r\beta|b|(p)_m \psi(p) \prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\psi(k)(q-1)\prod_{\varepsilon=0}^{q-2}(u+\varepsilon+k)b_k}|z|^{k-r}, \\
\left|f^{(r)}(z)\right| &\geq (p)_r|z|^{p-r} \\
&\quad - \frac{(k)_r\beta|b|(p)_m \psi(p) \prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{(k)_m\left[(k-m)_n-(p-m)_n+\beta|b|\right]\psi(k)(q-1)\prod_{\varepsilon=0}^{q-2}(u+\varepsilon+k)b_k}|z|^{k-r}, \\
&\qquad (p>r;\, r\in\mathbb{N}_0;\, z\in\mathbb{U}),
\end{aligned} \tag{2.14}$$

where ψ is defined by (2.2).

Proof. Suppose that a function $f \in \mathcal{T}_p$ is given by (1.2), and also let the function $h \in \mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n)$ be occurring in the nonhomogenous Cauchy-Euler differential equation (1.9) with of course

$$c_j \geq 0 \quad (j \geq k). \tag{2.15}$$

Then we readily see from (1.9) that

$$a_j = \frac{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+j)}c_j \quad (j \geq k), \tag{2.16}$$

so that

$$f(z) = z^p - \sum_{j=k}^{\infty} a_j z^j = z^p - \sum_{j=k}^{\infty} \frac{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+j)} c_j z^j, \tag{2.17}$$

$$\left|f(z)\right| \leq |z|^p + |z|^k \sum_{j=k}^{\infty} \frac{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+j)} c_j. \tag{2.18}$$

Moreover, since $h \in \mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n)$, the first assertion (2.3) of Lemma 2.3 yields the following inequality:

$$c_j \leq \frac{\beta|b|(p)_m \psi(p)}{(k)_m\left[(k-m)_n - (p-m)_n + \beta|b|\right]\psi(k) b_k}, \tag{2.19}$$

and together with (2.19) and (2.18) it yields that

$$\begin{aligned}\left|f(z)\right| \leq{}& |z|^p \\ &+ \frac{\beta|b|(p)_m \psi(p)\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{(k)_m\left[(k-m)_n - (p-m)_n + \beta|b|\right]\psi(k) b_k} |z|^k \sum_{j=k}^{\infty} \frac{1}{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+j)}.\end{aligned} \tag{2.20}$$

Finally, in view of the following sum:

$$\sum_{j=k}^{\infty} \frac{1}{\prod_{\varepsilon=0}^{q-1}(j+u+\varepsilon)} = \sum_{j=k}^{\infty}\left(\sum_{\varepsilon=0}^{q-1} \frac{(-1)^{\varepsilon}}{(q-1-\varepsilon)!\varepsilon!(j+u+\varepsilon)}\right) = \frac{1}{(q-1)\prod_{\varepsilon=0}^{q-2}(u+\varepsilon+k)}, \tag{2.21}$$

$$(u \in \mathbb{R} - \{-k, -k-1, -k-2, \ldots\}),$$

the assertion (2.12) of Theorem 2.7 follows at once from (2.20) together with (2.21). The assertion (2.13) can be proven by similarly applying (2.17), and (2.19)–(2.21). □

Remark 2.8. If we set $m = 0$, $n = 1$, $\mu = 0$, and $q = 2$ in Theorem 2.7, then we have [2, Theorem 1].

3. Neighborhoods for the Classes $S_g(p,k,\lambda,\mu,b,\beta,m,n)$ and $\mathcal{K}_g(p,k,\lambda,\mu,b,\beta,m,n;q,u)$

In this section, we determine inclusion relations for the classes

$$\mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n), \qquad \mathcal{K}_g(p,k,\lambda,\mu,b,\beta,m,n;q,u), \tag{3.1}$$

involving δ-neighborhoods defined by (1.11) and (1.13).

Theorem 3.1 (see [1]). *If* $b_j \geq b_k$ $(j \geq k)$ *and*

$$\delta = \frac{(k-m)!\beta|b|(p)_m\psi(p)}{(k-1)!\left[(k-m)_n-(p-m)_n+\beta|b|\right]\psi(k)b_k} \quad (p>|b|), \tag{3.2}$$

then

$$\mathcal{S}_g(p,k,\lambda,\mu,b,\beta,m,n) \subset \mathcal{N}_k^{\delta}(e), \tag{3.3}$$

where e *and* ψ *are given by* (1.12) *and* (2.2), *respectively.*

Remark 3.2. If we set $m=0$, $n=1$, and $\mu=0$ in Theorem 3.1, then we have [2, Theorem 2].

Theorem 3.3. *If* $b_j \geq b_k (j \geq k)$ *and*

$$\delta = \frac{(k-m)!\beta|b|(p)_m\psi(p)}{(k-1)!\left[(k-m)_n-(p-m)_n+\beta|b|\right]\psi(k)b_k}\left(1+\frac{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{(q-1)\prod_{\varepsilon=0}^{q-2}(u+\varepsilon+k)}\right) \quad (p>|b|), \tag{3.4}$$

then

$$\mathcal{K}_g(p,k,\lambda,\mu,b,\beta,m,n;q,u) \subset \mathcal{N}_k^{\delta}(h), \tag{3.5}$$

where h *and* ψ *are given by* (1.11) *and* (2.2), *respectively.*

Proof. Suppose that $f \in \mathcal{K}_g(p,k,\lambda,\mu,b,\beta,m,n;q,u)$. Then, upon substituting from (2.16) into the following coefficient inequality:

$$\sum_{j=k}^{\infty} j\left|c_j-a_j\right| \leq \sum_{j=k}^{\infty} jc_j + \sum_{j=k}^{\infty} ja_j \quad (c_j \geq 0; a_j \geq 0), \tag{3.6}$$

we obtain

$$\sum_{j=k}^{\infty} j\left|c_j-a_j\right| \leq \sum_{j=k}^{\infty} jc_j + \sum_{j=k}^{\infty} \frac{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+j)} jc_j. \tag{3.7}$$

Since $h \in \mathcal{S}_g(p,k,\lambda,b,\beta,m,n)$, the assertion (2.4) of Lemma 2.3 yields

$$jc_j \leq \frac{(k-m)!\beta|b|(p)_m\psi(p)}{(k-1)!\left[(k-m)_n-(p-m)_n+\beta|b|\right]\psi(k)b_k} \quad (p>|b|). \tag{3.8}$$

Finally, by making use of (2.4) as well as (3.8) on the right-hand side of (3.7), we find that

$$\sum_{j=k}^{\infty} j\left|c_j - a_j\right| \leq \frac{(k-m)!\beta|b|(p)_m \psi(p)}{(k-1)!\left[(k-m)_n - (p-m)_n + \beta|b|\right]\psi(k)b_k} \times \left(1 + \sum_{j=k}^{\infty} \frac{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+j)}\right), \tag{3.9}$$

which, by virtue of the sum in (2.21), immediately yields

$$\sum_{j=k}^{\infty} j\left|c_j - a_j\right| \leq \frac{(k-m)!\beta|b|(p)_m \psi(p)}{(k-1)!\left[(k-m)_n - (p-m)_n + \beta|b|\right]\psi(k)b_k} \times \left(1 + \frac{\prod_{\varepsilon=0}^{q-1}(u+\varepsilon+p)}{(q-1)\prod_{\varepsilon=0}^{q-2}(u+\varepsilon+k)}\right) =: \delta. \tag{3.10}$$

Thus, by applying the definition (1.11), we complete the proof of Theorem 3.3. □

Remark 3.4. If we set $m = 0$, $n = 1$, $\mu = 0$, and $q = 2$ in Theorem 3.3, then we have [2, Theorem 3].

Acknowledgment

The present paper was supported by the Kocaeli University under Grant no. HD 2011/22.

References

[1] S. Bulut, "Inclusion and neighborhood properties for certain classes of multivalently analytic functions," Submitted.

[2] A. O. Mostafa and M. K. Aouf, "Neighborhoods of certain p-valent analytic functions with complex order," *Computers & Mathematics with Applications*, vol. 58, no. 6, pp. 1183–1189, 2009.

[3] H. M. Srivastava, S. S. Eker, and B. Şeker, "Inclusion and neighborhood properties for certain classes of multivalently analytic functions of complex order associated with the convolution structure," *Applied Mathematics and Computation*, vol. 212, no. 1, pp. 66–71, 2009.

[4] J. K. Prajapat, R. K. Raina, and H. M. Srivastava, "Inclusion and neighborhood properties for certain classes of multivalently analytic functions associated with the convolution structure," *Journal of Inequalities in Pure and Applied Mathematics*, vol. 8, no. 1, article 7, 8 pages, 2007.

[5] H. M. Srivastava and S. Bulut, "Neighborhood properties of certain classes of multivalently analytic functions associated with the convolution structure," *Applied Mathematics and Computation*, vol. 218, no. 11, pp. 6511–6518, 2012.

[6] R. M. Ali, M. H. Khan, V. Ravichandran, and K. G. Subramanian, "A class of multivalent functions with negative coefficients defined by convolution," *Bulletin of the Korean Mathematical Society*, vol. 43, no. 1, pp. 179–188, 2006.

[7] H. M. Srivastava, O. Altıntaş, and S. K. Serenbay, "Coefficient bounds for certain subclasses of starlike functions of complex order," *Applied Mathematics Letters*, vol. 24, no. 8, pp. 1359–1363, 2011.

[8] A. W. Goodman, *Univalent Functions*, vol. 259, Springer, New York, NY, USA, 1983.

[9] S. Ruscheweyh, "Neighborhoods of univalent functions," *Proceedings of the American Mathematical Society*, vol. 81, no. 4, pp. 521–527, 1981.

[10] O. Altıntaş and S. Owa, "Neighborhoods of certain analytic functions with negative coefficients," *International Journal of Mathematics and Mathematical Sciences*, vol. 19, no. 4, pp. 797–800, 1996.

[11] O. Altintaş, Ö. Özkan, and H. M. Srivastava, "Neighborhoods of a certain family of multivalent functions with negative coefficients," *Computers & Mathematics with Applications*, vol. 47, no. 10-11, pp. 1667–1672, 2004.

[12] O. Altıntaş, "Neighborhoods of certain *p*-valently analytic functions with negative coefficients," *Applied Mathematics and Computation*, vol. 187, no. 1, pp. 47–53, 2007.