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Research Article

Generalization of Some Coupled Fixed Point Results on Partial Metric Spaces

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Using the setting of partial metric spaces, we prove some coupled fixed point results. Our results generalize several well-known comparable results of H. Aydi (2011). Also, we introduce an example to support our results.

1. Introduction and Preliminaries

The notion of coupled fixed point of a mapping $F : X \times X \to X$ was introduced by Gnana Bhaskar and Lakshmikantham in [1]. Later on, many authors investigated many coupled fixed point results in different spaces such as usual metric spaces, fuzzy metric spaces, generalized metric spaces, partial metric spaces, and partially ordered metric spaces (see [1–20]).

Definition 1.1 (see [1]). An element $(x, y) \in X \times X$ is called a coupled fixed point of a mapping $F : X \times X \to X$ if

$$F(x, y) = x, \qquad F(y, x) = y. \tag{1.1}$$

Matthews [21] in 1994 introduced the notion of partial metric spaces in such a way that each object does not necessarily have to have a zero distance from itself. Consistent with Matthews [21], the following definitions and results will be needed in the sequel.

Definition 1.2 (see [21]). A partial metric on a nonempty set X is a function $p : X \times X \to \mathbb{R}^+$ such that for all $x, y, z \in X$:

(p_1) $x = y \Leftrightarrow p(x,x) = p(x,y) = p(y,y)$,

(p_2) $p(x,x) \leq p(x,y)$,

(p_3) $p(x,y) = p(y,x)$,

(p_4) $p(x,y) \leq p(x,z) + p(z,y) - p(z,z)$.

A partial metric space is a pair (X,p) such that X is a nonempty set and p is a partial metric on X.

Each partial metric p on X generates a T_0 topology τ_p on X. The set $\{B_p(x,\varepsilon) : x \in X, \varepsilon > 0\}$, where $B_p(x,\varepsilon) = \{y \in X : p(x,y) < p(x,x)+\varepsilon\}$ for all $x \in X$ and $\varepsilon > 0$, forms the base of τ_p.

If p is a partial metric on X, then the function $p^s : X \times X \to \mathbb{R}^+$ given by

$$p^s(x,y) = 2p(x,y) - p(x,x) - p(y,y) \tag{1.2}$$

is a metric on X.

Definition 1.3 (see [21]). Let (X,p) be a partial metric space. Then:

(1) a sequence (x_n) in a partial metric space (X,p) converges, with respect to τ_p, to a point $x \in X$ if and only if $p(x,x) = \lim_{n\to\infty} p(x,x_n)$,

(2) a sequence (x_n) in a partial metric space (X,p) is called a Cauchy sequence if there exists (and is finite) $\lim_{n,m\to\infty} p(x_n,x_m)$,

(3) A partial metric space (X,p) is said to be complete if every Cauchy sequence (x_n) in X converges, with respect to τ_p, to a point $x \in X$ such that $p(x,x) = \lim_{n,m\to\infty} p(x_n,x_m)$.

Lemma 1.4 (see [21]). *Let (X,p) be a partial metric space.*

(1) *(x_n) is a Cauchy sequence in (X,p) if and only if it is a Cauchy sequence in the metric space (X,p^s).*

(2) *A partial metric space (X,p) is complete if and only if the metric space (X,p^s) is complete. Furthermore, $\lim_{n\to\infty} p^s(x_n,x) = 0$ if and only if*

$$p(x,x) = \lim_{n\to\infty} p(x_n,x) = \lim_{n,m\to\infty} p(x_n,x_m). \tag{1.3}$$

Abdeljawad et al. [22–24], Altun et al. [25], Karapinar and Erhan [26–28], Oltra and Valero [29] and Romaguera [30] studied fixed point theorems in partial metric spaces. For more works in partial metric spaces, we refer the reader to [31–40].

Aydi [2] proved the following coupled fixed point theorems in partial metric spaces.

Theorem 1.5. *Let (X,p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x,y,u,v \in X$:*

$$p(F(x,y),F(u,v)) \leq kp(x,u) + lp(y,v), \tag{1.4}$$

where k,l are nonnegative constants with $k+l<1$. Then F has a unique coupled fixed point.

Theorem 1.6. *Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies the following contractive condition for all $x, y, u, v \in X$:*

$$p(F(x, y), F(u, v)) \le kp(F(x, y), x) + lp(F(u, v), u), \tag{1.5}$$

where k, l are nonnegative constants with $k + l < 1$. Then F has a unique coupled fixed point.

In this paper, we prove some coupled fixed point results. Our results generalize Theorems 1.5 and 1.6. Also, we introduce an example to support our results.

2. The Main Result

Theorem 2.1. *Let (X, p) be a complete partial metric space. Suppose that the mapping $F : X \times X \to X$ satisfies*

$$p(F(x, y), F(u, v)) \le r \max\{p(x, u), p(y, v), p(F(x, y), x), p(F(u, v), u)\}, \tag{2.1}$$

for all $x, y, u, v \in X$. If $r \in [0, 1)$, then F has a unique coupled fixed point.

Proof. Choose $x_0, y_0 \in X$. Let $x_1 = F(x_0, y_0)$ and $y_1 = F(y_0, x_0)$. Again let $x_2 = F(x_1, y_1)$ and $y_2 = F(y_1, x_1)$. By continuing in the same way, we construct two sequences (x_n) and (y_n) in X such that

$$\begin{aligned} x_{n+1} &= F(x_n, y_n), \quad n = 0, 1, 2, 3, \dots, \\ y_{n+1} &= F(y_n, x_n), \quad n = 0, 1, 2, 3, \dots. \end{aligned} \tag{2.2}$$

Then by (2.1), we have

$$\begin{aligned} p(x_{n+1}, x_{n+2}) &= p(F(x_n, y_n), F(x_{n+1}, y_{n+1})) \\ &\le r \max\{p(x_n, x_{n+1}), p(y_n, y_{n+1}), p(F(x_n, y_n), x_n), \\ &\qquad p(F(x_{n+1}, y_{n+1}), x_{n+1})\} \\ &\le r \max\{p(x_n, x_{n+1}), p(y_n, y_{n+1}), p(x_{n+1}, x_n), p(x_{n+2}, x_{n+1})\} \\ &\le r \max\{p(x_n, x_{n+1}), p(y_n, y_{n+1})\} \\ p(y_{n+1}, y_{n+2}) &= p(F(y_n, x_n), F(y_{n+1}, x_{n+1})) \\ &\le r \max\{p(y_n, y_{n+1}), p(x_n, x_{n+1}), \\ &\qquad p(F(y_n, x_n), y_n), p(F(y_{n+1}, x_{n+1}), y_{n+1})\} \\ &\le r \max\{p(y_n, y_{n+1}), p(x_n, x_{n+1}), p(y_{n+1}, y_n), p(y_{n+2}, y_{n+1})\} \\ &\le r \max\{p(y_n, y_{n+1}), p(x_n, x_{n+1})\}. \end{aligned} \tag{2.3}$$

Thus from (2.3), we have

$$\max\{p(x_n, x_{n+1}), p(y_n, y_{n+1})\} \leq r \max\{p(x_{n-1}, x_n), p(y_{n-1}, y_n)\}. \tag{2.4}$$

By repeating (2.4) n-times, we get that

$$\begin{aligned}\max\{p(x_n, x_{n+1}), p(y_n, y_{n+1})\} &\leq r \max\{p(x_{n-1}, x_n), p(y_{n-1}, y_n)\} \\ &\leq r^2 \max\{p(x_{n-2}, x_{n-1}), p(y_{n-2}, y_{n-1})\} \\ &\ \vdots \\ &\leq r^n \max\{p(x_0, x_1), p(y_0, y_1)\}.\end{aligned} \tag{2.5}$$

Letting $n \to +\infty$ in (2.5), we get that

$$\lim_{n \to +\infty} \max\{p(x_n, x_{n+1}), p(y_n, y_{n+1})\} = 0. \tag{2.6}$$

Therefore, we have

$$\begin{aligned}\lim_{n \to +\infty} p(x_n, x_{n+1}) &= 0, \\ \lim_{n \to +\infty} p(y_n, y_{n+1}) &= 0.\end{aligned} \tag{2.7}$$

For $n, m \in \mathbb{N}$ with $m > n$, we have

$$\begin{aligned}p(x_n, x_m) &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_m) - p(x_{n+1}, x_{n+1}) \\ &\leq p(x_n, x_{n+1}) + p(x_{n+1}, x_{n+2}) + p(x_{n+2}, x_m) \\ &\quad - p(x_{n+1}, x_{n+1}) - p(x_{n+2}, x_{n+2}) \\ &\ \vdots \\ &\leq \sum_{i=n}^{m-1} p(x_i, x_{i+1}) - \sum_{i=n}^{m-2} p(x_{i+1}, x_{i+1}) \\ &\leq \sum_{i=n}^{m-1} p(x_i, x_{i+1}).\end{aligned} \tag{2.8}$$

By (2.5) and (2.8), we have

$$\begin{aligned} p(x_n, x_m) &\le \sum_{i=n}^{m-1} r^i \max\{p(x_0, x_1), p(y_0, y_1)\} \\ &\le \sum_{i=n}^{+\infty} r^i \max\{p(x_0, x_1), p(y_0, y_1)\} \\ &= \frac{r^n}{1-r} \max\{p(x_0, x_1), p(y_0, y_1)\}. \end{aligned} \tag{2.9}$$

Letting $n, m \to +\infty$ in (2.9), we get that

$$\lim_{n,m\to\infty} p(x_n, x_m) = 0. \tag{2.10}$$

Thus $\lim_{n,m\to\infty} p(x_n, x_m)$ exists and is finite. Hence (x_n) is a Cauchy sequence in (X, p). Similarly, we may show that

$$\lim_{n,m\to\infty} p(y_n, y_m) = 0, \tag{2.11}$$

and hence (y_n) is a Cauchy sequence in (X, p). By Lemma 1.4 there exist $x, y \in X$ such that $\lim_{n\to\infty} p^s(x_n, x) = 0$ (resp., $\lim_{n\to\infty} p^s(y_n, y) = 0$) if and only if

$$\begin{gathered} p(x, x) = \lim_{n\to\infty} p(x_n, x) = \lim_{n,m\to\infty} p(x_n, x_m) = 0, \\ \left(\text{resp., } p(y, y) = \lim_{n\to\infty} p(y_n, y) = \lim_{n,m\to\infty} p(y_n, y_m) = 0\right). \end{gathered} \tag{2.12}$$

Now, we prove that $x = F(x, y)$. By (2.1), we have

$$\begin{aligned} p(F(x, y), x) &\le p(F(x, y), x_{n+1}) + p(x_{n+1}, x) - p(x_{n+1}, x_{n+1}) \\ &\le p(F(x, y), x_{n+1}) + p(x_{n+1}, x) \\ &\le p(F(x, y), F(x_n, y_n)) + p(x_{n+1}, x) \\ &\le r \max\{p(x, x_n), p(y, y_n), p(F(x, y), x), p(F(x_n, y_n), x_n)\} + p(x_{n+1}, x) \\ &= r \max\{p(x, x_n), p(y, y_n), p(F(x, y), x), p(x_{n+1}, x_n)\} + p(x_{n+1}, x). \end{aligned} \tag{2.13}$$

Letting $n \to \infty$ in the above inequality and using (2.12), we get that

$$p(F(x, y), x) \le r p(F(x, y), x). \tag{2.14}$$

Since $r \in [0,1)$, we conclude that $p(F(x,y),x) = 0$. By (p_1) and (p_2), we have $F(x,y) = x$. Similarly, we may show that $F(y,x) = y$. Thus (x,y) is a coupled fixed point of F. To prove the uniqueness of the fixed point, we let (u,v) be a coupled fixed point of F. We will show that $x = u$ and $y = v$. By (2.1), we have

$$\begin{aligned} p(x,u) &= p(F(x,y),F(u,v)) \\ &\leq r\max\{p(x,u),p(F(x,y),x),p(y,v),p(F(u,v),u)\} \\ &= r\max\{p(x,u),p(y,v),p(x,x),p(u,u)\}. \end{aligned} \tag{2.15}$$

Since $p(x,x) \leq p(x,u)$ and $p(u,u) \leq p(x,u)$, we have

$$p(x,u) \leq r\max\{p(x,u),p(y,v)\}. \tag{2.16}$$

Also, from (2.1), we have

$$\begin{aligned} p(y,v) &= p(F(y,x),F(v,u)) \\ &\leq r\max\{p(y,v),p(F(y,x),y),p(x,u),p(F(v,u),v)\} \\ &= r\max\{p(x,u),p(y,v),p(y,y),p(v,v)\}. \end{aligned} \tag{2.17}$$

Since $p(y,y) \leq p(y,v)$ and $p(v,v) \leq p(y,v)$, we have

$$p(y,v) \leq r\max\{p(x,u),p(y,v)\}. \tag{2.18}$$

From (2.16) and (2.18), we have

$$\max\{p(x,u),p(y,v)\} \leq r\max\{p(x,u),p(y,v)\}. \tag{2.19}$$

Since $r < 1$, we have $\max\{p(x,u),p(y,v)\} = 0$. Hence $p(x,u) = 0$ and $p(y,v) = 0$. By (p_1) and (p_2), we have $x = u$ and $y = v$. □

Corollary 2.2. *Let (X,p) be a complete partial metric space. Suppose that there are $a,b,c,d \in [0,1)$ with $a+b+c+d < 1$ such that the mapping $F : X \times X \to X$ satisfies*

$$p(F(x,y),F(u,v)) \leq ap(x,u) + bp(y,v) + cp(F(x,y),x) + dp(F(u,v),u) \tag{2.20}$$

for all $x,y,u,v \in X$. Then F has a unique coupled fixed point.

Proof. The proof follows from Theorem 2.1 by noting that:

$$\begin{aligned} &ap(x,u) + bp(y,v) + cp(F(x,y),x) + dp(F(u,v),u) \\ &\quad \leq (a+b+c+d)\max\{p(x,u), p(y,v), p(F(x,y),x), p(F(u,v),u)\}. \end{aligned} \tag{2.21}$$

□

Remarks.

(1) Theorem 1.5 [2, Theorem 2.1] is a special case of Corollary 2.2.

(2) [2, Corollary 2.2] is a special case of Corollary 2.2.

(3) Theorem 1.6 [2, Theorem 2.4] is a special case of Corollary 2.2.

(4) [2, Corollary 2.6] is a special case of Corollary 2.2.

Now, we introduce an example satisfying the hypotheses of Theorem 2.1 but not the hypotheses of Theorems 2.1 and 2.4 of [2].

Example 2.3. Define $p : [0,1] \times [0,1] \to [0,1]$ by $p(x,y) = \max\{x,y\}$. Then $([0,1],p)$ is a complete partial metric space. Let $F : [0,1] \times [0,1] \to [0,1]$ be the mapping defined by

$$F(x,y) = \frac{|x-y|}{2}. \tag{2.22}$$

Then,

(a) $p(F(x,y),F(u,v)) \leq (1/2)\max\{p(x,u),p(y,v),p(F(x,y),x),p(F(u,v),u)\}$ for all $x,y,u,v \in [0,1]$,

(b) there are no $a,b \in [0,1)$ with $a+b<1$ such that $p(F(x,y),F(u,v)) \leq ap(x,u) + bp(y,v)$ for all $x,y,u,v \in [0,1]$.

(c) there are no $a,b \in [0,1)$ with $a+b<1$ such that $p(F(x,y),F(u,v)) \leq ap(F(x,y),x) + bp(F(u,v),u)$ for all $x,y,u,v \in [0,1]$.

Proof. To prove part (a), given $x,y,v,u \in [0,1]$. Then:

$$\begin{aligned} p(F(x,y),F(u,v)) &= \max\left\{\frac{|x-y|}{2}, \frac{|u-v|}{2}\right\} \\ &= \frac{1}{2}\max\{|x-y|,|u-v|\} \\ &= \frac{1}{2}\max\{x-y,y-x,u-v,v-u\} \\ &\leq \frac{1}{2}\max\{x,y,u,v\} \\ &= \frac{1}{2}\max\{p(x,u),p(y,v)\} \\ &\leq \frac{1}{2}\max\{p(x,u),p(y,v),p(F(x,y),x),p(F(u,v),u)\}. \end{aligned} \tag{2.23}$$

To prove part (b), suppose that there are $a, b \in [0,1)$ with $a + b < 1$ such that $p(F(x,y), F(u,v)) \leq ap(x,u) + bp(y,v)$ for all $x, y, u, v \in [0,1]$.

Since

$$\begin{aligned} p(F(1,0), F(0,0)) &= p\left(\frac{1}{2}, 0\right) = \frac{1}{2} \leq ap(1,0) + bp(0,0) = a, \\ p(F(0,1), F(0,0)) &= p\left(\frac{1}{2}, 0\right) = \frac{1}{2} \leq ap(0,0) + bp(1,0) = b, \end{aligned} \tag{2.24}$$

we have $a + b \geq 1$, which is a contradiction.

To prove part (c), suppose that there are $a, b \in [0,1)$ with $a + b < 1$ such that $p(F(x,y), F(u,v)) \leq ap(F(x,y), x) + bp(F(u,v), u)$ for all $x, y, u, v \in [0,1]$.

Since

$$\begin{aligned} p(F(1,0), F(0,0)) &= p\left(\frac{1}{2}, 0\right) = \frac{1}{2} \\ &\leq ap(F(1,0), 1) + bp(F(0,0), 0) \\ &= ap\left(\frac{1}{2}, 1\right) + bp(0,0) \\ &= a \\ p(F(0,0), F(1,0)) &= p\left(0, \frac{1}{2}\right) = \frac{1}{2} \\ &\leq ap(F(0,0), 0) + bp(F(1,0), 1) \\ &= ap(0,0) + bp\left(\frac{1}{2}, 1\right) \\ &= b, \end{aligned} \tag{2.25}$$

we have $a + b \geq 1$, which is a contradiction. □

Thus by Theorem 2.1, F has a unique coupled fixed point. Here, $(0,0)$ is the unique fixed point of F.

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References

[1] T. Gnana Bhaskar and V. Lakshmikantham, "Fixed point theorems in partially ordered metric spaces and applications," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 65, no. 7, pp. 1379–1393, 2006.

[2] H. Aydi, "Some coupled fixed point results on partial metric spaces," *International Journal of Mathematics and Mathematical Sciences*, vol. 2011, Article ID 647091, 11 pages, 2011.

[3] H. Aydi, B. Damjanović, B. Samet, and W. Shatanawi, "Coupled fixed point theorems for nonlinear contractions in partially ordered G-metric spaces," *Mathematical and Computer Modelling*, vol. 54, no. 9-10, pp. 2443–2450, 2011.

[4] H. Aydi, E. Karapınar, and W. Shatanawi, "Coupled fixed point results for (ψ, φ)-weakly contractive condition in ordered partial metric spaces," *Computers & Mathematics with Applications*, vol. 62, no. 12, pp. 4449–4460, 2011.

[5] Y. J. Cho, B. E. Rhoades, R. Saadati, B. Samet, and W. Shatanawi, "Nonlinear coupled fixed point theorems in ordered generalized metric spaces with integral type," *Fixed Point Theory and Applications*, vol. 2012, article 8, 2012.

[6] B. S. Choudhury and P. Maity, "Coupled fixed point results in generalized metric spaces," *Mathematical and Computer Modelling*, vol. 54, no. 1-2, pp. 73–79, 2011.

[7] E. Karapınar, "Couple fixed point theorems for nonlinear contractions in cone metric spaces," *Computers & Mathematics with Applications*, vol. 59, no. 12, pp. 3656–3668, 2010.

[8] V. Lakshmikantham and L. Ćirić, "Coupled fixed point theorems for nonlinear contractions in partially ordered metric spaces," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 70, no. 12, pp. 4341–4349, 2009.

[9] N. V. Luong and N. X. Thuan, "Coupled fixed points in partially ordered metric spaces and application," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 74, no. 3, pp. 983–992, 2011.

[10] H. K. Nashine and W. Shatanawi, "Coupled common fixed point theorems for a pair of commuting mappings in partially ordered complete metric spaces," *Computers & Mathematics with Applications*, vol. 62, no. 4, pp. 1984–1993, 2011.

[11] B. Samet, "Coupled fixed point theorems for a generalized Meir-Keeler contraction in partially ordered metric spaces," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 72, no. 12, pp. 4508–4517, 2010.

[12] S. Sedghi, I. Altun, and N. Shobe, "Coupled fixed point theorems for contractions in fuzzy metric spaces," *Nonlinear Analysis. Theory, Methods & Applications*, vol. 72, no. 3-4, pp. 1298–1304, 2010.

[13] W. Shatanawi and Z. Mustafa, "On coupled random fixed point results in partially ordered metric spaces," *Matematicki Vesnik*, vol. 64, no. 2, pp. 139–146, 2012.

[14] W. Shatanawi, "Coupled fixed point theorems in generalized metric spaces," *Hacettepe Journal of Mathematics and Statistics*, vol. 40, no. 3, pp. 441–447, 2011.

[15] W. Shatanawi, B. Samet, and M. Abbas, "Coupled fixed point theorems for mixed monotone mappings in ordered partial metric spaces," *Mathematical and Computer Modelling*, vol. 55, pp. 680–687, 2012.

[16] W. Shatanawi, "Some common coupled fixed point results in cone metric spaces," *International Journal of Mathematical Analysis*, vol. 4, no. 45–48, pp. 2381–2388, 2010.

[17] W. Shatanawi, "Partially ordered cone metric spaces and coupled fixed point results," *Computers & Mathematics with Applications*, vol. 60, no. 8, pp. 2508–2515, 2010.

[18] W. Shatanawi, "On w-compatible mappings and common coupled coincidence point in cone metric spaces," *Applied Mathematics Letters*, vol. 25, no. 6, pp. 925–931, 2012.

[19] W. Shatanawi, "Fixed point theorems for nonlinear weakly C-contractive mappings in metric spaces," *Mathematical and Computer Modelling*, vol. 54, no. 11-12, pp. 2816–2826, 2011.

[20] W. Shatanawia and H. K. Nashine, "A generalization of Banach's contraction principle for nonlinear contraction in a partial metric space," *Journal of Nonlinear Science and Applications*, vol. 2012, no. 5, pp. 37–43, 2012.

[21] S. G. Matthews, "Partial metric topology," in *Papers on General Topology and Applications*, vol. 728, pp. 183–197, The New York Academy of Sciences, New York, NY, USA, 1994.

[22] T. Abdeljawad, E. Karapınar, and K. Taş, "Existence and uniqueness of a common fixed point on partial metric spaces," *Applied Mathematics Letters*, vol. 24, no. 11, pp. 1900–1904, 2011.

[23] T. Abdeljawad, E. Karapınar, and K. Taş, "A generalized contraction principle with control functions on partial metric spaces," *Computers & Mathematics with Applications*, vol. 63, no. 3, pp. 716–719, 2012.

[24] T. Abdeljawad, "Fixed points for generalized weakly contractive mappings in partial metric spaces," *Mathematical and Computer Modelling*, vol. 54, no. 11-12, pp. 2923–2927, 2011.

[25] I. Altun, F. Sola, and H. Simsek, "Generalized contractions on partial metric spaces," *Topology and its Applications*, vol. 157, no. 18, pp. 2778–2785, 2010.

[26] E. Karapinar, "Weak φ-contraction on partial contraction," *Journal of Computational Analysis and Applications*. In press.

[27] E. Karapinar, "Generalizations of Caristi Kirk's theorem on partial metric spaces," *Fixed Point Theory and Applications*, vol. 2011, article 7, 2011.

[28] E. Karapınar and M. Erhan, "Fixed point theorems for operators on partial metric spaces," *Applied Mathematics Letters*, vol. 24, no. 11, pp. 1894–1899, 2011.
[29] S. Oltra and O. Valero, "Banach's fixed point theorem for partial metric spaces," *Rendiconti dell'Istituto di Matematica dell'Università di Trieste*, vol. 36, no. 1-2, pp. 17–26, 2004.
[30] S. Romaguera, "A Kirk type characterization of completeness for partial metric spaces," *Fixed Point Theory and Applications*, vol. 2010, Article ID 493298, 6 pages, 2010.
[31] I. Altun and A. Erduran, "Fixed point theorems for monotone mappings on partial metric spaces," *Fixed Point Theory and Applications*, Article ID 508730, 10 pages, 2011.
[32] I. Altun and H. Simsek, "Some fixed point theorems on dualistic partial metric spaces," *Journal of Advanced Mathematical Studies*, vol. 1, no. 1-2, pp. 1–8, 2008.
[33] H. Aydi, "Some fixed point results in ordered partial metric spaces," *The Journal of Nonlinear Science and Applications*, vol. 4, no. 2, pp. 1–12, 2011.
[34] H. Aydi, "Fixed point results for weakly contractive mappings in ordered partial metric spaces," *Journal of Advanced Mathematical Studies*, vol. 4, no. 2, pp. 1–12, 2011.
[35] H. Aydi, "Fixed point theorems for generalized weakly contractive condition in ordered partial metric spaces," *Journal of Nonlinear Analysis and Optimization*, vol. 2, no. 2, pp. 33–48, 2011.
[36] L. Ćirić, B. Samet, H. Aydi, and C. Vetro, "Common fixed points of generalized contractions on partial metric spaces and an application," *Applied Mathematics and Computation*, vol. 218, no. 6, pp. 2398–2406, 2011.
[37] R. Heckmann, "Approximation of metric spaces by partial metric spaces," *Applied Categorical Structures*, vol. 7, no. 1-2, pp. 71–83, 1999.
[38] O. Valero, "On Banach fixed point theorems for partial metric spaces," *Applied General Topology*, vol. 6, no. 2, pp. 229–240, 2005.
[39] S. Romaguera, "Fixed point theorems for generalized contractions on partial metric spaces," *Topology and its Applications*, vol. 159, no. 1, pp. 194–199, 2012.
[40] B. Samet, M. Rajović, R. Lazović, and R. Stoiljković, "Common fixed point results for nonlinear contractions in ordered partial metric spaces," *Fixed Point Theory and Applications*, vol. 2011, article 71, 2011.