Internat. J. Math. & Math. Sci.
VOL. 20 NO. 3 (1997) 451-456

COMMON FUZZY HYBRID FIXED POINT THEOREMS FOR A SEQUENCE OF FUZZY MAPPINGS

SHI CHUAN

Department of Applied Mathematics
Nanjing University of Science & Technology
Nanjing, 210014, PEOPLE'S REPUBLIC OF CHINA

(Received September 28, 1995 and in revised form May 20, 1996)

ABSTRACT. In this paper, we discuss the concepts of fuzzy hybrid fixed points, of g-Φ-contractive type fuzzy mappings and common fuzzy hybrid fixed point theorems of a sequence of fuzzy mappings. Our theorems improve and generalize the corresponding recent important results.

KEY WORDS AND PHRASES: Fuzzy hybrid fixed point, common fuzzy hybrid fixed point, g-Φ-contractive type fuzzy mapping, g-contractive type fuzzy mapping, sequence of fuzzy mappings.
1991 AMS SUBJECT CLASSIFICATION CODES: 54H25, 47H10.

1. INTRODUCTION

Heilpern [1] first introduced the concept of fuzzy mappings and proved fixed point theorems for contraction fuzzy mappings. Chang [2] introduced the concept of Φ-contraction type fuzzy mappings, and proved a fixed point theorem, which is an extension of the result of Heilpern. Also, he obtained common fixed point theorems for a sequence of fuzzy mappings. Lee, et al [3-4] introduced the concept of g-contractive type fuzzy mappings, and proved a common fixed point theorem for sequence of fuzzy mappings on a complete metric linear space.

In this paper, we introduced g-Φ-contractive type fuzzy mappings and defined the concept of the fuzz hybrid fixed point for fuzzy mappings, proved common fuzzy hybrid fixed point theorems for a sequence of fuzzy mappings on a complete metric space. Our theorems improve and generalize the recent important results of [1-4].

2. PRELIMINARIES

Throughout this paper let (E,d) be a complete metric space, $CB(E)$ be a collection of all non-empty bounded closed subsets of E and $C(E)$ be a collection of all non-empty compact subsets of E Let Z^+ be the set of all positive integers. A mapping $B : B \to [0,1]$ is called a fuzzy subset over E We denote by $W(E)$ the family of all fuzzy subsets over E. Let $A \in W(E)$, $\forall\, \alpha \in [0,1]$. Set $(A)_\alpha = \{x \in E : A(x) \geq \alpha\}$ is called the α-cut set of A. A mapping $T : E \to W(E)$ is called fuzzy mapping over E.

DEFINITION 2.1. Let the function $\Phi : [0, +\infty)^5 \to [0, +\infty)$. We say Φ satisfies the condition (Φ_1), (Φ_2) or (Φ_3), if $(\Phi_1)\Phi$ is upper semi-continuous and non-decreasing for each variable $(\Phi_2)\Phi(t,t,t,at,bt) \leq Q(t)$, $\forall\, t \geq 0, a, b = 0, 1, 2$, and $a + b = 2$, where $Q(t) : [0, +\infty) \to [0, +\infty)$, $Q(0) = 0, Q(t) < t$, $\forall\, t > 0$. $(\Phi_3)\Phi(t,t,t,at,bt) \leq rt$, where $r \in (0,1)$ is a constant, $a, b = 0, 1, 2$ and $a + b = 2$.

DEFINITION 2.2. Let $T : E \to W(E)$. We say that $T : E \to W(E)$ satisfies the condition A_1 (A_2). If there exists $\alpha(x) : E \to (0,1]$ such that $\forall x \in E, (Tx)_{\alpha(x)} \in CB(E)$ $(C(E))$.

Let $T_i : E \to W(E)(i = 1, 2, \ldots)$. We say $T_i : E \to W(E)(i = 1, 2, \ldots)$ satisfies the condition A_1 (A_2). If there exists a sequence of functions $a_i(x) : E \to (0,1]$ $(i = 1, 2, \ldots)$ such that $\forall x \in E$, $(Tix)_{\alpha_i(x)} \in CB(E)$ (or $C(E)$).

Let $T : E \to W(E)$ satisfies the condition A_1 (or A_2), $\forall\ x \in E, \tilde{T}x = (Tx)_{\alpha(x)} \in CB(E)$. $\tilde{T} : E \to CB(E)$ is called the set-valued mapping induced by T.

DEFINITION 2.3. Let $g : E \to E$ be a single-valued mapping, $F : E \to W(E)$ and $G : E \to W(E)$ be two fuzzy mappings satisfying condition A_1. If, $\forall\ x, y \in E$, $u_x \in \tilde{F}x$ $(\tilde{G}x)$ there exists $y_i \in \tilde{G}y$ $(\tilde{F}y)$ such that

$$d(u_x, v_y) \le \Phi(d(g(x), g(y)), d(g(x), g(u_x)), d(g(y), g(v_y)) \\ d(g(x), g(v_y)), d(g(y), g(u_x))). \tag{2.1}$$

Then, we say that F and G satisfy the condition B.

DEFINITION 2.4. Let $F : E \to W(E)$, $G : E \to W(E)$ be two fuzzy mappings satisfying the condition A_1. If for any $x, y \in E$, $u_x \in \tilde{F}x(\tilde{G}x)$ there exists $v_y \in \tilde{G}y(\tilde{F}y)$ such that

$$d(u_x, v_y) \le \Phi(d(x, y), d(x, u_x), d(y, v_y), d(x, v_y), d(y, u_x)). \tag{2.2}$$

Then, we say that F, G satisfy the condition C.

DEFINITION 2.5. Let $F : E \to W(E)$ and $G : E \to W(E)$ be two fuzzy mappings satisfying the condition A_1. If for any $x, y \in E$, $u_x \in \tilde{F}x(\tilde{G}x)$ there exists $v_y \in \tilde{G}y(\tilde{F}y)$ such that

$$H(\tilde{F}_x, \tilde{G}_y) \le \Phi(d(x, y), d(x, \tilde{F}_x), d(y, \tilde{G}y), d(x, \tilde{G}y), d(y, \tilde{F}x)), \tag{2.3}$$

where $d(x, \tilde{F}x) = \min_{p \in \tilde{F}x} d(x, p)$ and H is the Hausdorff metric induced by d, then, we say that F and G satisfy the condition D.

DEFINITION 2.6. Let $g : E \to E$ be a single-valued mapping, $F_i : E \to W(E)$ $(i = 1, 2, \ldots)$ be a sequence of fuzzy mappings, if for any $i, j \in Z^+$, F_i and F_j satisfy conditions A_1 and B. Moreover, Φ in condition B satisfies condition (Φ_1) and (Φ_2). Then we say $F_i : E \to W(E)(i = 1, 2, \ldots)$ be a g-Φ-contractive type sequence of fuzzy mappings. In particular, when $F_i = F_j = F$ $(\forall\ i, j \in Z^+)$ we say $F : E \to W(E)$ be a g-Φ-contractive type fuzzy mapping.

DEFINITION 2.7. Let $F : E \to W(E)$. If $P \in E$ such that $Fp(p) = \max_{x \in E} Fp(u)$, then P is called a fixed point of F. Let $F_i : E \to W(E)$ $(i = 1, 2, \ldots)$. If $P \in E$ such that $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) = \max_{x \in E}\left(\bigcap_{i=1}^{+\infty} Fip\right)(u)$ then P is called a common fixed point of $\{F_i\}$.

DEFINITION 2.8. Let $T : E \to E$ be a single-valued mapping and $F : E \to W(E)$ be a fuzzy mapping. If $P \in E$ such that $P = Tp$ and $Fp(p) = \max_{x \in E} Fp(u)$, then P is called a fuzzy hybrid fixed point of T and F.

Let $T : E \to E$ be a single-valued mapping and $F_i : E \to W(E)$ $(i = 1, 2, \ldots)$ be a sequence of fuzzy mappings. If $p \in E$ such that $p = T_p$ and $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) = \max_{x \in E}\left(\bigcap_{i=1}^{+\infty} Fip\right)(u)$, then p is called a common fuzzy hybrid fixed point of T and $\{F_i\}$.

3. MAIN RESULTS

THEOREM 3.1. Let (E, d) be a complete metric space. Let:

(1) $T : E \to E$ be a single-valued continuous mapping such that $\forall\, x, y \in E$

$$d(Tx, Ty) \le d(x, Ty) \tag{3.1}$$

(2) $F_i : E \to W(E)\,(i = 1,2,...)$ be a g-Φ-contractive type sequence of fuzzy mappings, where $g : E \to E$ is a non-expansive mapping, $\alpha_i(x) : E \to (0,1]\,(i = 1,2,...)$ such that $\forall\, x \in E, T(Fix)_{\alpha_i(x)} = (FiTx)_{\alpha_i(Tx)}\,(i = 1,2,...)$.

(3) Let $\delta > 1$, $x_0 \in T(E)$, $x_1 \in (F_1x_0)_{\alpha_1(x_0)}$, $\{t_x\}_{x=0}^{+\infty}$ be a sequence of nonnegative real numbers which is defined as follows

$$to = 0, t_1 > d(x_0, x_1), t_{k+1} = t_k + Q(\delta(t_k - t_{k-1})),\ k = 1,2,... \tag{3 2}$$

If $\lim\limits_{K\to\infty} t_k = t_* < +\infty$, then there exists $P \in E$ such that $P = Tp$ and $\left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(p) \geq \min\limits_{i\geq 1}\{\alpha_i(P)\}$, when $\alpha_i(x) = \max\limits_{x\in E} Fix(u)\,(i = 1,2,...)$ be a sequence of functions satisfying the condition (2). Then there exists $P \in E$ such that $P = Tp$ and $\left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(p) = \max\limits_{x\in E}\left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(u)$, i.e P be a common fuzzy hybrid fixed point of T and $\{F_i\}$.

PROOF. Let $T(E) = \{x \mid x = Tu, u \in E\}, F(E) = \{x \mid x = Tx, x \in E\}$. It is obvious that $F(E) \subseteq T(E)$. Next we prove that $T(E) \subseteq F(E)$, $\forall\, Z_1 \in T(E)$, $\exists\, u_1 \in E$ with $Z_1 = Tu_1$, by (3.1), $0 \leq d(Tz_1, Tu_1) \leq d(z_1, Tu_1) = d(z_1, z_1) = 0$, $\therefore Tz_1 = Tu_1 = z_1, z_1 \in F(E)$. Thus $T(E) \subseteq F(E)$, $T(E) = F(E)$.

We prove that $\forall\, x \in T(E)$, $\tilde{F}ix \subseteq T(E)\,(i = 1,2,...)$. In fact, for $x \in T(E) = F(E)$ by $x = Tx$, $T(Fix)_{\alpha_i(x)} = (FiTx)_{\alpha_i(Tx)}(i = 1,2,...)$, we have $\tilde{F}ix = \tilde{F}iTx = (FiTx)_{\alpha_i(Tx)} = T(Fix)_{\alpha_i(x)} = T\tilde{F}ix \subseteq T(E)(i = 1,2,...)$ take $x_0 \in T(E), x_1 \in \tilde{F}_1x_0 \subseteq T(E)$, by the condition B and $g : E \to E$ be a non-expansive mapping, $\exists\, x_2 \in \tilde{F}_2x_1$ such that

$$\begin{aligned} d(x_1,x_2) &\leq \Phi(d(g(x_0),g(x_1)), d(g(x_0),g(x_1)), d(g(x_1),g(x_2)) \\ &\quad d(g(x_0),g(x_2)), \alpha(g(x_1),g(x_1))) \\ &\leq \Phi(d(x_0,x_1), d(x_0,x_1), d(x_1,x_2), d(x_0,x_2), d(x_1,x_1)) \end{aligned}$$

for $x_2 \in \tilde{F}_2x_1$, $\exists\, x_3 \in \tilde{F}_3x_2$ such that

$$d(x_2,x_3) \leq \Phi(d(x_1,x_2), d(x_1,x_2), d(x_2,x_3), d(x_1,x_3)d(x_2,x_2)).$$

Taking this procedure repeatedly, we can define a sequence $\{x_s\}$ in $T(E)$, satisfying $x_s \in \tilde{F}_{sx_{s-1}} \subseteq T(E)$, $x_{s+1} \in \tilde{F}_{s+1}x_s \subseteq T(E)$, and

$$d(x_s,x_{s+1}) \leq \Phi(d(x_{s-1},x_s), d(x_{s-1},x_s), d(x_s,x_{s+1}), d(x_{s-1},x_{s+1}), d(x_s,x_s)). \tag{3.3}$$

We prove that $\{x_s\}_{s=0}^{+\infty}$ be convergent. First we prove the following inequality

$$d(x_n,x_{n-1}) \leq \delta(t_n - t_{n-1})(n = 1,2,...) \tag{3.4}$$

for $n = 1, d(x_1,x_0) < t_1 = t_1 - t_0 < \delta(t_1 - t_0)$, (3.4) is true. Suppose that $n = k$. (3.4) is true, i.e. $d(x_k,x_{k-1}) \leq \delta(t_k - t_{k-1})$ We prove that it remains true for $n = k+1$, when $n = k+1$, by $(\Phi_1), (\Phi_2)$, (3.2), (3.3), $d(x_{k-1},x_{k+1}) \leq d(x_{k-1},x_k) + d(x_k,x_{k+1})$, and it is easy to prove that $d(x_{k+1},x_k) \leq d(x_{k-1},x_k)$, we have

$$\begin{aligned} d(x_{k+1},x_k) &\leq \Phi(d(x_k,x_{k-1}), d(x_k,x_{k-1}), d(x_k,x_{k+1}), d(x_{k-1},x_{k+1}), d(x_k,x_k)) \\ &\leq \Phi(d(x_k,x_{k-1}), d(x_k,x_{k-1}), d(x_{k-1},x_k), 2d(x_{k-1},x_k), 0) \\ &\leq \Phi(\delta(t_k - t_{k-1}), \delta(t_k - t_{k-1}), \delta(t_k - t_{k-1}), 2\delta(t_k - t_{k-1}), 0) \\ &\leq Q(\delta(t_k - t_{k-1})) = t_{k+1} - t_k < \delta(t_{k+1} - t_k). \end{aligned}$$

Thus (3.4) remains true for $n = k+1$. This completes the proof of (3.4).

By $\lim\limits_{k\to\infty} = t_* < +\infty$ and (3.4) $d(x_{k+m},x_k) \leq \sum\limits_{j=k}^{k+m-1} d(x_{j+1},x_j) \leq \delta \sum\limits_{j=k}^{k+m-1}(t_{j+1} - t_j) = \delta(t_{k+m} - t_k)$.

Thus $\{xs\}_{s=0}^{+\infty}$ be a Cauchy sequence in $T(E)$. Since $(T(E),d)$ is a complete metric space, therefore $\exists$ $P \in E$ such that $\lim\limits_{s\to\infty} x_s = P$, $\because P \in T(E) = F(E)$, $\therefore P = Tp$. Next, we prove that $P \in \bigcap\limits_{i=1}^{+\infty} \tilde{F}ip$, $\forall$

$m \in Z^+$, $\because x_s \in \tilde{F}sx_{s-1}(n = 1, 2, \ldots)$ by the condition B and $g : E \to E$ be a nonexpansive mapping, $\exists v_s \in \tilde{F}mp$ such that

$$d(x_s, v_s) \le \phi(d(x_{s-1}, p), d(x_{s-1}, x_s), d(p, v_s), d(x_{s-1}, v_s), d(p, x_s)) \tag{3.5}$$

by the condition A_1, $\tilde{F}mp = (Fmp)_{\alpha_m(p)} \in CB(E)$, $\tilde{F}mp$ be a non-empty bounded closet set of $E, v_s \in \tilde{F}mp$. Thus $\{d(v_s, p)\}$ be a bounded sequence of real numbers. Therefore, there exists $\{d(v_{s_j}, p)\} \subseteq \{d(v_s, p)\}$ satisfies $\lim\limits_{j\to\infty} d(v_{s_j}, p) = d$, by (3.5) and $d(v_{s_j}, x_{s_j-1}) \le d(v_{s_j}, p) + d(x_{s_j-1}, p)$, we have

$$d(v_{s_j}, p) \le d(p, x_{s_j}) + \Phi(d(x_{s_j-1}, p), d(x_{s_j-1}, x_{s_j}), d(p, v_{s_j}), d(x_{s_j-1}, p) + d(p, v_{s_j}), d(p, x_{s_j})).$$

Let $j \to +\infty$, by $d(v_{s_j}, p) \to d$, $x_s \to p$, (Φ_1), (Φ_2), we have, when $d \ne 0$

$$d \le +\Phi(0, 0, d, 0 + d, 0) \le Q(d) < d.$$

This is a contradiction, therefore, $d = 0$, i.e. $\lim\limits_{j\to\infty} v_{s_j} = p$, by $v_{s_j} \in \tilde{F}mp$ and $v_{s_j} \to p$, $\therefore\ p \in \tilde{F}mp = (Fmp)_{\alpha_m(p)} (\forall m \in z^+)$ i.e. $Fmp(p) \ge \alpha_m(p) (m = 1, 2, \ldots)$. Thus $Fmp(p) \ge \min\limits_{i\ge1}\{\alpha_i(p)\} (m = 1, 2, \ldots)$, $\left(\bigcap\limits_{m=1}^{+\infty} Fmp\right)(p) = \min\limits_{i\ge1}\{\alpha_i(p)\}$.

When $\alpha_i(x) = \max\limits_{x\in E} Fix(u) (i = 1, 2, \ldots)$. Then $\left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(p) \ge \min\limits_{i\ge1}\{\alpha_i(p)\} \ge \min\limits_{i\ge1} \max\limits_{\mu\in E} Fip(u) \ge \min\limits_{i\ge1} Fip(u) = \left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(u)$, $\forall u \in E$. Thus $\left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(p) \ge \max\limits_{\mu\in E} \left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(u) \ge \left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(p)$. $\therefore \left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(p) = \max\limits_{\mu\in E} \left(\bigcap\limits_{i=1}^{+\infty} Fip\right)(u)$, i.e. p be common fixed point of $\{F_i\}$, by $p \in T(E) = F(E)$, $p = Tp$. Thus p be a common fuzzy hybrid fixed point of T and $\{F_i\}$.

COROLLARY 3.1. Let $(E, d), T : E \to E$ and $F_i : E \to W(E)\, (i = 1, 2, \ldots)$ satisfy the conditions of Theorem 3.1. Moreover Φ satisfies the condition (Φ_3), then the conclusion of Theorem 3.1 remains true.

PROOF. Taking $t_0 = 0$, $x_0 \in T(E)$, $x_1 \in \tilde{F}_1 x_0$, $t_1 > d(x_0, x_1)$. We define a sequence of non-negative real numbers $\{t_k\}_{k=0}^{+\infty}$ as follows:

$$t_{k+1} = t_k + r\delta(t_k - t_{k-1}),\ k = 1, 2, \ldots \tag{3.6}$$

where $\delta > 1$ and $\delta r < 1$, r be a constant in the condition (Φ_3). It follows from (3.6)

$$t_{k+1} - t_k = r\delta(t_k - t_{k-1}) = \cdots = (r\delta)^k t_1.$$

Therefore we have $\lim\limits_{k\to\infty} t_k = \lim\limits_{k\to\infty} \sum\limits_{i=1}^{k} (t_i - t_{i-1}) = \frac{t_1}{1 - r\delta} < +\infty$. The conclusion of Corollary 3 1 follows from Theorem 3.1 immediately.

COROLLARY 3.2. Let $(E, d), T : E \to E$ satisfies the condition of Theorem 3.1 Let $F_i : E \to W(E)\, (i = 1, 2, \ldots)$ for $\alpha_i(x) : E \to (0, 1]\, (i = 1, 2, \ldots)$ satisfies the condition A_i and $\forall$ $x \in E$, $T(Fix)_{\alpha_i(x)} = (FiTx)_{\alpha_i(Tx)} (i = 1, 2, \ldots)$. Moreover, for any $i, j \in z^+, x, y, \in E, u_x \in \tilde{F}ix$, $\exists$ $v_y \in Fjy$ such that

$$\begin{aligned} d(u_x, v_y) \le q \max\{&d(g(x), g(y)), d(g(x), g(u_x)) \\ &d(g(y), g(v_y)), \frac{1}{2}[d(g(x), g(v_y)) + d(g(y), g(u_x))]\} \end{aligned} \tag{3.7}$$

where $q \in (0, 1)$ is a constant, $g : E \to E$ be a non-expansive mapping. Then the conclusion of Theorem 3.1 remains true.

PROOF. Taking $\Phi(t_1,t_2,t_3,t_4,t_5) = q\max\{t_1,t_2,t_3,\frac{1}{2}(t_4+t_5)\}$, we have $\Phi(t,t,t,at,bt) = qt$, where $a,b = 0,1,2$ and $a+b=2$. It is easy to see that Φ satisfies the condition (Φ_1) and (Φ_3), therefore the conclusion follows from Corollary 3.1 directly.

THEOREM 3.2. Let (E,d) and $T: E \to E$ satisfy the condition of Theorem 3.1. Let $F_i : E \to W(E)\,(i=1,2,\ldots)$ for $\alpha_i(x) : E \to (0,1]\,(i=1,2,\ldots)$ satisfy the condition A_1 and $\forall$ $x \in E, T(Fix)_{\alpha_i(x)} = (FiTx)_{\alpha_i(Tx)}(i=1,2,\ldots)$. Moreover for any $i,j \in Z^+, x,y \in E, u_x \in \tilde{F}ix$, $\exists$ $v_y \in \tilde{F}jy$ such that

$$d(u_x,v_y) \le \alpha_1 d(g(x),g(u_x)) + \alpha_2 d(g(y),g(v_y)) + \alpha_3 d(g(y),g(u_x)) + \alpha_r d(g(x),g(v_y)) + \alpha_5 d(g(x),g(y)) \tag{3 8}$$

where $g: E \to E$ be a non-expansive mapping, $\alpha_i > 0\,(i=1,2,\ldots,5), \alpha_1+\alpha_2+\ldots+\alpha_5 < 1$ and $\alpha_3 \ge \alpha_4$. Then the conclusion of Theorem 3.1 remains true.

PROOF. By proof of Theorem 3.1, $T(E) = F(E)$, and $\forall\ x \in T(E), \tilde{F}ix \subseteq T(E)(i=1,2,\ldots)$, by (3.8) and $g: E \to E$ be a non-expansive mapping, the same as the proof of Theorem 3.1 We can define a sequence $\{x_s\} \subseteq T(E)$, such that $x_{s+1} \subseteq \tilde{F}_{s+1}x_s \subseteq T(E)$. Moreover

$$\begin{aligned} d(x_s,x_{s+1}) &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2(x_s,x_{s+1}) + \alpha_3 d(x_s,x_s) \\ &\quad + \alpha_4 d(x_{s-1},x_{s+1}) + \alpha_5 d(x_{s-1},x_s) \\ &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2 d(x_s,x_{s+1}) + \alpha_4 d(x_{s-1},x_s) \\ &\quad + \alpha_4 d(x_s,x_{s+1}) + \alpha_5 d(x_{s-1},x_s). \end{aligned}$$

Therefore

$$\alpha(x_s,x_{s+1}) \le \frac{\alpha_1+\alpha_4+\alpha_5}{1-\alpha_2-\alpha_4} d(x_{s-1},x_s) \tag{3.9}$$

$\because\ \alpha_3 \ge \alpha_4 > 0$, $\alpha_1 + \cdots + \alpha_5 < 1$. Thus $r = \frac{\alpha_1+\alpha_4+\alpha_5}{1-\alpha_2-\alpha_4} < 1$, we have

$$d(x_s,x_{s+1}) \le rd(x_{s-1},x_s) \le r^2 d(x_{s-2},x_s) \le \cdots \le r^s d(x_0,x_1). \tag{3.10}$$

By (3.10), it is easy to see that $\{x_s\}_{s=1}^{+\infty}$ is a Cauchy sequence in $T(E)$. Thus $\exists\ p \in T(E)$, such that $\lim_{s\to\infty} x_s = p$. Next, we prove that $p \in \bigcap_{m=1}^{+\infty} \tilde{F}_{mp}$, $\forall\ m \in Z^+$, for $x_s \in \tilde{F}_s x_{s-1}(n=1,2,\ldots)$, by assumption, $\exists\ v_s \in \tilde{F}mp$ such that

$$\begin{aligned} d(x_s,v_s) &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2 d(p,v_s) + \alpha_3 d(p,x_s) \\ &\quad + \alpha_4 d(x_{s-1},v_s) + \alpha_5 d(x_{s-1},p) \\ &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2 d(p,x_s) + \alpha_2 d(x_s,v_s) \\ &\quad \alpha_3 d(p,x_s) + \alpha_4 d(x_{s-1},x_s) + \alpha_4 d(x_s,v_s) + \alpha_5 d(x_{s-1},p). \end{aligned}$$

Thus we have

$$\begin{aligned} (1-\alpha_2-\alpha_4)d(x_s,v_s) &\le \alpha_1 d(x_{s-1},x_s) + \alpha_2 d(p,x_s) \\ &\quad + \alpha_3 d(p,x_s) + \alpha_4 d(x_{s-1},x_s) + \alpha_5 d(x_{s-1},p). \end{aligned}$$

We have $d(x_s,v_s) \to 0 (n \to +\infty)$. Thus $d(v_s,p) \le d(v_s,x_s) + d(x_s,p) \to 0(n \to +\infty)$, $\therefore \lim_{s\to\infty} v_s = p$, by $v_s \in \tilde{F}mp \in CB(E)$. Therefore $p \in \tilde{F}mp(\forall\ m \in z^+)$. By $p = Tp$ and $p \in \bigcap_{m=1}^{+\infty} \tilde{F}mp$, the same as the proof of Theorem 3.1, we obtain the conclusion of Theorem 3.1.

When $T = I$ is the identity operator on E, we obtain the following result.

COROLLARY 3.3. Let (E,d) and $F_i : E \to W(E)\,(i=1,2,\ldots)$ satisfy the conditions of Theorem 3.2. Then there exists $p \in E$ such that $\left(\bigcap_{i=1}^{+\infty} Fip\right)(p) \ge \min_{i\ge 1}\{\alpha_i(p)\}$, when $\alpha_i(x) = \max_{x\in E} Fix(u)$ $(i=1,2,\ldots)$ satisfies corresponding conditions, p is a common fixed point of $\{F_i\}$.

REMARK 3.1. When $\alpha_3 = \alpha_4$ in the condition (3.8) of Theorem 3.2, Theorem 3.2 is a special case of Corollary 3.2. Corollary 3.3 is an improvement and generalized version of Theorem 3.1 of [4] and Theorem 3.10 of [3]. In Theorem 3 of [2], if $\{F_i\}$ for $\{\alpha_i(x)\}$ satisfy condition A_2, then Theorem 3 of [2] is a special case of Theorem 3.1 of this paper. In fact, when $T = I$ and $g = I$ are identity operators on E, by the theorem of Nadler [5], it is easy to see the condition D implies the condition C.

REFERENCES

[1] HEILPERN, S., Fuzzy mappings and fixed point theorem, *J. Math. Anal. Appl.* **83** (1981), 566-569.

[2] SHIH-SEN, C., Fixed degree for fuzzy mappings and a generalization of Ky Fan's theorem, *Fuzzy Sets and Systems*, **24** (1984), 103-112.

[3] LEE, B.S. and CHO, S.J., Common fixed point theorems for sequences of fuzzy mappings, *Internat. J. Math. & Math. Sci.*, **17**, 3 (1994), 423-428.

[4] LEE, B.S., LEE, G.M., CHO, S.J. and KIM, D.S., Generalized common fixed point theorems for a sequence of fuzzy mappings, *Internat. J. Math. & Math. Sci.* **17**, 3 (1994), 437-440.

[5] NADLER, S.B. Jr., Muli-valued contraction mappings, *Pacific J. Math.* **30** (1969), 475-488.