

COOLING OF A PLATE WITH GENERAL BOUNDARY CONDITIONS

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ABSTRACT. We consider steady state temperature distribution in a homogeneous rectangular infinite plate the lower part of which is cooled by a fluid flowing at a constant velocity while the upper part satisfies the general mixed boundary conditions. The Wiener-Hopf method has been used to obtain the solution in the infinite series form and some special cases have been discussed.

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1. Introduction. The problem of heat conduction and cooling of plates and rods are important due to their industrial applications. The simplest problem is that of the one-dimensional heat conduction or linear heat flow. Carslaw and Jaeger [4] have discussed different aspects of linear heat flow in plates and rods having homogeneous or composite structure. In such problems, the boundary of the body under consideration is either assumed to be insulated or kept at a constant temperature. Some problems of practical interest however, require imposition of mixed boundary conditions. The classical transform or Fourier series techniques are then no longer applicable. An ingenious method of dealing with such problems is use of the Wiener-Hopf technique.

One of the early studies in this regard is by Caflisch and Keller [3] who have studied the problem of steady state heat conduction in a sufficiently hot plate being cooled by water flowing over its upper surface while its lower surface is insulated. The water adjacent to the hotter part is converted to steam, while the water adjacent to the cooler part is still in the liquid form. This situation results in the mixed boundary condition as the part which is covered by liquid satisfies the cooling condition while the remaining part may be treated as being insulated. The solution in terms of an infinite product involving the roots of a certain transcendental equation is then obtained using the Wiener-Hopf technique. More details about this technique may be found in the treatise by Noble [8].

Levine [7] also considered the same problem, but assumed a simpler representation of the sputtering temperature. In both cases, the authors first obtain an appropriate Green's function for the problem, thus reducing the problem to a singular integral equation which is then solved using the Wiener-Hopf technique.

Evans [5] considered the problem of lowering a long circular cylinder.

In this paper, we study a more general model of the steady state cooling problem by assuming general mixed boundary conditions over the upper surface of an infinite

rectangular plate. We suppose that half of the upper surface is kept at a prescribed temperature while the other half is allowed a heat flux at a variable but prescribed rate. The lower surface of the plate is cooled by a fluid moving with a uniform speed. Some cases of practical interest can be obtained as a limiting case from this general situation.

2. Formulation of the problem. We consider an infinite rectangular plate of uniform thickness h , composed of uniform and homogeneous material possessing constant thermal diffusivity k with the lower surface of the plate coinciding with $y = 0$. We suppose that half of the upper surface ($y = h, -\infty < x < 0$) has been prescribed temperature $f(x)$, while the remaining half ($y = h, 0 \leq x < \infty$) has heat flux given by $g(x)$. The lower surface of the plate is cooled by fluid moving with a uniform speed v having a constant rate of cooling λ . Moreover temperature as ($x \rightarrow \pm\infty$) is assumed to have a particular level of difference (see [1, 6]). The physical considerations lead to the following mixed boundary value problem:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2s \frac{\partial u}{\partial x}, \quad 0 \leq y \leq h, \quad -\infty < x < \infty, \quad (2.1)$$

where $s = v/2k$.

(i) On the lower surface $y = 0$,

$$\frac{\partial u}{\partial y} + \lambda u = 0, \quad \text{on } y = 0, \quad -\infty < x < \infty. \quad (2.2)$$

(ii) On the upper surface $y = h$,

$$\begin{aligned} u(x, y) &= f(x), \quad \text{on } y = h, \quad 0 \leq x < \infty, \\ \frac{\partial u}{\partial y} &= g(x), \quad \text{on } y = h, \quad -\infty < x < 0. \end{aligned} \quad (2.3)$$

(iii) In addition, we assume

$$\begin{aligned} u &\rightarrow 1, & x &\rightarrow \infty, \\ u &\rightarrow 0, & x &\rightarrow -\infty, \end{aligned} \quad (2.4)$$

$$\begin{aligned} u &\sim O(1), & x &\rightarrow +0, \\ \frac{\partial u}{\partial y} &\sim O(1), & x &\rightarrow -0. \end{aligned} \quad (2.5)$$

The boundary data is assumed to be of exponential order, i.e.,

$$\begin{aligned} |f(x)| &< c_1 \exp(\tau_- x), & x &\rightarrow \infty, \\ |g(x)| &< c_2 \exp(\tau_+ x), & x &\rightarrow -\infty, \end{aligned} \quad (2.6)$$

where c_1, c_2, τ_- , and τ_+ are constants and $-s < \tau_- < \tau_+ < s$.

The above boundary value problem can be reduced to a more suitable form by setting

$$u(x, y) = \phi(x, y) \exp(sx). \quad (2.7)$$

This yields,

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} - s^2 \phi = 0. \quad (2.8)$$

The boundary conditions now take the form

$$\frac{\partial \phi}{\partial y} + \lambda \phi = 0, \quad \text{on } y = 0, \quad -\infty < x < \infty, \quad (2.9)$$

$$\phi(x, y) = f(x) \exp(-sx), \quad \text{on } y = h, \quad 0 \leq x < \infty, \quad (2.10)$$

$$\frac{\partial \phi}{\partial y} = g(x) \exp(-sx), \quad \text{on } y = h, \quad -\infty < x < 0. \quad (2.11)$$

The behavior of the solution at $\pm\infty$ is now given by

$$\begin{aligned} \phi &\sim \exp -sx, \quad \text{as } x \rightarrow \infty, \\ \phi &\sim \epsilon \exp -sx \rightarrow 0, \quad \text{as } x \rightarrow -\infty, \end{aligned} \quad (2.12)$$

where ϵ is an arbitrary small real number. This condition can be derived from the behavior of the general solution of (2.8) as $x \rightarrow -\infty$ (see [2]).

The edge conditions at the point of separation of the two parts of the upper surface are

$$\phi, \frac{\partial \phi}{\partial y} \sim O(1), \quad \text{as } x \rightarrow 0 \quad \text{on } y = h. \quad (2.13)$$

3. Reduction to the Wiener-Hopf equation. We define Fourier transform in x and its inverse as

$$f^*(\alpha) = \int_{-\infty}^{\infty} f(x) \exp(i\alpha x) dx, \quad (3.1)$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty+id}^{\infty+id} f^*(\alpha) \exp(-i\alpha x) d\alpha, \quad (3.2)$$

where d is a constant chosen in the domain of analyticity of $f^*(\alpha)$ in the region $\tau_- \leq \text{Im}(\alpha) \leq \tau_+$.

The half-range Fourier transforms are defined by

$$f_+^*(\alpha) = \int_0^{\infty} f(x) \exp(i\alpha x) dx, \quad (3.3)$$

$$f_-^*(\alpha) = \int_{-\infty}^0 f(x) \exp(i\alpha x) dx,$$

so that

$$f^*(\alpha) = f_+^*(\alpha) + f_-^*(\alpha). \quad (3.4)$$

It may be noted that if $f(x) = 0(e^{\tau-x})$ as $x \rightarrow \infty$ and $f(x) = 0(e^{\tau+x})$ as $x \rightarrow -\infty$, then $f_+^*(\alpha)$ is an analytic function of α in the upper half-plane $\text{Im}(\alpha) > \tau_-$, while

$f_{\pm}^*(\alpha)$ is thus in the lower half-plane $\text{Im}(\alpha) < \tau_+$. From (2.9), $f^*(\alpha)$ is then analytic in $\tau_- < \text{Im}(\alpha) < \tau_+$ (Noble [8])

With these preliminaries, we apply the Fourier transform to the boundary value problem (2.8), (2.9), (2.10), and (2.11) to obtain

$$\frac{d^2 \phi^*(\alpha, y)}{dy^2} - y^2 \phi^*(\alpha, y) = 0, \tag{3.5}$$

where $y(\alpha) = \sqrt{s^2 + \alpha^2}$. The branch cuts of y are chosen such that $y(0) = +s$.

$$\phi^{*\prime}(\alpha, 0) + \lambda \phi^*(\alpha, 0) = 0, \tag{3.6}$$

$$\phi_+^*(\alpha, h) = f_+^*(\alpha + is), \quad \text{Im}(\alpha) > \tau_- - s, \tag{3.7}$$

$$\phi_-^{*\prime}(\alpha, h) = g_-^*(\alpha + is), \quad \text{Im}(\alpha) < \tau_+ - s. \tag{3.8}$$

The solution of the differential equation (3.5) that satisfies condition (3.6) is

$$\phi^*(\alpha, y) = A(\alpha)(y \cosh y - \lambda \sinh y). \tag{3.9}$$

Eliminating the unknown $A(\alpha)$ and using the decomposition (3.4) and the boundary conditions (3.7) and (3.8), we get, after some manipulation,

$$\phi_-^*(\alpha, h) + f_+^*(\alpha + is) = K(\alpha)(\phi_+^{*\prime}(\alpha, h) + g_-^*(\alpha + is)), \tag{3.10}$$

where

$$K(\alpha) = \frac{\cosh y h - \lambda(\sinh y h / y)}{y \sinh y h - \lambda \cosh y h} = \frac{G_1(\alpha)}{G_2(\alpha)}. \tag{3.11}$$

4. Solution of the Wiener-Hopf equation. Since $G_1(\alpha)$ and $G_2(\alpha)$ are entire functions of α having infinite number of zeros, $K(\alpha)$ being free of any branch points, we may use Weirstrass theorem to write $K(\alpha) = K_+(\alpha)K_-(\alpha)$, where

$$K_{\pm}(\alpha) = A^{1/2} \prod_{n=1}^{\infty} \left(\frac{\alpha \pm i\alpha_n}{\alpha \pm i\beta_n} \right), \tag{4.1}$$

where

$$A = \prod_{n=1}^{\infty} \left(\frac{\beta_n^2}{\alpha_n^2} \right) \left(\frac{\cosh s h - \lambda(\sinh s h / s)}{s \sinh s h - \lambda \cosh s h} \right), \tag{4.2}$$

where $\pm i\alpha_n, \pm i\beta_n, n = 1, 2, 3, \dots$ are the simple zeros of $G_1(\alpha)$ and $G_2(\alpha)$, respectively. $K_+(\alpha)$ is free from zeros and poles in the upper half-plane and $K_-(\alpha)$ is free from zeros and poles in the lower half-plane.

Thus the Wiener-Hopf equation can be written as

$$J(\alpha) = \frac{\phi_-^*(\alpha, h)}{K_-(\alpha)} - L_-(\alpha) = K_+(\alpha)\phi_+^{*\prime}(\alpha) + L_+(\alpha), \tag{4.3}$$

where

$$L(\alpha) = L_+(\alpha) + L_-(\alpha) = K_+(\alpha)g_-^*(\alpha + is) - \frac{f_+^*(\alpha + is)}{K_-(\alpha)}. \tag{4.4}$$

The explicit expressions for $L_{\pm}(\alpha)$ can be obtained by using the factorization theorem, Noble [8] as

$$L_+(\alpha) = \frac{+1}{2\pi i} \int_{-\infty+ic}^{\infty+ic} \left(K_+(\eta)g_-^*(\eta+is) - \frac{f_+^*(\eta+is)}{K_-(\eta)} \right) \frac{d\eta}{\eta-\alpha}, \tag{4.5}$$

$$L_-(\alpha) = \frac{-1}{2\pi i} \int_{-\infty+ic}^{\infty+ic} \left(K_+(\eta)g_-^*(\eta+is) - \frac{f_+^*(\eta+is)}{K_-(\eta)} \right) \frac{d\eta}{\eta-\alpha},$$

We can argue that equation (4.3) defines an entire function since left-hand side is analytic in the lower half plane while right-hand side is analytic in the upper half plane, both sides being equal on the common strip of analyticity, are analytic continuation of each other. Using the asymptotic behavior of the functions and the infinite products, we can show that $J(\alpha) = 0$. Hence

$$\phi_-^*(\alpha, h) = K_-(\alpha)L_-(\alpha), \tag{4.6}$$

$$\phi_+^{*'}(\alpha) = -\frac{L_+(\alpha)}{K_+(\alpha)}. \tag{4.7}$$

Using the factorization theorem of Noble [8], explicit forms for $L_{\pm}(\alpha)$ can be obtained as

$$L_+(\alpha) = -\frac{f_+^*(\alpha+is)}{K_-(\alpha)} + \sum_{j=1}^{\infty} a_j \frac{g_-^*(-i\beta_j+is)}{-i\beta_j-\alpha} + \sum_{j=1}^{\infty} a'_j \frac{f_+^*(i\alpha_j+is)}{i\alpha_j-\alpha}, \tag{4.8}$$

$$L_-(\alpha) = K_+(\alpha)g_-^*(\alpha+is) - \sum_{j=1}^{\infty} a_j \frac{g_-^*(-i\beta_j+is)}{-i\beta_j-\alpha} - \sum_{j=1}^{\infty} a'_j \frac{f_+^*(i\alpha_j+is)}{i\alpha_j-\alpha},$$

where

$$a_j = A^{1/2}i(\beta_j-\alpha_j) \prod_{n=1, n \neq j}^{\infty} \left(\frac{\beta_j-\alpha_n}{\beta_j-\beta_n} \right), \tag{4.9}$$

$$a'_j = A^{-1/2}i(\alpha_j-\beta_j) \prod_{n=1, n \neq j}^{\infty} \left(\frac{\alpha_j-\beta_n}{\alpha_j-\alpha_n} \right).$$

5. Fourier transform inversion. The solution of the heat problem in the (x, y) -plane can be obtained by using the inversion formula (3.2) as

$$\phi(x, y) = \frac{1}{2\pi} \int_{-\infty+id}^{\infty+id} \left(\frac{\phi_-^*(\alpha, h) + f_+^*(\alpha+is)}{\cosh y h - \lambda(\sinh y h/y)} \right) \left(\cosh y y - \lambda \frac{\sinh y y}{y} \right) \exp(-i\alpha x) d\alpha, \tag{5.1}$$

where d is chosen such that the contour of integration lie in the domain of convergence of the transform integral.

Noting that $\cosh y y - \lambda(\sinh y y/y)$ has no branch points and so using its infinite product representation, we may write

$$\frac{\cosh y \gamma - \lambda(\sinh y \gamma / \gamma)}{\cosh y h - \lambda(\sinh y h / \gamma)} = DN(\gamma) \prod_{n=1}^{\infty} \left(\frac{\alpha^2 + \rho_n^2}{\alpha^2 + \alpha_n^2} \right), \tag{5.2}$$

where

$$D = \prod_{n=1}^{\infty} \left(\frac{\alpha_n^2}{\rho_n^2} \right), \tag{5.3}$$

$$N(\gamma) = \frac{\cosh s \gamma - \lambda(\sinh s \gamma / s)}{\cosh s h - \lambda(\sinh s h / s)}, \tag{5.4}$$

where $\pm i\rho_n$ are the simple zeros of $\cosh y \gamma - \lambda(\sinh y \gamma / \gamma)$. Thus

$$\begin{aligned} \phi(x, y) &= \frac{1}{2\pi} DN(\gamma) \int_{-\infty+id}^{\infty+id} (\phi_-^*(\alpha, h) + f_+^*(\alpha + is)) \prod_{n=1}^{\infty} \left(\frac{\alpha^2 + \rho_n^2}{\alpha^2 + \alpha_n^2} \right) \exp(-i\alpha x) d\alpha \\ &= \frac{D}{2\pi} N(\gamma) (I_1(x) + I_2(x)), \end{aligned} \tag{5.5}$$

where

$$\begin{aligned} I_1(x) &= \int_{-\infty+id}^{\infty+id} f_+^*(\alpha + is) \prod_{n=1}^{\infty} \left(\frac{\alpha^2 + \rho_n^2}{\alpha^2 + \alpha_n^2} \right) \exp(-i\alpha x) d\alpha, \\ I_2(x) &= \int_{-\infty+id}^{\infty+id} K_-(\alpha) H_-(\alpha) \prod_{n=1}^{\infty} \left(\frac{\alpha^2 + \rho_n^2}{\alpha^2 + \alpha_n^2} \right) \exp(-i\alpha x) d\alpha. \end{aligned} \tag{5.6}$$

If $x < 0$, we close the contour of integration in the upper half-plane and for $x \geq 0$, the contour is chosen to be closed in the lower half-plane so that the above integrals are defined. This gives

$$\begin{aligned} I_3(x) &= \begin{cases} 2\pi - \sum_{j=1}^{\infty} b_j f_+^*(-i\alpha_j + is) \exp(-\alpha_j x) \\ \quad + \text{the contribution of the poles of } f_+^*(\alpha + is) \text{ if any, } & x \geq 0, \\ 2\pi - \sum_{j=1}^{\infty} b_j f_+^*(i\alpha_j + is) \exp(\alpha_j x), & x < 0, \end{cases} \\ I_4(x) &= \begin{cases} 2\pi - \sum_{j=1}^{\infty} b_j K_-(i\alpha_j) L_-(i\alpha_j) \exp(-\alpha_j x), & x \geq 0, \\ 2\pi - \sum_{j=1}^{\infty} b_j K_-(\alpha_j) L_-(\alpha_j) \\ \quad + \text{the contribution of the poles of } K_-(\alpha) H_-(\alpha), & x < 0, \end{cases} \end{aligned} \tag{5.7}$$

where

$$b_j = \frac{(\alpha_j^2 - \rho_j^2)}{2\alpha_j} \prod_{n=1, n \neq j}^{\infty} \left(\frac{\alpha_j^2 - \rho_n^2}{(\alpha_j + \alpha_n)(\alpha_j - \alpha_n)} \right). \tag{5.8}$$

6. Discussion and limiting cases. The temperature distribution in the plate can be obtained using (2.7) and (5.5). In a similar way the heat flux at the upper surface of the plate, which is of interest in many situations, can be obtained using expressions for $\phi_+^*(\alpha)$ obtainable from (4.7).

(a) Let us assume that the left half of the upper surface of the plate is insulated, i.e., $g(x) = 0$, thus

$$\phi(x, y) = \begin{cases} DN(y) \sum_{j=1}^{\infty} \left[-b_j f_+^*(-i\alpha_j + is) + \sum_{l=1}^{\infty} a_l c_j d_{jl} f_+^*(i\alpha_l + is) \right] \exp(-\alpha_j x) \\ \quad + \text{the contribution of the poles of } f_+^*(\alpha + is) \text{ if any, } x \geq 0, \\ DN(y) \sum_{j=1}^{\infty} \left[-b_j f_+^*(i\alpha_j + is) + \sum_{l=1}^{\infty} a_l c'_j d'_{jl} f_+^*(i\alpha_l + is) \right] \exp(\alpha_j x) \\ \quad + \sum_{j=1}^{\infty} e_j f_+^*(i\beta_j + is) \exp(\beta_j x), \quad x < 0, \end{cases} \tag{6.1}$$

where

$$c_j = b_j K_-(-i\alpha_i), \quad c'_j = b_j K_-(i\alpha_i) + \prod_{n=1, n \neq j}^{\infty} \frac{\alpha_j^2 - \rho_n^2}{(\alpha_l + \beta_n)(\alpha_l - \alpha_n)},$$

$$d_{jl} = \frac{1}{i(\alpha_j + \alpha_l)}, \quad d'_{jl} = \frac{1}{i(\alpha_l - \alpha_j)}, \tag{6.2}$$

$$e_j = \prod_{n=1, n \neq j}^{\infty} \frac{\beta_j^2 - \rho_n^2}{(\beta_l + \alpha_n)(\beta_l - \beta_n)}.$$

(b) If the right half of the upper surface was heated uniformly from the point $x = 0$ to ∞ , then

$$f(x) = aH(x), \tag{6.3}$$

where a is a constant and $H(x)$ is the Heaviside unit function. With this choice

$$f_+^*(\alpha + is) = \frac{ai}{\alpha + is}. \tag{6.4}$$

$f_+^*(\alpha + is)$ has a simple pole at $\alpha = -is$ in the lower half-plane, then for $s \neq \alpha_n, n = 1, 2, \dots$ we get

$$\phi(x, y) = \begin{cases} aDN(y) \sum_{j=1}^{\infty} \left[\frac{b_i}{\alpha_j - s} + \sum_{l=1}^{\infty} \frac{a_l c_j d_{jl}}{\alpha_l + s} \right] \exp(-\alpha_j x) \\ \quad + \prod_{n=1}^{\infty} \frac{s^2 - \rho_n^2}{s^2 - \alpha_n^2} \exp(-sx), \quad x \geq 0, \\ aDN(y) \sum_{j=1}^{\infty} \left[\frac{-b_j}{\alpha_j + s} + \sum_{l=1}^{\infty} \frac{a_l c'_j d'_{jl}}{\alpha_l + s} \right] \exp(\alpha_j x) \\ \quad + \sum_{j=1}^{\infty} \frac{e_j}{\beta_j + s} \exp(\beta_j x), \quad x < 0. \end{cases} \tag{6.5}$$

(c) If the right upper-half ($y = h, 0 \leq x < \infty$) was heated by an exponentially decaying behavior, i.e.,

$$f(x) = b \exp(-ax), \tag{6.6}$$

where $a > 0$ and b are constants.

Then

$$f_+^*(\alpha + is) = \frac{ib}{\alpha + i(s+a)}. \tag{6.7}$$

Thus $f_+^*(\alpha + is)$ has simple pole at $\alpha = -i(s+a)$ on the lower half-plane, then for $s+a \neq \alpha_n, n = 1, 2, \dots$, we get

$$\phi(x, y) = \begin{cases} bDN(y) \sum_{j=1}^{\infty} \left[\frac{b_i}{\alpha_j - (s+a)} + \sum_{l=1}^{\infty} \frac{a_l c_j d_{jl}}{\alpha_l + (s+a)} \right] \exp(-\alpha_j x) \\ \quad + \prod_{n=1}^{\infty} \frac{s^2 - \rho_n^2}{s^2 - \alpha_n^2} \exp(-sx), \quad x \geq 0, \\ bDN(y) \sum_{j=1}^{\infty} \left[\frac{-b_j}{\alpha_j + (s+a)} + \sum_{l=1}^{\infty} \frac{a_l c'_j d'_{jl}}{\alpha_l + (s+a)} \right] \exp(\alpha_j x) \\ \quad + \sum_{j=1}^{\infty} \frac{e_i}{\beta_j + s} \exp(\beta_j x), \quad x < 0. \end{cases} \tag{6.8}$$

In a similar way, we may consider different forms of $f(x)$ and $g(x)$ to fit many physically interesting situations and obtain the solution in the infinite series form.

7. Numerical results. The numerical results of (6.5) and (6.8) for the temperature $u(0, y)$ at the midpoint $x = 0$ are tabulated in Tables 7.1, 7.2, and 7.3. u_0 was calculated for suitable choices of the parameters Ω, s_1 , and h_1 where $\Omega = \lambda y, s_1 = sy, h_1 = y/h$. Table 7.1 shows that for fixed s_1 and h_1 , the temperature u_0 of the uniformly heated plate with $a = 1$, increase with increase of Ω . Also for fixed Ω and h_1, u_0 decreases with increase of s_1 . If we increase Ω more than 0.6, the temperature u_0 begins to decrease until it reaches a constant value for example at $s_1 = 0.02, h_1 = 0.8$, we find that u_0 has nearly the same value ($u_0 = 0.398$) for large values of Ω ($\Omega = 300, 400, 500$).

TABLE 7.1.

$s_1 = 0.001, h_1 = 0.8$						
Ω	0.00	0.04	0.08	0.2	0.4	0.6
$u_0(y)$	0.4998	0.5050	0.5109	0.5331	0.5996	0.7993
$s_1 = 0.2, h_1 = 0.8$						
$u_0(y)$	0.4577	0.4584	0.4628	0.4801	0.5218	0.6540

Tables 7.2 and 7.3 shows that the temperature of the exponentially heated plate with $b = 1$ depends on the exponent a . The temperature u_0 decreases with increase of a which is clear from comparison of the two tables.

TABLE 7.2.

$s_1 = 0.001, h_1 = 0.8, a = 0.6$						
Ω	0.00	0.04	0.08	0.2	0.4	0.6
$u_0(\gamma)$	0.3206	0.3225	0.3249	0.3347	0.3433	0.4087
$s_1 = 0.02, h_1 = 0.8, a = 0.6$						
$u_0(\gamma)$	0.3192	0.3210	0.3234	0.3332	0.3415	0.4063

TABLE 7.3.

$s_1 = 0.001, h_1 = 0.8, a = 1.0$						
Ω	0.00	0.04	0.08	0.2	0.4	0.6
$u_0(\gamma)$	0.2638	0.2651	0.2670	0.2747	0.2771	0.3252
$s_1 = 0.02, h_1 = 0.8, a = 1.0$						
$u_0(\gamma)$	0.2629	0.2642	0.2660	0.2737	0.2760	0.3238

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