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ON ALMOST PRECONTINUOUS FUNCTIONS

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Abstract. Nasef and Noiri (1997) introduced and investigated the class of almost precontinuous functions. In this paper, we further investigate some properties of these functions.

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1. Introduction. Singal and Singal [24] introduced the notion of almost continuity. Feeble continuity was introduced by Maheshwari et al. [8]. As a generalization of almost continuity and feeble continuity, Maheshwari et al. [7] introduced the notion of almost feeble continuity. Nasef and Noiri [12] introduced a new class of functions called almost precontinuous functions. They showed that almost precontinuity is a generalization of each of almost feeble continuity and almost α-continuity [17].

The purpose of this paper is to investigate some more properties of almost precontinuous functions. It turns out that almost precontinuity is stronger than almost weak continuity introduced by Jankovič [5].

2. Preliminaries. Throughout this paper, (X,τ) and (Y,σ) (or X and Y) are always topological spaces. A set A in a space X is called *preopen* [11] (respectively, *semi-open* [6] and *α-open* [13]) if $A \subset \mathring{\bar{A}}$ (respectively, $A \subset \bar{A^{\circ}}$ and $A \subset \bar{A^{\circ}}^{\circ}$). The complement of a preopen set is called *preclosed*.

The intersection of all preclosed sets containing a subset A is called the *preclosure* [2] of A and is denoted by Pcl(A). The *preinterior* of A is the union of all preopen sets of X contained in A. The family of all preopen sets of X will be denoted by $\mathrm{PO}(X)$. For a point x of X, we put $\mathrm{PO}(X,x) = \{U \mid x \in U \in \mathrm{PO}(X)\}$. A set A is called *regular open* (respectively, *regular closed*) if $A = \mathring{\bar{A}}$ (respectively, $A = \bar{A^{\circ}}$).

Definition 2.1. A function $f : X \to Y$ is called *almost continuous* [24] (in the sense of Singal) at $x \in X$ if for every open set V in Y containing $f(x)$, there is an open set U in X containing x such that $f(U) \subset \bar{V}^{\circ}$. If f is almost continuous at every point of X, then it is called almost continuous.

Definition 2.2. A function $f : X \to Y$ is called *almost weakly continuous* [5] (briefly a.w.c.) if $f^{-1}(V) \subset \overline{f^{-1}(\bar{V})}^{\circ}$ for every open set V of Y.

Remark 2.3. In [20, Theorem 3.1] Popa and Noiri have defined the following pointwise description of almost weak continuity: a function $f : X \to Y$ is a.w.c. if and only if for each point $x \in X$ and every open set V in Y containing $f(x)$, there exists $U \in \mathrm{PO}(X,x)$ such that $f(U) \subset \bar{V}$. The referee has given a global description as follows: a function $f : X \to Y$ is a.w.c. if and only if for each open set V of Y, there exists $U \in \mathrm{PO}(X)$ such that $f^{-1}(V) \subset U \subset f^{-1}(\bar{V})$.

Definition 2.4. A function $f : X \to Y$ is called *almost precontinuous* [12] (briefly a.p.c.) at $x \in X$ if for each regular open set $V \subset Y$ containing $f(x)$, there exists $U \in \mathrm{PO}(X,x)$ such that $f(U) \subset V$. If f is almost precontinuous at every point of X, then it is called almost precontinuous.

Definition 2.5. A function $f : X \to Y$ is said to be *weakly α-continuous* [16] (briefly w.α.c.) if for each $x \in X$ and each open set $V \subset Y$ containing $f(x)$, there exists an α-open set U containing x such that $f(U) \subset \bar{V}$.

Definition 2.6. A function $f : X \to Y$ is said to be *precontinuous* [11] if for every open set V of Y, the inverse image of V is preopen in X.

Remark 2.7. Between almost precontinuity and precontinuity, we have the following relationship: a function $f : X \to Y$ is a.p.c. if and only if $f : X \to Y_s$ is precontinuous, where Y_s denotes the semi-regularization of Y.

Remark 2.8. It easily follows from [20, Theorem 3.1] that precontinuity implies almost precontinuity and almost precontinuity implies almost weak continuity. However, the converses are not true as the following examples show.

Example 2.9. Let $X = \{a,b,c\}$, $\tau = \{X,\emptyset,\{a\},\{c\},\{a,c\}\}$ and $\sigma = \{X,\emptyset,\{a\},\{b\},\{a,b\},\{b,c\}\}$. Define a function $f : (X,\tau) \to (X,\sigma)$ as follows: $f(a) = f(b) = b$ and $f(c) = c$. Then f is an almost continuous and hence a.p.c. function which is not precontinuous. Because, there exists $\{b\} \in \sigma$ such that $f^{-1}(\{b\}) \notin \mathrm{PO}(X,\tau)$.

Example 2.10. Let $X = \{a,b,c,d\}$ and $\tau = \{X,\emptyset,\{b\},\{c\},\{a,b\},\{b,c\},\{a,b,c\},\{b,c,d\}\}$. Define a function $f : (X,\tau) \to (X,\tau)$ as follows: $f(a) = c$, $f(b) = d$, $f(c) = b$, and $f(d) = a$. Then f is a.w.c. However, f is not a.p.c. because there exists a regular open set $\{c\}$ of (X,τ) such that $f^{-1}(\{c\}) \notin \mathrm{PO}(X,\tau)$.

Recall that a filter base $\mathcal{F}$ is called *δ-convergent* [25] (respectively, *p-convergent* [4]) to a point x in X if for any open set U containing x (respectively, any $U \in \mathrm{PO}(X,x)$), there exists $B \in \mathcal{F}$ such that $B \subset \bar{U}^\circ$ (respectively, $B \subset U$).

3. Some properties.

In [9], Mashhour et al. introduced the following notion.

Definition 3.1. A function $f : X \to Y$ is called *M-preopen* if the image of each preopen set is preopen.

We have the following result.

Theorem 3.2. *If $f : X \to Y$ is M-preopen a.w.c., then f is a.p.c.*

Proof. Suppose that $x \in X$ and V is any open set containing $f(x)$. Since f is a.w.c., then there exists $U \in \mathrm{PO}(X,x)$ such that $f(U) \subset \bar{V}$ [20, Theorem 3.1]. Since f is

M-preopen, $f(U)$ is preopen in Y and hence $f(U) \subset \overline{f(U)}^{\circ} \subset \overline{\bar{V}}^{\circ} = \bar{V}^{\circ}$. It follows that $f(U) \subset \bar{V}^{\circ}$. Hence f is a.p.c. □

Recall that a space X is called *submaximal* if every dense subset of X is open in X. It is shown in [22, Theorem 4] that a space X is submaximal if and only if every preopen set of X is open in X.

Theorem 3.3. *If a function $f : X \to Y$ is a.p.c., then for each point $x \in X$ and each filter base $\mathcal{F}$ in X p-converging to x, the filter base $f(\mathcal{F})$ is δ-convergent to $f(x)$. If X is submaximal, then the converse also holds.*

Proof. Suppose that x belongs to X and $\mathcal{F}$ is any filter base in X p-converging to x. By the almost precontinuity of f, for any regular open set V in Y containing $f(x)$, there exists $U \in \mathrm{PO}(X,x)$ such that $f(U) \subset V$. But $\mathcal{F}$ is p-convergent to x in X, then there exists $B \in \mathcal{F}$ such that $B \subset U$. It follows that $f(B) \subset V$. This means that $f(\mathcal{F})$ is δ-convergent to $f(x)$.

Now suppose that X is submaximal. Let x be a point in X and V any regular open set containing $f(x)$. Since X is submaximal, every preopen set of X is open [22, Theorem 4]. If we set $\mathcal{F} = \mathrm{PO}(X,x)$, then $\mathcal{F}$ will be a filter base which p-converges to x. So there exists U in $\mathcal{F}$ such that $f(U) \subset V$. This completes the proof. □

The following corollary is suggested by the referee.

Corollary 3.4. *Let X be a submaximal space. Then a function $f : X \to Y$ is a.p.c. if and only if $f : X \to Y_s$ is continuous.*

Definition 3.5. A space X is called *pre-T_2* [18] if for every pair of distinct points x and y in X, there exist preopen sets U and V containing x and y, respectively, such that $U \cap V = \emptyset$.

Theorem 3.6. *If $f : X \to Y$ is an a.p.c. injection and Y is Hausdorff, then X is pre-T_2.*

Proof. Since $f : X \to Y$ is a.p.c. injective, $f : X \to Y_s$ is a precontinuous injection and Y_s is Hausdorff. Let x and y be any distinct points of X. Since f is injective, $f(x) \neq f(y)$ and hence there exist disjoint open sets V and W of Y_s such that $f(x) \in V$ and $f(y) \in W$. Therefore, we obtain $f^{-1}(V) \in \mathrm{PO}(X,x)$, $f^{-1}(W) \in \mathrm{PO}(X,y)$, and $f^{-1}(V) \cap f^{-1}(W) = \emptyset$. This shows that X is pre-T_2. □

Recall that a space X is called a *door space* if every subset of X is either open or closed. Reilly and Vamanamurthy proved the following result in [22, Theorem 2].

Lemma 3.7. *If X is a door space, then every preopen set in X is open.*

Theorem 3.8. *Let $f, g : X \to Y$ be functions, Y Hausdorff and X a door space. If f and g are a.p.c. functions, then the set $E = \{x \in X \mid f(x) = g(x)\}$ is closed in X.*

Proof. Let $x \in X - E$. It follows that $f(x) \neq g(x)$. Since Y is Hausdorff, then there exist open sets V_1 and V_2 in Y such that $f(x) \in V_1$, $g(x) \in V_2$, and $V_1 \cap V_2 = \emptyset$. Since V_1 and V_2 are disjoint, we obtain $\bar{V}_1^{\circ} \cap \bar{V}_2^{\circ} = \emptyset$. Since f and g are a.p.c., there exist preopen sets U_1 and U_2 in X containing x such that $f(U_1) \subset \bar{V}_1^{\circ}$ and $g(U_2) \subset \bar{V}_2^{\circ}$. Put

$U = U_1 \cap U_2$, so, by Lemma 3.7, U is an open set in X containing x. Thus we have $f(U) \cap g(U) = \emptyset$. It follows that $x \notin \bar{E}$. Hence $\bar{E} \subset E$ and E is closed in X. □

Lemma 3.9 (Popa and Noiri [20]). *If A is an α-open set of a space X and $B \in \mathrm{PO}(X)$, then $A \cap B \in \mathrm{PO}(X)$.*

Theorem 3.10. *Let $f, g : X \to Y$ be functions and Y Hausdorff. If f is w.α.c. and g is a.p.c., then the set $E = \{x \in X \mid f(x) = g(x)\}$ is preclosed in X.*

Proof. Suppose that $x \notin E$. Then $f(x) \neq g(x)$. Since Y is Hausdorff, there exist open sets V and W of Y such that $f(x) \in V$, $g(x) \in W$, and $V \cap W = \emptyset$; hence $\bar{V} \cap \bar{W}^\circ = \emptyset$. Since f is w.α.c., there exists an α-open set U containing x such that $f(U) \subset \bar{V}$. Since g is a.p.c., there exists $G \in \mathrm{PO}(X,x)$ such that $g(G) \subset \bar{W}^\circ$. Put $O = U \cap G$, then $O \in \mathrm{PO}(X,x)$ by Lemma 3.9 and $O \cap E = \emptyset$. Therefore, we obtain $x \notin \mathrm{Pcl}(E)$. This shows that E is preclosed in X. □

Corollary 3.11 (Popa [19]). *Let $f, g : X \to Y$ be functions and Y Hausdorff. If f is continuous and g is precontinuous, then the set $E = \{x \in X \mid f(x) = g(x)\}$ is preclosed in X.*

Theorem 3.12. *Let $f : X_1 \to Y$ and $g : X_2 \to Y$ be two a.p.c. functions. If Y is a Hausdorff space, then the set $\{(x_1 \times x_2) \in X_1 \times X_2 \mid f(x_1) = g(x_2)\}$ is preclosed in $X_1 \times X_2$.*

Proof. Let $(x_1, x_2) \notin E$. Then $f(x_1) \neq g(x_2)$. Since Y is Hausdorff, there exist disjoint open neighborhoods V and W of $f(x_1)$ and $g(x_2)$, respectively. Since V and W are disjoint, we have $\bar{V}^\circ \cap \bar{W}^\circ = \emptyset$. Since f and g are a.p.c., there exist $U \in \mathrm{PO}(X_1, x_1)$ and $G \in \mathrm{PO}(X_2, x_2)$ such that $f(U) \subset \bar{V}^\circ$ and $g(G) \subset \bar{W}^\circ$, respectively. Put $O = U \times G$, then $(x_1, x_2) \in O$, O is preopen in $X_1 \times X_2$ and $O \cap E = \emptyset$. Therefore, we obtain $(x_1, x_2) \in \mathrm{Pcl}(E)$. This shows that E is preclosed in $X_1 \times X_2$. □

Corollary 3.13. *If Y is Hausdorff and $f : X \to Y$ is an a.p.c. function, then the set $E = \{(x, y) \mid f(x) = f(y)\}$ is preclosed in $X \times X$.*

Proof. By setting $X = X_1 = X_2$ and $g = f$ in Theorem 3.12, the result follows. □

Corollary 3.14 (Mashhour et al. [11]). *If $f : X \to Y$ is a precontinuous function and Y is Hausdorff, then the set $\{(x, y) \mid f(x) = f(y)\}$ is preclosed in $X \times X$.*

Corollary 3.15 (Popa [19]). *Let $f : X_1 \to Y$ and $g : X_2 \to Y$ be two precontinuous functions. If Y is a Hausdorff space, then the set $\{(x, y) \mid f(x) = g(y)\}$ is preclosed in $X_1 \times X_2$.*

We introduce the following concept.

Definition 3.16. For a function $f : X \to Y$, the graph $G(f) = \{(x, f(x)) \mid x \in X\}$ is called *strongly almost preclosed* if for each $(x, y) \in X \times Y - G(f)$, there exist $U \in \mathrm{PO}(X, x)$ and a regular open set V containing y such that $(U \times V) \cap G(f) = \emptyset$.

Lemma 3.17. *A function $f : X \to Y$ has the strongly almost preclosed graph if and only if for each $x \in X$ and $y \in Y$ such that $f(x) \neq y$, there exist $U \in \mathrm{PO}(X, x)$ and a regular open set V containing y such that $f(U) \cap V = \emptyset$.*

Proof. It is an immediate consequence of the above definition. □

Theorem 3.18. *If $f : X \to Y$ is a.w.c. and Y is Hausdorff, then $G(f)$ is strongly almost preclosed.*

Proof. Suppose that (x,y) is any point of $X \times Y - G(f)$. Then $y \neq f(x)$. But Y is Hausdorff and hence there exist open sets G_1 and G_2 in Y such that $y \in G_1$, $f(x) \in G_2$, and $G_1 \cap G_2 = \emptyset$. Since G_1 and G_2 are disjoint, we obtain $\bar{G}_1^\circ \cap \bar{G}_2 = \emptyset$. And since f is a.w.c., then there exists $U \in \mathrm{PO}(X,x)$ such that $f(U) \subset \bar{G}_2$. Hence, $f(U) \cap \bar{G}_1^\circ = \emptyset$. It follows from Lemma 3.17 that $G(f)$ is strongly almost preclosed. □

Recall that a subset A of a space X is said to be *strongly compact relative to* X [9] (respectively, *N-closed relative to* X [1]) if every cover of A by preopen (respectively, regular open) sets of X has a finite subcover.

Definition 3.19. A space X is called *strongly compact* [10] (respectively, *nearly compact* [23]) if every preopen (respectively, regular open) cover of X has a finite subcover.

Theorem 3.20. *If $f : X \to Y$ is a.p.c. and K is a strongly compact relative to X, then $f(K)$ is N-closed relative to Y.*

Proof. Let $\{G_\alpha \mid \alpha \in A\}$ be any cover of $f(K)$ by regular open sets of Y. Then, $\{f^{-1}(G_\alpha) \mid \alpha \in A\}$ is a cover of K by preopen sets of X [12, Theorem 3.1]. Since K is strongly compact relative to X, there exists a finite subset $A_\circ$ of A such that $K \subset \cup\{f^{-1}(G_\alpha) \mid \alpha \in A_\circ\}$. Therefore, we obtain $f(K) \subset \cup\{G_\alpha \mid \alpha \in A_\circ\}$. This shows that $f(K)$ is N-closed relative to Y. □

Corollary 3.21. *If $f : X \to Y$ is an a.p.c. surjection and X is strongly compact, then Y is nearly compact.*

Definition 3.22. A function $f : X \to Y$ is said to be *δ-continuous* [14] if for each $x \in X$ and each open set V of Y containing $f(x)$, there exists an open set U in X containing x such that $f(\bar{U}^\circ) \subset \bar{V}^\circ$.

Theorem 3.23. *If $f : X \to Y$ is a.p.c. and $g : Y \to Z$ is δ-continuous, then $g \circ f : X \to Z$ is a.p.c.*

Proof. The proof is obvious and is omitted. □

Theorem 3.24. *If $f : X \to Y$ is an M-preopen surjection and $g : Y \to Z$ is a function such that $g \circ f : X \to Z$ is a.p.c., then g is a.p.c.*

Proof. Let $y \in Y$ and $x \in X$ such that $f(x) = y$. Let G be a regular open set containing $(g \circ f)(x)$. Then there exists $U \in \mathrm{PO}(X,x)$ such that $g(f(U)) \subset G$. Since f is M-preopen, $f(U) \in \mathrm{PO}(Y,y)$ such that $g(f(U)) \subset G$. This shows that g is a.p.c. at y. □

Theorem 3.25. *If $f : X \to Y$ is a.p.c. and A is a semi-open set of X, then the restriction $f \mid A : A \to Y$ is a.p.c.*

Proof. Let V be any regular open set of Y. Since f is a.p.c., the inverse image of V is preopen in X [12, Theorem 3.1] and $(f \mid A)^{-1}(V) = A \cap f^{-1}(V)$. Since A is semi-open in X, it follows from [11, Lemma 2.1] that $A \cap f^{-1}(V) \in \mathrm{PO}(A)$. Therefore, $f \mid A$ is a.p.c. □

Remark 3.26. It should be noted that every restriction of an a.p.c. function is not necessarily a.p.c. In [15, proof of Theorem 6.2.5], it is pointed out that there is a precontinuous function whose restriction to a not semi-open set is not even a.w.c. It might also be noted that neither is almost precontinuity for a function $f : X \to Y$ preserved by restriction of the codomain to $f(X)$. The following example is due to referee.

Example 3.27. Let $f : \mathbb{Q} \to \mathbb{R}$ be the inclusion map of the rationals into the reals. Let the domain have the usual subspace topology and let the nonempty open sets in the codomain have the form $P \cup A$, where $P = \mathbb{R} - \mathbb{Q}$ is the set of irrationals and where $A \subseteq \mathbb{Q}$. Then $\mathbb{R}_s$ is indiscrete so that f is a.p.c. Yet, $f(\mathbb{Q})$ is a discrete subspace of $\mathbb{R}$ so that $f : \mathbb{Q} \to f(\mathbb{Q})$ is not a.p.c. since not every subset of the domain space is preopen.

Theorem 3.28. *Let $f : X \to Y$ be a function and $x \in X$. If there exists $U \in \mathrm{PO}(X,x)$ such that the restriction of f to U is a.p.c. at x, then f is a.p.c. at x.*

Proof. Suppose that V_2 is any regular open set containing $f(x)$. Since $f \mid U$ is a.p.c. at x, there exists $V_1 \in \mathrm{PO}(U,x)$ such that $f(V_1) = (f \mid U)(V_1) \subset V_2$. Since $U \in \mathrm{PO}(X,x)$, it follows from [11, Lemma 2.2] that $V_1 \in \mathrm{PO}(X,x)$. This shows clearly that f is a.p.c. at x. □

Definition 3.29. Let $A \subset X$. The *preboundary* $\mathrm{pFr}(A)$ of A is defined by $\mathrm{pFr}(A) = \mathrm{Pcl}(A) \cap \mathrm{Pcl}(X - A)$.

Theorem 3.30. *The set of all points x of X at which $f : X \to Y$ is not a.p.c. is identical with the union of the preboundaries of the inverse images of regular open subsets of Y containing $f(x)$.*

Proof. If f is not a.p.c. at $x \in X$, then there exists a regular open set V containing $f(x)$ such that for every $U \in \mathrm{PO}(X,x)$, $f(U) \cap (Y - V) \neq \emptyset$. This means that for every $U \in \mathrm{PO}(X,x)$, we must have $U \cap (X - f^{-1}(V)) \neq \emptyset$. Hence, it follows from [2, Lemma 2.2] that $x \in \mathrm{Pcl}(X - f^{-1}(V))$. But $x \in f^{-1}(V)$ and hence $x \in \mathrm{Pcl}(f^{-1}(V))$. This means that x belongs to the preboundary of $f^{-1}(V)$. Suppose that x belongs to the preboundary of $f^{-1}(V_1)$ for some regular open subset V_1 of Y such that $f(x) \in V_1$. Suppose that f is a.p.c. at x. Then there exists $U \in \mathrm{PO}(X,x)$ such that $f(U) \subset V_1$. Then, we have: $x \in U \subset f^{-1}(f(U)) \subset f^{-1}(V_1)$. This shows that x is a preinterior point of $f^{-1}(V_1)$. Therefore, we have $x \notin \mathrm{Pcl}(X - f^{-1}(V_1))$ and $x \notin \mathrm{pFr}(f^{-1}(V_1))$. But this is a contradiction. This means that f is not a.p.c. □

Recall that a subset A of a space X is said to be *H-set* [25] or *quasi H-closed relative to X* [21] if for every cover $\{U_i \mid i \in I\}$ of A by open sets of X, there exists a finite subset I_0 of I such that $A \subset \cup\{\bar{U}_i \mid i \in I_0\}$.

Theorem 3.31. *If $f : X \to Y$ is a.w.c. and K is strongly compact relative to X, then $f(K)$ is quasi H-closed relative to Y.*

Proof. The proof is similar to the one of Theorem 3.20. □

Recall that a function $f : X \to Y$ is called *r-preopen* [3] if the image of a preopen set in X is open in Y.

Theorem 3.32. *Let $f : X \to Y$ be an a.w.c. bijection. If X is strongly compact and Y is Hausdorff, then f is r-preopen.*

Proof. Suppose that U is a preopen subset of X. Then $X - U$ is preclosed subset of the strongly compact space X. This means that $X - U$ is strongly compact relative to X. By Theorem 3.31, $f(X - U)$ is quasi H-closed relative to Y. Since f is bijective, we have $f(X - U) = Y - f(U)$, where $Y - f(U)$ is quasi H-closed relative to Y. Since Y is Hausdorff, therefore $Y - f(U)$ is closed in Y. Hence $f(U)$ is open in Y. □

Corollary 3.33. *Let $f : X \to Y$ be an a.p.c. bijection. If X is strongly compact and Y is Hausdorff, then f is r-preopen.*

Proof. Since every a.p.c. function is a.w.c., hence the proof follows from Theorem 3.32. □

Definition 3.34. Let E and F be any two subsets of X. E and F are called *strongly p-separated* if there exist disjoint preopen sets U and V such that $E \subset U$ and $F \subset V$.

Definition 3.35. A function $f : X \to Y$ is said to be *strongly preclosed* [18] if the image of a preclosed set in X is preclosed in Y.

Definition 3.36. A space X is called *strongly prenormal* [18] if for disjoint preclosed subsets E and F of X, there exist disjoint preopen sets U and V such that $E \subset U$ and $F \subset V$.

Theorem 3.37. *If f is an a.p.c., strongly preclosed function of strongly pre-normal space X onto a space Y, then any two disjoint regular closed subsets of Y can be strongly p-separated.*

Proof. Let F and D be two disjoint regular closed subsets of Y. Then $f^{-1}(F)$ and $f^{-1}(D)$ are disjoint, preclosed subsets of the strongly prenormal space X and therefore there exist preopen sets U and W such that $U \cap W = \varnothing$, $f^{-1}(F) \subset U$, and $f^{-1}(D) \subset W$. Suppose that

$$P_1 = \{y \mid f^{-1}(y) \subset U\}, \qquad P_2 = \{y \mid f^{-1}(y) \subset W\}. \tag{3.1}$$

Since f is strongly preclosed, then P_1 and P_2 are preopen sets. Then we have

$$F \subset P_1, \quad D \subset P_2, \quad P_1 \cap P_2 = \varnothing. \tag{3.2}$$

□

Now we obtain the following results whose proofs are omitted since they are straightforward.

Recall that a space X is said to be *extremally disconnected* if the closure of each open set of X is open in X.

Theorem 3.38. *If $f : X \to Y$ is a.w.c. and Y is extremally disconnected, then f is a.p.c.*

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