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ON A CLASS OF CONTACT RIEMANNIAN MANIFOLDS

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ABSTRACT. We determine a locally symmetric or a Ricci-parallel contact Riemannian manifold which satisfies a D-homothetically invariant condition.

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1. Introduction. In [8] Tanno proved that a locally symmetric K-contact Riemannian manifold is of constant curvature 1, which generalizes the corresponding result for a Sasakian manifold due to Okumura [6]. For dimensions greater than or equal to 5 it was proved by Olszak [7] that there are no contact Riemannian structures of constant curvature unless the constant is 1 and in which case the structure is Sasakian. Further, Blair and Sharma [4] proved that a 3-dimensional locally symmetric contact Riemannian manifold is either flat or is Sasakian and of constant curvature 1. By the recent result [5] and private communication with Blair we know that the simply connected covering space of a complete 5-dimensional locally symmetric contact Riemannian manifold is either $S^5(1)$ or $E^3 \times S^2(4)$. The question of the classification of locally symmetric contact Riemannian manifolds in higher dimensions is still open.

On the other hand, recently, Blair, Koufogiorgos and Papantoniou [3] introduced a class of contact Riemannian manifolds which is characterized by the equation

$$R(X,Y)\xi = \kappa\big(\eta(Y)X - \eta(X)Y\big) + \mu\big(\eta(Y)hX - \eta(X)hY\big), \tag{1.1}$$

where κ, μ are constant and $2h$ is the Lie derivative of ϕ in the direction ξ. It is remarkable that this class of spaces is invariant under D-homothetic deformations (see [3]). It was also proved in [3] that a Sasakian manifold, in particular, is determined by $\kappa = 1$ and further that this class contains the tangent sphere bundle of Riemannian manifolds of constant curvature. In this paper, we determine a locally symmetric or a Ricci-parallel contact Riemannian manifold which satisfies (1.1). More precisely, we prove the following two Theorems 1.1 and 1.2 in Sections 3 and 4.

Theorem 1.1. *Let M be a contact Riemannian manifold satisfying (1.1). Suppose that M is locally symmetric. Then M is the product of flat $(n+1)$-dimensional manifold and an n-dimensional manifold of positive constant curvature equal to 4, or a space of constant curvature 1 and in which case the structure is Sasakian.*

Theorem 1.2. *Let M be a contact Riemannian manifold satisfying (1.1). Suppose that M is Ricci-parallel. Then M is the product of flat $(n+1)$-dimensional manifold and* *an n-dimensional manifold of positive constant curvature equal to 4 or an Einstein-Sasakian manifold.*

2. Preliminaries. All manifolds in the present paper are assumed to be connected and of class C^∞. A $(2n+1)$-dimensional manifold M^{2n+1} is said to be *a contact manifold* if it admits a global 1-form η such that $\eta\wedge(d\eta)^n \neq 0$ everywhere. Given a contact form η, we have a unique vector field ξ, which is called the characteristic vector field, satisfying $\eta(\xi)=1$ and $d\eta(\xi,X)=0$ for any vector field X. It is well known that there exists an associated Riemannian metric g and a $(1,1)$-type tensor field ϕ such that

$$\eta(X)=g(X,\xi),\qquad d\eta(X,Y)=g(X,\phi Y),\qquad \phi^2X=-X+\eta(X)\xi, \tag{2.1}$$

where X and Y are vector fields on M. From (2.1) it follows that

$$\phi\xi=0,\qquad \eta\circ\phi=0,\qquad g(\phi X,\phi Y)=g(X,Y)-\eta(X)\eta(Y). \tag{2.2}$$

A Riemannian manifold M equipped with structure tensors (η,g) satisfying (2.1) is said to be *a contact Riemannian manifold* and is denoted by $M=(M;\eta,g)$. Given a contact Riemannian manifold M, we define a $(1,1)$-type tensor field h by $h=L_\xi\phi/2$, where L denotes Lie differentiation. Then we may observe that h is symmetric and satisfies

$$h\xi=0,\qquad h\phi=-\phi h, \tag{2.3}$$

$$\nabla_X\xi=-\phi X-\phi hX, \tag{2.4}$$

where ∇ is Levi-Civita connection. From (2.3) and (2.4), we see that each trajectory of ξ is a geodesic.

A contact Riemannian manifold for which ξ is Killing is called a *K-contact Riemannian manifold*. It is easy to see that a contact Riemannian manifold is K-contact if and only if $h=0$. For a contact Riemannian manifold M one may define naturally an almost complex structure J on $M\times\mathbb{R}$;

$$J\Big(X,f\frac{d}{dt}\Big)=\Big(\phi X-f\xi,\eta(X)\frac{d}{dt}\Big), \tag{2.5}$$

where X is a vector field tangent to M, t the coordinate of $\mathbb{R}$, and f a function on $M\times\mathbb{R}$. If the almost complex structure J is integrable, M is said to be *normal or Sasakian*. It is known that M is normal if and only if M satisfies

$$[\phi,\phi]+2d\eta\otimes\xi=0, \tag{2.6}$$

where $[\phi,\phi]$ is the Nijenhuis torsion of ϕ. A Sasakian manifold is characterized by a condition

$$(\nabla_X\phi)Y=g(X,Y)\xi-\eta(Y)X \tag{2.7}$$

for all vector fields X and Y on the manifold. We denote by R the Riemannian curvature tensor of M defined by

$$R(X,Y)Z=\nabla_X(\nabla_YZ)-\nabla_Y(\nabla_XZ)-\nabla_{[X,Y]}Z \tag{2.8}$$

for all vector fields X, Y, Z on M. It is well known that M is Sasakian if and only if

$$R(X,Y)\xi = \eta(Y)X - \eta(X)Y \tag{2.9}$$

for all vector fields X and Y. For a contact Riemannian manifold M, the tangent space T_pM of M at each point $p \in M$ is decomposed as $T_pM = D_p \oplus \{\xi\}_p$ (direct sum), where we denote $D_p = \{v \in T_pM \mid \eta(v) = 0\}$. Then $D : p \to D_p$ defines a distribution orthogonal to ξ. The $2n$-dimensional distribution D is called the *contact distribution*. A contact Riemannian manifold is said to be *η-Einstein* if

$$Q = aI + b\eta \otimes \xi, \tag{2.10}$$

where Q is the Ricci operator and a, b are smooth functions on M.

For more details about the fundamental properties on contact Riemannian manifolds we refer to [1, 2]. Blair [2] proved the following theorem.

Theorem 2.1. *Let $M = (M;\eta,g)$ be a contact Riemannian manifold and suppose that $R(X,Y)\xi = 0$ for all vector fields X, Y on M. Then M is locally the product of $(n+1)$-dimensional flat manifold and an n-dimensional manifold of positive constant curvature 4.*

Recently, Blair, Koufogiorgos, and Papantoniou [3] introduced a class of contact Riemannian manifolds which are characterized by equation (1.1). A D-homothetic deformation (cf. [9]) is defined by a change of structure tensors of the form

$$\bar{\eta} = a\eta, \qquad \bar{\xi} = \frac{1}{a}\xi, \qquad \bar{\phi} = \phi, \qquad \bar{g} = ag + a(a-1)\eta \otimes \eta, \tag{2.11}$$

where a is a positive constant. It was shown that [3] a contact Riemannian manifold M satisfying (1.1) is obtained by applying a D-homothetic deformation on a contact Riemannian manifold with $R(X,Y)\xi = 0$ and that the property (1.1) is invariant under the D-homothetic deformation. It is well known that the tangent sphere bundle of a flat Riemannian manifold admits a contact Riemannian structure satisfying $R(X,Y)\xi = 0$ [1, page 137]. In [3] the authors classified the 3-dimensional case and showed that this class contains the tangent sphere bundles of Riemannian manifolds of constant sectional curvature. Furthermore in the same paper they showed that M satisfies

$$(\nabla_Z h)X = (1-\kappa)\{(1-\kappa)g(Z,\phi X) + g(Z,h\phi X)\}\xi + \eta(X)(h\phi + h\phi h)Z - \mu\eta(Z)\phi hX \tag{2.12}$$

for any vector fields X, Z on M. Here, we state some useful results in [3] to prove our Theorems 1.1 and 1.2.

Proposition 2.2. *Let $M = (M;\eta,g)$ be a contact Riemannian manifold which satisfies (1.1), where $\kappa < 1$.*

(i) *If $X, Y \in D(\lambda)$ (respectively, $D(-\lambda)$), then $\nabla_X Y \in D(\lambda)$ (respectively, $D(-\lambda)$).*

(ii) *If $X \in D(\lambda)$, $Y \in D(-\lambda)$, then $\nabla_X Y$ (respectively, $\nabla_Y X$) $\in D(-\lambda) \oplus D(0)$ (respectively, $D(\lambda) \oplus D(0)$).*

Theorem 2.3. *Let $M = (M;\eta,g)$ be a contact Riemannian manifold which satisfies (1.1), then $\kappa \le 1$. If $\kappa = 1$, then $h = 0$ and M is a Sasakian manifold. If $k < 1$, then M admits three mutually orthogonal and integrable distributions $D(0)$, $D(\lambda)$, and $D(-\lambda)$, defined by the eigenspaces of h, where $\lambda = \sqrt{1-\kappa}$. Moreover*

$$\begin{aligned}
&R(X_\lambda,Y_\lambda)Z_{-\lambda} = (\kappa-\mu)\{g(\phi Y_\lambda,Z_{-\lambda})\phi X_\lambda - g(\phi X_\lambda,Z_{-\lambda})\phi Y_\lambda\},\\
&R(X_{-\lambda},Y_{-\lambda})Z_{\lambda} = (\kappa-\mu)\{g(\phi Y_{-\lambda},Z_{\lambda})\phi X_{-\lambda} - g(\phi X_{-\lambda},Z_{\lambda})\phi Y_{-\lambda}\},\\
&R(X_\lambda,Y_{-\lambda})Z_{-\lambda} = \kappa g(\phi X_\lambda,Z_{-\lambda})\phi Y_{-\lambda} + \mu g(\phi X_\lambda,Y_{-\lambda})\phi Z_{-\lambda},\\
&R(X_\lambda,Y_{-\lambda})Z_{\lambda} = -\kappa g(\phi Y_{-\lambda},Z_{\lambda})\phi X_\lambda - \mu g(\phi Y_{-\lambda},X_\lambda)\phi Z_\lambda,\\
&R(X_\lambda,Y_\lambda)Z_\lambda = \{2(1+\lambda)-\mu\}\{g(Y_\lambda,Z_\lambda)X_\lambda - g(X_\lambda,Z_\lambda)Y_\lambda\},\\
&R(X_{-\lambda},Y_{-\lambda})Z_{-\lambda} = \{2(1-\lambda)-\mu\}\{g(Y_{-\lambda},Z_{-\lambda})X_{-\lambda} - g(X_{-\lambda},Z_{-\lambda})Y_{-\lambda}\},
\end{aligned} \tag{2.13}$$

where X_λ, Y_λ, $Z_\lambda \in D(\lambda)$ and $X_{-\lambda}$, $Y_{-\lambda}$, $Z_{-\lambda} \in D(-\lambda)$.

Theorem 2.4. *For a contact Riemannian manifold satisfying (1.1) with $\kappa < 1$, the Ricci operator Q is given by*

$$Q = \{2(n-1)-n\mu\}I + \{2(n-1)+\mu\}h + \{2(1-n)+n(2\kappa+\mu)\}\eta\otimes\xi. \tag{2.14}$$

For more results about a contact Riemannian manifold satisfying (1.1), we refer to [3].

3. Proof of Theorem 1.1.

Let M^{2n+1} be a $(2n+1)$-dimensional contact Riemannian manifold which satisfies (1.1). Suppose that M is locally symmetric, that is, $\nabla R = 0$. In view of the results of the Sasakian case [6] and the 3-dimensional contact Riemannian case [4], we now assume that $n > 1$ and M is non-Sasakian ($\kappa \ne 1$). From $h\xi = 0$, with (2.4) we have

$$(\nabla_Z h)\xi = \nabla_Z(h\xi) - h\nabla_Z\xi = (h\phi + h\phi h)Z. \tag{3.1}$$

If we differentiate (1.1) covariantly, then using (2.4) we get

$$\begin{aligned}
R(X,Y)(-\phi Z - \phi hZ) = \kappa\{&g(-\phi Z-\phi hZ,Y)X - g(-\phi Z-\phi hZ,X)Y\}\\
&+\mu\{g(-\phi Z-\phi hZ,Y)hX + \eta(Y)(\nabla_Z h)X\\
&\quad - g(-\phi Z-\phi hZ,X)hY - \eta(X)(\nabla_Z h)Y\}
\end{aligned} \tag{3.2}$$

for any vector fields X,Y on M. Putting $Y = \xi$, then with (2.2), (2.3), and (3.1) we have

$$R(X,\xi)(-\phi Z-\phi hZ) = \kappa g(\phi Z+\phi hZ,X)\xi + \mu\{(\nabla_Z h)X - \eta(X)(h\phi+h\phi h)Z\}. \tag{3.3}$$

Together with (1.1) we have

$$\mu(\nabla_Z h)X = \mu\{\eta(X)(h\phi+h\phi h)Z + g((h\phi+h\phi h)Z,X)\xi\}. \tag{3.4}$$

From (2.12) and (3.4) we have

$$\begin{aligned}
&\mu\{\eta(X)(h\phi+h\phi h)Z + g((h\phi+h\phi h)Z,X)\xi\}\\
&\quad = \mu\{(1-\kappa)\{(1-\kappa)g(Z,\phi X) + g(Z,h\phi X)\}\xi + \eta(X)(h\phi+h\phi h)Z - \mu\eta(Z)\phi hX\}
\end{aligned} \tag{3.5}$$

for any vector fields X, Z in M. If we put $Z = \xi$, then we have

$$\mu^2 \phi h X = 0. \tag{3.6}$$

Since M is not Sasakian, we have $\mu = 0$. Now, we consider the following equation in Theorem 2.3:

$$R(X_\lambda, Y_\lambda)Z_\lambda = 2(1+\lambda)\{g(Y_\lambda, Z_\lambda)X_\lambda - g(X_\lambda, Z_\lambda)Y_\lambda\}, \tag{3.7}$$

where $X_\lambda, Y_\lambda, Z_\lambda \in D(\lambda)$. Differentiating (3.7) covariantly with respect to $V_{-\lambda} \in D(-\lambda)$, then since M is locally symmetric we have

$$\begin{aligned} &R(\nabla_{V_{-\lambda}} X_\lambda, Y_\lambda)Z_\lambda + R(X_\lambda, \nabla_{V_{-\lambda}} Y_\lambda)Z_\lambda + R(X_\lambda, Y_\lambda)\nabla_{V_{-\lambda}} Z_\lambda \\ &\quad = 2(1+\lambda)\{g(\nabla_{V_{-\lambda}} Y_\lambda, Z_\lambda)X_\lambda + g(Y_\lambda, \nabla_{V_{-\lambda}} Z_\lambda)X_\lambda + g(Y_\lambda, Z_\lambda)\nabla_{V_{-\lambda}} X_\lambda \\ &\qquad - g(\nabla_{V_{-\lambda}} X_\lambda, Z_\lambda)Y_\lambda - g(X_\lambda, \nabla_{V_{-\lambda}} Z_\lambda)Y_\lambda - g(X_\lambda, Z_\lambda)\nabla_{V_{-\lambda}} Y_\lambda\}. \end{aligned} \tag{3.8}$$

Together with Proposition 2.2 and using (3.7) again we get

$$\begin{aligned} &g(\nabla_{V_{-\lambda}} X_\lambda, \xi)R(\xi, Y_\lambda)Z_\lambda + g(\nabla_{V_{-\lambda}} Y_\lambda, \xi)R(X_\lambda, \xi)Z_\lambda + g(\nabla_{V_{-\lambda}} Z_\lambda, \xi)R(X_\lambda, Y_\lambda)\xi \\ &\quad = 2(1+\lambda)\{g(Y_\lambda, Z_\lambda)g(\nabla_{V_{-\lambda}} X_\lambda, \xi)\xi - g(X_\lambda, Z_\lambda)g(\nabla_{V_{-\lambda}} Y_\lambda, \xi)\xi\}. \end{aligned} \tag{3.9}$$

From (1.1), by using the property of the curvature tensor, we get

$$R(\xi, X)Y = \kappa(g(Y, X)\xi - \eta(Y)X) + \mu(g(hY, X)\xi - \eta(Y)hX). \tag{3.10}$$

By using (1.1), (2.1), and (3.10) we have

$$(\kappa - 2\lambda - 2)\{g(Y_\lambda, Z_\lambda)g(X_\lambda, \phi V_{-\lambda} + \phi h V_{-\lambda})\xi - g(X_\lambda, Z_\lambda)g(Y_\lambda, \phi V_{-\lambda} + \phi h V_{-\lambda})\xi\} = 0, \tag{3.11}$$

and thus we have

$$(1-\lambda)(\kappa - 2\lambda - 2)\{g(Y_\lambda, Z_\lambda)g(X_\lambda, \phi V_{-\lambda})\xi - g(X_\lambda, Z_\lambda)g(Y_\lambda, \phi V_{-\lambda})\xi\} = 0. \tag{3.12}$$

We may take an adapted orthonormal basis $\{\xi, e_i, \phi e_i\}$ such that $h\xi = 0$, $he_i = \lambda_i e_i$ and $h\phi e_i = -\lambda_i \phi e_i$, $i = 1, 2, \ldots, n$ at any point $p \in M$. Since $g(\phi e_i, \phi V_{-\lambda}) = 0$ and $g(Y_\lambda, \xi)g(\xi, \phi V_{-\lambda}) = 0$, from (3.12) we have

$$\begin{aligned} (1-\lambda)(\kappa - 2\lambda - 2)\Big\{ &\sum_1^n g(Y_\lambda, e_i)g(e_i, \phi V_{-\lambda})\xi \\ &+ \sum_1^n g(Y_\lambda, \phi e_i)g(\phi e_i, \phi V_{-\lambda})\xi + g(Y_\lambda, \xi)g(\xi, \phi V_{-\lambda})\xi \\ &- \sum_1^n g(e_i, e_i)g(Y_\lambda, \phi V_{-\lambda})\xi \Big\} = 0. \end{aligned} \tag{3.13}$$

And hence, we obtain

$$(1-n)(1-\lambda)(\kappa - 2\lambda - 2)g(Y_\lambda, \phi V_{-\lambda})\xi = 0. \tag{3.14}$$

If we put $\phi V_{-\lambda} = Y_\lambda$ in (3.14), then it follows that

$$(1-n)(1-\lambda)(\kappa-2\lambda-2) = 0, \tag{3.15}$$

where X,Y are vector fields on M. Since $n > 1$ and $\kappa = 1-\lambda^2$, we conclude that $\kappa = \mu = 0$, that is, M satisfies $R(X,Y)\xi = 0$ for any vector fields X,Y in M. Therefore by the results in [4, 6] and Theorem 2.1 we have proved Theorem 1.1.

4. Proof of Theorem 1.2. Let M be a contact Riemannian manifold which satisfies (1.1). Suppose that M is Ricci-parallel, that is, $\nabla Q = 0$. From (1.1) and (2.3) we have

$$Q\xi = 2n\kappa\xi. \tag{4.1}$$

From (2.4) and (4.1), we have

$$(\nabla_Z Q)\xi = -2n\kappa(\phi+\phi h)Z + Q(\phi+\phi h)Z. \tag{4.2}$$

Since M is Ricci-parallel, we have

$$Q(\phi+\phi h)Z = 2n\kappa(\phi+\phi h)Z \tag{4.3}$$

for any vector field Z on M. If we substitute Z with ϕZ, then by using (2.1) and (4.1), we obtain that

$$Q(I-h) = 2n\kappa(I-h). \tag{4.4}$$

If $\kappa = 1$ ($h \equiv 0$), then from (4.4) we see that M is Einstein-Sasakian and the scalar curvature $\tau = 2n(2n+1)$.

Now, we assume that $\kappa \neq 1$, that is, M is non-Sasakian. Differentiating (2.14) covariantly, then it follows that

$$\begin{aligned}(\nabla_Z Q)X =&\{2(n-1)+\mu\}(\nabla_Z h)X - \{2(1-n)+n(2\kappa+\mu)\}g((\phi+\phi h)Z,X)\xi \\ &- \{2(1-n)+n(2\kappa+\mu)\}\eta(X)(\phi+\phi h)Z,\end{aligned} \tag{4.5}$$

and thus we get

$$\{2(n-1)+\mu\}(\nabla_Z h)X = \{2(1-n)+n(2\kappa+\mu)\}\{g((\phi+\phi h)Z,X)\xi+\eta(X)(\phi+\phi h)Z\}. \tag{4.6}$$

Together with (2.12) we have

$$\begin{aligned}&\{2(n-1)+\mu\}[(1-\kappa)\{(1-\kappa)g(Z,\phi X)+g(Z,h\phi X)\}\xi+\eta(X)(h\phi+h\phi h)Z-\mu\eta(Z)\phi hX] \\ &\quad = \{2(1-n)+n(2\kappa+\mu)\}\{g((\phi+\phi h)Z,X)\xi+\eta(X)(\phi+\phi h)Z\}.\end{aligned} \tag{4.7}$$

If we put $Z = \xi$ in (4.7), then we have

$$\mu\{2(n-1)+\mu\}\phi h = 0, \tag{4.8}$$

and hence we see that $\mu = 0$ or $2(n-1)+\mu = 0$. Now, we discuss our arguments divided into two cases: (i) $\mu = 0$, (ii) $2(n-1)+\mu = 0$.

The case (i) $\mu = 0$. Then (4.7) becomes

$$\begin{aligned}&2(n-1)[(1-\kappa)\{(1-\kappa)g(Z,\phi X)+g(Z,h\phi X)\}\xi+\eta(X)(h\phi+h\phi h)Z]\\&\quad=\{2(1-n)+2n\kappa\}\{g((\phi+\phi h)Z,X)\xi+\eta(X)(\phi+\phi h)Z\}.\end{aligned}\tag{4.9}$$

Putting $X = \xi$, then by using (2.2) and (2.3) we get

$$2(1-n)(\phi h+\phi h^2)Z = \{2(1-n)+2n\kappa\}(\phi+\phi h)Z.\tag{4.10}$$

We apply ϕ and use (2.2), then we have

$$2(n-1)h^2Z+2n\kappa hZ+\{2(1-n)+2n\kappa\}(Z-\eta(Z)\xi) = 0.\tag{4.11}$$

Since the trace of $h^2 = 2n(1-\kappa)$ and the trace of $h = 0$, we have $\kappa = 0$. Thus, M satisfies $R(X,Y)\xi = 0$. By Theorem 2.1 we conclude that M is locally the product of $(n+1)$-dimensional manifold and an n-dimensional manifold of positive constant curvature 4.

The case (ii) $2(n-1)+\mu = 0$. Then (2.14) is reduced to

$$Q = \{2(n-1)-n\mu\}I+\{2(1-n)+n(2\kappa+\mu)\}\eta\otimes\xi,\tag{4.12}$$

that is, M is η-Einstein. From (4.7) we get

$$\{2(1-n)+n(2\kappa+\mu)\}\{g(-(\phi+\phi h)Z,X)\xi+\eta(X)(\phi+\phi h)Z\} = 0\tag{4.13}$$

for any vector field X,Z on M. Putting $X = \xi$ in (4.13), then we have

$$\{2(1-n)+n(2\kappa+\mu)\}(\phi+\phi h)Z = 0.\tag{4.14}$$

If $2(1-n)+n(2\kappa+\mu) = 0$, since $\mu = 2(1-n)$ we have

$$\kappa = \frac{n^2-1}{n}.\tag{4.15}$$

But we know that $\kappa < 1$, and thus we see that n must be equal to 1 and hence $\kappa = \mu = 0$. Otherwise, $2(1-n)+n(2\kappa+\mu) \neq 0$, then (4.14) becomes

$$\phi+\phi h = 0,\tag{4.16}$$

which is impossible. Therefore, summing up all the arguments in this section we have Theorem 1.2. □

Remark 4.1. $\mathbb{R}^3(x^1,x^2,x^3)$ or T^3 (torus) with $\eta = 1/2(\cos x^3dx^1+\sin x^3dx^2)$ and $g_{ij} = 1/4\delta_{ij}$ is an η-Einstein, non-Sasakian, contact Riemannian manifold (cf. [1]).

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