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ON THE DIOPHANTINE EQUATION $Ax^2+2^{2m}=y^n$

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ABSTRACT. Let h denote the class number of the quadratic field $\mathbb{Q}(\sqrt{-A})$ for a square free odd integer $A>1$, and suppose that $n>2$ is an odd integer with $(n,h)=1$ and $m>1$. In this paper, it is proved that the equation of the title has no solution in positive integers x and y if n has any prime factor congruent to 1 modulo 4. If n has no such factor it is proved that there exists at most one solution with x and y odd. The case $n=3$ is solved completely. A result of E. Brown for $A=3$ is improved and generalized to the case where A is a prime $\not\equiv 7 \pmod 8$.

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1. Introduction. Let A,m,n denote positive integers where n is odd >1 and A square free odd integer. Let $K=\mathbb{Q}(\sqrt{-A})$, where $\mathbb{Q}$ is the field of rational numbers, let further h denote the number of classes of ideals in K and suppose $(h,n)=1$. In this paper, we consider the Diophantine equation $Ax^2=2^{2m}=y^n$, where x and y are integers. The case $A=1$ was studied in [1] so we will assume that $A>1$. The first result regarding this equation is due to Nagell [5] who proved that when $m=0,1$, this equation has no solutions in integers x and y under the above assumptions about A and n so we will suppose that $m>0$. Since n is odd, there is no loss of generality in considering only odd primes p and x,y positive integers, so we will assume this in what follows.

We start by proving the main result of this paper.

THEOREM 1.1. *Let $A>1$ be a square free odd integer, p an odd prime with $(h,p)=1$ and $m\geq 1$. Then the Diophantine equation*

$$Ax^2+2^{2m}=y^p \tag{1.1}$$

has no solution with x odd in any of the following cases:
(i) *if $A=3$;*
(ii) *if $p\equiv 1 \pmod 4$;*
(iii) *if $A\equiv 3 \pmod 4$ and $p>3$.*
For $p=3$, such a solution exists if and only if A is the square-free part of either $(1/3)(1+2^{m+3})$ with m even or of $(1/3)(2^{2m}-1)$, although in these cases, there might be other solutions if $3\mid h$.

PROOF. We factorize (1.1) in the field K,

$$\left(2^m+x\sqrt{-A}\right)\left(2^m-x\sqrt{-A}\right)=y^p. \tag{1.2}$$

Now the principal ideal $\lfloor 2^m + x\sqrt{-A}\rfloor$ and its conjugate ideal are coprime, so $\lfloor 2^m + x\sqrt{-A}\rfloor = \pi^p$ for some ideal π in K. It follows that π^p is a principal ideal and since $(h,p) = 1$, therefore π is a principal ideal, say $\pi = [\xi]$ for some element ξ in K. So we get the equation

$$\left\lfloor 2^m + x\sqrt{-A}\right\rfloor = [\xi]^p, \tag{1.3}$$

and, consequently,

$$\left(2^m + x\sqrt{-A}\right) = \varepsilon\xi^p, \tag{1.4}$$

for some unit ε in K. Therefore we have the following three cases:

$$\begin{aligned} x\sqrt{-3}+2^m &= \left(\frac{1\pm\sqrt{-3}}{2}\right)\left(\frac{a+b\sqrt{-3}}{2}\right)^3, \quad a \equiv b \pmod 2, \\ x\sqrt{-A}+2^m &= \left(\frac{a+b\sqrt{-A}}{2}\right)^p, \quad a \equiv b \equiv 1 \pmod 2, \\ x\sqrt{-A}+2^m &= \left(a+b\sqrt{-A}\right)^p, \end{aligned} \tag{1.5}$$

for some rational integers a and b.

Equating the imaginary parts in the first case we get

$$16x = \pm(a^3 - 9ab^2) + (3a^2b - 3b^3), \tag{1.6}$$

and we can absorb the lower sign into a. Then

$$16x = (a+b)^3 - 12ab^2 - 4b^3. \tag{1.7}$$

Since a and b have the same parity, we write $2c = a+b$, and obtain

$$2x = c^3 - 3cb^2 + b^3. \tag{1.8}$$

Equation (1.8) is impossible, since the right-hand side is odd unless both b and c are even, and then this side is divisible by 8 if they are which is not possible since x is odd. So this case does not arise.

The second case arises only if $A \equiv 3 \pmod 4$, and we will prove that in this case $p = 3$ and $A \neq 3$.

Observe that $((a+b\sqrt{-A})/2)^p \in Z[\sqrt{-A}]$ only if $A \equiv 3 \pmod 8$ and $p = 3$ and then equating the real parts in this case, we get

$$2^{m+3} = a(a^2 - 3Ab^2). \tag{1.9}$$

Since a is odd we get $a = \pm 1$ and then

$$\pm 2^{m+3} = 1 - 3Ab^2. \tag{1.10}$$

Now $A > 1$, so only the negative sign holds and then

$$Ab^2 = \frac{1+2^{m+3}}{3}. \tag{1.11}$$

Considering this equation modulo 3 we deduce that m should be even. If $A=3$, then we get

$$-2^{m+3}=(1-3b)(1+3b). \tag{1.12}$$

So

$$2^t=1+3b, \qquad -2^k=1-3b, \tag{1.13}$$

where $t+k=m+3$. By adding these two equations we get $t=2$, $k=1$, which is impossible since $m\geq 1$.

Finally the third case can occur for all A, and we will prove that there is no solution when either $p\equiv 1 \pmod 4$ or $A\equiv 3 \pmod 4$.

Since x is odd it follows that $y=a^2+Ab^2$ is odd, so a and b have opposite parity. On equating the real parts we get

$$2^m=a\sum_{r=0}^{(p-1)/2}\binom{p}{2r}a^{p-2r-1}(-Ab^2)^r. \tag{1.14}$$

Here $\sum$ is odd, since the first and the last terms have opposite parity and the rest are all even. So $a=\pm 2^m$, b is odd and from (1.14) we get

$$\pm 1=\sum_{r=0}^{(p-1)/2}\binom{p}{2r}2^{m(p-2r-1)}(-Ab^2)^r. \tag{1.15}$$

Then $\pm 1\equiv 2^{m(p-1)} \pmod p$ and so the lower sign is impossible. That is,

$$1=\sum_{r=0}^{(p-1)/2}\binom{p}{2r}2^{m(p-2r-1)}(-Ab^2)^r, \tag{1.16}$$

and $a=2^m$. So $y=2^{2m}+Ab^2$.

Now suppose that $p\equiv 1 \pmod 4$, say $p=1+2^k u$, where $(u,2)=1$ and $k\geq 2$. Since both b and A are odd,

$$b^{p-1}=(b^u)^{2^k}\equiv 1 \pmod{2^{k+2}}, \qquad (-A)^{(p-1)/2}=(A^u)^{2^{k-1}}\equiv 1 \pmod{2^{k+1}}. \tag{1.17}$$

Then from (1.16) we get

$$\begin{aligned} 1&\equiv\binom{p}{3}2^{2m}(-Ab^2)^{(p-3)/2}+pb^{p-1}(-A)^{(p-1)/2} \pmod{2^{k+1}}\\ &\equiv\frac{p(p-2)}{3}\cdot 2^{k+2m-1}u(-Ab^2)^{(p-1)/3}+p \pmod{2^{k+1}}, \end{aligned} \tag{1.18}$$

since $m\geq 1$, therefore $k+2m-1\geq k+1$, so from (1.18) we get $3\equiv 3p \pmod{2^{k+1}}$. Hence $p\equiv 1 \pmod{2^{k+1}}$ which is not possible. We conclude that there is no solution when $p\equiv 1 \pmod 4$. If $A\equiv 3 \pmod 4$, then considering (1.16) modulo 4 we get $p\equiv 1 \pmod 4$ hence there is no solution when $A\equiv 3 \pmod 4$.

Now let $p=3$ in (1.16) then

$$1=2^{2m}-3Ab^2 \tag{1.19}$$

or $Ab^2=(2^{2m}-1)/3$. This completes the proof. □

Remark 1.2. From Theorem 1.1, we note that to solve (1.1) it is sufficient to consider (1.16) where b is odd, $p \equiv 3 \pmod 4$, and $A \equiv 1 \pmod 4$. If there is a solution then $y = 2^{2m} + Ab^2$.

Now we prove the following theorem which gives us the number of solutions of our equation.

Theorem 1.3. *For a given A, if (1.1) has a solution in x odd where $(h,p) = 1$, then it is unique.*

Proof. If $A \equiv 3 \pmod 4$, we have proved that there is a solution only if $p = 3$, and we have found this unique solution. If $A \equiv 1 \pmod 4$, then from the last proof it is sufficient to consider (1.16), where b is odd and $p \equiv 3 \pmod 4$. Suppose $b_1 > b > 0$ is another solution, then from (1.16) we obtain

$$1 = \sum_{r=0}^{(p-1)/2} \binom{p}{2r} 2^{m(p-2r-1)} \left(-Ab_1^2\right)^r. \tag{1.20}$$

Subtracting (1.20) from (1.16) and dividing by $b_1^2 - b^2$, we get

$$\begin{aligned} 0 &= \sum_{r=0}^{(p-1)/2} \binom{p}{2r} \frac{b_1^{2r} - b^{2r}}{b_1^2 - b^2} \cdot 2^{m(p-2r-1)} (-A)^r \\ &\equiv p \cdot \frac{b_1^{p-1} - b^{p-1}}{b_1^2 - b^2} \pmod 2. \end{aligned} \tag{1.21}$$

Since $p \equiv 3 \pmod 4$, the number $(b_1^{p-1} - b^{p-1})/(b_1^2 - b^2)$ is odd, so (1.21) is impossible and the solution is unique as required. □

Now we prove that to solve (1.1) it is sufficient to consider only x odd. First we need the following lemma.

Lemma 1.4 [4]. *The Diophantine equations*

$$Ax^2 + 1 = 2y^n, \quad A \equiv 1 \pmod 4, \qquad Ax^2 + 1 = 4y^n, \quad A \equiv 3 \pmod 4, \tag{1.22}$$

have no solutions in positive integers with $y > 1$, $n > 2$, $2 \nmid ny$ and $(n,h) = 1$.

Theorem 1.5. *If $A = 3$, equation (1.1) has a solution with x even only if $m \equiv -1 \pmod p$, and this solution is given by $x = 2^m$; for all other $A \not\equiv 7 \pmod 8$ with $(h,p) = 1$ there exists a solution with x even of the form $x = 2^u X$ with X odd, if and only if there is a solution of the equation $AX^2 + 2^{2(m-u)} = Y^p$.*

Proof. If x is even then y is even, so let $x = 2^u X$, $y = 2^v \cdot Y$, where $u > 0$, $v > 0$, $(2,X) = (2,Y) = 1$. Then (1.1) becomes

$$A\left(2^u X\right)^2 + 2^{2m} = 2^{vp} Y^p. \tag{1.23}$$

We have three cases:

(1) $pv > 2u = 2m$. Then cancelling 2^{2m} in (1.23) we get

$$AX^2 + 1 = 2^{vp-2m} Y^p, \tag{1.24}$$

where X is odd. Now $A \not\equiv 7 \pmod 8$, so $vp - 2m = 1$ or 2.

If $A \equiv 1 \pmod 4$ then $\nu p - 2m = 1$ and so $AX^2+1=2Y^p$. This equation has no solution from Lemma 1.4. If $A \equiv 3 \pmod 8$, then $\nu p - 2m = 2$, so $AX^2+1=4Y^p$, and again from Lemma 1.4 this equation has no solution in integers with $Y>1$. Let $Y=1$, then $AX^2+1=4Y^p$ implies that $A=3$, $X=1$ and hence $x=2^m$, also $\nu p = 2m+2$ implies that $m \equiv -1 \pmod p$,

(2) $2u > 2m = \nu p$. Then cancelling 2^{2m} in (1.23) we get $A(2^{u-m}X)^2+1=Y^p$. This equation has no solution [5, Theorem 25].

(3) $2m > 2u = p\nu$. Then

$$AX^2+2^{2(m-u)}=Y^p, \tag{1.25}$$

and this is (1.1) with x odd and smaller m. □

Remark 1.6. From the proof of the last theorem we deduce that to solve (1.1) in even integers when $A \neq 3$ and $A \not\equiv 7 \pmod 8$, it is sufficient to consider the equation

$$AX^2+2^{2(m-u)}=Y^p, \tag{1.26}$$

where $x=2^uX$, $y=2^\nu \cdot Y$, $m>u>0$, $\nu>0$, $(2,X)=(2,Y)=1$, and $2u=p\nu$.

Summarizing the above we give the following theorem.

Theorem 1.7. *The Diophantine equation (1.1) where $A \not\equiv 7 \pmod 8$ and $(h,p)=1$ has no integer solution if $p \equiv 1 \pmod 4$. In particular, the equation $px^2+2^{2m}=y^p$ has no solution for all $p>3$ and $p \not\equiv 7 \pmod 8$.*

Proof. If x is odd, then from Theorem 1.1, equation (1.1) has no solution when $p \equiv 1 \pmod 4$. Now let x be even then from Theorem 1.5 it is sufficient to consider the equation

$$AX^2+2^{2(m-u)}=Y^p, \tag{1.27}$$

where X is odd and $0<u<m$. Since $p \equiv 1 \pmod 4$ then again Theorem 1.1 implies that there is no solution.

Now the class number of the field $\mathbb{Q}(\sqrt{-p})$ is less than p, so as above the equation $px^2+2^{2m}=y^p$ has no solution if $p \equiv 1 \pmod 4$. Let $p \equiv 3 \pmod 4$, since $p>3$, therefore the equation has no solution in odd integers from Theorem 1.1(iii). If x is even then we have

$$pX^2+2^{2(m-u)}=Y^p, \tag{1.28}$$

where X is odd. Equation (1.28) has no solution in odd integers from the first part. □

Brown [2, Theorem 3] considered the Diophantine equation (1.1) when $A=3$, but he did not solve it completely. In the following we give the complete solution.

Theorem 1.8. *The Diophantine equation $3x^2+2^{2m}=y^p$ has a solution only if $m \equiv -1 \pmod p$, and this solution is given by $x=2^m$, $y=2^{(2m+2)/p}$.*

Proof. Now $A=3$ and the field $\mathbb{Q}(\sqrt{-3})$ is a unique prime factorization domain, so from Theorem 1.1 this equation has no solution for all p if x is odd. If x is even then from Theorem 1.5 we have $x=2^m$, $y=2^{(2m+2)/p}$. Also the equation

$$3X^2+2^{2(m-u)}=Y^p, \tag{1.29}$$

where X is odd, has no solution from the first part of this proof. □

Combining the two last theorems we can generalize Brown's result [2] for any odd prime p as follows.

Theorem 1.9. *The Diophantine equation* $px^2+2^{2m}=y^p$, *where* $p\not\equiv 7 \pmod 8$, *has a solution only if* $p=3$ *and* $m\equiv 2 \pmod 3$ *and this solution is given by* $x=2^m$, $y=2^{(2m+2)/3}$.

Considering (1.16) modulo 8 it is easy to prove the following.

Corollary 1.10. *For a given* A, *in (1.16) where* $m\ge 2$, *if* $A\equiv 1 \pmod 8$ *then* $p\equiv 7 \pmod 8$ *and if* $A\equiv 5 \pmod 8$ *then* $p\equiv 3 \pmod 8$.

As a special case we consider $A=q$ an odd prime and prove the following theorem.

Theorem 1.11. *The Diophantine equation* $qx^2+2^{2m}=y^3$, *where* $q\equiv 1 \pmod 4$ *is a prime integer and* $(3,h)=1$, *has a solution only if* $q=5$ *and* $m=2+3M$, *and the unique solution is given by* $x=43\cdot 2^{3M}$ *and* $y=21\cdot 2^{2M}$.

Proof. First suppose that x is odd, since $q\equiv 1 \pmod 4$ and $p=3$, therefore it is sufficient to consider (1.16), then $y=2^{2m}+qb^2$ and

$$3qb^2=2^{2m}-1=(2^m-1)(2^m+1). \tag{1.30}$$

From [5] it is sufficient to consider $m\ge 2$ and from Corollary 1.10 we have $q\equiv 5 \pmod 8$. Now $(2^m-1,2^m+1)=1$, let $b=cd$, where $(c,d)=1$ and both c and d are odd, then from (1.30) we have only the following possibilities:

(1) $2^m-1=3qc^2$, $2^m+1=d^2$, subtracting these two equations, we get $2=d^2-3qc^2$ which is not possible modulo 3.

(2) $2^m-1=3c^2$, $2^m+1=qd^2$, considering the first equation modulo 8, we get $m=2$ and hence $q=5$. Therefore $y=2^{2m}+qb^2=2^4+5(1)=21$ and so $x=43$.

(3) $2^m-1=d^2$, $2^m+1=3qc^2$, again considering the first equation modulo 8, we get $m=1$ and then $q=1$ which is not our case.

(4) $2^m-1=qc^2$, $2^m+1=3d^2$, considering the first equation modulo 8, we get a contradiction.

Now suppose that x is even, then we have only the following equation:

$$qX^2+2^{2(m-u)}=Y^3, \tag{1.31}$$

where $x=2^uX$, $y=2^v\cdot Y$, $m>u>0$, $v>0$, $(2,X)=(2,Y)=1$, and $2u=3v$. From the first part of this proof, equation (1.31) has a unique solution given by $q=5$, $m-u=2$, $X=43$, and $Y=21$. Since $2u=3v$ we get $3\mid u$, let $u=3M$ then $m=2+3M$ and $v=2M$. Hence $x=43\cdot 2^{3M}$ and $y=21\cdot 2^{2M}$. □

We are unable to solve (1.1) completely when $A\equiv 1 \pmod 4$ but we are able to solve it for many particular values of A for all p as we will show in the following example. But before this we give a corollary which will help us.

Corollary 1.12. *If* m *is odd then the Diophantine equation (1.1) has no solution in* x *odd when* $5\mid A$.

Proof. Since m is odd, therefore from the proof of Theorem 1.1, it is sufficient to consider (1.16), where $p\equiv 3 \pmod 4$. If $5\mid A$ in (1.16), then we get $1\equiv 2^{m(p-1)} \pmod 5$ which implies that $4\mid m(p-1)$ and this is not possible. □

Example 1.13. Consider the Diophantine equation $5x^2+2^{10}=y^p$.

Here $m=5$, $A=5$, $h=2$, so from Corollary 1.12, this equation has no solution in x odd for all p. If x is even then it is sufficient to consider the equation

$$5X^2+2^{2(5-u)}=Y^p, \tag{1.32}$$

where $x=2^uX$, $y=2^v\cdot Y$, $5>u>0$, $v>0$, $(2,X)=(2,Y)=1$, and $2u=pv$. Since p is an odd prime, the only possibility is $u=3$, $p=3$, and (1.32) becomes $5X^2+2^4=Y^3$, which has a unique solution from Theorem 1.11, given by $X=43$ and $Y=21$, so the given equation has a unique solution, $x=8.43$, $y=4.21$, and $p=3$.

By using the method similar to [3, Lemma 3] we can prove the following lemma.

Lemma 1.14. *If q is any odd prime which divides the integer b defined in (1.16), then*

$$2^{m(q-1)}\equiv 1 \pmod{q^2}. \tag{1.33}$$

Considering (1.16) modulo 3 we are able to prove the following theorem.

Theorem 1.15. *If $3\mid b$ in (1.16), then $m=3^k\cdot m'$, where $k\geq 1$, $(3,m')=1$ and either*

(1) *$3\nmid A$ and then there is no solution if k even, or*

(2) *$3\mid A$ and then there is no solution if k odd.*

Proof. Let $3\mid b$ then from Lemma 1.14, $2^{2m}\equiv 1 \pmod 9$ which implies that $3\mid m$. Let $m=3^k\cdot m'$, where $(3,m')=1$, $k\geq 1$. Since $p\equiv 3 \pmod 4$, put $p-1=2\cdot 3^t\cdot p'$, where $(2,p')=(3,p')=1$, $t\geq 0$ and put $b=3^s\cdot b'$, where $(3,b')=1$, $s\geq 1$. Rewrite (1.16) as

$$1-2^{m(p-1)}=\sum_{r=1}^{(p-1)/2}\binom{p}{2r}2^{m(p-2r-1)}(-Ab^2)^r. \tag{1.34}$$

The general term in the right-hand side is

$$\binom{p}{2r}2^{m(p-2r-1)}(-Ab^2)^r=\binom{p-2}{2r-2}2^{m(p-2r-1)}\times\frac{pb^{2r-2}}{r(2r-1)}\cdot b^2(-A)^r. \tag{1.35}$$

Since $3^{2r-2}\geq r(2r-1)$ for $r\geq 1$, this right-hand side is divisible at least by 3^{2S+t} if $(3,A)=1$, so from (1.34) we get

$$2^{m(p-1)}\equiv 1 \pmod{3^{2s+t}}. \tag{1.36}$$

Since 2 is a primitive root of 3^{2s+t}, therefore $\phi(3^{2s+t})\mid m(p-1)$ which implies that $3^{2s+t-1}\mid 2\cdot 3^k m'\cdot 3^t p'$, hence $3^{2s-k-1}\mid m'p'$. But $(3,m')=(3,p')=1$, so $2s-k-1=0$ which implies that k is odd.

Now if $3\mid A$, then the right-hand side in (1.34) is divisible at least by 3^{2s+t+1} and as above we get $k=2s$, implying k even. □

We are unable to solve (1.1) completely when $p = 7$, but as a special case we prove the following theorem.

Theorem 1.16. *The Diophantine equation (1.1), where* $(7,h) = 1$*, has no solution in* x *odd when* $p = 7$*,* $A \equiv 1 \pmod{12}$*, and* $m = 3^{2k} \cdot m'$*, where* $k \geq 1$*,* $(3,m') = 1$*.*

Proof. Here $p = 7$, so from Theorem 1.1(iii) we get $A \equiv 1 \pmod 4$. Put $p = 7$ in (1.16), then

$$1 = 2^{6m} - 21Ab^2 2^{4m} + 35A^2 b^4 2^{2m} - 7A^3 b^6. \tag{1.37}$$

If $3 \mid b$ in (1.37) then from Theorem 1.15(1), this equation has no solution. So $(3,b) = 1$, and then considering (1.37) modulo 3 we get

$$2A^2 - A^3 \equiv 0 \pmod 3 \tag{1.38}$$

which is not true since $A \equiv 1 \pmod 3$. □

Theorem 1.17. *If* $(3,m) = 1$*, then the Diophantine equation (1.1), where* $(p,h) = 1$ *has no solution in* x *odd when* $A \equiv 1 \pmod{24}$*.*

Proof. The case $m = 1$, the Diophantine equation (1.1) has no solution [5]. Let $m \geq 2$, since $A \equiv 1 \pmod 8$ then from Corollary 1.10, $p = 7 + 8H$. Since $(3,m) = 1$, then from Theorem 1.15, $(3,b) = 1$ so $b^2 \equiv 1 \pmod 3$. Considering (1.16) modulo 3, where $A \equiv 1 \pmod 3$ we get

$$1 \equiv \sum_{r=0}^{(p-1)/2} \binom{p}{2r} (-1)^r \pmod 3 \equiv \frac{(1+i)^p + (1-i)^p}{2} \pmod 3. \tag{1.39}$$

But $(1 \pm i)^8 \equiv 1 \pmod 3$, so (1.39) implies that

$$\begin{aligned} 1 &\equiv \frac{\{(1+i)^{8(1+H)}(1-i) + (1-i)^{8(1+H)}(1+i)\}}{2(1+i)(1-i)} \pmod 3 \\ &\equiv 4^{1+H} \times \frac{1}{2} \pmod 3 \\ &\equiv 2 \pmod 3 \end{aligned} \tag{1.40}$$

which is a contradiction. □

Example 1.18. Consider the Diophantine equation $73x^2 + 2^{14} = y^p$.

Here $m = 7$, $A = 73$, $h = 4$ so from Theorem 1.17, this equation has no solution in x odd. If x is even then it is sufficient to consider the equation

$$73X^2 + 2^{2(7-u)} = Y^p, \tag{1.41}$$

where $x = 2^u X$, $y = 2^v \cdot Y$, $7 > u > 0$, $v > 0$, $(2,X) = (2,Y) = 1$, and $2u = pv$. If $(7-u,3) = 1$, then (1.41) has no solution from Theorem 1.17. If $3 \mid 7-u$ then $u = 1$ or 4, which is not possible since $2u = pv$. So the given equation has no solution in integers.

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