Internat. J. Math. & Math. Sci.
Vol. 3 No. 3 (1980) 433-443

PROPERTIES OF COMPLEMENTS IN THE LATTICE OF CONVERGENCE STRUCTURES

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(Received October 11, 1979)

ABSTRACT. Relative complements and differences are investigated for several convergence structure lattices, especially the lattices of Kent convergence structures and the lattice of pretopologies. Convergence space properties preserved by relative complementation are studied. Mappings of some convergence structure lattices into related lattices of lattice homomorphisms are considered.

KEY WORDS AND PHRASES. *Convergence structure, pretopology, limitierung, relative pseudo-complement, pseudo-difference, continuous lattice.*

1980 MATHEMATICS SUBJECT CLASSIFICATION CODES. 54A05, 06D15.

1. INTRODUCTION.

The author classified the relative complements and differences for the lattice of Kent convergence structures on a nonempty set in [8]. This paper investigates further the convergence space properties preserved by these complements and their relationships to some of the standard lattice operations such as products and quotients.

The definitions used are essentially those of [3], [4] and [11] with a convergence structure on X considered as a map $q : X \to \beta(F(X))$ (the power set of the set of filters on X). U(X) and $\dot{x}$ are the set of ultrafilters on X and the principal ultrafilter generated by $\{x\}$. For a convergence space (X,q), let λq, ϕq and ωq be the *topological, pretopological,* and *completely regular topological modifications* of q. The *q-limit set* of a filter F is $ad\, q(F) = \{x \mid F \in q(x)\}$ and the *closure* cl(A) of a subset A is $\{x \mid F \in q(x)$ for some filter F with $A \in F\}$.

An element z of lattice L is the *pseudo-complement of x relative to y* ($x{*}y$) if z is the greatest element with $x \wedge z \leq y$ and the *pseudo-difference of y and x* (y-x) if z is the least element with $y \leq x \vee z$. If L is a complete lattice with 0 and 1 the least and greatest elements, then the *pseudo-complement of x* is $x^* = x{*}0$ and the *pseudo-difference of x* is $-x = 1-x$ (a change of notation from [8]).

2. RELATIVE COMPLEMENTS.

The relative pseudo-complement and pseudo-difference of two convergence structures in C(X) were described in [8] as:

$$q{*}r(x) = \{F \mid F = \dot{x} \text{ or } G \cap \dot{x} \subseteq F \text{ for some } G \in r(x) \setminus q(x)\}$$

$$q-r(x) = \{F \mid F + G = \beta(X) \text{ or is in } q(x) \text{ for all } G \in r(x)\}$$

For q and r limitierungs of Fischer [3] or pseudo-topologies the same descriptions hold for relative pseudo-complements and pseudo-differences in the lattices of limitierungs or pseudotopologies. In P(X), the lattices of pretopologies on X, pseudo-differences do not exist from [9]. Relative pseudo-complements also fail to exist in P(X) even though, from [9], P(X) is pseudo-complemented.

EXAMPLE 2.1: On an infinite set X, let A be an infinite subset with infinite complement and $x \in X$. If q(r) is the finest pretopology on X such that an ultrafilter F q(r)-converges to x if and only if $F = \dot{x}$ or F is free and contains A(X-A), then $q{*}r$ does not exist in P(X).

Many properties of convergence structures are preserved by relative pseudocomplementation. If r is a limitierung (resp. pseudotopology, pretopology, topology), then from [8], $q*r$ is the same type of structure for any convergence structure q.

PROPOSITION 2.2: For any convergence structures q and r on X:

(i) $q*r$ is T_1 (Hausdorff) if r is T_1 (Hausdorff).

(ii) $q*r$ is T_3 if r is T_3.

(iii) $q*r$ is compact if and only if r is compact and for any ultrafilter $\mathcal{F}$, $ad_r(\mathcal{G}) \not\subset ad_q(\mathcal{G})$ for some $\mathcal{G} \subset \mathcal{F}$.

(iv) $q*r$ is T-regular [5] if r is T-regular.

(v) $q*r$ is first countable ($\aleph$-countable, [2]) if r is first countable ($\aleph$-countable).

(vi) $q*r$ is second countable if r is second countable and $q*r$ has at most countably many discrete points.

In addition, for any pretopology q on X:

(vii) $q*r$ is a completely regular topology if r is a completely regular topology.

(viii) $q*r$ is ω-regular [6] if r is ω-regular.

(ix) $q*r$ is C-embedded [6] if r is C-embedded.

PROOF: The proof of (i) is in [8].

(ii) If $\mathcal{F} \in q*r(x)$ with $\mathcal{G} \leq \mathcal{F}$ for some $\mathcal{G} \in r(x) \setminus q(x)$ then $cl_r\mathcal{G} \in r(x) \setminus q(x)$ since r is regular so $cl_r\mathcal{G} \leq cl_{q*r}\mathcal{G} \leq cl_{q*r}\mathcal{F}$ and $cl_{q*r}\mathcal{F}$ $q*r$-converges to x. If $q*r(x) = \{\dot{x}\}$ then $q*r$ is T_1 so $cl_{q*r}\dot{x} = \dot{x}$.

(iii) is obvious.

(iv) Suppose $\mathcal{F} \in q*r(x)$ and $\mathcal{G} \leq \mathcal{F}$ for some $\mathcal{G} \in r(x) \setminus q(x)$. Then $cl_{\lambda r}\mathcal{G} \in r(x) \setminus q(x)$ so $cl_{\lambda r}\mathcal{G} \leq cl_{q*\lambda r}\mathcal{G} \leq cl_{\lambda(q*r)}\mathcal{G} \leq cl_{\lambda(q*r)}\mathcal{F}$ since $q*\lambda r \leq \lambda(q*r)$. If $q*r(x) = \{\dot{x}\}$, $\lambda(q*r)(x) = \{\dot{x}\}$ so $q*r$ is T-regular.

(v) If $\mathcal{F} \in q*r(x)$ with $\mathcal{G} \subseteq \mathcal{F}$ and $\mathcal{G} \in r(x) \setminus q(x)$, then $\mathcal{H} \in r(x) \setminus q(x)$ for some $\mathcal{H}$ with filterbase of cardinality less than $\aleph$ for any cardinal $\aleph$.

(vi) Let $\mathcal{B}$ be a countable basis for (X,r). Then $\mathcal{B}' = \mathcal{B} \cup \{x \mid q*r(x) = \{\dot{x}\}\}$ is a countable basis for $(X,q*r)$.

(vii) Since $q*r$ is topological from [8], suppose A is $q*r$-closed and $x \notin A$. Then if $x \notin cl_r(A)$, any real valued continuous function on (X,r) which separates x and A is also $q*r$-continuous. If $x \in cl_r(A)$, then $q*r$ is discrete at x so x and A can be separated by a $q*r$-continuous, real-valued function.

(viii) If $\mathcal{F} \in q*r(x)$ and $\mathcal{G} \in r(x) \setminus q(x)$ with $cl_{\omega r}\mathcal{G} \in r(x) \setminus q(x)$ then from (vii), if ωr is the completely regular modification of r, $cl_{\omega r}\mathcal{G} \subseteq cl_{q*\omega r}\mathcal{G} \subseteq cl_{\omega(q*r)}\mathcal{G} \subseteq cl_{\omega(q*r)}\mathcal{F}$ and if $q*r$ is discrete at x, so is $\omega(q*r)$ and the conclusion follows.

(ix) From [8], $q*r$ is pseudo-topological if r is pseudo-topological. If r is Hausdorff and ω-regular, then $q*r$ has the same properties from (i) and (viii) so by [6], $q*r$ is C-embedded if r is C-embedded.

COROLLARY 2.3: (i) If r^o is the finest first countable structure coarser than r, then $(q*r)^o = q*r^o$ for every convergence structure q.

(ii) If Rr, the finest regular structure coarser than r [7], is T_1, then $q*Rr \leq R(q*r)$.

PROOF: Since $r^o \leq r$, $q*r^o \leq q*r$ and $q*r^o$ being first countable implies $q*r^o \leq (q*r)^o$. Conversely, if $\mathcal{F} \in q*r^o(x)$ then $\mathcal{G} \subseteq \mathcal{F}$ for some $\mathcal{G} \in r(x) \setminus q(x)$ with countable filterbase. Then $\mathcal{G} \in q*r(x)$ so $\mathcal{F} \in (q*r)^o(x)$.

As $q*Rr$ is T_3 from (ii) of Proposition 2.2 and $q*Rr \leq q*r$, then $q*Rr \leq R(q*r)$.

The converses of the statements in Proposition 2.2 fail to be true since if $q \leq r$, $q*r$ is discrete. In (ii) of Proposition 2.2, one cannot substitute regular for T_3.

EXAMPLE 2.4: (i) Let $X = \{x,y\}$ and q be the finest convergence structure on X for which the principal filter F_{xy} generated by $\{x,y\}$ converges to x. Then 0 is regular but not T_1 and q^* is not regular.

(ii) Let r be a convergence structure on an infinite set X for which $Rr \neq r$ and Rr is T_1, such as a non-regular T_2-convergence structure which is finer than some T_2, regular topology. Then $1 = R(r*r) \neq r*Rr$.

The following description of the convergent ultrafilters of the pseudo-difference q-r of two convergence structures is given in [8]:

LEMMA 2.5: An ultrafilter F q-r converges to x if and only if F q-converges to x or does not r-converge to x.

Because q-r can have so many convergent ultrafilters, most convergence space properties are not preserved. This can also be observed from the result of [9] that the image of the map $q \to 1-q$ is the lattice of pseudotopologies. For example, one can readily show that q-r is not pretopological if $q \nleq r$, r is T_1 and q is not discrete and 1-q is not regular if q is T_1 and not discrete. A few properties can be easily seen to be preserved.

PROPOSITION 2.6: For any convergence structures q and r on X,

(i) q-r is T_1 if and only if q is T_1 and the pretopological modification ϕr of r is indiscrete.

(ii) q-r is Hausdorff if and only if $1-q \leq r$.

(iii) q-r is compact if q is compact.

(iv) q-r is compact if and only if no ultrafilter F $(1-r)\wedge q$-converges to every point.

For complements of product convergence structures there exist relationships to the complements in the original spaces. If $\{(X_\alpha,q_\alpha) \mid \alpha \in \Gamma\}$ is a family of nondegenerate convergence spaces with products $(\Pi X_\alpha, \Pi q_\alpha)$, let $\Pi_w q_\alpha$ denote the convergence structure defined on ΠX_α by: F $\Pi_w q_\alpha$-converges to $x = (x_\alpha)$ if and only if the projection $p_\gamma(F)$ q_γ-converges to x_γ for some $\gamma \in \Gamma$. $\Pi_w q_\alpha$ will be called the *weak product convergence structure*. In the subsequent four propositions, p or p_α will denote the appropriate projection or quotient map.

PROPOSITION 2.7: If q_α and r_α are convergence structures on X_α for $\alpha \in \Gamma$ with $|\Gamma| > 1$, then in $C(\Pi X_\alpha)$:

(i) $(\Pi q_\alpha)*(\Pi r_\alpha) \leq \Pi(q_\alpha * r_\alpha)$.

(ii) F converges to $x = (x_\alpha)$ with respect to $(\Pi q_\alpha)*(\Pi r_\alpha)$ if and only if $p_\gamma(F)$ $q_\gamma * r_\gamma$-converges to x_γ for some $\gamma \in \Gamma$.

(iii) $(\Pi q_\alpha)*(\Pi r_\alpha) = \Pi_w(q_\alpha * r_\alpha)$.

(iv) $\Pi(q_\alpha * r_\alpha) = (\Pi q_\alpha)*(\Pi r_\alpha)$ if and only if each r_α is indiscrete.

(v) $(\Pi q_\alpha)^* = \Pi_w q_\alpha^*$.

PROOF: (i) If F $\Pi(q_\alpha * r_\alpha)$-converges to $x = (x_\alpha)$ then each $p_\alpha(F)$ $q_\alpha * r_\alpha$-converges to x_α so for each α there exists a filter G_α on X_α with $G_\alpha = p_\alpha(F) = \dot{x}_\alpha$ or $G_\alpha \subseteq p_\alpha(F)$ and $G_\alpha \in r_\alpha(x_\alpha) \setminus q_\alpha(x_\alpha)$. Then ΠG_α is Πr_α-convergent to x and $\Pi G_\alpha \in (\Pi r_\alpha)(x) \setminus (\Pi q_\alpha)(x)$ or $\Pi G_\alpha = \dot{x}$ and F $(\Pi q_\alpha)*(\Pi r_\alpha)$-converges to x since $\Pi G_\alpha \subseteq \Pi p_\alpha(F) \subseteq F$.

(ii) Suppose F $(\Pi q_\alpha)*(\Pi r_\alpha)$-converges to $x = (x_\alpha)$. Then $F = \dot{x}$ or $G \subseteq F$ for some $G \in (\Pi r_\alpha)(x) \setminus (\Pi q_\alpha)(x)$. In the latter case, $p_\gamma(G) \in r_\gamma(x_\gamma) \setminus q_\gamma(x_\gamma)$ for some γ so $p_\gamma(F) \in q_\gamma * r_\gamma(x_\gamma)$. The converse is similar.

(iii) follows immediately from (ii) and the definition of a weak product.

(iv) If $|\Gamma| > 1$ and $\Pi q_\alpha \leq \Pi_w q_\alpha$ then for F_γ any filter on X_γ and $x_\gamma \in X_\gamma$, let $G = \Pi G_\alpha$ where $G_\alpha = \dot{x}_\alpha$ for some $x_\alpha \in X_\alpha$ if $\alpha \neq \gamma$ and $G_\gamma = F_\gamma$. Then G $(\Pi_w q_\alpha)$-converges to $x = (x_\alpha)$ so must Πq_α-converge to x and $F_\gamma = p_\gamma(G)$ q_γ-converges to x_γ and q_γ is indiscrete. Thus if $\Pi q_\alpha * r_\alpha \leq \Pi_w q_\alpha * r_\alpha$, each $q_\alpha * r_\alpha$ is indiscrete and it follows that each r_α is indiscrete (since $\{X_\alpha\} \in r_\alpha(x_\alpha) \setminus q_\alpha(x_\alpha)$ for each $x_\alpha \in X_\alpha$).

(v) is a direct consequence of (iv) since $q^* = q*0$.

PROPOSITION 2.8: If q_α and r_α are convergence structures on X_α for $\alpha \in \Gamma$, then in $C(\Pi X_\alpha)$:

(i) $(\Pi q_\alpha)-(\Pi r_\alpha) \leq \Pi(q_\alpha - r_\alpha)$

(ii) $-(\Pi r_\alpha)$ and $\Pi_w(-r_\alpha)$ agree on convergent ultrafilters.

PROOF: (i) If $x = (x_\alpha)$ and $p_\alpha(F)$ $q_\alpha - r_\alpha$ converges to x_α for all α, let $G \in (\Pi r_\alpha)(x)$. Then $p_\alpha(G) \in r_\alpha(x_\alpha)$ for all α, so $p_\alpha(F)+p_\alpha(G) \in q_\alpha(x_\alpha)$ or $p_\alpha(F)+p_\alpha(G) = \beta(X_\alpha)$. In either case, $p_\alpha(F)+p_\alpha(G) \subseteq p_\alpha(F+G)$ and $F+G \in [(\Pi q_\alpha)-(\Pi r_\alpha)](x)$.

(ii) Let F be an ultrafilter in $[-(\Pi r_\alpha)](x)$. Then $p_\alpha(F) \notin r_\alpha(x_\alpha)$ for some α so $p_\alpha(F) \in (-r_\alpha)(x_\alpha)$ and F $\Pi_w(-r_\alpha)$-converges to x. The reverse inequality is similar.

From Lemma 2.5, one can show that equality does not hold in Proposition 2.8(i) even for q topological.

The product operation can also be viewed as a lattice operation on C(X).

PROPOSITION 2.9: The map Π: $\Pi[C(X_\alpha)] \to C(\Pi X_\alpha)$ defined by $\Pi[(X_\alpha, q_\alpha)(\alpha \in \Gamma)] = (\Pi X_\alpha, \Pi q_\alpha)$ is a complete join homomorphism.

If (X,q) is a convergence space with ~ an equivalence relation on X, let X/~ be the quotient space with quotient structure $\overline{q}$, $\overline{A} = \{\overline{x} \mid x \in A\}$ for $A \subset X$ and, for F a filter on X, $\overline{F} = \{\overline{A} \mid A \in F\}$. Let f: $C(X) \to C(X/\sim)$ be the map $f(q) = \overline{q}$. The subsequent propositions are readily established.

PROPOSITION 2.10: f is a complete meet homomorphism.

PROPOSITION 2.11: For q and r in C(X), in C(X/~):

(i) $\overline{q*r} \leq \overline{q}*\overline{r}$

(ii) $\overline{r^*} = (\overline{r})^*$ if and only if for each x and F in r(x), there does not exist $A \in F$ with $A \cap \overline{y} \neq \phi$ for all y in X.

(iii) $-\overline{q} \leq (\overline{-q})$ with equality if and only if for each x, $F \neq \dot{x}$ in q(x) and $A \in F$, $A \notin \overline{x}$.

(iv) If q and r are pretopologies, $\bar{q}*\bar{r} = \overline{q*r}$ if $y \sim x$ and $\overline{N_r(y)} = \overline{N_q(x)}$ implies $N_r(y) = N_q(x)$.

For A a nonempty subset of X and $\mathcal{F}$ a filter on X with $A \in \mathcal{F}$, let $\mathcal{F}_A$ be the filter on A where $\mathcal{F}_A = \{A \cap B \mid B \in \mathcal{F}\}$ and f_A: $C(X) \to C(A)$ be $f_A(q)(x) = \{\mathcal{F}_A \mid A \in \mathcal{F}$ and $\mathcal{F} \in q(x)\}$, i.e., $f_A(q)$ is the subspace structure on A.

PROPOSITION 2.12: (i) f_A is a complete lattice epimorphism.

(ii) For any q and r in C(X), $f_A(q*r) = f_A(q)*f_A(r)$ and $f_A(q-r) = f_A(q)-f_A(r)$.

As one would expect, Proposition 2.12 establishes that the restriction of the relative complements to a subspace are the complements of the restrictions.

3. LATTICE OPERATORS INDUCED BY RELATIVE COMPLEMENTS.

The relative pseudo-complement and pseudo-difference induce four obvious self-maps of C(X) for each convergence structure q:

(i) $f^*(q)$: $f^*(q)(r) = q*r$

(ii) $f_*(q)$: $f_*(q)(r) = r*q$

(iii) $f^-(q)$: $f^-(q)(r) = q-r$

(iv) $f_-(q)$: $f_-(q)(r) = r-q$

Of these maps, (i) and (iv) were considered in [8]. Only (i) and (iv) will be considered here since (ii) and (iii) have similar lattice properties if considered as maps of C(X) into its dual.

If Γ is a cardinal, a subset A of a lattice L is *prime with respect to Γ-joins in L* if for any subset $\{x_\gamma \mid \gamma \in \Gamma\}$ with $\vee x_\gamma \in A$, some $x_\gamma \in A$. A convergence structure q of C(X) is *join prime* if each $q(x) \setminus \{\dot{x}\}$ is prime with respect to finite joins in $\{r(x) \setminus \{\dot{x}\} \mid r \in C(X)\}$. As an extension of a result in [8] one has:

PROPOSITION 3.1: For any convergence structure q on X:

(i) $f^*(q)$ is a complete meet homomorphism.

(ii) $f^*(q)$ is a Γ-join homomorphism if and only if $q(x) \setminus \{\dot{x}\}$ is prime with respect to Γ-joins in F(X) for any cardinal Γ.

(iii) $f^*(q)$ is bijective if and only if q is discrete.

PROOF: (i) is a result of [8] while the proof of (ii) parallels the result of [8] for finite joins. (iii) is a property of complete lattices.

PROPOSITION 3.2: (i) $f_-(q)$ is a complete join homomorphism.

(ii) $f_-(q)$ is complete with respect to Γ-meets for a cardinal Γ if and only if each q(x) is complete with respect to Γ-meets in C(X) for each x.

(iii) $f_-(q)$ is bijective if and only if q is indiscrete.

PROOF: (i) is from [8] while the proof of (ii) is similar to Theorem 4.2 of [8]. (iii) is dual to Proposition 3.1(iii).

From Proposition 3.2 one can observe that $f_-(q)$ is a complete lattice homomorphism if and only if q is a pretopology. If Γ and Ω are infinite cardinals with $\Gamma < \Omega$, by choosing the cardinal of X large enough so that if $y \in X$ and q(x) is discrete for $y \neq x$ and q(y) is closed with respect to Γ-meets but not Ω-meets, then $f_-(q)$ is a Γ-homomorphism that is not an Ω-homomorphism.

Using the given four lattice operators, one can construct maps of certain sublattices of C(X) into the duals of their lattices of homomorphisms (with coordinatewise order). For example, if L(X) is the lattice of limitierungs on X and P(X) the lattice of pretopologies, one can define $f_L^*\colon L(X) \to L^L$ by $f_L^*(q)(r) = q*r$ and $f_P^*\colon P(X) \to P^P$ similarly, where $L^L(P^P)$ is the dual of the lattice of homomorphisms of L(X) and q*r is the relative pseudo-complement in C(X). The succeeding two propositions follow directly from the definitions and properties of pseudo-complements and differences.

PROPOSITION 3.3: (i) f_L^* is a lattice embedding and a complete join homomorphism.

(ii) f_P^* is a complete lattice embedding.

If $C^C(X,\vee)$ denotes the join semilattice of join-homomorphisms of C(X) then the map f_-: $C(X) \to C^C(X,\vee)$ is $f_-(q)(r) = r-q$.

PROPOSITION 3.4: (i) f_- is a complete meet homomorphism of $C(X)$ into the dual of $C^C(X,\vee)$.

(ii) f_- is an embedding.

In a partially ordered set $(L,\leq)$, let $x \ll y$ if and only if for every up-directed set D, $y \leq \sup D$ implies $x \leq d$ for some d in D. Then from Scott [10], a *complete lattice L is continuous* if $x = \sup\{y \in L \mid y \ll x\}$ for all x in L. The induced topology is that topology for which $U \subseteq L$ is open if U is a terminal set and if $S \subseteq L$ is directed, sup S exists and is in U, then $S \cap U \neq \phi$. Since a compactly generated lattice is continuous, we have from [8] that C(X), L(X) and P(X) are continuous with C(X) and L(X) having continuous duals. Since also from [10], a function between complete lattices is continuous in the induced topologies if and only if it is join-preserving, one has immediately from Propositions 3.1 and 3.2:

PROPOSITION 3.5: For any convergence structure q on X:

(i) $f^*(q)$ is continuous if and only if each $q(x) \setminus \{\dot{x}\}$ is prime with respect to joins in F(X).

(ii) $f_-(q)$ is continuous.

PROPOSITION 3.6: f_L^* and f_P^* are continuous in the induced topologies.

Since join-prime elements q determine when $f^*(q)$ is a homomorphism, one may note that if q(x) is join-prime, there exists at most one ultrafilter $\mathcal{F}$ not q-convergent to x. Also, the join-prime elements of C(X) form a meet-sublattice of C(X) but not a join-sublattice.

A number of special types of convergence structure lattices are continuous lattices by virtue of being retracts of C(X) in the induced topologies and Proposition 2.10 of [10]. Some examples are the lattices of T_1-structures, pseudotopologies, locally bounded structures and locally compact structures which can be shown to be continuous by virtue of the standard modification maps.

In [8], Theorem 5.1, the incorrect statement is made that the map ϕ of pretopological modification is a join homomorphism. If q is the cofinite topology and r is the finest convergence structure for which each principal ultrafilter converges to each point, then $\phi(q \vee r) \neq \phi q \vee \phi r$. Therefore ϕ cannot be used to show P(X) is a continuous lattice.

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