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SOME GENERATING FUNCTIONS OF MODIFIED BESSEL POLYNOMIALS FROM THE VIEW POINT OF LIE GROUP

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ABSTRACT. In this paper we have derived a class of bilateral generating relation for modified Bessel polynomials from the view point of Lie group. An application of our theorem is also pointed out.

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1. INTRODUCTION.

In [1], the modified Bessel polynomials are defined by

$$y_n(x;\alpha-n,\beta) = {}_2F_0(-n,\alpha-1;-;-x/\beta), \tag{1.1}$$

where $y_n(x;\alpha,\beta)$ denotes the Bessel polynomials introduced by H. L. Krall and O. Frink [2].

The object of the present paper is to derive the following theorem on bilateral generating relation for modified Bessel polynomials from the view point of Lie-group.

THEOREM. If there exists a generating relation of the form,

$$G(x,w) = \sum_{n=o}^{\infty} a_n w^n \, y_n(x;\alpha-n,\beta) \tag{1.2}$$

then

$$(1-nw)^{1-\alpha}\, e^{\beta w}\left(G\,\frac{x}{1-xw}\,,wz\right) = \sum_{n=o}^{\infty} w^n\, g_n(z)\, y_n(x;\alpha-n,\beta) \tag{1.3}$$

where

$$g_n(z) = \sum_{m=o}^{n} \frac{a_m}{m!}(\beta z)^m.$$

The importance of our result lies in the fact that if one knows a generating relation of the type (1.2) for a particular value of a_n, then the corresponding bilateral generating relation follows atonce from (1.3).

2. PROOF OF THE THEOREM.

From [1] we observe

$$\exp(wR)\, f(x,y) = (1-wxy)^{1-\alpha}\, e^{\beta wy}\, f\left(\frac{x}{1-wxy}\,,y\right) \tag{2.1}$$

where

$$R = x^2 y \frac{\partial}{\partial x} + \beta y + (\alpha-1)xy$$

and

$$R(F_n(x, y, \alpha-n,\beta)) = \beta F_{n+1}(x,y, \alpha-n-1,\beta) \qquad (2.2)$$

where

$$F_n(x, y,\alpha-n,\beta) = y^n y_n(x; \alpha-n,\beta).$$

In the formula:

$$G(x,w) = \sum_{n=o}^{\infty} a_n w^n y_n(x; \alpha-n,\beta)$$

replacing wby wyz and then operating both sides by exp(wR), we get

$$\exp(wR) G(x,wyz) = \exp(wR) \sum_{n=o}^{\infty} a_n (wz)^n F_n(x, y,\alpha-n,\beta).$$

The first member of (2.3) is equal to

$$(1-wxy)^{1-\alpha} e^{\beta wy} G(\frac{x}{1-wxy}, wyz),$$

and the second member of (2.3) is equal to

$$\sum_{n=o}^{\infty} \sum_{m=o}^{\infty} a_n \frac{w^{m+n}}{m!} z^n R^m (F_n(x, y,\alpha-n,\beta))$$

$$= \sum_{n=o}^{\infty} \sum_{m=o}^{\infty} a_n \frac{w^{m+n}}{m!} z^n \beta^m F_{n+m}(x, y,\alpha-n-m,\beta)$$

$$= \sum_{n=o}^{\infty} (wy)^n g_n(z) y_n(x; \alpha-n,\beta)$$

where

$$g_n(z) = \sum_{m=o}^{\infty} a_m \frac{(\beta z)^m}{m!} .$$

Equating the above two results and then putting y=1, we get

$$(1-wx)^{1-\alpha} e^{\beta w} G(\frac{x}{1-wx}, wz) = \sum_{n=o}^{\infty} w^n g_n(z) y_n(x; \alpha-n,\beta)$$

where

$$g_n(x) = \sum_{m=o}^{n} a_m \frac{(\beta z)^m}{m!} ,$$

this completes the proof of our theorem.

3. APPLICATION.

As an application of our theorem we consider the following generating relation [3].

$$\sum_{n=o}^{\infty} \frac{w^n}{n!} y_n(x; \alpha-n,\beta) = e^w(1- \frac{wx}{\beta})^{1-\alpha} . \qquad (3.1)$$

If, in our theorem, we put $a_n = 1/n!$ we obtain

$$e^{(\beta+z)w}(1- \frac{(\beta+z)w}{\beta} x)^{1-\alpha} = \sum_{n=o}^{\infty} w^n g_n(z) y_n(x; \alpha-n,\beta) \qquad (3.2)$$

where

$$g_n(z) = \sum_{m=o}^{\infty} \frac{(\beta z)^m}{(m!)^n} .$$

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