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Research Article

Robust Stability Criteria for Uncertain Neutral Systems with Interval Nondifferentiable Time-Varying Delay and Nonlinear Perturbations

W. Weera[1, 2] and P. Niamsup[1, 2]

[1] *Department of Mathematics, Chiang Mai University, Chiang Mai 50200, Thailand*
[2] *Center of Excellence in Mathematics, CHE, Si Ayutthaya Road, Bangkok 10400, Thailand*

Correspondence should be addressed to P. Niamsup, scipnmsp@chiangmai.ac.th

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We study the robust stability criteria for uncertain neutral systems with interval time-varying delays and time-varying nonlinear perturbations simultaneously. The constraint on the derivative of the time-varying delay is not required, which allows the time-delay to be a fast time-varying function. Based on the Lyapunov-Krasovskii theory, we derive new delay-dependent stability conditions in terms of linear matrix inequalities (LMIs) which can be solved by various available algorithms. Numerical examples are given to demonstrate that the derived conditions are much less conservative than those given in the literature.

1. Introduction

It is well known that the existence of time delay in a system may cause instability and oscillations. Example, of time-delay systems are chemical engineering systems, biological modeling, electrical networks, physical networks, and many others, [7–16]. The stability criteria for system with time delays can be classified into two categories: delay-independent and delay-dependent. Delay-independent criteria do not employ any information on the size of the delay; while delay-dependent criteria make use of such information at different levels. Delay-dependent stability conditions are generally less conservative than delay-independent ones especially when the delay is small.

In many practical systems, models of system are described by neutral differential equations, in which the models depend on the delays of state and state derivatives. Heat exchanges, distributed networks containing lossless transmission lines and population ecology are examples of neutral systems because of its wider application. Therefore, several researchers have studied neutral systems and provided sufficient conditions to guarantee the asymptotic stability of neutral time delay systems, see [5, 9, 11–14, 16, 17] and references cited therein.

Well-known nonlinearities, as the delays, may cause instability and poor performance of practical systems, which have driven many researchers to study the problem of nonlinear perturbed systems with state delays during the recent years [5, 7, 9, 18]. In [18], the delay-dependent robust stability for linear time-varying systems with nonlinear perturbations is given, by using the Newton-Leibniz formula which has been taken into account instead of applying an integral inequality. In [7], a model transformation technique is used to deal with the stability of system with time varying for delays and nonlinear perturbations. In [9], based on a descriptor model transformation combined with a matrix decomposition approach, the robust stability of uncertain systems with time varying discrete delay is studied by applying an integral inequality. However, these model transformations often introduce additional dynamics which leads to relatively conservative results. In [5], the neutral delay and the discrete delay are all time-varying, while the derivative of discrete delay is less than 1 which limits its bigger application. In most studies the time-varying delays are required to be differentiable [1–5, 7, 9, 11–14, 16, 18]. Therefore their methods have a conservatism which can be improved upon. However, in most cases, these conditions are difficult to satisfy. From these reasons, the conditions are interesting to study, but there are fewer results for removing restriction to the derivative of interval time-varying delays. Therefore, in this paper we will employ some new techniques so that the above conditions can be removed.

In this paper, the problem of delay-dependent criterion for asymptotic stability for uncertain neutral system is studied with interval time-varying delay and time-varying nonlinear perturbations simultaneously. The restriction to the derivative of the interval time-varying delays is removed, which means that a fast interval time-varying delay is allowed. Based on the Lyapunov-Krasovskii theory, we derive new delay-dependent stability conditions in terms of linear matrix inequalities (LMIs) which can be solved by various available algorithms. The new stability condition is much less conservative and is more general than some existing results. Numerical examples are given to illustrate the effectiveness of our theoretical results.

2. Problem Formulation and Preliminaries

The following notations will be used in this paper: $\mathbb{R}^+$ denotes the set of all real nonnegative numbers; $\mathbb{R}^n$ denotes the n-dimensional space and the vector norm $\|\cdot\|$; $M^{n\times r}$ denotes the space of all matrices of $(n\times r)$-dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{\operatorname{Re}\lambda; \lambda \in \lambda(A)\}$. $x_t := \{x(t+s) : s \in [-h, 0]\}$, $\|x_t\| = \sup_{s\in[-h,0]}\|x(t+s)\|$; $C([0,t],\mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$-valued continuous functions on $[0,t]$; Matrix A is called semipositive definite ($A \geq 0$) if $x^T Ax \geq 0$, for all $x \in \mathbb{R}^n$; A is positive definite ($A > 0$) if $x^T Ax > 0$, for all $x \neq 0$; $A > B$ means $A - B > 0$. The symmetric term in a matrix is denoted by $*$.

Consider the following neutral system with time-varying delay:

$$\begin{gathered}\dot{x}(t) - C\dot{x}(t-d(t)) = A(t)x(t) + B(t)x(t-\tau(t)) + D_1(t)f_1(t,x(t)) + D_2(t)f_2(t,x(t-\tau(t))),\\ x(t_0+\theta) = \phi(\theta), \quad \theta \in [-h, 0],\end{gathered} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $d(t)$ is a neutral delay, $\tau(t)$ is a time-varying continuous function which satisfies

$$0 \leq \tau_m \leq \tau(t) \leq \tau_M, \quad 0 \leq d(t) \leq d, \quad \dot{d}(t) \leq \delta, \tag{2.2}$$

where τ_m, τ_M, d, δ are constants and $h = \max\{d, \tau_M\}$; the initial condition function $\phi(t)$ denotes a continuous vector-valued initial function of $t \in [-h, 0]$, $f_1(t, x(t))$ and $f_2(t, x(t - \tau(t)))$ are unknown nonlinear perturbations satisfying $f_1(t, 0) = 0$, $f_2(t, 0) = 0$ and

$$\begin{gathered} f_1^T(t, x(t)) f_1(t, x(t)) \leq \alpha^2 x^T(t) x(t), \\ f_2^T(t, x(t-\tau(t))) f_2(t, x(t-\tau(t))) \leq \beta^2 x^T(t-\tau(t)) x(t-\tau(t)), \end{gathered} \tag{2.3}$$

where α and β are positive real numbers.

The uncertain matrices $A(t)$, $B(t)$, $D_1(t)$, and $D_2(t)$ satisfy

$$\begin{aligned} A(t) &= A + \Delta A(t), & B(t) &= B + \Delta B(t), \\ D_1(t) &= D_1 + \Delta D_1(t), & D_2(t) &= D_2 + \Delta D_2(t), \end{aligned} \tag{2.4}$$

where A, B, D_1, $D_2 \in \mathbb{R}^{n \times n}$ are constant matrices with appropriate dimension, and $\Delta A(t)$, $\Delta B(t)$, $\Delta D_1(t)$, and $\Delta D_2(t)$ are unknown real matrices of appropriate dimension representing the systems time-varying parameter uncertainties which satisfy

$$\begin{aligned} \Delta A(t) &= G_1 F(t) E_A, & \Delta B(t) &= G_2 F(t) E_B, \\ \Delta D_1(t) &= G_3 F(t) E_{D_1}, & \Delta D_2(t) &= G_4 F(t) E_{D_2}, \end{aligned} \tag{2.5}$$

where G_1, G_2, G_3, G_4, E_A, E_B, E_{D_1}, and E_{D_2} are known real constant matrices of appropriate dimension. $F(t)$ is unknown time-varying matrix satisfying

$$F^T(t) F(t) \leq I. \tag{2.6}$$

For simplicity, we denote $f_1(t, x(t))$, $f_2(t, x(t - \tau(t)))$, by f_1, f_2, respectively.

Let $\tau_e = (1/2)(\tau_M + \tau_m)$ and $\rho = (1/2)(\tau_M - \tau_m)$. Then $\tau(t)$ can be expressed as

$$\tau(t) = \tau_e + \rho \xi(t), \tag{2.7}$$

where

$$\xi(t) = \begin{cases} \dfrac{2\tau(t) - (\tau_M + \tau_m)}{\tau_M - \tau_m}, & \tau_M > \tau_m, \\ 0, & \tau_M = \tau_m. \end{cases} \tag{2.8}$$

Obviously, $|\xi(t)| \leq 1$. For this case, $\tau(t)$ is a function belonging to the interval $[\tau_e - \rho, \tau_e + \rho]$, where ρ can be taken as the range of variation of the time-varying delay $\tau(t)$. Using the fact that

$$x(t-\tau_e) - x(t-\tau(t)) = \int_{t-\tau(t)}^{t-\tau_e} \dot{x}(s)ds \tag{2.9}$$

system (2.1) can be rewritten as

$$\begin{aligned} \dot{x}(t) - C\dot{x}(t-d(t)) = A(t)x(t) + B(t)x(t-\tau_e) - B(t)\int_{t-\tau(t)}^{t-\tau_e} \dot{x}(s)ds \\ + D_1(t)f_1 + D_2(t)f_2. \end{aligned} \tag{2.10}$$

Lemma 2.1 (see [17]). *There exists a symmetric matrix X such that*

$$\begin{bmatrix} P_1 - LXL^T & Q_1 \\ Q_1^T & R_1 \end{bmatrix} < 0, \qquad \begin{bmatrix} P_2 + X & Q_2 \\ Q_2^T & R_2 \end{bmatrix} < 0 \tag{2.11}$$

if and only if

$$\begin{bmatrix} P_1 + LP_2L^T & Q_1 & LQ_2 \\ Q_1^T & R_1 & 0 \\ Q_2^T L^T & 0 & R_2 \end{bmatrix} < 0. \tag{2.12}$$

Lemma 2.2 (see [3]). *For any constant symmetric matrix $M \in R^{n\times n}$, $M = M^T > 0$, $0 \leq h_m \leq h(t) \leq h_M$, $t \geq 0$, and any differentiable vector function $x(t) \in R^n$, we have*

$$\begin{aligned} &\text{(a)} \left[\int_{t-h_m}^{t} \dot{x}(s)ds\right]^T M \left[\int_{t-h_m}^{t} \dot{x}(s)ds\right] \leq h_m \int_{t-h_m}^{t} \dot{x}^T(s)M\dot{x}(s)ds, \\ &\text{(b)} \left[\int_{t-h(t)}^{t-h_m} \dot{x}(s)ds\right]^T M \left[\int_{t-h(t)}^{t-h_m} \dot{x}(s)ds\right] \leq (h(t)-h_m)\int_{t-h(t)}^{t-h_m} \dot{x}^T(s)M\dot{x}(s)ds \\ &\qquad \leq (h_M - h_m)\int_{t-h(t)}^{t-h_m} \dot{x}^T(s)M\dot{x}(s)ds. \end{aligned} \tag{2.13}$$

Lemma 2.3 (see [19]). *Given matrices $Q = Q^T$, H, E, and $R = R^T > 0$ with appropriate dimensions. Then*

$$Q + HFE + E^T F^T H^T < 0 \tag{2.14}$$

for all F satisfying $F^T F \leq R$, if and only if there exists an $\epsilon > 0$ such that

$$Q + \epsilon HH^T + \epsilon^{-1} E^T RE < 0. \tag{2.15}$$

Proposition 2.4 (Cauchy inequality). *For any symmetric positive definite matrix $N \in M^{n\times n}$ and $x, y \in \mathbb{R}^n$, we have*

$$\pm 2x^T y \leq x^T Nx + y^T N^{-1} y. \tag{2.16}$$

3. Main Results

Now we present a new delay-dependent condition for the asymptotic stability of system (2.1).

Assumption 3.1. All the eigenvalues of matrix C are inside the unit circle.

First, we study the problem of stability for nominal system of (2.10) with $\Delta A(t) = 0$, $\Delta B(t) = 0$, $\Delta D_1(t) = 0$, and $\Delta D_2(t) = 0$.

Theorem 3.2. *Under Assumption 3.1, nominal system of* (2.10) *with time-varying delay satisfying* (2.2) *is asymptotically stable if there exist positive definite matrices P, Q, Q_1, R, S, W, matrices K_1, K_2, L_i, M_i, $i = 1, 2, \ldots, 7$ of appropriate dimension and $\delta_1, \delta_2 > 0$ such that*

$$\Sigma_1 = \begin{bmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \tau_e L_1^T & \rho M_1 \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \tau_e L_2^T & \rho M_2 \\ * & * & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \tau_e L_3^T & \rho M_3 \\ * & * & * & \phi_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \tau_e L_4^T & \rho\left(K_1^T B + M_4\right) \\ * & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} & \tau_e L_5^T & \rho\left(K_2^T B + M_5\right) \\ * & * & * & * & * & \phi_{66} & 0 & \tau_e L_6^T & \rho M_6 \\ * & * & * & * & * & * & \phi_{77} & \tau_e L_7^T & \rho M_7 \\ * & * & * & * & * & * & * & -\tau_e R & 0 \\ * & * & * & * & * & * & * & * & -\rho S \end{bmatrix} < 0, \tag{3.1}$$

where

$$\phi_{11} = Q + L_1 + L_1^T + \varepsilon_1 \alpha^2 I,$$

$$\phi_{12} = M_1^T + L_2,$$

$$\phi_{13} = -L_1^T + L_3 + M_1,$$

$$\phi_{14} = P + A^T K_1 + L_4,$$

$$\phi_{15} = A^T K_2 + L_5,$$

$$\phi_{16} = L_6,$$

$$\phi_{17} = L_7,$$

$$
\begin{aligned}
\phi_{22} &= M_2^T + M_2 + \varepsilon_2\beta^2 I - W,\\
\phi_{23} &= -L_2^T - M_2^T + M_3 + W,\\
\phi_{24} &= M_4,\\
\phi_{25} &= M_5,\\
\phi_{26} &= M_6,\\
\phi_{27} &= M_7,\\
\phi_{33} &= -Q - L_3^T - L_3 - M_3^T - M_3 - W,\\
\phi_{34} &= B^T K_1 - L_4 - M_4,\\
\phi_{35} &= B^T K_2 - L_5 - M_5,\\
\phi_{36} &= -L_6 - M_6,\\
\phi_{37} &= -L_7 - M_7,\\
\phi_{44} &= Q_1 + \tau_e R + \rho S - K_1^T - K_1 + \rho^2 W,\\
\phi_{45} &= K_1^T C - K_2,\\
\phi_{46} &= K_1^T D_1,\\
\phi_{47} &= K_1^T D_2,\\
\phi_{55} &= -(1-\delta)Q_1 + K_2^T C + C^T K_2,\\
\phi_{56} &= K_2^T D_1,\\
\phi_{57} &= K_2^T D_2,\\
\phi_{66} &= -\delta_1 I,\\
\phi_{77} &= -\delta_2 I.
\end{aligned}
\tag{3.2}
$$

Proof. We prove that Theorem 3.2 is true for three cases, namely, $\tau_m \leq \tau(t) < \tau_e$; $\tau(t) = \tau_e$; $\tau_e < \tau(t) \leq \tau_M$.

Case 1 ($\tau_m \leq \tau(t) < \tau_e$). Choose a Lyapunov-Krasovskii functional candidate as

$$
\begin{aligned}
V_1(x_t) = {}& x^T(t)Px(t) + \int_{t-\tau_e}^{t} x^T(s)Qx(s)ds\\
&+ \int_{t-d(t)}^{t} \dot{x}^T(s)Q_1\dot{x}(s)ds \int_{-\tau_e}^{0}\int_{t+s}^{t} \dot{x}^T(\theta)R\dot{x}(\theta)d\theta ds\\
&+ \int_{-\tau_e}^{-\tau_m}\int_{t+s}^{t} \dot{x}^T(\theta)S\dot{x}(\theta)d\theta ds + \rho\int_{-\tau_e}^{-\tau_m}\int_{t+s}^{t} \dot{x}^T(\theta)W\dot{x}(\theta)d\theta ds,
\end{aligned}
\tag{3.3}
$$

where P, Q, Q_1, R, S, and W are positive definite matrices. Taking the derivative of $V_1(x_t)$ with respect to t along the trajectory of (2.10) yields

$$
\begin{aligned}
\dot{V}_1(x_t) &= 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau_e)Qx(t-\tau_e) \\
&\quad + \dot{x}^T(t)Q_1\dot{x}(t) - \left(1-\dot{d}(t)\right)\dot{x}^T(t-d(t))Q_1\dot{x}^T(t-d(t)) \\
&\quad + \dot{x}^T(t)\left(\tau_e R + \rho^2 W + \rho S\right)\dot{x}(t) \\
&\quad - \int_{t-\tau_e}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s)S\dot{x}(s)ds \\
&\quad - \rho\int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s)W\dot{x}(s)ds \\
&\le 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau_e)Qx(t-\tau_e) \\
&\quad + \dot{x}^T(t)Q_1\dot{x}(t) - (1-\delta)\dot{x}^T(t-d(t))Q_1\dot{x}^T(t-d(t)) \\
&\quad + \dot{x}^T(t)\left(\tau_e R + \rho^2 W + \rho S\right)\dot{x}(t) \\
&\quad - \int_{t-\tau_e}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s)S\dot{x}(s)ds \\
&\quad - \rho\int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s)W\dot{x}(s)ds,
\end{aligned}
\tag{3.4}
$$

since

$$
\begin{aligned}
-\int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s)S\dot{x}(s)ds &\le -\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)S\dot{x}(s)ds, \\
-\rho\int_{t-\tau_e}^{t-\tau_m} \dot{x}^T(s)W\dot{x}(s)ds &\le -\rho\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)W\dot{x}(s)ds.
\end{aligned}
\tag{3.5}
$$

Based on Lemma 2.2, we obtain

$$
\begin{aligned}
-\rho\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)W\dot{x}(s)ds &\le -(\tau_e - \tau(t))\int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)W\dot{x}(s)ds \\
&\le -x^T(t-\tau_e)Wx(t-\tau_e) + 2x^T(t-\tau_e)Wx(t-\tau(t)) \\
&\quad - x^T(t-\tau(t))Wx(t-\tau(t)),
\end{aligned}
\tag{3.6}
$$

and from the following equalities:

$$
\begin{aligned}
&2\left[\dot{x}^T(t)K_1^T + \dot{x}^T(t-d(t))K_2^T\right] \times \left[Ax(t) + Bx(t-\tau_e) - B\int_{t-\tau(t)}^{t-\tau_e} \dot{x}(s)ds\right. \\
&\qquad\qquad \left. + D_1 f_1 + D_2 f_2 + C\dot{x}(t-d(t)) - \dot{x}(t)\right] = 0,
\end{aligned} \tag{3.7}
$$

$$
\begin{aligned}
&2\left[x^T(t)L_1^T + x^T(t-\tau(t))L_2^T + x^T(t-\tau_e)L_3^T + \dot{x}^T(t)L_4^T + \dot{x}^T(t-d(t))L_5^T\right. \\
&\quad \left. + f_1^T L_6^T + f_2^T L_7^T\right] \times \left[x(t) - x(t-\tau_e) - \int_{t-\tau_e}^{t} \dot{x}(s)ds\right] = 0,
\end{aligned} \tag{3.8}
$$

$$
\begin{aligned}
&2\left[x^T(t)M_1^T + x^T(t-\tau(t))M_2^T + x^T(t-\tau_e)M_3^T + \dot{x}^T(t)M_4^T + \dot{x}^T(t-d(t))M_5^T\right. \\
&\quad \left. + f_1^T M_6^T + f_2^T M_7^T\right] \times \left[x(t-\tau(t)) - x(t-\tau_e) - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}(s)ds\right] = 0,
\end{aligned} \tag{3.9}
$$

where K_1, K_2, and L_i, M_i, $i = 1, 2, \ldots, 7$ are some matrices of appropriate dimension. Next, from (4.5), for any scalars $\delta_1 > 0$ and $\delta_2 > 0$, we obtain

$$
\begin{aligned}
&\delta_1\left[\alpha^2 x^T(t)x(t) - f_1^T f_1\right] \geq 0, \\
&\delta_2\left[\beta^2 x^T(t-\tau(t))x(t-\tau(t)) - f_2^T f_2\right] \geq 0.
\end{aligned} \tag{3.10}
$$

By adding the terms on left of (3.7)–(3.10) to $\dot{V}_1(x_t)$, we may express $\dot{V}_1(x_t)$ as

$$
\begin{aligned}
\dot{V}_1(x_t) \leq\; & 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau_e)Qx(t-\tau_e) + \dot{x}^T(t)Q_1\dot{x}(t) \\
& - (1-\delta)\dot{x}^T(t-d(t))Q_1\dot{x}(t-d(t)) + \dot{x}^T(t)\left(\tau_e R + \rho S + \rho^2 W\right)\dot{x}(t) \\
& - \int_{t-\tau_e}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)S\dot{x}(s)ds + 2\left[\dot{x}^T(t)K_1^T + \dot{x}^T(t-d(t))K_2^T\right] \\
& \times \left[Ax(t) + Bx(t-\tau_e) + B\int_{t-\tau_e}^{t-\tau(t)} \dot{x}(s)ds + D_1 f_1 + D_2 f_2 + C\dot{x}(t-d(t)) - \dot{x}(t)\right] \\
& + 2\left[x^T(t)L_1^T + x^T(t-\tau(t))L_2^T + x^T(t-\tau_e)L_3^T + \dot{x}^T(t)L_4^T + \dot{x}^T(t-d(t))L_5^T\right. \\
& \qquad \left. + f_1^T L_6^T + f_2^T L_7^T\right] \times \left[x(t) - x(t-\tau_e) - \int_{t-\tau_e}^{t} \dot{x}(s)ds\right] \\
& + 2\left[x^T(t)M_1^T + x^T(t-\tau(t))M_2^T\right. \\
& \qquad \left. + x^T(t-\tau_e)M_3^T + \dot{x}^T(t)M_4^T + \dot{x}^T(t-d(t))M_5^T + f_1^T M_6^T + f_2^T M_7^T\right] \\
& \times \left[x(t-\tau(t)) - x(t-\tau_e) - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}(s)ds\right] + \delta_1\left[\alpha^2 x^T(t)x(t) - f_1^T f_1\right]
\end{aligned}
$$

$$
\begin{aligned}
&- x^T(t-\tau_e)Wx(t-\tau_e) + 2x^T(t-\tau_e)Wx(t-\tau(t)) \\
&- x^T(t-\tau(t))Wx(t-\tau(t)) + \delta_2\left[\beta^2 x^T(t-\tau(t))x(t-\tau(t)) - f_2^T f_2\right] \\
= &\frac{1}{\tau_e}\int_{t-\tau_e}^{t} \omega^T(t,s)\phi_1\omega(t,s)ds + \frac{1}{\tau_e-\tau(t)}\int_{t-\tau_e}^{t-\tau(t)} \omega^T(t,s)\phi_2\omega(t,s)ds,
\end{aligned}
\tag{3.11}
$$

where

$$
\omega^T(t,s) = \left[x^T(t)x^T(t-\tau(t))x^T(t-\tau_e)\dot{x}^T(t)\dot{x}^T(t-d(t))f_1^T f_2^T - \dot{x}^T(s)\right],
$$

$$
\phi_1 = \begin{bmatrix}
\phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \tau_e L_1^T \\
* & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \tau_e L_2^T \\
* & * & \phi_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \tau_e L_3^T \\
* & * & * & \phi_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \tau_e L_4^T \\
* & * & * & * & \phi_{55} & \phi_{56} & \phi_{57} & \tau_e L_5^T \\
* & * & * & * & * & \phi_{66} & 0 & \tau_e L_6^T \\
* & * & * & * & * & * & \phi_{77} & \tau_e L_7^T \\
* & * & * & * & * & * & * & -\tau_e R
\end{bmatrix} + Z,
$$

$$
\phi_2 = \begin{bmatrix}
-Z_{11} & -Z_{12} & -Z_{13} & -Z_{14} & -Z_{15} & -Z_{16} & -Z_{17} & (\tau_e-\tau(t))M_1 \\
* & -Z_{22} & -Z_{23} & -Z_{24} & -Z_{25} & -Z_{26} & -Z_{27} & (\tau_e-\tau(t))M_2 \\
* & * & -Z_{33} & -Z_{34} & -Z_{35} & -Z_{36} & -Z_{37} & (\tau_e-\tau(t))M_3 \\
* & * & * & -Z_{44} & -Z_{45} & -Z_{46} & -Z_{47} & \varphi_1 \\
* & * & * & * & -Z_{55} & -Z_{56} & -Z_{57} & \varphi_2 \\
* & * & * & * & * & -Z_{66} & -Z_{67} & (\tau_e-\tau(t))M_6 \\
* & * & * & * & * & * & -Z_{77} & (\tau_e-\tau(t))M_7 \\
* & * & * & * & * & * & * & -|\tau_e-\tau(t)|S
\end{bmatrix},
$$

$$
Z = \begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} & Z_{14} & Z_{15} & Z_{16} & Z_{17} & 0 \\
* & Z_{22} & Z_{23} & Z_{24} & Z_{25} & Z_{26} & Z_{27} & 0 \\
* & * & Z_{33} & Z_{34} & Z_{35} & Z_{36} & Z_{37} & 0 \\
* & * & * & Z_{44} & Z_{45} & Z_{46} & Z_{47} & 0 \\
* & * & * & * & Z_{55} & Z_{56} & Z_{57} & 0 \\
* & * & * & * & * & Z_{66} & Z_{67} & 0 \\
* & * & * & * & * & * & Z_{77} & 0 \\
* & * & * & * & * & * & * & 0
\end{bmatrix},
\tag{3.12}
$$

$\varphi_i = (\tau_e - \tau(t))(-K_i^T B + M_i)$, $i = 1,2$, $Z_{ii} > 0$, $i = 1,2,\ldots,7$, Z_{ij}, $i = 1,2,\ldots,7$, $j = i+1,\ldots,7$ are some parameter matrices of appropriate dimensions. From (3.11) if $\phi_1 < 0$ and $\phi_2 < 0$, then $\dot{V}_1(x_t) \leq -\lambda_1 \|x(t)\|^2$ for some $\lambda_1 > 0$. Pre and postmultiplying both sides of $\phi_2 < 0$ by $\mathrm{diag}\{I, I, I, I, I, I, \mathrm{sgn}(\tau(t) - \tau_e)\}$, we get that

$$\phi_2 \leq \begin{bmatrix} -Z_{11} & -Z_{12} & -Z_{13} & -Z_{14} & -Z_{15} & -Z_{16} & -Z_{17} & \rho M_1 \\ * & -Z_{22} & -Z_{23} & -Z_{24} & -Z_{25} & -Z_{26} & -Z_{27} & \rho M_2 \\ * & * & -Z_{33} & -Z_{34} & -Z_{35} & -Z_{36} & -Z_{37} & \rho M_3 \\ * & * & * & -Z_{44} & -Z_{45} & -Z_{46} & -Z_{47} & \rho\left(-K_1^T B + M_4\right) \\ * & * & * & * & -Z_{55} & -Z_{56} & -Z_{57} & \rho\left(-K_2^T B + M_5\right) \\ * & * & * & * & * & -Z_{66} & -Z_{67} & \rho M_6 \\ * & * & * & * & * & * & -Z_{77} & \rho M_7 \\ * & * & * & * & * & * & * & -\rho S \end{bmatrix} < 0. \tag{3.13}$$

By Schur complement lemma, this implies $\phi_2 < 0$. In light of Lemma 2.1, (3.1) holds if and only if $\phi_1 < 0$ and (3.13) simultaneously hold. Then (3.1) holds if and only if there exists a symmetric matrix Z, $\phi_1 < 0$ and (3.13) simultaneously hold. Therefore, nominal system of (2.10) is asymptotically stable.

Case 2 ($\tau(t) = \tau_e$). For this case, we choose a Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V_2(x_t) = {} & x^T(t) P x(t) + \int_{t-\tau_e}^{t} x^T(s) Q x(s) ds + \int_{t-d(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds \\ & + \int_{-\tau_e}^{0} ds \int_{t+s}^{t} \dot{x}^T(\theta) R \dot{x}(\theta) d\theta, \end{aligned} \tag{3.14}$$

where P, Q, Q_1, and R positive definite matrices are the same as those in $V_1(x_t)$.

Case 3 ($\tau_e < \tau(t) < \tau_M$). For this case, we choose the Lyapunov-Krasovskii functional candidate as

$$\begin{aligned} V_3(x_t) = {} & x^T(t) P x(t) + \int_{t-\tau_e}^{t} x^T(s) Q x(s) ds \\ & + \int_{t-d(t)}^{t} \dot{x}^T(s) Q_1 \dot{x}(s) ds \int_{-\tau_e}^{0} \int_{t+s}^{t} \dot{x}^T(\theta) R \dot{x}(\theta) d\theta ds \\ & + \int_{-\tau_M}^{-\tau_e} \int_{t+s}^{t} \dot{x}^T(\theta) S \dot{x}(\theta) d\theta ds + \rho \int_{-\tau_M}^{-\tau_e} \int_{t+s}^{t} \dot{x}^T(\theta) W \dot{x}(\theta) d\theta ds, \end{aligned} \tag{3.15}$$

where P, Q, Q_1, R, S, and W are positive definite matrices and are the same as those in $V_1(x_t)$.

By similar arguments used in proof of Theorem 3.2, we conclude that the nominal system of (2.10) is robustly asymptotically stable. The proof is complete. □

Based on Theorem 3.2, we can perform the robust stability analysis for system (2.10) with uncertainties (2.5) and (2.6).

Theorem 3.3. *Under Assumption 3.1, system* (2.10) *with time-varying delay satisfying* (2.2) *and uncertainties* (2.5) *and* (2.6) *is asymptotically stable if there exist positive definite matrices* P, Q, Q_1, R, S, W *and* K_1, K_2, L_i, M_i, $i = 1, 2, \ldots, 7$ *of appropriate dimension and scalars* $\epsilon_i > 0$, $i = 1, 2, \ldots, 10$ *such that*

$$\mathcal{M} = \begin{bmatrix} M_{11} & \phi_{12} & \phi_{13} & \phi_{14} & \phi_{15} & \phi_{16} & \phi_{17} & \tau_e L_1^T & \rho M_1 \\ * & \phi_{22} & \phi_{23} & \phi_{24} & \phi_{25} & \phi_{26} & \phi_{27} & \tau_e L_2^T & \rho M_2 \\ * & * & M_{33} & \phi_{34} & \phi_{35} & \phi_{36} & \phi_{37} & \tau_e L_3^T & \rho M_3 \\ * & * & * & M_{44} & \phi_{45} & \phi_{46} & \phi_{47} & \tau_e L_4^T & \rho\left(K_1^T B + M_4\right) \\ * & * & * & * & M_{55} & \phi_{56} & \phi_{57} & \tau_e L_5^T & \rho\left(K_2^T B + M_5\right) \\ * & * & * & * & * & M_{66} & 0 & \tau_e L_6^T & \rho M_6 \\ * & * & * & * & * & * & M_{77} & \tau_e L_7^T & \rho M_7 \\ * & * & * & * & * & * & * & -\tau_e R & 0 \\ * & * & * & * & * & * & * & * & M_{99} \end{bmatrix} < 0,$$

$$\mathcal{M}_1 = \begin{bmatrix} -0.1K_1^T - 0.1K_1 & K_1^T G_1 & K_1^T G_2 & K_1^T G_2 & K_1^T G_3 & K_1^T G_4 \\ G_1^T K_1 & -\epsilon_1 I & 0 & 0 & 0 & 0 \\ G_2^T K_1 & 0 & -\epsilon_2 I & 0 & 0 & 0 \\ G_2^T K_1 & 0 & 0 & -\epsilon_3 I & 0 & 0 \\ G_3^T K_1 & 0 & 0 & 0 & -\epsilon_4 I & 0 \\ G_4^T K_1 & 0 & 0 & 0 & 0 & -\epsilon_5 I \end{bmatrix} < 0, \tag{3.16}$$

$$\mathcal{M}_2 = \begin{bmatrix} -0.1Q_1 + 0.1K_2^T C + 0.1C^T K_2 & K_2^T G_1 & K_2^T G_2 & K_2^T G_2 & K_2^T G_3 & K_2^T G_4 \\ G_1^T K_2 & -\epsilon_6 I & 0 & 0 & 0 & 0 \\ G_2^T K_2 & 0 & -\epsilon_7 I & 0 & 0 & 0 \\ G_2^T K_2 & 0 & 0 & -\epsilon_8 I & 0 & 0 \\ G_3^T K_2 & 0 & 0 & 0 & -\epsilon_9 I & 0 \\ G_4^T K_2 & 0 & 0 & 0 & 0 & -\epsilon_{10} I \end{bmatrix} < 0,$$

where

$$M_{11} = \phi_{11} + \epsilon_1 E_A^T E_A + \epsilon_6 E_A^T E_A,$$

$$M_{33} = \phi_{33} + \epsilon_2 E_B^T E_B + \epsilon_7 E_B^T E_B,$$

$$M_{44} = Q_1 + \tau_e R + \rho S - 0.9K_1^T - 0.9K_1 + \rho^2 W,$$

$$M_{55} = -0.9Q_1 + \delta Q_1 + 0.9K_2^T C + 0.9C^T K_2,$$

$$M_{66} = \phi_{66} + \epsilon_4 E_{D_1}^T E_{D_1} + \epsilon_9 \rho^2 E_B^T E_B,$$
$$M_{77} = \phi_{77} + \epsilon_5 E_{D_2}^T E_{D_2} + \epsilon_{10} \rho^2 E_B^T E_B,$$
$$M_{99} = -\rho S + \epsilon_3 \rho^2 E_B^T E_B + \epsilon_8 \rho^2 E_B^T E_B. \tag{3.17}$$

Proof. We choose Lyapunov-Krasovskii functional as in Theorem 3.2, we may proof this Theorem by using a similar arguments as in the proof of Theorem 3.2. By replacing A, B, D_1, and D_2 in (3.11) with $A + GF(t)E_A$, $B + GF(t)E_B$, $D_1 + GF(t)E_{D_1}$ and $D_2 + GF(t)E_{D_2}$, respectively. For Case 1

$$\begin{aligned}
\dot{V}_1(x_t) \leq{} & 2x^T(t)P\dot{x}(t) + x^T(t)Qx(t) - x^T(t-\tau_e)Qx(t-\tau_e) + \dot{x}^T(t)Q_1\dot{x}(t) \\
& - (1-\delta)\dot{x}^T(t-d(t))Q_1\dot{x}(t-d(t)) + \dot{x}^T(t)\left(\tau_e R + \rho S + \rho^2 W\right)\dot{x}(t) \\
& - \int_{t-\tau_e}^{t} \dot{x}^T(s)R\dot{x}(s)ds - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}^T(s)S\dot{x}(s)ds + 2\left[\dot{x}^T(t)K_1^T + \dot{x}^T(t-d(t))K_2^T\right] \\
& \times \Bigg[(A + G_1F(t)E_A)x(t) + (B + G_2F(t)E_B)x(t-\tau_e) + (B + G_2F(t)E_B) \\
& \quad \times \int_{t-\tau_e}^{t-\tau(t)} \dot{x}(s)ds + (D_1 + G_3F(t)E_{D_1})f_1 + (D_2 + G_4F(t)E_{D_2})f_2 + C\dot{x}(t-d(t)) - \dot{x}(t)\Bigg] \\
& + 2\Big[x^T(t)L_1^T + x^T(t-\tau(t))L_2^T + x^T(t-\tau_e)L_3^T + \dot{x}^T(t)L_4^T + \dot{x}^T(t-d(t))L_5^T \\
& \quad + f_1^T L_6^T + f_2^T L_7^T\Big] \times \left[x(t) - x(t-\tau_e) - \int_{t-\tau_e}^{t} \dot{x}(s)ds\right] \\
& + 2\Big[x^T(t)M_1^T + x^T(t-\tau(t))M_2^T + x^T(t-\tau_e)M_3^T + \dot{x}^T(t)M_4^T + \dot{x}^T(t-d(t))M_5^T \\
& \quad + f_1^T M_6^T + f_2^T M_7^T\Big]\left[x(t-\tau(t)) - x(t-\tau_e) - \int_{t-\tau_e}^{t-\tau(t)} \dot{x}(s)ds\right] + \delta_1\left[\alpha^2 x^T(t)x(t) - f_1^T f_1\right] \\
& - x^T(t-\tau_e)Wx(t-\tau_e) + 2x^T(t-\tau_e)Wx(t-\tau(t)) \\
& - x^T(t-\tau(t))Wx(t-\tau(t)) + \delta_2\left[\beta^2 x^T(t-\tau(t))x(t-\tau(t)) - f_2^T f_2\right].
\end{aligned} \tag{3.18}$$

Applying Lemmas 2.3. and 2.4., the following estimations hold:

$$\begin{aligned}
2\dot{x}^T(t)K_1^T(A + G_1F(t)E_A)x(t) \leq{} & 2\dot{x}^T(t)K_1^T Ax(t) \\
& + \epsilon_1^{-1}\dot{x}^T(t)K_1^T G_1 G_1^T K_1\dot{x}(t) \\
& + \epsilon_1 x^T(t)E_A^T E_A x(t),
\end{aligned} \tag{3.19}$$

$$\begin{aligned}2\dot{x}^T(t)K_1^T(B+G_2F(t)E_B)x(t-\tau_e) \leq{} & 2\dot{x}^T(t)K_1^TBx(t-\tau_e)\\ & + \epsilon_2^{-1}\dot{x}^T(t)K_1^TG_2G_2^TK_1\dot{x}(t)\\ & + \epsilon_2 x^T(t-\tau_e)E_B^TE_Bx(t-\tau_e),\end{aligned} \tag{3.20}$$

$$\begin{aligned}2\dot{x}^T(t)K_1^T(B+G_2F(t)E_B)\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds \leq{} & 2\dot{x}^T(t)K_1^TB\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds\\ & + \epsilon_3^{-1}\dot{x}^T(t)K_1^TG_2G_2^TK_4\dot{x}^T(t)\\ & + \epsilon_3\rho\left(\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)E_B^TE_B\dot{x}(s)ds\right),\end{aligned} \tag{3.21}$$

$$\begin{aligned}2\dot{x}^T(t)K_1^T(D_1+G_3F(t)E_{D_1})f_1 \leq{} & 2\dot{x}^T(t)K_1^TD_1f_1\\ & + \epsilon_4^{-1}\dot{x}^T(t)K_1^TG_3G_3^TK_1\dot{x}(t)\\ & + \epsilon_4 f_1^TE_{D_1}^TE_{D_1}f_1,\end{aligned} \tag{3.22}$$

$$\begin{aligned}2\dot{x}^T(t)K_1^T(D_2+G_4F(t)E_{D_2})f_2 \leq{} & 2\dot{x}^T(t)K_1^TD_2f_2\\ & + \epsilon_5^{-1}\dot{x}^T(t)K_1^TG_4G_4^TK_1\dot{x}(t)\\ & + \epsilon_5 f_2^TE_{D_2}^TE_{D_2}f_2,\end{aligned} \tag{3.23}$$

$$\begin{aligned}2\dot{x}^T(t-d(t))K_2^T(A+G_1F(t)E_A)x(t) \leq{} & 2\dot{x}^T(t-d(t))K_2^TAx(t)\\ & + \epsilon_6^{-1}\dot{x}^T(t-d(t))K_2^TG_1G_1^TK_2\dot{x}(t-d(t))\\ & + \epsilon_6 x^T(t-d(t))E_A^TE_Ax(t),\end{aligned} \tag{3.24}$$

$$\begin{aligned}2\dot{x}^T(t-d(t))K_2^T(B+G_2F(t)E_B)x(t-\tau_e) \leq{} & 2\dot{x}^T(t-d(t))K_1^TBx(t-\tau_e)\\ & + \epsilon_7^{-1}\dot{x}^T(t-d(t))K_1^TG_2G_2^TK_1\dot{x}(t-d(t))\\ & + \epsilon_7 x^T(t-\tau_e)E_B^TE_Bx(t-\tau_e),\end{aligned} \tag{3.25}$$

$$\begin{aligned}2\dot{x}^T(t-d(t))K_2^T(B+G_2F(t)E_B)\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds \leq{} & 2\dot{x}^T(t-d(t))K_2^TB\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)ds\\ & + \epsilon_8^{-1}\dot{x}^T(t-d(t))K_2^TG_2G_2^TK_2\dot{x}^T(t-d(t))\\ & + \epsilon_8\rho\left(\int_{t-\tau_e}^{t-\tau(t)}\dot{x}(s)E_B^TE_B\dot{x}(s)ds\right),\end{aligned} \tag{3.26}$$

$$
\begin{aligned}
2\dot{x}^T(t-d(t))K_2^T\left(D_1+G_3F(t)E_{D_1}\right)f_1 \le{}& 2\dot{x}^T(t-d(t))K_2^TD_1f_1 \\
&+\epsilon_9^{-1}\dot{x}^T(t-d(t))K_2^TG_3G_3^TK_2\dot{x}(t-d(t)) \\
&+\epsilon_9 f_1^TE_{D_1}^TE_{D_1}f_1,
\end{aligned}
\tag{3.27}
$$

$$
\begin{aligned}
2\dot{x}^T(t-d(t))K_2^T\left(D_2+G_4F(t)E_{D_2}\right)f_2 \le{}& 2\dot{x}^T(t-d(t))K_2^TD_2f_2 \\
&+\epsilon_{10}^{-1}\dot{x}^T(t-d(t))K_2^TG_4G_4^TK_2\dot{x}(t-d(t)) \\
&+\epsilon_{10} f_2^TE_{D_2}^TE_{D_2}f_2.
\end{aligned}
\tag{3.28}
$$

Therefore, from (3.18)–(3.28), it follows that

$$
\dot{V}_1(x_t) \le \omega^T(t,s)\mathcal{M}\omega(t,s) + \dot{x}^T(t)\Omega_1\dot{x}(t) + \dot{x}^T(t-d(t))\Omega_2\dot{x}(t-d(t)),
\tag{3.29}
$$

where

$$
\begin{aligned}
\Omega_1 ={}& -0.1K_1^T - 0.1K_1 + \epsilon_1^{-1}K_1^TG_1G_1^TK_1 + \epsilon_2^{-1}K_1^TG_2G_2^TK_1 \\
&+ \epsilon_3^{-1}K_1^TG_2G_2^TK_1 + \epsilon_4^{-1}K_1^TG_3G_3^TK_1 + \epsilon_5^{-1}K_1^TG_4G_4^TK_1, \\
\Omega_2 ={}& -0.1Q_1^T + 0.1K_2^TC + 0.1C^TK_2 + \epsilon_6^{-1}K_2^TG_1G_1^TK_2 + \epsilon_7^{-1}K_2^TG_2G_2^TK_2 \\
&+ \epsilon_8^{-1}K_2^TG_2G_2^TK_2 + \epsilon_9^{-1}K_2^TG_3G_3^TK_2 + \epsilon_{10}^{-1}K_2^TG_4G_4^TK_2.
\end{aligned}
\tag{3.30}
$$

Applying Schur complement lemma, the inequalities $\Omega_1 < 0$ and $\Omega_2 < 0$ are equivalent to $\mathcal{M}_1 < 0$ and $\mathcal{M}_2 < 0$, respectively. Therefore, system (2.10) is robust asymptotically stable if the condition (3.16) holds.

By using arguments similar to the proof of Case 1 for Case 2 and Case 3, we may conclude that the close-loop system (2.10) is robust asymptotically stable. □

Remark 3.4. In this paper, the restriction that the state delay is differentiable is not required, which allows state delay to be fast time varying. Meanwhile, this restriction is required in some existing results, see [1–5, 7, 9, 11–14, 16, 18].

Remark 3.5. In the proof of Theorem 3.3, we need negative definiteness of matrices $\mathcal{M}$, Ω_1 and Ω_2 simultaneously. In order to do so, we need to have certain diagonal terms of matrices $\mathcal{M}$, Ω_1 and Ω_2 being negative. This leads to the splitting of the term K_1 as $(0.1+0.9)K_1$ which is one possibility to achieve such goal.

4. Numerical Examples

In this section, we provide numerical examples to show the effectiveness of our theoretical results.

Table 1: Comparison of the maximum value τ_M for $\gamma = 0.1$.

	$\alpha = 0,\ \beta = 0.1$	
	$\dot{\tau}(t) = 0.5$	No restriction on $\dot{\tau}(t)$
Zhang et al. [1]	0.704	—
Shen and Zhong [2]	1.060	—
Ours	—	1.0762
	$\alpha = 0.1,\ \beta = 0.1$	
	$\dot{\tau}(t) = 0.5$	No restriction on $\dot{\tau}(t)$
Zhang et al. [1]	0.689	—
Shen and Zhong [2]	1.040	—
Ours	—	1.0638

Example 4.1. Consider the following uncertain neutral system with time-varying delay and nonlinear uncertainties which is studied in [1, 2]:

$$\begin{aligned}\dot{x}(t) - C\dot{x}(t-d) &= (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t-\tau(t)) + (D_1 + \Delta D_1(t))f_1(t, x(t)) \\ &\quad + (D_2 + \Delta D_2(t))f_2(t, x(t-\tau(t))),\end{aligned} \tag{4.1}$$

where

$$\begin{gathered}A = \begin{bmatrix}-1.2 & -0.1\\ -0.1 & -1\end{bmatrix}, \qquad B = \begin{bmatrix}-0.6 & 0.7\\ -1 & -0.8\end{bmatrix}, \qquad C = \begin{bmatrix}-0.2 & 0\\ 0.2 & -0.1\end{bmatrix}, \\ D_1 = \begin{bmatrix}-0.1 & 0\\ 0 & -0.1\end{bmatrix}, \qquad D_2 = \begin{bmatrix}-0.1 & 0\\ 0 & -0.1\end{bmatrix}, \\ \Delta A(t) = GF(t)E_A, \qquad \Delta B(t) = GF(t)E_B, \qquad \Delta D_1(t) = GF(t)E_{D_1}, \\ \Delta D_2(t) = GF(t)E_{D_2}, \qquad F^T(t)F(t) \le I, \qquad G = \gamma I, \qquad E_A = I, \\ E_B = I, \qquad E_{D_1} = I, \qquad E_{D_2} = I.\end{gathered} \tag{4.2}$$

It is assumed that the nonlinear uncertainties satisfy

$$\left\| f_1(t, x(t)) \right\| \le \alpha \|x(t)\|, \quad \left\| f_2(t, x(t-\tau(t))) \right\| \le \beta \|x(t-\tau(t))\|, \quad \alpha > 0,\ \beta > 0. \tag{4.3}$$

Applying Theorem 3.3, the maximum allowable value of τ_M is given in Table 1 when $\gamma = 0.1$ and in Table 2 for $\gamma = 0.5$. The results obtained in [1, 2] may not be used for the case when $\tau_m \neq 0$. Moreover, the differentiability of the time delay $\tau(t)$ is not required in Theorem 3.3. Tables 1 and 2 show that our results significantly improve the results of [1, 2].

Table 2: Comparison of the maximum value τ_M for $\gamma = 0.5$.

	$\alpha = 0,\ \beta = 0.1$	
	$\dot{\tau}(t) = 0.5$	No restriction on $\dot{\tau}(t)$
Zhang et al. [1]	0.259	—
Shen and Zhong [2]	0.450	—
Ours	—	0.5333
	$\alpha = 0.1,\ \beta = 0.1$	
	$\dot{\tau}(t) = 0.5$	No restriction on $\dot{\tau}(t)$
Zhang et al. [1]	0.240	—
Shen and Zhong [2]	0.429	—
Ours	—	0.5207

Moreover, it should be pointed out that if we let $\tau_m = 0.1$ and $\tau_M = 0.85$, then from Theorem 3.3, the solutions of LMI (3.16) are given as follows:

$$P = \begin{bmatrix} 2.4597 & 0.3509 \\ 0.3509 & 1.9870 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.9955 & 0.1967 \\ 0.1967 & 1.0236 \end{bmatrix},$$

$$Q_1 = \begin{bmatrix} 0.5615 & -0.0545 \\ -0.0545 & 0.2292 \end{bmatrix}, \qquad S = \begin{bmatrix} 1.4211 & 0.1224 \\ 0.1224 & 1.1393 \end{bmatrix},$$

$$R = \begin{bmatrix} 1.8164 & 0.2718 \\ 0.2718 & 1.7699 \end{bmatrix}, \qquad W = \begin{bmatrix} 0.4131 & 0.0520 \\ 0.0520 & 0.3768 \end{bmatrix},$$

$$L_1 = \begin{bmatrix} -1.8399 & -0.0430 \\ -0.1215 & -1.9400 \end{bmatrix}, \qquad L_2 = \begin{bmatrix} 1.5341 & 1.0797 \\ -1.1061 & 1.2226 \end{bmatrix},$$

$$L_3 = \begin{bmatrix} -0.1656 & -0.0202 \\ -0.1437 & -0.3157 \end{bmatrix}, \qquad L_4 = \begin{bmatrix} -1.3144 & -0.4567 \\ 0.1806 & -1.3580 \end{bmatrix},$$

$$L_5 = \begin{bmatrix} -0.1514 & -0.0592 \\ -0.0410 & 0.0423 \end{bmatrix}, \qquad L_6 = \begin{bmatrix} -0.0047 & 0.0064 \\ -0.0036 & -0.0045 \end{bmatrix},$$

$$L_7 = \begin{bmatrix} -0.0047 & 0.0064 \\ -0.0037 & -0.0046 \end{bmatrix}, \qquad K_1 = \begin{bmatrix} 1.7652 & 0.2207 \\ 0.0928 & 1.4950 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.2816 & 0.0203 \\ 0.0408 & -0.1346 \end{bmatrix}, \qquad M_1 = \begin{bmatrix} -1.4428 & -0.0867 \\ -0.0535 & -1.3096 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} -1.2780 & -1.0258 \\ 1.1266 & -0.9849 \end{bmatrix}, \qquad M_3 = \begin{bmatrix} 0.2312 & -0.1717 \\ 0.0660 & 0.4381 \end{bmatrix},$$

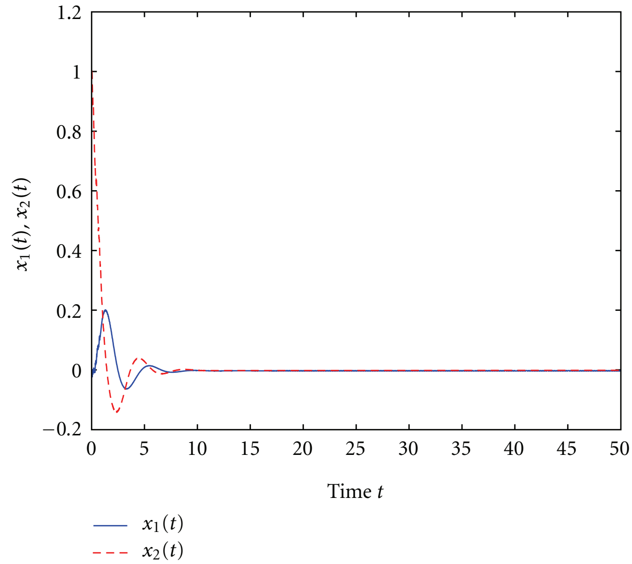

Figure 1: The trajectories of $x_1(t)$ and $x_2(t)$ of the system (4.1) with time-varying delay $\tau(t) = 0.1 + 0.75|\sin 10t|$.

$$M_4 = \begin{bmatrix} 0.3926 & -1.0687 \\ 0.9108 & 0.5805 \end{bmatrix}, \qquad M_5 = \begin{bmatrix} 0.3601 & 0.1664 \\ -0.1928 & 0.1095 \end{bmatrix},$$
$$M_6 = \begin{bmatrix} 0.0032 & -0.0046 \\ 0.0110 & 0.0081 \end{bmatrix}, \qquad M_7 = \begin{bmatrix} 0.0032 & -0.0047 \\ 0.0111 & 0.0081 \end{bmatrix},$$
$$\delta_1 = 2.0156, \qquad \delta_2 = 1.9922, \qquad \epsilon_1 = 0.4383,$$
$$\epsilon_2 = 0.4197, \qquad \epsilon_3 = 0.5513, \qquad \epsilon_4 = 0.9140,$$
$$\epsilon_5 = 0.9042, \qquad \epsilon_6 = 0.3288, \qquad \epsilon_7 = 0.3143,$$
$$\epsilon_8 = 0.4376, \qquad \epsilon_9 = 0.9021, \qquad \epsilon_{10} = 0.9003. \tag{4.4}$$

Figure 1 shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the system (4.1) with time-varying delay $\tau(t) = 0.1 + 0.75|\sin 10t|$, $d = 1$, $\phi(t) = [\sin t, \cos t)]$, for all $t \in [-1, 0]$, $f_1(t, x(t)) = [0.1\sin|x_1(t)|, 0.1\cos|x_2(t)|]^T$, $f_2(t, x(t-\tau(t))) = [0.1e^{-\sin^2 x_1(t-\tau(t))}, 0.1e^{-\cos^2 x_2(t-\tau(t))}]^T$ and $F(t) = \mathrm{diag}\{\sin^2(t), \sin^2(t)\}$. Since the time-delay $\tau(t)$ is not differentiable, the stability criterion in [1, 2] cannot be applied to this case because it is only applicable to the system with the differentiable delay.

Example 4.2. Consider the following uncertain neutral system with time-varying delay in [3, 4]:

$$\dot{x}(t) - C\dot{x}(t - d(t)) = (A + \Delta A(t))x(t) + (B + \Delta B(t))x(t - \tau(t)), \tag{4.5}$$

Table 3: Maximum allowable upper bounds τ_M of the time-varying delay for different values of the lower bounds τ_m and $c = 0.1$.

	$\tau_m = 0$	$\tau_m = 0.5$
Yu and Lien [3]	0.658	0.793
Kwon et al. [4]	0.852	0.894
Ours	0.873	0.951

Table 4: Maximum allowable upper bounds τ_M of the time-varying delay for different values of the lower bounds τ_m, $c = 0.1$ and $\delta = 0.1$.

$\tau_m = 0$	$\dot{\tau}(t) = 0.5$	$\dot{\tau}(t) = 0.9$	$\dot{\tau}(t) \geq 1$	No restriction on $\dot{\tau}(t)$
Han and Yu [5]	0.57	0.17	—	—
Qiu et al. [6]	0.7890	0.7199	0.7216	—
Ours	—	—	—	0.8676
$\tau_m = 0.5$	$\dot{\tau}(t) = 0.5$	$\dot{\tau}(t) = 0.9$	$\dot{\tau}(t) \geq 1$	No restriction on $\dot{\tau}(t)$
Ours	—	—	—	0.9460

where

$$
\begin{gathered}
A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix}, \qquad C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \\
\Delta A(t) = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}, \qquad \Delta B(t) = \begin{bmatrix} \gamma_3 & 0 \\ 0 & \gamma_4 \end{bmatrix},
\end{gathered}
\tag{4.6}
$$

where $0 \leq |c| < 1$, and γ_i, $i = 1, 2, \ldots, 4$ are unknown parameter satisfying $|\gamma_1| \leq 1.6$, $|\gamma_2| \leq 0.05$, $|\gamma_3| < 0.1$, and $|\gamma_4| < 0.3$.

Case 1. For $c = 0.1$, $\delta = 0$, the maximum values of τ_M are listed in Table 3 for $c = 0.1$ by applying criteria in [3, 4] and in this paper. We see that the maximum allowable bounds for τ_M obtained from Theorem 3.3 are much better than that obtained in [3, 4].

Case 2. For $c = 0.1$, $\delta = 0.1$, the maximum value τ_M obtained form Theorem 3.3 is listed in Table 4. In [3, 4] the neutral delay is constant, then its stability criterion cannot be applied to systems with time-varying neutral delay. Furthermore, the stability criterion in [5, 6] cannot be applied to this case because Theorem 3.3 does not have restriction on the derivative of time-varying delay. It is obvious that the obtained results are significantly better than those in [3–6].

Figure 2 shows the trajectories of solutions $x_1(t)$ and $x_2(t)$ of the system (4.5) with time-varying delay $\tau(t) = 0.3 + 0.5|\cos 10t|$, $d(t) = 0.1\sin^2 t$, $\phi(t) = [\sin t, \cos t)]$, for all $t \in [-0.8, 0]$.

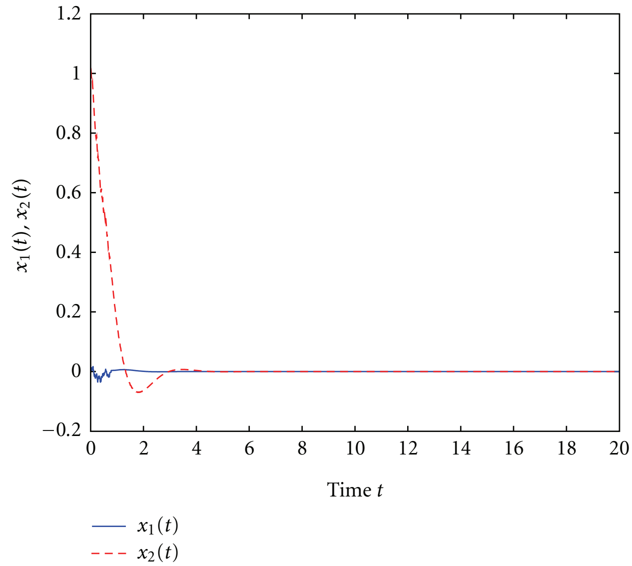

Figure 2: The trajectories of $x_1(t)$ and $x_2(t)$ of the system (4.5) with time-varying delay $\tau(t) = 0.3 + 0.5|\cos 10t|$.

5. Conclusions

In this paper, we have investigated the delay-dependent robust stability criteria for uncertain neutral systems with interval time-varying delays and time-varying nonlinear perturbations simultaneously. Based on Lyapunov-krasovskii theory, new delay-dependent sufficient conditions for robust stability have been derived in terms of LMIs. The interval time-varying delay function is not required to be differentiable, which allows time-delay function to be a fast time-varying function. Numerical examples are given to illustrate the effectiveness of the theoretic results which show that our results are much less conservative than some existing results in the literature.

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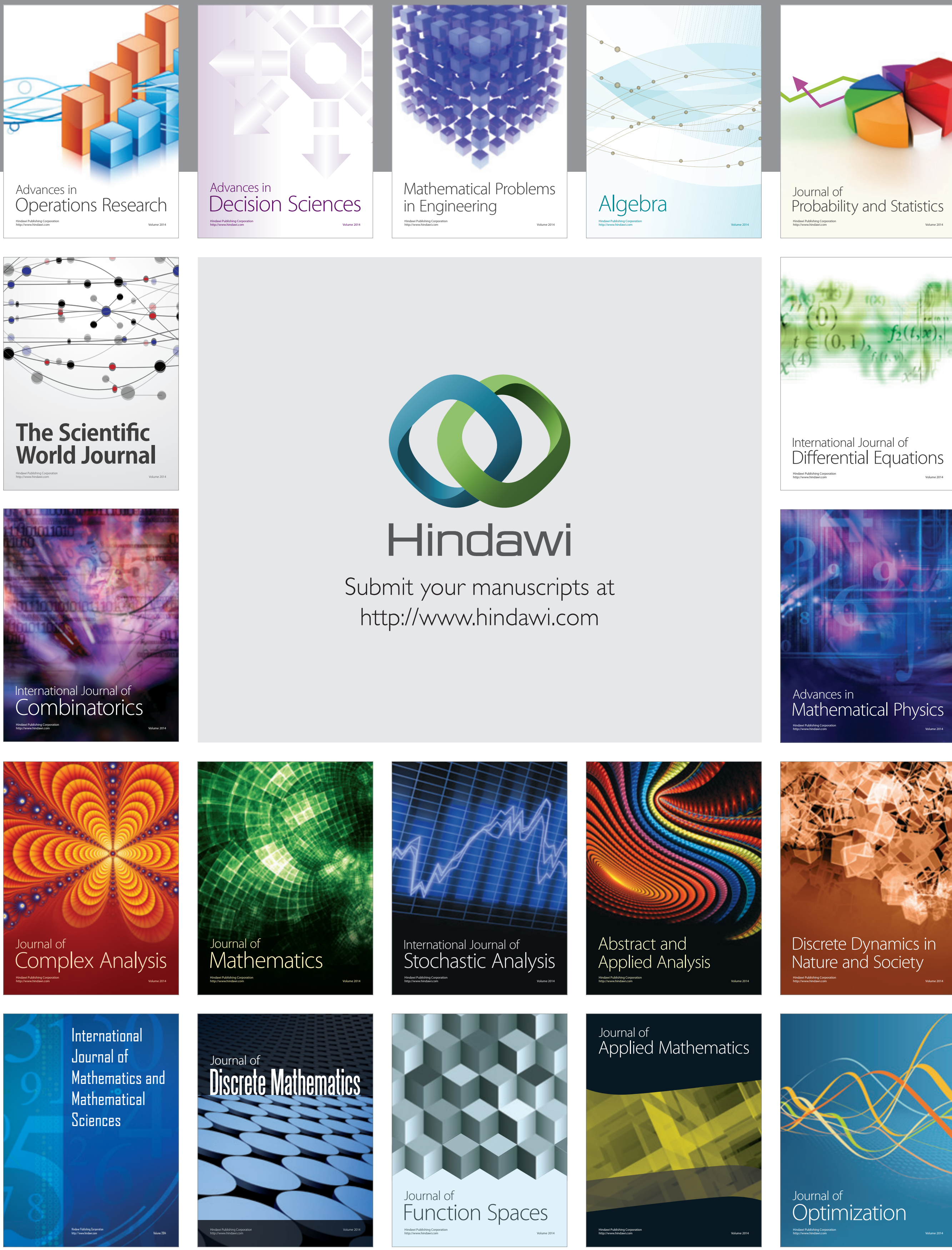
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