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Research Article

Lyapunov-Type Inequalities for Some Quasilinear Dynamic System Involving the $(p_1, p_2, \ldots, p_m)$-Laplacian on Time Scales

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We establish several new Lyapunov-type inequalities for some quasilinear dynamic system involving the $(p_1, p_2, \ldots, p_m)$-Laplacian on an arbitrary time scale $\mathbb{T}$, which generalize and improve some related existing results including the continuous and discrete cases.

1. Introduction

In recent years, the theory of time scales (or measure chains) has been developed by several authors with one goal being the unified treatment of differential equations (the continuous case) and difference equations (the discrete case). A time scale is an arbitrary nonempty closed subset of the real numbers $\mathbb{R}$. We assume that $\mathbb{T}$ is a time scale and $\mathbb{T}$ has the topology that it inherits from the standard topology on the real numbers $\mathbb{T}$. The two most popular examples are $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$. In Section 2, we will briefly introduce the time scale calculus and some related basic concepts of Hilger [1–3]. For further details, we refer the reader to the books independently by Kaymakcalan et al. [4] and by Bohner and Peterson [5, 6].

Consider the following quasilinear dynamic system involving the $(p_1, p_2, \ldots, p_m)$-Laplaci-an on an arbitrary time scale $\mathbb{T}$:

$$-\left(r_1(t)\left|u_1^{\Delta}(t)\right|^{p_1-2}u_1^{\Delta}(t)\right)^{\Delta} = f_1(t)|u_1(\sigma(t))|^{\alpha_1-2}|u_2(\sigma(t))|^{\alpha_2}\cdots|u_m(\sigma(t))|^{\alpha_m}u_1(\sigma(t)),$$

$$-\left(r_2(t)\left|u_2^{\Delta}(t)\right|^{p_2-2}u_2^{\Delta}(t)\right)^{\Delta} = f_2(t)|u_1(\sigma(t))|^{\alpha_1}|u_2(\sigma(t))|^{\alpha_2-2}\cdots|u_m(\sigma(t))|^{\alpha_m}u_2(\sigma(t)),$$

$$\vdots$$

$$-\left(r_m(t)\left|u_m^{\Delta}(t)\right|^{p_m-2}u_m^{\Delta}(t)\right)^{\Delta}=f_m(t)|u_1(\sigma(t))|^{\alpha_1}|u_2(\sigma(t))|^{\alpha_2}\cdots|u_m(\sigma(t))|^{\alpha_m-2}u_m(\sigma(t)). \tag{1.1}$$

It is obvious that system (1.1) covers the continuous quasilinear system and the corresponding discrete case, respectively, when $\mathbb{T}=\mathbb{R}$ and $\mathbb{T}=\mathbb{Z}$; that is,

$$\begin{aligned}
&-\left(r_1(t)\left|u_1'(t)\right|^{p_1-2}u_1'(t)\right)'=f_1(t)|u_1(t)|^{\alpha_1-2}|u_2(t)|^{\alpha_2}\cdots|u_m(t)|^{\alpha_m}u_1(t),\\
&-\left(r_2(t)\left|u_2'(t)\right|^{p_2-2}u_2'(t)\right)'=f_2(t)|u_1(t)|^{\alpha_1}|u_2(t)|^{\alpha_2-2}\cdots|u_m(t)|^{\alpha_m}u_2(t),\\
&\vdots\\
&-\left(r_m(t)\left|u_m'(t)\right|^{p_m-2}u_m'(t)\right)'=f_m(t)|u_1(t)|^{\alpha_1}|u_2(t)|^{\alpha_2}\cdots|u_n(t)|^{\alpha_m-2}u_m(t),\\
&-\Delta\left(r_1(n)|\Delta u_1(n)|^{p_1-2}\Delta u_1(n)\right)=f_1(n)|u_1(n+1)|^{\alpha_1-2}|u_2(n+1)|^{\alpha_2}\cdots|u_m(n+1)|^{\alpha_m}u_1(n+1),\\
&-\Delta\left(r_2(n)|\Delta u_2(n)|^{p_2-2}\Delta u_2(n)\right)=f_2(n)|u_1(n+1)|^{\alpha_1}|u_2(n+1)|^{\alpha_2-2}\cdots|u_m(n+1)|^{\alpha_m}u_2(n+1),\\
&\vdots\\
&-\Delta\left(r_m(n)|\Delta u_m(n)|^{p_m-2}\Delta u_m(n)\right)=f_m(n)|u_1(n+1)|^{\alpha_1}|u_2(n+1)|^{\alpha_2}\cdots|u_m(n+1)|^{\alpha_m-2}u_m(n+1).
\end{aligned} \tag{1.2}$$

In 1907, Lyapunov [7] established the first so-called Lyapunov inequality

$$(b-a)\int_a^b\left|q(t)\right|dt>4, \tag{1.3}$$

if the Hill equation

$$x''(t)+q(t)x(t)=0 \tag{1.4}$$

has a real solution $x(t)$ such that

$$x(a)=x(b)=0,\quad x(t)\not\equiv 0,\ t\in[a,b]. \tag{1.5}$$

Moreover the constant 4 in (1.3) cannot be replaced by a larger number, where $q(t)$ is a piecewise continuous and nonnegative function defined on $\mathbb{R}$.

It is a classical topic for us to study Lyapunov-type inequalities which have proved to be very useful in oscillation theory, disconjugacy, eigenvalue problems, and numerous other applications in the theory of differential and difference equations. So far, there are many literatures which improved and extended the classical Lyapunov including continuous and discrete cases. For example, inequality (1.3) has been generalized to discrete linear Hamiltonian system by Zhang and Tang [8], to second-order nonlinear differential equations by Eliason [9] and by Pachpatte [10], to second-order nonlinear difference system by He and Zhang [11], to the second-order delay differential equations by Eliason [12] and by Dahiya and Singh [13], to higher-order differential equations by Pachpatte [14], Yang [15, 16], Yang and Lo [17] and Cakmak and Tiryaki [18, 19]. Lyapunov-type inequalities for the Emden-Fowler-type equations can be found in Pachpatte [10], and for the half-linear equations can be found in Lee et al. [20] and Pinasco [21]. Recently, there has been much attention paid to Lyapunov-type inequalities for dynamic systems on time scales and some authors including Agarwal et al. [22], Jiang and Zhou [23], He [24], He et al. [25], Saker [26], Bohner et al. [27], and Ünal and Cakmak [28] have contributed the above results.

In this paper, we use the methods in [29] to establish some Lyapunov-type inequalities for system (1.1) on an arbitrary time scale $\mathbb{T}$.

2. Preliminaries about the Time Scales Calculus

We introduce some basic notions connected with time scales.

Definition 2.1 (see [6]). Let $t \in \mathbb{T}$. We define *the forward jump operator* $\sigma : \mathbb{T} \to \mathbb{T}$ by

$$\sigma(t) := \inf\{s \in \mathbb{T} : s > t\}, \quad \forall t \in \mathbb{T}, \tag{2.1}$$

while *the backward jump operator* $\rho : \mathbb{T} \to \mathbb{T}$ by

$$\rho(t) := \sup\{s \in \mathbb{T} : s < t\}, \quad \forall t \in \mathbb{T}. \tag{2.2}$$

In this definition, we put $\inf \emptyset = \sup \mathbb{T}$ (i.e., $\sigma(M) = M$ if $\mathbb{T}$ has a maximum M) and $\sup \emptyset = \inf \mathbb{T}$ (i.e., $\rho(m) = m$ if $\mathbb{T}$ has a minimum m), where $\emptyset$ denotes the empty set. If $\sigma(t) > t$, we say that t is *right-scattered*, while if $\rho(t) < t$, we say that t is *left-scattered*. Also, if $t < sup\mathbb{T}$ and $\sigma(t) = t$, then t is called *right-dense*, and if $t > inf\mathbb{T}$ and $\rho(t) = t$, then t is called *left-dense*. Points that are right-scattered and left-scattered at the same time are called *isolated*. Points that are right-dense and left-dense at the same time are called *dense*. If $\mathbb{T}$ has a left-scattered maximum M, then we define $\mathbb{T}^k = \mathbb{T} - \{M\}$ otherwise; $\mathbb{T}^k = \mathbb{T}$. The *graininess function* $u : \mathbb{T} \to [0, \infty)$ is defined by

$$\mu(t) := \sigma(t) - t, \quad \forall t \in \mathbb{T}. \tag{2.3}$$

We consider a function $f : \mathbb{T} \to \mathbb{R}$ and define so-called delta (or Hilger) derivative of f at a point $t \in \mathbb{T}^k$.

Definition 2.2 (see [6]). Assume that $f : \mathbb{T} \to \mathbb{R}$ is a function, and let $t \in \mathbb{T}^k$. Then, we define $f^{\Delta}(t)$ to be the number (provided it exists) with the property that given any $\varepsilon > 0$, there is a neighborhood U of t (i.e., $U = (t - \delta, t + \delta) \cap \mathbb{T}$ for some $\delta > 0$) such that

$$\left| f(\sigma(t)) - f(s) - f^{\Delta}(t)(\sigma(t) - s) \right| \leq \varepsilon |\sigma(t) - s|, \quad \forall s \in U. \tag{2.4}$$

We call $f^{\Delta}(t)$ the delta (or Hilger) derivative of f at t.

Lemma 2.3 (see [6]). *Assume that $f, g : \mathbb{T} \to \mathbb{R}$ are differential at $t \in \mathbb{T}^k$, then,*

(i) *for any constant a and b, the sum $af + bg : \mathbb{T} \to \mathbb{R}$ is differential at t with*

$$(af + bg)^{\Delta}(t) = af^{\Delta}(t) + bg^{\Delta}(t), \tag{2.5}$$

(ii) *if $f^{\Delta}(t)$ exists, then f is continuous at t,*

(iii) *if $f^{\Delta}(t)$ exists, then $f(\sigma(t)) = f(t) + \mu(t) f^{\Delta}(t)$,*

(iv) *the product $fg : \mathbb{T} \to \mathbb{R}$ is differential at t with*

$$(fg)^{\Delta}(t) = f^{\Delta}(t)g(t) + f(\sigma(t))g^{\Delta}(t) = f(t)g^{\Delta}(t) + f^{\Delta}(t)g(\sigma(t)), \tag{2.6}$$

(v) *if $g(t)g(\sigma(t)) \neq 0$, then f/g is differential at t and*

$$\left(\frac{f}{g}\right)^{\Delta}(t) = \frac{f^{\Delta}(t)g(t) - f(t)g^{\Delta}(t)}{g(t)g(\sigma(t))}. \tag{2.7}$$

Definition 2.4 (see [6]). A function $f : \mathbb{T} \to \mathbb{R}$ is called rd-continuous, provided it is continuous at right-dense points in $\mathbb{T}$ and left-sided limits exist (finite) at left-dense points in $\mathbb{T}$ and denotes by $C_{\mathrm{rd}} = C_{\mathrm{rd}}(\mathbb{T}) = C_{\mathrm{rd}}(\mathbb{T}, \mathbb{R})$.

Definition 2.5 (see [6]). A function $F : \mathbb{T} \to \mathbb{R}$ is called an antiderivative of $f : \mathbb{T} \to \mathbb{R}$, provided $F^{\Delta}(t) = f(t)$ holds for all $t \in \mathbb{T}^k$. We define the Cauchy integral by

$$\int_{\tau}^{s} f(t)\Delta t = F(s) - F(\tau), \quad \forall s, \tau \in \mathbb{T}. \tag{2.8}$$

The following lemma gives several elementary properties of the delta integral.

Lemma 2.6 (see [6]). *If $a, b, c \in \mathbb{T}, k \in \mathbb{R}$ and $f, g \in C_{\mathrm{rd}}$, then*

(i) $\int_a^b [f(t) + g(t)]\Delta t = \int_a^b f(t)\Delta t + \int_a^b g(t)\Delta t$,

(ii) $\int_a^b (kf)(t)\Delta t = k\int_a^b f(t)\Delta t$,

(iii) $\int_a^b f(t)\Delta t = \int_a^c f(t)\Delta t + \int_c^b f(t)\Delta t$,

(iv) $\int_a^b f(\sigma(t))g^\Delta(t)\Delta t = (fg)(b) - (fg)(a) - \int_a^b f^\Delta(t)g(t)\Delta t$,

(v) $\int_t^{\sigma(t)} f(s)\Delta s = \mu(t)f(t)$ for $t \in \mathbb{T}^k$,

(vi) *if* $|f(t)| \le g(t)$ on $[a,b)$, *then*

$$\left|\int_a^b f(t)\Delta t\right| \le \int_a^b g(t)\Delta t. \tag{2.9}$$

The notation $[a,b]$, $[a,b)$ and $[a,+\infty)$ will denote time scales intervals. For example, $[a,b) = \{t \in \mathbb{T} \mid a \le t < b\}$. To prove our results, we present the following lemma.

Lemma 2.7 (see [6]). *Let $a,b \in \mathbb{T}$ and $1 < p,q < \infty$ with $1/p + 1/q = 1$. For $f,g \in C_{\mathrm{rd}}$, one has*

$$\int_a^b |f(t)g(t)|\Delta t \le \left\{\int_a^b |f(t)|^p \Delta t\right\}^{1/p} \left\{\int_a^b |g(t)|^q \Delta t\right\}^{1/q}. \tag{2.10}$$

Lemma 2.8 (see [6]). *Let $a,b \in \mathbb{T}$ and $1 < r_k < \infty$ with $\sum_{k=1}^m (1/r_k) = 1$ for $k = 1,2,\ldots,m$. For $f_k \in C_{\mathrm{rd}}$, $k = 1,2,\ldots,m$, one has*

$$\int_a^b \prod_{k=1}^m |f_k(t)|\Delta t \le \prod_{k=1}^m \left\{\int_a^b |f_k(t)|^{r_k} \Delta t\right\}^{1/r_k}. \tag{2.11}$$

3. Lyapunov-Type Inequalities

Denote

$$\zeta_i(t) := \left(\int_a^{\sigma(t)} [r_i(\tau)]^{1/(1-p_i)} \Delta\tau\right)^{p_i-1}, \quad i = 1,2,\ldots,m, \tag{3.1}$$

$$\eta_i(t) := \left(\int_{\sigma(t)}^b [r_i(\tau)]^{1/(1-p_i)} \Delta\tau\right)^{p_i-1}, \quad i = 1,2,\ldots,m. \tag{3.2}$$

First, we give the following hypothesis.

(H1) $r_i(t)$ and $f_i(t)$ are rd-continuous real functions and $r_i(t) > 0$ for $i = 1,2,\ldots,m$ and $t \in \mathbb{T}$. Furthermore, $1 < p_i < \infty$ and $\alpha_i > 0$ satisfy $\sum_{i=1}^m (\alpha_i/p_i) = 1$ for $i = 1,2,\ldots,m$.

Theorem 3.1. *Let $a,b \in \mathbb{T}^k$ with $\sigma(a) \le b$. Suppose that hypothesis (H1) is satisfied. If* (1.1) *has a real solution $(u_1(t),u_2(t),\ldots,u_m(t))$ satisfying the boundary value conditions*

$$u_i(a) = u_i(b) = 0, \quad u_i(t) \not\equiv 0, \ \forall t \in [a,b], \ i = 1,2,\ldots,m, \tag{3.3}$$

then one has

$$\prod_{i=1}^{m}\prod_{j=1}^{m}\left(\int_a^b \frac{\zeta_i(t)\eta_i(t)}{\zeta_i(t)+\eta_i(t)} f_j^+(t)\Delta t\right)^{\alpha_i\alpha_j/p_ip_j} \geq 1, \tag{3.4}$$

where and in what follows $f_i^+(t) = \max\{f_i(t), 0\}$ *for* $i = 1, 2, \ldots, m$.

Proof. By (1.1) and Lemma 2.3(iv), we obtain

$$-\left(r_i(t)\left|u_i^{\Delta}(t)\right|^{p_i-2}u_i^{\Delta}(t)u_i(t)\right)^{\Delta} + r_i(t)\left|u_i^{\Delta}(t)\right|^{p_i} = f_i(t)\prod_{k=1}^{m}|u_k(\sigma(t))|^{\alpha_k}, \tag{3.5}$$

where $i = 1, 2, \ldots, m$. From Definition 2.5, integrating (3.5) from a to b, together with (3.3), we get

$$\int_a^b r_i(t)\left|u_i^{\Delta}(t)\right|^{p_i}\Delta t = \int_a^b f_i(t)\prod_{k=1}^{m}|u_k(\sigma(t))|^{\alpha_k}\Delta t, \quad i = 1, 2, \ldots, m. \tag{3.6}$$

It follows from (3.1), (3.3), and Lemma 2.7 that

$$\begin{aligned} |u_i(\sigma(t))|^{p_i} &= \left|\int_a^{\sigma(t)} u_i^{\Delta}(\tau)\Delta\tau\right|^{p_i} \\ &\leq \left(\int_a^{\sigma(t)} [r_i(\tau)]^{1/(1-p_i)}\Delta\tau\right)^{p_i-1}\int_a^{\sigma(t)} r_i(\tau)\left|u_i^{\Delta}(\tau)\right|^{p_i}\Delta\tau \\ &= \zeta_i(t)\int_a^{\sigma(t)} r_i(\tau)\left|u_i^{\Delta}(\tau)\right|^{p_i}\Delta\tau, \quad a \leq t \leq b,\ i = 1, 2, \ldots, m. \end{aligned} \tag{3.7}$$

Similarly, it follows from (3.2), (3.3), and Lemma 2.7 that

$$\begin{aligned} |u_i(\sigma(t))|^{p_i} &= \left|\int_{\sigma(t)}^{b} u_i^{\Delta}(\tau)\Delta\tau\right|^{p_i} \\ &\leq \left(\int_{\sigma(t)}^{b} [r_i(\tau)]^{1/(1-p_i)}\Delta\tau\right)^{p_i-1}\int_{\sigma(t)}^{b} r_i(\tau)\left|u_i^{\Delta}(\tau)\right|^{p_i}\Delta\tau \\ &= \eta_i(t)\int_{\sigma(t)}^{b} r_i(\tau)\left|u_i^{\Delta}(\tau)\right|^{p_i}\Delta\tau, \quad a \leq t \leq b,\ i = 1, 2, \ldots, m. \end{aligned} \tag{3.8}$$

From (3.7) and (3.8), we have

$$|u_i(\sigma(t))|^{p_i} \leq \frac{\zeta_i(t)\eta_i(t)}{\zeta_i(t)+\eta_i(t)}\int_a^b r_i(\tau)\left|u_i^{\Delta}(\tau)\right|^{p_i}\Delta\tau, \quad a \leq t \leq b,\ i = 1, 2, \ldots, m. \tag{3.9}$$

So, from (3.3), (3.6), (3.9), (H1), and Lemma 2.8, we have

$$\begin{aligned}\int_a^b f_i^+(t)|u_i(\sigma(t))|^{p_i}\Delta t &\le \int_a^b \frac{\zeta_i(t)\eta_i(t)}{\zeta_i(t)+\eta_i(t)} f_i^+(t)\Delta t \int_a^b r_i(t)\left|u_i^{\Delta}(t)\right|^{p_i}\Delta t \\ &= M_{ij}\int_a^b f_i(t)\prod_{k=1}^{m}|u_k(\sigma(t))|^{\alpha_k}\Delta t \\ &\le M_{ij}\int_a^b f_i^+(t)\prod_{k=1}^{m}|u_k(\sigma(t))|^{\alpha_k}\Delta t \\ &\le M_{ij}\prod_{k=1}^{m}\left(\int_a^b f_i^+(t)|u_k(\sigma(t))|^{p_k}\Delta t\right)^{\alpha_k/p_k},\end{aligned} \tag{3.10}$$

where

$$M_{ij} = \int_a^b \frac{\zeta_i(t)\eta_i(t)}{\zeta_i(t)+\eta_i(t)} f_j^+(t)\Delta t, \quad i,j = 1,2,\ldots,m. \tag{3.11}$$

Next, we prove that

$$\int_a^b f_i^+(t)|u_k(\sigma(t))|^{p_k}\Delta t > 0. \tag{3.12}$$

If (3.12) is not true, there exist $i_0, k_0 \in \{1,2,\ldots,m\}$ such that

$$\int_a^b f_{i_0}^+(t)|u_{k_0}(\sigma(t))|^{p_{k_0}}\Delta t = 0. \tag{3.13}$$

From (3.6), (3.13), and Lemma 2.8, we have

$$\begin{aligned}0 \le \int_a^b r_{i_0}(t)\left|u_{i_0}^{\Delta}(t)\right|^{p_{i_0}}\Delta t &= \int_a^b f_{i_0}(t)\prod_{k=1}^{m}|u_k(\sigma(t))|^{\alpha_k}\Delta t \\ \le \prod_{k=1}^{m}\left(\int_a^b f_{i_0}^+(t)|u_k(\sigma(t))|^{p_k}\Delta t\right)^{\alpha_k/p_k} &= 0.\end{aligned} \tag{3.14}$$

It follows from the fact that $r_{i_0}(t) > 0$ that

$$u_{i_0}^{\Delta}(t) \equiv 0, \quad a \le t \le b. \tag{3.15}$$

Combining (3.7) with (3.15), we obtain that $u_{i_0}(t) \equiv 0$ for $a \le t \le b$, which contradicts (3.3). Therefore, (3.12) holds. From (3.10), (3.12), and (H1), we have

$$\prod_{i=1}^{m}\prod_{j=1}^{m} M_{ij}^{\alpha_i\alpha_j/p_ip_j} \ge 1. \tag{3.16}$$

It follows from (3.11) and (3.16) that (3.4) holds. □

Corollary 3.2. *Let $a, b \in \mathbb{T}^k$ with $\sigma(a) \le b$. Suppose that hypothesis (H1) is satisfied. If* (1.1) *has a real solution* $(u_1(t), u_2(t), \dots, u_m(t))$ *satisfying the boundary value conditions* (3.3)*, then one has*

$$\prod_{i=1}^{m}\prod_{j=1}^{m}\left(\int_a^b f_j^+(t)\left[\zeta_i(t)\eta_i(t)\right]^{1/2}\Delta t\right)^{\alpha_i\alpha_j/p_ip_j} \ge 2. \tag{3.17}$$

Proof. Since

$$\zeta_i(t) + \eta_i(t) \ge 2\left[\zeta_i(t)\eta_i(t)\right]^{1/2}, \quad i = 1, 2, \dots, m, \tag{3.18}$$

it follows from (3.4) and (H1) that (3.17) holds. □

Corollary 3.3. *Let $a, b \in \mathbb{T}^k$ with $\sigma(a) \le b$. Suppose that hypothesis (H1) is satisfied. If* (1.1) *has a real solution* $(u_1(t), u_2(t), \dots, u_m(t))$ *satisfying the boundary value conditions* (3.3)*, then one has*

$$\prod_{i=1}^{m}\left(\int_a^b \left[r_i(t)\right]^{1/(1-p_i)}\Delta t\right)^{\alpha_i(p_i-1)/p_i}\prod_{j=1}^{m}\left(\int_a^b f_i^+(t)\Delta t\right)^{\alpha_j/p_j} \ge 2^{\mathcal{A}}, \tag{3.19}$$

where $\mathcal{A} = \sum_{i=1}^{m}\alpha_i$.

Proof. Since

$$\begin{aligned}\left[\zeta_i(t)\eta_i(t)\right]^{1/2} &= \left(\int_a^{\sigma(t)}\left[r_i(\tau)\right]^{1/(1-p_i)}\Delta\tau\int_{\sigma(t)}^b\left[r_i(\tau)\right]^{1/(1-p_i)}\Delta\tau\right)^{(p_i-1)/2} \\ &\le \frac{1}{2^{p_i-1}}\left(\int_a^b\left[r_i(\tau)\right]^{1/(1-p_i)}\Delta\tau\right)^{p_i-1}, \quad i = 1, 2, \dots, m,\end{aligned} \tag{3.20}$$

it follows from (3.20) and (H1) that (3.19) holds. □

When $m = 1$, $p_1 = \alpha_1 = \gamma > 1$, $r_1(t) = r(t) > 0$, $u_1(\sigma(t)) = u(\sigma(t))$, $u_1(t) = u(t)$, and $f_1(t) = \varrho(t)$, system (1.1) reduces to a second-order half-linear dynamic equation, and denote by

$$\left(r(t)\left|u^{\Delta}(t)\right|^{\gamma-2}u^{\Delta}(t)\right)^{\Delta} + \varrho(t)|u(\sigma(t))|^{\gamma-2}u(\sigma(t)) = 0. \tag{3.21}$$

We can easily derive the following corollary for (3.21).

Corollary 3.4. *Let $a, b \in \mathbb{T}^k$ with $\sigma(a) \leq b$. If* (3.21) *has a solution $u(t)$ satisfying*

$$u(a) = u(b) = 0, \quad u(t) \not\equiv 0, \ \forall t \in [a, b], \tag{3.22}$$

then

$$\int_a^b \frac{\left(\int_a^{\sigma(t)} [r(\tau)]^{1/(1-\gamma)} \Delta\tau\right)^{\gamma-1} \left(\int_{\sigma(t)}^b [r(\tau)]^{1/(1-\gamma)} \Delta\tau\right)^{\gamma-1}}{\left(\int_a^{\sigma(t)} [r(\tau)]^{1/(1-\gamma)} \Delta\tau\right)^{\gamma-1} + \left(\int_{\sigma(t)}^b [r(\tau)]^{1/(1-\gamma)} \Delta\tau\right)^{\gamma-1}} \varrho^+(t) \Delta t \geq 1. \tag{3.23}$$

Especially, while $m = 1$, $p_1 = \alpha_1 = 2$, $r_1(t) = 1$, $u_1(\sigma(t)) = u(\sigma(t))$, $u_1(t) = u(t)$, and $f_1(t) = \varrho(t)$, system (1.1) reduces to a second-order linear dynamic equation and denote by

$$\left(u^\Delta(t)\right)^\Delta + \varrho(t) u(\sigma(t)) = 0. \tag{3.24}$$

Obviously, (3.24) is a special case of (3.21). One can also obtain a corollary immediately.

Corollary 3.5. *Let $a, b \in \mathbb{T}^k$ with $\sigma(a) \leq b$. If* (3.24) *has a solution $u(t)$ satisfying*

$$u(a) = u(b) = 0, \quad u(t) \not\equiv 0, \ \forall t \in [a, b], \tag{3.25}$$

then

$$(b - a) \int_a^b \varrho^+(t) \Delta t \geq 4. \tag{3.26}$$

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References

[1] S. Hilger, *Einßmakettenkalküul mit anwendung auf zentrumsmannigfaltigkeiten*, Ph.D. thesis, Universitäat Wüurzburg, 1988.
[2] S. Hilger, "Analysis on measure chains-a unified approach to continuous and discrete calculus," *Results in Mathematics*, vol. 18, no. 1-2, pp. 18–56, 1990.
[3] S. Hilger, "Differential and difference calculus-unified!," vol. 30, no. 5, pp. 2683–2694.
[4] B. Kaymakcalan, V. Lakshmikantham, and S. Sivasundaram, *Dynamic Systems on Measure Chains*, vol. 370 of *Mathematics and its Applications*, Kluwer Academic, Dordrecht, The Netherlands, 1996.
[5] M. Bohner and A. Peterson, *Advances in Dynamic Equations on Time Scales*, Birkhäuser Boston, Boston, Mass, USA, 2003.

[6] C. D. Ahlbrandt and A. C. Peterson, *Discrete Hamiltonian systems: Difference Equations, Continued Fractions, and Riccati Equations*, vol. 16 of *Kluwer Texts in the Mathematical Sciences*, Kluwer Academic, Boston, Mass, USA, 1996.

[7] A. M. Lyapunov, “Problème général de la stabilité du mouvement,” *Annde la Faculté*, vol. 2, no. 9, pp. 203–474, 1907.

[8] Q. Zhang and X. H. Tang, “Lyapunov inequalities and stability for discrete linear Hamiltonian system,” *Applied Mathematics and Computation*, vol. 218, pp. 574–582, 2011.

[9] S. B. Eliason, “A Lyapunov inequality for a certain second order nonlinear differential equation,” *Journal of the London Mathematical Society*, vol. 2, pp. 461–466, 1970.

[10] B. G. Pachpatte, “Inequalities related to the zeros of solutions of certain second order differential equations,” *Facta Universitatis. Series: Mathematics and Informatics*, no. 16, pp. 35–44, 2001.

[11] X. He and Q. Zhang, “A discrete analogue of Lyapunov-type inequalities for nonlinear difference systems,” *Computers & Mathematics with Applications*, vol. 62, pp. 677–684, 2011.

[12] S. B. Eliason, “Lyapunov type inequalities for certain second order functional differential equations,” *SIAM Journal on Applied Mathematics*, vol. 27, pp. 180–199, 1974.

[13] R. S. Dahiya and B. Singh, “A Lyapunov inequality and nonoscillation theorem for a second order non-linear differential-difference equation,” vol. 7, pp. 163–170, 1973.

[14] B. G. Pachpatte, “On Lyapunov-type inequalities for certain higher order differential equations,” *Journal of Mathematical Analysis and Applications*, vol. 195, no. 2, pp. 527–536, 1995.

[15] X. Yang, “On Liapunov-type inequality for certain higher-order differential equations,” *Applied Mathematics and Computation*, vol. 134, no. 2-3, pp. 307–317, 2003.

[16] X. Yang, “On inequalities of Lyapunov type,” *Applied Mathematics and Computation*, vol. 134, no. 2-3, pp. 293–300, 2003.

[17] X. Yang and K. Lo, “Lyapunov-type inequality for a class of even-order differential equations,” *Applied Mathematics and Computation*, vol. 215, no. 11, pp. 3884–3890, 2010.

[18] D. Cakmak and A. Tiryaki, “On Lyapunov-type inequality for quasilinear systems,” *Applied Mathematics and Computation*, vol. 216, no. 12, pp. 3584–3591, 2010.

[19] D. Cakmak and A. Tiryaki, “Lyapunov-type inequality for a class of Dirichlet quasilinear systems involving the $p_1, p_2, \ldots, p_n$-Laplacian,” *Journal of Mathematical Analysis and Applications*, vol. 369, no. 1, pp. 76–81, 2010.

[20] C. Lee, C. Yeh, C. Hong, and R. P. Agarwal, “Lyapunov and Wirtinger inequalities,” *Applied Mathematics Letters*, vol. 17, no. 7, pp. 847–853, 2004.

[21] J. P. Pinasco, “Lower bounds for eigenvalues of the one-dimensional p-Laplacian,” *Abstract and Applied Analysis*, vol. 2004, no. 2, pp. 147–153, 2004.

[22] R. P. Agarwal, M. Bohner, and P. Rehak, “Half-linear dynamic equations,” *Nonlinear Analysis and Applications*, vol. 1, pp. 1–56, 2003.

[23] L. Q. Jiang and Z. Zhou, “Lyapunov inequality for linear Hamiltonian systems on time scales,” *Journal of Mathematical Analysis and Applications*, vol. 310, no. 2, pp. 579–593, 2005.

[24] Z. He, “Existence of two solutions of m-point boundary value problem for second order dynamic equations on time scales,” *Journal of Mathematical Analysis and Applications*, vol. 296, no. 1, pp. 97–109, 2004.

[25] X. He, Q. Zhang, and X. H. Tang, “On inequalities of Lyapunov for linear Hamiltonian systems on time scales,” *Journal of Mathematical Analysis and Applications*, vol. 381, pp. 695–705, 2011.

[26] S. H. Saker, “Oscillation of nonlinear dynamic equations on time scales,” *Applied Mathematics and Computation*, vol. 148, no. 1, pp. 81–91, 2004.

[27] M. Bohner, S. Clark, and J. Ridenhour, “Lyapunov inequalities for time scales,” *Journal of Inequalities and Applications*, vol. 7, no. 1, pp. 61–77, 2002.

[28] M. Ünal and D. Cakmak, “Lyapunov-type inequalities for certain nonlinear systems on time scales,” *Turkish Journal of Mathematics*, vol. 32, no. 3, pp. 255–275, 2008.

[29] X. H. Tang and X. He, “Lower bounds for generalized eigenvalues of the quasilinear systems,” *Journal of Mathematical Analysis and Applications*, vol. 385, pp. 72–85, 2012.