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Research Article

Pseudo Almost-Periodic Solution of Shunting Inhibitory Cellular Neural Networks with Delay

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Shunting inhibitory cellular neural networks are studied. Some sufficient criteria are obtained for the existence and uniqueness of pseudo almost-periodic solution of this system. Our results improve and generalize those of the previous studies. This is the first paper considering the pseudo almost-periodic SICNNs. Furthermore, several methods are applied to establish sufficient criteria for the globally exponential stability of this system. The approaches are based on constructing suitable Lyapunov functionals and the well-known Banach contraction mapping principle.

1. Introduction

It is well known that the cellular neural networks (CNNs) are widely applied in signal processing, image processing, pattern recognition, and so on. The theoretical and applied studies of CNNs have been a new focus of studies worldwide (see [1–12]). Bouzerdoum and Pinter in [1] have introduced a new class of CNNs, namely, the shunting inhibitory CNNs (SICNNs). Shunting neural networks have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. Recently, Chen and Cao [9] have studied the existence of almost-periodic solutions of the following system of SICNNs:

$$\dot{x}_{ij}(t) = -a_{ij}x_{ij}(t) - \sum_{C^{kl}\in N_r(i,j)} c_{ij}^{kl} f\left(x_{ij}(t-\tau)\right)x_{ij}(t) + L_{ij}(t), \tag{1.1}$$

where C_{ij} denotes the cell at the (i,j) position of the lattice, the r-neighborhood $N_r(i,j)$ of C_{ij} is

$$N_r(i,j) = \left\{C_{ij}^{kl} : \max(|k-l|, |l-j| \le r), 1 \le k \le m, 1 \le l \le n\right\}, \tag{1.2}$$

x_{ij} is the activity of the cell C_{ij}, $L_{ij}(t)$ is the external input to C_{ij}, the constant $a_{ij} > 0$ represents the passive decay rate of the cell activity, $C_{ij}^{kl} \ge 0$ is the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij}, and the activation function $f(x_{kl})$ is a positive continuous function representing the output or firing rate of the cell C_{ij}. Since studies on neural dynamic systems not only involve a discussion of stability properties, but also involve many dynamic properties such as periodic oscillatory behavior, almost-periodic oscillatory properties, chaos, and bifurcation. To the best of our knowledge, few authors have studied almost-periodic solutions for SINNs with delays and variable coefficients, and most of them discuss the stability, periodic oscillation in the case of constant coefficients. In their paper, they investigated the existence and stability of periodic solutions of SINNs with delays and variable coefficients. They considered the SICNNs with delays and variable coefficients

$$\begin{aligned}\dot{x}_{ij}(t) = -a_{ij}(t)x_{ij}(t) &- \sum_{B^{kl}\in N_r(i,j)} B_{ij}^{kl}(t)f_{ij}(x_{kl}(t))x_{ij}(t) \\ &- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)g_{ij}(x_{kl}(t-\tau_{kl}))x_{ij}(t) + L_{ij}(t),\end{aligned} \tag{1.3}$$

where, for each $i = 1,2,\ldots,n$, $j = 1,2,\ldots,m$, $a_{ij}(t), B_{ij}^{kl}(t), C_{ij}^{kl}(t)$,and $L_{ij}(t)$ are all continuous $\omega-$ periodic functions and $a_{ij}(t) > 0, B_{ij}^{kl}(t) \ge 0, C_{ij}^{kl}(t) \ge 0$, and τ_{ij} is a positive constant.

In this paper, we consider the following more general SICNNs:

$$\begin{aligned}\dot{x}_{ij}(t) = -a_{ij}(t)x_{ij}(t) &- \sum_{B^{kl}\in N_r(i,j)} B_{ij}^{kl}(t)f_{ij}(x_{kl}(t))x_{ij}(t) \\ &- \sum_{C^{kl}\in N_r(i,j)} C_{ij}^{kl}(t)g_{ij}(x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) + L_{ij}(t).\end{aligned} \tag{1.4}$$

By using the Lyapunov functional and contraction mapping, a set of criteria are established for the globally exponential stability, the existence, and uniqueness of pseudo almost-periodic solution for the SICNNs. This is the first paper considering the pseudo almost-periodic solution of SICNNs. Since the nature is full of all kinds of tiny perturbations, either the periodicity assumption or the almost-periodicity assumption is just approximation of some degree of the natural perturbations. A well-known extension of almost periodicity is the asymptotically almost periodicity, which was introduced by Frechet. In 1992, Zhang [13, 14] introduced a more general extension of the concept of asymptotically almost periodicity, the so-called pseudo almost periodicity, which has been widely applied in the theory of ODEs and PDEs. However, it is rarely applied in the theory of neural networks or mathematical biology. This paper is expected to establish criteria that provide much flexibility in the designing and training of neural networks and to shed some new light on the application of pseudo almost periodicity in neural networks, population dynamics, and the theory of differential equations.

Throughout this paper, we will use the notations $g^M = \sup_{t\in R} g(t), g^L = \sup_{t\in R} g(t)$, where $g(t)$ is a bounded continuous function on R.
In this paper, we always use $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n;$ unless otherwise stated.

In this paper, we always consider system (1.4) together with the following assumptions.

(A_1) $a_{ij}(t)$ are almost periodic on R with $a_{ij}(t) > 0$, and $L_{ij}(t)$ and $\tau_{ij}(t)$ are pseudo almost periodic on R with $L_{ij} > 0$.

(A_2) $\tau_{ij}(t)$ is bounded, continuous and differentiable with $0 \le \tau_{ij} \le \tau, 1 - \tau > 0$ for $t \in R$, where τ is constant.

(A_3) $f_{ij}, g_{ij} \in C(R, R)$ are bounded, and continuous, and there exist positive numbers μ_{ij}, ν_{ij} such that $|f_{ij}(x) - f_{ij}(y)| \le \mu_{ij}|x - y|, |g_{ij}(x) - g_{ij}(y)| \le \nu_{ij}|x - y|$, for all $x, y \in R$.

2. Preliminaries and Basic Results of Pseudo Almost-Periodic Function

In this section, we explore the existence of pseudo almost-periodic solution of (1.4). First, we would like to recall some basic notations and results of almost periodicity and pseudo almost periodicity [15, 16] which will come into play later on.

Let $\Omega \subset C^n$ be close and let $\mathcal{L}(R)$ (resp., $\mathcal{L}(R \times \Omega)$) denote the C^*-algebra of bounded continuous complex-valued functions on R (respectively, $R \times \Omega$) with supremum norm $|\cdot|_\infty$ denoting the Euclidean norm in C^n; that is, $|x|_\infty = \max_{1\le i\le n}|x_i|$, $x \in C^n$.

Definition 2.1. A function $g \in \mathcal{L}(R)$ is called almost periodic if, for each $\epsilon > 0$, there exists an $l_\epsilon > 0$ such that every interval of length l_ϵ contains a number τ with the property that $|g(t + \tau) - g(t)| < \epsilon, t \in R$.

Definition 2.2. A function $g \in \mathcal{L}(R \times \Omega)$ is called almost periodic in $t \in R$, uniformly in $Z \in \Omega$, if, for each $\epsilon > 0$ and any compact set M of Ω, there exists an $l_\epsilon > 0$ such that every interval of length l_ϵ contains a number τ with the property that $|g(t + \tau, z) - g(t, z)| < \epsilon, t \in R, Z \in \Omega$. The number τ is called an ϵ-translation number of g. Denote by $\mathcal{AP}(R)(\mathcal{AP}(R \times \Omega))$ the set of all such function.

Set

$$\begin{aligned}\mathcal{PAP}_0(R) &= \left\{\varphi \in \mathcal{L}(R) : \lim_{t\to\infty}\frac{1}{2t}\int_{-t}^{t}|\varphi(s)|ds = 0\right\},\\ \mathcal{PAP}_0(R\times\Omega) &= \left\{\varphi \in \mathcal{L}(R\times\Omega) : \lim_{t\to\infty}\frac{1}{2t}\int_{-t}^{t}|\varphi(s,Z)|ds = 0, \text{uniformly in } Z\in\Omega\right\}.\end{aligned} \tag{2.1}$$

Definition 2.3. A function $f \in \mathcal{L}(R)(\mathcal{L}(R \times \Omega))$ is called pseudo almost periodic (pseudo almost periodic in $t \in R$, uniformly in $Z \in \Omega$), if $f = g+\varphi$, where $g \in \mathcal{PA}(R)(\mathcal{PA}(R\times\Omega))$ and $\varphi \in \mathcal{PAP}_0(R)(\mathcal{PAP}_0(R\times\Omega))$. The function g and φ are called the almost periodic component and the erigodic perturbation, respectively, of the function f. Denote by $\mathcal{PAP}(R)(\mathcal{PAP}(R \times \Omega))$ the set of all such functions f.

Define $\|x\|_\infty = \sup_{t\in R}|x(t)|_\infty, x \in \mathcal{PAP}$. It is trivial to show that $\mathcal{PAP}$ is a Banach space with $\|\cdot\|_\infty$.

Let $A(t) = (a_{ij}(t))$ be a complex $n \times n$ matrix-valued function with elements (entries) which are continuous on R. We consider the homogeneous linear ODE and nonhomogeneous linear ODE as follows:

$$\frac{dx}{dt} = A(t)x, \tag{2.2}$$

$$\frac{dx}{dt} = A(t)x + f(t), \tag{2.3}$$

where x denotes an n-column vector.

Definition 2.4 (see [15, 16]). The homogeneous linear ODE (2.2) is said to admit an exponential dichotomy if there exist a linear projection p (i.e., $p^2 = P$) on C^n and positive constants k, α, β such that

$$\begin{aligned} \left\|X(t)PX^{-1}(s)\right\| &\le ke^{-\alpha(t-s)}, \quad t \ge s, \\ \left\|X(t)(I-P)X^{-1}(s)\right\| &\le ke^{-\beta(s-t)}, \quad t \le s, \end{aligned} \tag{2.4}$$

where $X(t)$ is a fundamental matrix of (2.2) with $X(0) = E$; E is the $n \times n$ identity matrix.

Definition 2.5 (see [15, 16]). The matrix $A(t)$ is said to be row dominant if there exists a number $\delta > 0$ such that $|\operatorname{Re} a_{ij}(t)| \ge \sum_{j=1,j\ne i}^{n} |a_{ij}(t)| + \delta$ for all $t \in (-\infty, \infty)$ and $i = 1, 2, \dots, n$.

Lemma 2.6 (see [15, 16]). *If $A(t)$ is a bounded, continuous, and row-dominant $n\times n$ matrix function on R, and there exists $k \le n$ such that $\operatorname{Re} a_{ij} < 0$ $(i = 1, 2, \dots, k)$. Then (2.2) has a fundamental matrix solution $X(t)$ satisfying*

$$\begin{aligned} \left\|X(t)PX^{-1}(s)\right\| &\le ke^{-\delta(t-s)}, \quad t \ge s, \\ \left\|X(t)(I-P)X^{-1}(s)\right\| &\le ke^{-\delta(s-t)}, \quad t \le s, \end{aligned} \tag{2.5}$$

where K is a positive constant, and $P = \operatorname{diag}(E_k, 0)$ with E_k being a $k \times k$ identity matrix.

For $H = (h_1, h_2, \dots, h_n) \in \mathcal{L}(R)^n$, suppose that $H(t) \in \Omega$ for all $t \in R$. Define $H \times l : R \to \Omega \times R$ by $H \times l(t) = (h_1(t), h_2(t), \dots, h_n(t), t)$ $(t \in R)$.

Lemma 2.7 (see [15, 16]). *Assume that the function $f(t,z) \in \mathcal{PAP}(R \times \Omega)$ is continuous in $z \in M$ uniformly in $t \in R$ for all compact subsets $M \subset \Omega$ and $F \in \mathcal{PAP}(R)^n$ such that $F(R) \subset \Omega$. Then $f \circ (F \times l) \in \mathcal{PAP}(R)$.*

It is obvious that if f satisfies a Lipschitz condition; that is, there is an $L > 0$ such that

$$\left|f(z',t) - f(z,t)\right| \leq L\left|z' - z\right| \quad (z', z \in M, t \in R), \tag{2.6}$$

then f is continuous in $z \in M$ uniformly in $t \in R$. Obviously, if $f(t) \in \mathcal{PAP}(R)$ is uniformly continuous in $t \in R$ and $\varphi \in \mathcal{PAP}(R)$ such that $\varphi(R) \subset \operatorname{Im} f$, then $f \circ \varphi \in \mathcal{PAP}(R)$.

Lemma 2.8 (see [15, 16]). *Assume that $A(t)$ is an almost-periodic matrix function and $f(t) \in \mathcal{PAP}(R^n)$. If* (2.2) *satisfies an exponential dichotomy, then* (2.3) *has unique pseudo almost periodic solution $x(t)$ reading*

$$x(t) = \int_{-\infty}^{t} X(t)PX^{-1}(s)f(s)ds - \int_{t}^{\infty} X(t)(E-P)X^{-1}(s)f(s)ds \tag{2.7}$$

and satisfying $\|x\| \leq (K/\alpha + K/\beta)\|f\|$, where $X(t)$ is a fundamental matrix solution of (2.2).

Definition 2.9. System (1.4) is said to be globally exponentially stable (GES), if for any two solutions $x(t)$ and $y(t)$ of (1.4), there exist positive numbers M and ε such that

$$\left|x(t) - y(t)\right|_p \leq Me^{-\varepsilon(t-t_0)}\left\|\varphi - \psi\right\|_p, \quad t > t_0, \tag{2.8}$$

where $x(t) = x(t,\varphi)$ and $y(t) = y(t,\psi)$ denoting the solution of (1.4) through (t_0,φ) and (t_0,ψ) respectively. Here ε is called the Lyapunov exponent of (1.4).

3. Existence and Stability of Pseudo Almost-Periodic Solution

Theorem 3.1. *Assume that (A_1)-(A_3) hold and*

(A_4)

$$r = \sup_{t\in R}\max_{(i,j)}\left\{\int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta}\left(\sum_{B^{hl}\in N_r(i,j)}\mu_{ij}B_{ij}^{hl}(s) + \sum_{C^{hl}\in N_r(i,j)}\nu_{ij}C_{ij}^{hl}(s)\right)ds\right\} < 1. \tag{3.1}$$

Then (1.4) *has a unique pseudo almost-periodic solution, say $x^*(t)$, satisfying $\|x^*\|_\infty \leq L/(1-r)$.*

Proof. For any $\varphi \in \mathcal{PAP}(R)$, consider

$$\begin{aligned} \dot{x}_{ij} = -a_{ij}(t)x_{ij}(t) - \sum_{B^{hl}\in N_r(ij)} B_{ij}^{hl}(t) f_{ij}(\varphi_{hl}(t))\varphi_{ij}(t) \\ - \sum_{C^{hl}\in N_r(ij)} C_{ij}^{hl}(t) g_{ij}(\varphi_{hl}(t-\tau_{hl}(t)))\varphi_{ij}(t) + L_{ij}(t). \end{aligned} \tag{3.2}$$

Since $-a_{ij}(t) < 0$, from Lemmas 2.6, 2.7, and 2.8, it follows that (3.2) has a unique pseudo almost-periodic solution, which is given by

$$\begin{aligned} x_\varphi(t) = \Bigg(\int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta}\Bigg[-\sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(s) f_{ij}(\varphi_{hl}(s))\varphi_{ij}(s) \\ - \sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(s) g_{ij}(\varphi_{hl}(s-\tau_{hl}(s)))\varphi_{ij}(s) + L_{ij}(s)\Bigg] ds\Bigg)_{mn\times 1}. \end{aligned} \tag{3.3}$$

Define the mapping $\mathcal{T} : \mathcal{PAP}(R) \to \mathcal{PAP}(R)$ by $\mathcal{T}(\varphi)(t) = x_\varphi(t), \varphi \in \mathcal{PAP}(R)$.

Let $B^* = \{\varphi \mid \varphi \in \mathcal{PAP}(R), \|\varphi - \varphi_0\|_\infty \le (\gamma/(1-\gamma))L\}$, where $\varphi_0(t) = (\int_{-\infty}^{t} e^{-\int_s^t a_{11}(\eta)d\eta} L_{11}(s)ds, \ldots, \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta} L_{ij}(s)ds, \ldots, \int_{-\infty}^{t} e^{-\int_s^t a_{mn}(\eta)d\eta} L_{mn}(s)ds)^T$.

Clearly, B^* is closed and convex in B. Note that

$$\begin{aligned} \|\varphi_0\|_\infty &= \sup_{t\in R}\max_{(i,j)}\left\{\left|\int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta} L_{ij}(s)ds\right|\right\} \\ &\le \sup_{t\in R}\max_{(i,j)}\left\{\left|\int_{-\infty}^{t} e^{-a_{ij}^{-}(s-t)} L_{ij}^{+}ds\right|\right\} \\ &\le \sup_{t\in R}\max_{(i,j)}\left\{\frac{L_{ij}^{+}}{a_{ij}^{-}}\right\} = \max_{(i,j)}\left\{\frac{L_{ij}^{+}}{a_{ij}^{-}}\right\} = L. \end{aligned} \tag{3.4}$$

Therefore, for any $\varphi \in B^*$, we have

$$\|\varphi\| \le \|\varphi - \varphi_0\| + \|\varphi_0\| \le \frac{\gamma}{1-\gamma}L + L = \frac{L}{1-\gamma}. \tag{3.5}$$

Now, we will show that $\mathcal{T}$ maps B^* into itself. In fact, for any $\varphi \in B^*$, by using $L/(1-\gamma) \le 1$, we have

$$\begin{aligned}
\|\mathcal{T}\varphi - \varphi_0\|_\infty &= \sup_{t\in R} \max_{(i,j)} \left\{ \left| \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta} \left[- \sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(s) f_{ij}(\varphi_{hl}(s))\varphi_{ij}(s) \right.\right.\right. \\
&\qquad \left.\left.\left. - \sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(s) g_{ij}(\varphi_{hl}(s-\tau_{hl}(s)))\varphi_{ij}(s) \right] ds \right| \right) \\
&\le \sup_{t\in R} \max_{(i,j)} \left\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta} \left[\mu_{ij} \sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(s) \left|(\varphi_{hl}(s))\right| \, |\varphi_{ij}(s)| \right.\right. \\
&\qquad \left.\left. + \nu_{ij} \sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(s) |\varphi_{hl}(s-\tau_{hl}(s))| |\varphi_{ij}(s)| \right] ds \right) \\
&\le \max_{(i,j)} \left\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta} \left(\mu_{ij} \sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(s) + \nu_{ij} \sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(s) \right) ds \right\} \|\varphi\|^2. \\
&\le \gamma \|\varphi\| \le \gamma \|\varphi\| \le \frac{\gamma L}{1-\gamma}.
\end{aligned} \tag{3.6}$$

For any $\varphi, \phi \in B^*$, it follows from $L/(1-\gamma) \le 1$ that

$$\begin{aligned}
&\|\mathcal{T}\varphi - \mathcal{T}\psi\|_\infty \\
&= \sup_{t\in R} \max_{(i,j)} \left\{ \left| \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta} \right.\right. \\
&\qquad \times \left[\sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(s) \left| f_{ij}(\varphi_{hl}(s))\varphi_{ij}(s) - f_{ij}(\psi_{hl}(s))\psi_{ij}(s) \right| \right. \\
&\qquad \left.\left.\left. + \sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(s) \left| g_{ij}(\varphi_{hl}(s-\tau_{hl}(s)))\varphi_{ij}(s) - g_{ij}(\psi_{hl}(s-\tau_{hl}(s)))\psi_{ij}(s) \right| \right] ds \right| \right) \\
&\le \sup_{t\in R} \max_{(i,j)} \left\{ \int_{-\infty}^{t} e^{-\int_s^t a_{ij}(\eta)d\eta} \right. \\
&\qquad \times \left[\sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(s) \left(\left| f_{ij}(\varphi_{hl}(s)) \right| \cdot \left| \varphi_{ij}(s) - \psi_{ij}(s) \right| \right. \right.
\end{aligned}$$

$$
\begin{aligned}
&+\left|f_{ij}(\varphi_{hl}(s))-f_{ij}(\psi_{hl}(s))\right|\cdot\left|\psi_{ij}(s)\right|\Big)+\sum_{C^{hl}\in N_r(i,j)}C_{ij}^{hl}(s)\Big(\left|g_{ij}(\varphi_{hl}(s-\tau_{hl}(s)))\right|\\
&\times\left|\varphi_{ij}(s)-\psi_{ij}(s)\right|+\left|g_{ij}(\varphi_{hl}(s-\tau_{hl}(s)))-g_{ij}(\psi_{hl}(s-\tau_{hl}(s)))\right|\cdot\left|\psi_{ij}(s)\right|\Big)\Big]ds\Bigg\}\\
&\le\max_{(i,j)}\left\{\int_{-\infty}^{t}e^{-\int_s^t a_{ij}(\eta)d\eta}\left[\mu_{ij}\sum_{B^{hl}\in N_r(i,j)}B_{ij}^{hl}(s)+\nu_{ij}\sum_{C^{hl}\in N_r(i,j)}C_{ij}^{hl}(s)\right]\left(\|\varphi\|_\infty+\|\psi\|_\infty\right)\|\varphi-\psi\|ds\right\}\\
&=\gamma\cdot\frac{2L}{1-\gamma}\|\varphi-\psi\|=\frac{2L\gamma}{1-\gamma}\|\varphi-\psi\|=\delta\|\varphi-\psi\|.
\end{aligned}
\tag{3.7}
$$

Since $\delta<1$, $\mathcal{T}$ is a contraction mapping. Therefore, there exists a unique fixed point $x^*\in B^*$ such that $\mathcal{T}x^*=x^*$. That is, system (1.4) has a unique pseudo almost-periodic solution $x^*\in B^*$ with $\|x^*-\varphi_0\|\le(\gamma/(1-\gamma))L$. □

Now we go ahead with the GES of (1.4). The approaches involve constructing suitable Lyapunov functions and application of a generalized Halanay's delay differential inequality. We will stop here to see our first criteria for the globally exponential stability of (1.4), which is delay dependent.

Theorem 3.2. *In addition to* (A_1)*–*(A_4)*, if one further assumes that*

(A_5)

$$
\begin{aligned}
c=\min_{(i,j)}\inf_{(i,j)}\Bigg\{&\beta_{ij}\left(2a_{ij}(t)-3\beta_{ij}\frac{L}{1-\gamma}\left[\mu_{ij}\sum_{B^{hl}\in N_r(i,j)}B_{ij}^{hl}(t)+\nu_{ij}\sum_{C^{hl}\in N_r(i,j)}C_{ij}^{hl}(t)\right]\right)\\
&+\beta_{hl}\left(\mu_{hl}\frac{L}{1-\gamma}\sum_{B^{ij}\in N_r(h,l)}B_{hl}^{ij}(t)+\frac{L}{1-\gamma}\nu_{hl}\sum_{C^{hl}\in N_r(i,j)}\frac{C_{hl}^{ij}\left(\xi_{ij}^{-1}(t)\right)}{1-\tau_{ij}\left(\xi_{ij}^{-1}(t)\right)}\right)\Bigg\}>0
\end{aligned}
\tag{3.8}
$$

or

(A_6)

$$
\begin{aligned}
c=\inf_{(i,j)}\Bigg\{&\beta_{ij}\left(a_{ij}(t)-\beta_{ij}\frac{L}{1-\gamma}\left[\mu_{ij}\sum_{B^{hl}\in N_r(i,j)}B_{ij}^{hl}(t)+\nu_{ij}\sum_{C^{hl}\in N_r(i,j)}C_{ij}^{hl}(t)\right]\right)\\
&-\beta_{hl}\left(\mu_{hl}\frac{L}{1-\gamma}\sum_{B^{ij}\in N_r(h,l)}B_{hl}^{ij}(t)+\frac{L}{1-\gamma}\nu_{hl}\sum_{C^{hl}\in N_r(i,j)}\frac{C_{hl}^{ij}\left(\xi_{ij}^{-1}(t)\right)}{1-\tau_{ij}\left(\xi_{ij}^{-1}(t)\right)}\right)\Bigg\}>0.
\end{aligned}
\tag{3.9}
$$

Then there exists a unique pseudo-almost periodic solution of system (1.4) *and all other solutions converge exponentially to the (pseudo) almost-periodic attractor.*

Proof. By Theorem (3.2), there exists a unique pseudo almost-periodic solution, namely, $x(t) = x(t,\varphi)$. Let $y = y(t,\varphi)$ be any other solution of (1.4) through (t_0,φ). Assume that (A_5) is satisfied and consider the auxiliary functions $F_{ij}(\varepsilon)$ defined on $[0,+\infty]$ as follows:

$$F_{ij}(\varepsilon) = \inf_{t\in R}\left\{\beta_{ij}\left(2a_{ij}(t)-\varepsilon\right) - 3\beta_{ij}\frac{L}{1-\gamma}\left[\mu_{ij}\sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(t) + \nu_{ij}\sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(t)\right]\right.$$
$$\left. +\beta_{hl}\left(\mu_{hl}\frac{L}{1-\gamma}\sum_{B^{ij}\in N_r(h,l)} B_{hl}^{ij}(t) + \frac{L}{1-\gamma}\nu_{hl}\sum_{C^{hl}\in N_r(i,j)}\frac{C_{hl}^{ij}\left(\xi_{ij}^{-1}(t)\right)}{1-\tau_{ij}\left(\xi_{ij}^{-1}(t)\right)}\right)\right\}. \tag{3.10}$$

From(A_1)–(A_3), one can easily show that $F_{ij}(\varepsilon)$ is well defined and is continuous. From (A_5), it follows that $F_{ij}(0) > 0, F_{ij}(\varepsilon) \to -\infty$ as $\varepsilon \to \infty$ it follows that there exists an $\varepsilon_{ij} > 0$ such that $F_{ij}(\varepsilon_{ij}) > 0$. Let $\varepsilon = \min_{i,j}\varepsilon_{ij}$. Then we have $F_{ij}(\varepsilon) > 0, 1 \le i \le n, 1 \le j \le m$.

Consider the Lyapunov functional defined by

$$V(t) = \frac{1}{2}\sum_{(i,j)}\beta_{ij}\left[\left(x_{ij}(t)-y_{ij}(t)\right)^2 e^{\varepsilon t}\right.$$
$$\left. + \sum_{C^{hl}\in N_r(i,j)} V_{ij}\frac{L}{1-\gamma}\int_{t-\tau_{hl}(t)}^{t}\frac{C^{hl}\left(\xi_{hl}^{-1}(t)\right)}{1-\tau_{hl}\left(\xi_{hl}^{-1}\right)(t)}\left(x_{hl}(s)-y_{hl}(s)\right)^2 e^{\varepsilon(s+\tau_{hl}^M)}ds\right]. \tag{3.11}$$

Calculating the upper-right derivative of $V(t)$ and using the inequality $2ab \le a^2 + b^2$, one has

$$V'(t) \le \sum_{(i,j)}\beta_{ij}\left\{\frac{1}{2}\left(x_{ij}(t)-y_{ij}(t)\right)^2\varepsilon e^{\varepsilon t} - e^{\varepsilon t}a_{ij}(t)\left(x_{ij}-y_{ij}(t)\right)^2\right.$$
$$+ e^{\varepsilon t}\sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(t)\left|x_{ij}(t)-y_{ij}(t)\right|\cdot\left|f_{ij}(x_{hl}(t))x_{ij}(t) - f_{ij}\left(y_{hl}(t)\right)y_{ij}(t)\right|$$
$$+ e^{\varepsilon t}\sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(t)\left|x_{ij}(t)-y_{ij}(t)\right|$$
$$\cdot\left|g_{ij}(x_{hl}(t-\tau_{hl}(t)))x_{ij}(t) - g_{ij}\left(y_{hl}(t-\tau_{hl}(t))\right)y_{ij}(t)\right|$$
$$+\frac{1}{2}\sum_{C^{hl}\in N_r(i,j)}\nu_{ij}\frac{L}{1-\gamma}\frac{C_{ij}^{hl}\left(\xi_{hl}^{-1}(t)\right)}{1-\tau_{hl}\left(\xi_{hl}^{-1}(t)\right)}\left(x_{hl}(t)-y_{hl}(t)\right)^2 e^{\varepsilon(t+\tau_{hl}^M)}$$
$$\left.-\frac{1}{2}\sum_{C^{hl}\in N_r(i,j)}\nu_{ij}\frac{L}{1-\gamma}C_{ij}^{hl}(t)(x_{hl}(t-\tau_{hl}(t)) - y_{hl}(t-\tau_{hl}(t)))^2 e^{\varepsilon(t-\tau_{hl}(t)+\tau_{hl}^M)}\right\}$$

$$
\begin{aligned}
&\le \sum_{(i,j)} \beta_{ij} \Bigg\{ \frac{1}{2}\left(x_{ij}(t)-y_{ij}(t)\right)^2 \varepsilon e^{\varepsilon t} - e^{\varepsilon t} a_{ij}(t)\left(x_{ij}-y_{ij}(t)\right)^2 \\
&\qquad + e^{\varepsilon t} \sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(t)\left|x_{ij}(t)-y_{ij}(t)\right| \cdot \left| f_{ij}(x_{hl}(t))x_{ij}(t) - f_{ij}(y_{hl}(t))y_{ij}(t)\right| \\
&\qquad + e^{\varepsilon t} \sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(t)\left|x_{ij}(t)-y_{ij}(t)\right| \\
&\qquad\qquad\qquad \cdot \left| g_{ij}(x_{hl}(t-\tau_{hl}(t)))x_{ij}(t) - g_{ij}(y_{hl}(t-\tau_{hl}(t)))y_{ij}(t)\right| \\
&\qquad + \frac{1}{2} \sum_{C^{hl}\in N_r(i,j)} \nu_{ij} \frac{L}{1-\gamma} \frac{C_{ij}^{hl}(\xi_{hl}^{-1}(t))}{1-\tau_{hl}(\xi_{hl}^{-1}(t))} \left(x_{hl}(t)-y_{hl}(t)\right)^2 e^{\varepsilon(t+\tau_{hl}^{M})} \\
&\qquad - \frac{1}{2} \sum_{C^{hl}\in N_r(i,j)} \nu_{ij} \frac{L}{1-\gamma} C_{ij}^{hl}(t) \left(x_{hl}(t-\tau_{hl}(t)) - y_{hl}(t-\tau_{hl}(t))\right)^2 e^{\varepsilon(t)} \Bigg\} \\
&\le e^{\varepsilon(t)} \sum_{(i,j)} \beta_{ij} \Bigg\{ \left(\frac{\varepsilon}{2} - a_{ij}(t)\right)\left(x_{ij}(t)-y_{ij}(t)\right)^2 \\
&\qquad + \sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(t)\left|x_{ij}(t)-y_{ij}(t)\right| \left[\left|f_{ij}(x_{hl}(t))\right| \left|x_{ij}(t)-y_{ij}(t)\right| \right. \\
&\qquad\qquad\qquad \left. + \left| f_{ij}(x_{hl}(t)) - f_{ij}(y_{hl}(t))\right| \left|y_{ij}(t)\right| \right] \\
&\qquad + \sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(t)\left|x_{ij}(t)-y_{ij}(t)\right| \\
&\qquad\quad \times \left[\left| g_{ij}(x_{hl}(t-\tau_{hl}(t)))\right| \left|x_{ij}(t)-y_{ij}(t)\right| \right. \\
&\qquad\qquad \left. + \left| g_{ij}(x_{hl}(t-\tau_{hl}(t))) - g_{ij}(y_{hl}(t-\tau_{hl}(t)))\right| \left|y_{ij}(t)\right| \right] \\
&\qquad + \frac{1}{2} \sum_{C^{hl}\in N_r(i,j)} \nu_{ij} \frac{L}{1-\gamma} \frac{C_{ij}^{hl}(\xi_{hl}^{-1}(t))}{1-\tau_{hl}(\xi_{hl}^{-1}(t))} \left(x_{hl}(t)-y_{hl}(t)\right)^2 e^{\varepsilon\tau_{hl}^{M}} \\
&\qquad - \frac{1}{2} \sum_{C^{hl}\in N_r(i,j)} \nu_{ij} \frac{L}{1-\gamma} C_{ij}^{hl}(t) \left(x_{hl}(t-\tau_{hl}(t)) - y_{hl}(t-\tau_{hl}(t))\right)^2 \Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&\le e^{\varepsilon(t)}\sum_{(i,j)}\beta_{ij}\Bigg\{\left(\frac{\varepsilon}{2}-a_{ij}(t)\right)\left(x_{ij}(t)-y_{ij}(t)\right)^2\\
&\qquad+\sum_{B^{hl}\in N_r(i,j)}B_{ij}^{hl}(t)\mu_{ij}\frac{L}{1-\gamma}\\
&\qquad\qquad\times\left[\left(x_{ij}(t)-y_{ij}(t)\right)^2+\left|x_{ij}(t)-y_{ij}(t)\right|\left|x_{hl}(t)-y_{hl}(t)\right|\right]\\
&\qquad+\sum_{C^{hl}\in N_r(i,j)}C_{ij}^{hl}(t)\nu_{ij}\frac{L}{1-\gamma}\\
&\qquad\qquad\times\left[\left(x_{ij}(t)-y_{ij}(t)\right)^2+\left|x_{ij}(t)-y_{ij}(t)\right|\right.\\
&\qquad\qquad\quad\left.\times\left|x_{hl}(t-\tau_{hl}(t))-y_{hl}(t-\tau_{hl}(t))\right|\right]\\
&\qquad+\frac{1}{2}\sum_{C^{hl}\in N_r(i,j)}\nu_{ij}\frac{L}{1-\gamma}\frac{C_{ij}^{hl}\left(\xi_{hl}^{-1}(t)\right)}{1-\tau_{hl}\left(\xi_{hl}^{-1}(t)\right)}\left(x_{ij}(t)-y_{ij}(t)\right)^2e^{\varepsilon\tau_{hl}^M}\\
&\qquad-\frac{1}{2}\sum_{C^{hl}\in N_r(i,j)}\nu_{ij}\frac{L}{1-\gamma}C_{ij}^{hl}(t)\left(x_{hl}(t-\tau_{hl}(t))-y_{hl}(t-\tau_{hl}(t))\right)^2\Bigg\}\\
&\le e^{\varepsilon(t)}\sum_{(i,j)}\beta_{ij}\Bigg\{\left(\frac{\varepsilon}{2}-a_{ij}(t)\right)\left(x_{ij}(t)-y_{ij}(t)\right)^2\\
&\qquad+\sum_{B^{hl}\in N_r(i,j)}B_{ij}^{hl}(t)\mu_{ij}\frac{L}{1-\gamma}\\
&\qquad\qquad\times\left[\left(x_{ij}(t)-y_{ij}(t)\right)^2+\frac{1}{2}\left(x_{ij}(t)-y_{ij}(t)\right)^2\left(x_{hl}(t)-y_{hl}(t)\right)^2\right]\\
&\qquad+\sum_{C^{hl}\in N_r(i,j)}C_{ij}^{hl}(t)\nu_{ij}\frac{L}{1-\gamma}\\
&\qquad\qquad\times\left[\left(x_{ij}(t)-y_{ij}(t)\right)^2+\frac{1}{2}\left(x_{ij}(t)-y_{ij}(t)\right)^2\right.\\
&\qquad\qquad\quad\left.\times\left(x_{hl}(t-\tau_{hl}(t))-y_{hl}(t-\tau_{hl}(t))\right)^2\right]\\
&\qquad+\frac{1}{2}\sum_{C^{hl}\in N_r(i,j)}\nu_{ij}\frac{L}{1-\gamma}\frac{C_{ij}^{hl}\left(\xi_{hl}^{-1}(t)\right)}{1-\tau_{hl}\left(\xi_{hl}^{-1}(t)\right)}\left(x_{ij}(t)-y_{ij}(t)\right)^2e^{\varepsilon\tau_{hl}^M}\\
&\qquad-\frac{1}{2}\sum_{C^{hl}\in N_r(i,j)}\nu_{ij}\frac{L}{1-\gamma}C_{ij}^{hl}(t)\left(x_{hl}(t-\tau_{hl}(t))-y_{hl}(t-\tau_{hl}(t))\right)^2\Bigg\}
\end{aligned}
$$

$$
\begin{aligned}
&\le e^{\varepsilon(t)} \sum_{(i,j)} \Bigg\{ \beta_{ij}\left(\frac{\varepsilon}{2} - a_{ij}(t)\right)\left(x_{ij}(t) - y_{ij}(t)\right)^2 + \frac{3}{2}\sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(t)\mu_{ij}\frac{L}{1-\gamma}\left(x_{ij}(t) - y_{ij}(t)\right)^2 \\
&\qquad + \frac{3}{2}\beta_{ij}\sum_{C^{hl}\in N_r(i,j)} C_{\mathrm{ij}}^{hl}(t)\nu_{ij}\frac{L}{1-\gamma}\left(x_{ij}(t) - y_{ij}(t)\right)^2 \\
&\qquad + \frac{1}{2}\sum_{B^{ij}\in N_r(h,l)} \beta_{hl}B_{hl}^{ij}(t)\mu_{hl}\frac{L}{1-\gamma}\left(x_{ij}(t) - y_{ij}(t)\right)^2 \\
&\qquad + \frac{1}{2}\sum_{C^{ij}\in N_r(h,l)} \nu_{hl}\frac{L}{1-\gamma}\frac{C_{hl}^{ij}\left(\xi_{ij}^{-1}(t)\right)}{1-\tau_{ij}\left(\xi_{ij}^{-1}(t)\right)}\left(x_{ij}(t) - y_{ij}(t)\right)^2 e^{\varepsilon\tau_{hl}^{M}} \Bigg\} \\
&\le -\frac{c_0}{2}e^{\varepsilon t}\left(x_{ij}(t) - y_{ij}(t)\right)^2 \le 0,
\end{aligned}
\tag{3.12}
$$

where $c_0 > 0$ is defined by

$$
\begin{aligned}
c_0 = \min_{(i,j)}\inf_{t\in R}\Bigg\{ &\beta_{ij}\left(2a_{ij}(t) - \varepsilon\right) - 3\beta_{ij}\frac{L}{1-\gamma}\left[\mu_{ij}\sum_{B^{hl}\in N_r(i,j)} B_{ij}^{hl}(t) + \nu_{ij}\sum_{C^{hl}\in N_r(i,j)} C_{ij}^{hl}(t)\right] \\
&+\beta_{hl}\left(\mu_{hl}\frac{L}{1-\gamma}\sum_{B^{ij}\in N_r(h,l)} B_{hl}^{ij}(t) + \frac{L}{1-\gamma}\nu_{hl}\sum_{C^{hl}\in N_r(i,j)}\frac{C_{hl}^{ij}\left(\xi_{ij}^{-1}(t)\right)}{1-\tau_{ij}\left(\xi_{ij}^{-1}(t)\right)}\right)\Bigg\} > 0.
\end{aligned}
\tag{3.13}
$$

From the above, we have $V(t) \le V(t_0), t \ge t_0$, and

$$
\begin{gathered}
\frac{1}{2}e^{\varepsilon t}\min_{h,l}\beta_{hl}\sum_{i=1}^{n}\left(x_{ij}(t) - y_{ij}(t)\right)^2 \le V(t), \quad t \ge t_0, \\
V(t_0) \le \frac{1}{2}e^{\varepsilon t_0}\left[\sum_{(i,j)}\beta_{ij} + \sum_{(i,j)}\sum_{(h,l)}\beta_{ij}\nu_{ij}\frac{L}{1-\gamma}\tau_{hl}^{\mu}e^{\varepsilon\tau_{hl}^{\mu}}\sup_{t\in[t_0-\tau,t_0]}\frac{C^{hl}\left(\xi_{hl}^{-1}(t)\right)}{1-\tau_{hl}\left(\xi_{hl}^{-1}(t)\right)}\right]\|\varphi - \psi\|_2^2.
\end{gathered}
\tag{3.14}
$$

Thus, it follows that there exists a positive constant $M > 1$ such that

$$
\left|x(t) - y(t)\right|_2 \le Me^{-(\varepsilon/2)(t-t_0)}\|\varphi - \psi\|_2, \quad t \ge t_0,
\tag{3.15}
$$

which implies that (1.4) is GES.

Now we assume that (A_6) is satisfied. By carrying out similar arguments as above, one can easily show that there exists an $\varepsilon > 0$ such that

$$\inf_{(i,j)}\Bigg\{\beta_{ij}\left(a_{ij}(t)-\varepsilon\right)-\beta_{ij}\frac{L}{1-\gamma}\left[\mu_{ij}\sum_{B^{hl}\in N_r(i,j)}B_{ij}^{hl}(t)+\nu_{ij}\sum_{C^{hl}\in N_r(i,j)}C_{ij}^{hl}(t)\right] \\ -\beta_{hl}\left(\mu_{hl}\frac{L}{1-\gamma}\sum_{B^{ij}\in N_r(h,l)}B_{hl}^{ij}(t)+\frac{L}{1-\gamma}\nu_{hl}\sum_{C^{hl}\in N_r(i,j)}\frac{C_{hl}^{ij}\left(\xi_{ij}^{-1}(t)\right)}{1-\tau_{ij}\left(\xi_{ij}^{-1}(t)\right)}\right)\Bigg\}>0. \tag{3.16}$$

Consider the Lyapunov function

$$V(t)=\sum_{(i,j)}\beta_{ij}\Bigg\{\left|x_{ij}(t)-y_{ij}(t)\right|e^{\varepsilon t}+\sum_{C^{hl}\in N_r(i,j)}\nu_{ij}\frac{L}{1-\gamma} \\ \times\int_{t-\tau_{hl}(t)}^{t}\frac{C_{ij}^{hl}\left(\xi_{hl}^{-1}(s)\right)}{1-\tau_{hl}\left(\xi_{hl}^{-1}(s)\right)}\left|x_{hl}(s)-y_{hl}(s)\right|e^{\varepsilon(t+\tau_{hl}^{\mu})}ds\Bigg\}. \tag{3.17}$$

Similar to the above arguments, calculating the upper-right derivative $D^+V(t)$ produces

$$D^+V(t)\le -c_2e^{\varepsilon t}\sum_{(i,j)}\left|x_{ij}(t)-y_{ij}(t)\right|\le 0, \tag{3.18}$$

where

$$c_2=\min_{(i,j)}\inf_{t\in R}\Bigg\{\beta_{ij}\left(a_{ij}(t)-\varepsilon\right)-\beta_{ij}\frac{L}{1-\gamma}\left[\mu_{ij}\sum_{B^{hl}\in N_r(i,j)}B_{ij}^{hl}(t)+\nu_{ij}\sum_{C^{hl}\in N_r(i,j)}C_{ij}^{hl}(t)\right] \\ -\beta_{hl}\left(\mu_{hl}\frac{L}{1-\gamma}\sum_{B^{ij}\in N_r(h,l)}B_{hl}^{ij}(t)+\frac{L}{1-\gamma}\nu_{hl}\sum_{C^{hl}\in N_r(i,j)}\frac{C_{hl}^{ij}\left(\xi_{ij}^{-1}(t)\right)}{1-\tau_{ij}\left(\xi_{ij}^{-1}(t)\right)}\right)\Bigg\}>0. \tag{3.19}$$

Then we have

$$e^{\varepsilon t}\left(\min_{(h,l)}\beta_{hl}\right)\sum_{(i,j)}\left|x_{ij}(t)-y_{ij}(t)\right|\le V(t)\le V(t_0),\quad t\ge t_0. \tag{3.20}$$

Note that

$$V(t_0)\le e^{\varepsilon t_0}\left[\sum_{(i,j)}\beta_{ij}+\sum_{(i,j)}\beta_{ij}\sum_{C^{ij}\in N_r(h,l)}\tau_{hl}^{\mu}e^{\varepsilon\tau_{hl}^{\mu}}\sup_{t\in[t_0-\tau,t_0]}\frac{C_{ij}^{hl}\left(\xi_{hl}^{-1}(t)\right)}{1-\tau_{hl}\left(\xi_{hl}^{-1}(t)\right)}\right]\|\varphi-\psi\|_1. \tag{3.21}$$

Then there exists a positive constant $M > 1$ such that

$$\left|x(t) - y(t)\right|_1 = \sum_{i=1}^{n}\left|x_i(t) - y_i(t)\right| \leq M\|\varphi - \psi\|_1 e^{\varepsilon(t-t_0)}, \quad t \geq t_0. \tag{3.22}$$

The proof is complete. □

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