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Research Article

Stability and Superstability of Generalized (θ,ϕ)-Derivations in Non-Archimedean Algebras: Fixed Point Theorem via the Additive Cauchy Functional Equation

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Let A be an algebra, and let θ, ϕ be ring automorphisms of A. An additive mapping $H : A \to A$ is called a (θ,ϕ)-derivation if $H(xy) = H(x)\theta(y) + \phi(x)H(y)$ for all $x, y \in A$. Moreover, an additive mapping $F : A \to A$ is said to be a generalized (θ,ϕ)-derivation if there exists a (θ,ϕ)-derivation $H : A \to A$ such that $F(xy) = F(x)\theta(y) + \phi(x)H(y)$ for all $x, y \in A$. In this paper, we investigate the superstability of generalized (θ,ϕ)-derivations in non-Archimedean algebras by using a version of fixed point theorem via Cauchy's functional equation.

1. Introduction and Preliminaries

In 1897, Hensel [1] has introduced a normed space which does not have the Archimedean property. It turned out that non-Archimedean spaces have many nice applications [2, 3].

A non-Archimedean field is a field $\mathbb{K}$ equipped with a function (valuation) $|\cdot|$ from $\mathbb{K}$ into $[0,\infty)$ such that $|r| = 0$ if and only if $r = 0$, $|rs| = |r||s|$, and $|r + s| \leq \max\{|r|, |s|\}$ for all $r, s \in \mathbb{K}$. An example of a non-Archimedean valuation is the mapping $|\cdot|$ taking everything but 0 into 1 and $|0| = 0$. This valuation is called trivial (see [4]).

Definition 1.1. Let X be a vector space over a scalar field $\mathbb{K}$ with a non-Archimedean nontrivial valuation $|\cdot|$. A function $\|\cdot\| : X \to \mathbb{R}$ is a non-Archimedean norm (valuation) if it satisfies the following conditions:

(NA_1) $\|x\| = 0$ if and only if $x = 0$,

(NA_2) $\|rx\| = |r|\|x\|$ for all $r \in \mathbb{K}$ and $x \in X$,

(NA_3) $\|x + y\| \leq \max\{\|x\|, \|y\|\}$ for all $x, y \in X$ (the strong triangle inequality).

A sequence $\{x_m\}$ in a non-Archimedean space is Cauchy's if and only if $\{x_{m+1} - x_m\}$ converges to zero. By a complete non-Archimedean space, we mean one in which every Cauchy's sequence is convergent. A non-Archimedean-normed algebra is a non-Archimedean-normed space A with a linear associative multiplication, satisfying $\|xy\| \leq \|x\|\|y\|$ for all $x, y \in A$. A non-Archimedean complete normed algebra is called a non-Archimedean Banach's algebra (see [5]).

Definition 1.2. Let X be a nonempty set, and let $d : X \times X \to [0, \infty]$ satisfy the following properties:

(D_1) $d(x, y) = 0$ if and only if $x = y$,

(D_2) $d(x, y) = d(y, x)$ (symmetry),

(D_3) $d(x, z) \leq \max\{d(x, y), d(y, z)\}$ (strong triangle inequality),

for all $x, y, z \in X$. Then (X, d) is called a non-Archimedean generalized metric space. (X, d) is called complete if every d-Cauchy's sequence in X is d-convergent.

Definition 1.3. Let A be a non-Archimedean algebra, and let θ, ϕ be ring automorphisms of A. An additive mapping $H : A \to A$ is called a (θ, ϕ)-derivation in case $H(xy) = H(x)\theta(y) + \phi(x)H(y)$ holds for all $x, y \in A$. An additive mapping $F : A \to A$ is said to be a generalized (θ, ϕ)-derivation if there exists a (θ, ϕ)-derivation $H : A \to A$ such that

$$F(xy) = F(x)\theta(y) + \phi(x)H(y) \tag{1.1}$$

for all $x, y \in A$.

We need the following fixed point theorem (see [6, 7]).

Theorem 1.4 (Non-Archimedean Alternative Contraction Principle). *Suppose (X, d) is a non-Archimedean generalized complete metric space and $\Lambda : X \to X$ is a strictly contractive mapping; that is,*

$$d(\Lambda x, \Lambda y) \leq L d(x, y) \quad (x, y \in X) \tag{1.2}$$

for some $L < 1$. If there exists a nonnegative integer k such that $d(\Lambda^{k+1}x, \Lambda^k x) < \infty$ for some $x \in X$, then the followings are true.

(a) *The sequence $\{\Lambda^n x\}$ converges to a fixed point x^* of Λ.*

(b) *x^* is a unique fixed point of Λ in*

$$X^* = \left\{y \in X \mid d\left(\Lambda^k x, y\right) < \infty\right\}. \tag{1.3}$$

(c) *If* $y \in X^*$, *then*

$$d(y, x^*) \leq d(\Lambda y, y). \tag{1.4}$$

A functional equation (ξ) is *superstable* if every approximately solution of (ξ) is an exact solution of it.

The stability of functional equations was first introduced by Ulam [8] during his talk before a mathematical colloquium at the University of Wisconsin in 1940. In 1941, Hyers [9] gave a first affirmative answer to the question of Ulam for Banach spaces. In 1978, Rassias [10] generalized the theorem of Hyers by considering the stability problem with unbounded Cauchy's differences $\|f(x+y) - f(x) - f(y)\| \leq \epsilon(\|x\|^p + \|y\|^p)$, $(\epsilon > 0, p \in [0,1))$. Moreover, John Rassias [11–13] investigated the stability of some functional equations when the control function is the product of powers of norms. In 1991, Gajda [14] answered the question for the case $p > 1$, which was raised by Rassias. This new concept is known as the Hyers-Ulam-Rassias or the generalized Hyers-Ulam stability of functional equations ([11–13, 15–35]).

In 1992, Găvruţa [36] generalized the Rassias theorem as follows.

Suppose $(G, +)$ is an ablian group, X is a Banach space, $\varphi : G \times G \to [0, \infty)$ satisfies

$$\tilde{\varphi}(x, y) = \frac{1}{2}\sum_{n=0}^{\infty} 2^{-n}\varphi(2^n x, 2^n y) < \infty, \tag{1.5}$$

for all $x, y \in G$. If $f : G \to X$ is a mapping with

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(x, y), \tag{1.6}$$

for all $x, y \in G$, then there exists a unique mapping $T : G \to X$ such that $T(x+y) = T(x)+T(y)$ and $\|f(x) - T(x)\| \leq \tilde{\varphi}(x, x)$ for all $x, y \in G$.

In 1949, Bourgin [37] proved the following result, which is sometimes called the superstability of ring homomorphisms: suppose that A and B are Banach algebras with unit. If $f : A \to B$ is a surjective mapping such that

$$\begin{aligned} &\|f(x+y) - f(x) - f(y)\| \leq \epsilon, \\ &\|f(xy) - f(x)f(y)\| \leq \delta, \end{aligned} \tag{1.7}$$

for some $\epsilon \geq 0$, $\delta \geq 0$ and for all $x, y \in A$, then f is a ring homomorphism.

The first superstability result concerning derivations between operator algebras was obtained by Šemrl in [38]. Badora [39] proved the superstability of the functional equation $f(xy) = xf(y) + f(x)y$, where f is a mapping on normed algebra A with unit. Ansari-Piri and Anjidani [40] discussed the superstability of generalized derivations on Banach's algebras. Recently, Eshaghi Gordji et al. [41] investigated the stability and superstability of higher ring derivations on non-Archimedean Banach's algebras (see also [42]). In this paper, we investigate the superstability of generalized (θ, ϕ)-derivations on non-Archimedean Banach algebras by using the fixed point methods.

2. Non-Archimedean Superstability of Generalized (θ, ϕ)-Derivations

In this paper, we assume that A is a non-Archimedean Banach's algebra, with unit over a non-Archimedean field $\mathbb{K}$, and θ, ϕ are ring automorphisms of A.

Theorem 2.1. *Let $\varphi, \psi : A \times A \to [0, \infty)$ be functions. Suppose that $f : A \to A$ is a mapping such that*

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(x, y), \tag{2.1}$$

$$\|f(xy) - f(x)\theta(y) - \phi(x)g(y)\| \leq \psi(x, y), \tag{2.2}$$

for all $x, y \in A$. If there exist constants $K, L < 1$ and a natural number $k \in \mathbb{K}$,

$$|k|^{-1}\varphi(kx, ky) \leq L\varphi(x, y), \qquad |k|^{-1}\psi(kx, y), \qquad |k|^{-1}\psi(x, ky) \leq K\psi(x, y), \tag{2.3}$$

for all $x, y \in A$, then f is a generalized (θ, ϕ)-derivation and g is a (θ, ϕ)-derivation.

Proof. By induction on i, we prove that for each $n \in \mathbb{N}_0$, for all $x \in A$ and $i \geq 2$,

$$\|f(ix) - if(x)\| \leq \max\{\varphi(0,0), \varphi(x,x), \varphi(2x,x), \ldots, \varphi((i-1)x, x)\}. \tag{2.4}$$

Let $x = y$ in (2.1), then

$$\|f(2x) - 2f(x)\| \leq \max\{\varphi(0,0), \varphi(x,x)\}, \quad n \in \mathbb{N}_0, \ x \in A. \tag{2.5}$$

This proves (2.4) for $i = 2$. Let (2.4) hold for $i = 1, 2, \ldots, J$. Replacing x by jx and y by x in (2.1) for each $n \in \mathbb{N}_0$, and for all $x \in A$, we get

$$\|f((j+1)x) - f(jx) - f(x)\| \leq \max\{\varphi(0,0), \varphi(jx, x)\}. \tag{2.6}$$

Since

$$\begin{aligned} f((j+1)x) - f(jx) - f(x) &= f((j+1)x) - (j+1)f(x) + (j+1)f(x) - f(jx) - f(x) \\ &= f((j+1)x) - (j+1)f(x) + jf(x) - f(jx), \end{aligned} \tag{2.7}$$

for all $x \in A$, it follows from induction hypothesis and (2.6) that, for all $x \in A$,

$$\begin{aligned} \|f((j+1)x) - (j+1)f(x)\| &\leq \max\{\|f((j+1)x) - f(jx) - f(x)\|, \|jf(x) - f(jx)\|\} \\ &\leq \max\{\varphi(0,0), \varphi(x,x), \varphi(2x,x), \ldots, \varphi((j)x, x)\}. \end{aligned} \tag{2.8}$$

This proves (2.4) for all $i \geq 2$. In particular, for all $x \in A$,

$$\left\| f(kx) - kf(x) \right\| \leq \Phi(x), \tag{2.9}$$

where

$$\Phi(x) = \max\left\{\varphi(0,0), \varphi(x,x), \varphi(2x,x), \dots, \varphi((k-1)x,x)\right\} \quad (x \in A). \tag{2.10}$$

Let us define a set X of all functions $r : A \to A$ by

$$X = \{r : A \longrightarrow A\} \tag{2.11}$$

and introduce d on X as follows:

$$d(r,s) = \inf\{\alpha > 0 : \|r(x) - s(x)\| \leq \alpha\Phi(x) \forall x \in A\}. \tag{2.12}$$

It is easy to see that d defines a generalized complete metric on X. Define $J : X \to X$ by $J(r)(x) = k^{-1}r(kx)$. Then J is strictly contractive on X, in fact if

$$\|r(x) - s(x)\| \leq \alpha\Phi(x) \quad (x \in A), \tag{2.13}$$

then, by (2.3),

$$\|J(r)(x) - J(s)(x)\| = |k|^{-1}\|r(kx) - s(kx)\| \leq \alpha|k|^{-1}\Phi(kx) \leq L\alpha\Phi(x) \quad (x \in A). \tag{2.14}$$

It follows that

$$d(J(r), J(s)) \leq Ld(r,s) \quad (g, h \in X). \tag{2.15}$$

Hence, J is strictly contractive mapping with the Lipschitz constant L. By (2.9),

$$\begin{aligned} \left\| (Jf)(x) - f(x) \right\| &= \left\| k^{-1} f(kx) - f(x) \right\|, \\ |k|^{-1} \left\| f(kx) - kf(x) \right\| &\leq |k|^{-1}\Phi(x) \quad (x \in A). \end{aligned} \tag{2.16}$$

This means that $d(J(f), f) \leq 1/|k|$. By Theorem 1.4, J has a unique fixed point $h : A \to A$ in the set

$$U = \left\{r \in X : d\left(r, J(f)\right) < \infty\right\}, \tag{2.17}$$

and, for each $x \in A$,

$$h(x) = \lim_{m \to \infty} J^m\left(f(x)\right) = \lim k^{-m} f(k^m x). \tag{2.18}$$

Therefore, each $x, y \in A$,

$$\begin{aligned}\|h(x+y)-h(x)-h(y)\| &= \lim_{m\to\infty}|k|^{-m}\|f(k^m(x+y))-f(k^m x)-f(k^m y)\| \\ &\le \lim_{m\to\infty}|k|^{-m}\max\{\varphi(0,0),\varphi(k^n x,k^n y)\} \\ &\le \lim_{m\to\infty}L^m\varphi(x,y)=0.\end{aligned} \tag{2.19}$$

This shows that h is additive.

Replacing y by $k^n y$ in (2.2), we get

$$\|f(k^n xy)-f(x)\theta(k^n y)-\phi(x)g(k^n y)\| \le \varphi(x,k^n y), \tag{2.20}$$

and so

$$\left\|\frac{f(k^n xy)}{k^n}-f(x)\theta(y)-\phi(x)\frac{g(k^n y)}{k^n}\right\| \le \frac{1}{|k|^n}\varphi(x,k^n y) \le K^n\varphi(x,y), \tag{2.21}$$

for all $x, y \in A$ and each $n \in \mathbb{N}$. By taking $n \to \infty$, we have

$$h(xy)=f(x)\theta(y)+\lim_{n\to\infty}\phi(x)\frac{g(k^n y)}{k^n}, \tag{2.22}$$

for all $x, y \in A$.

Fix $m \in \mathbb{N}$. By (2.22), we have

$$\begin{aligned}f(k^m x)\theta(y) &= h(k^m xy)-\lim_{n\to\infty}\phi(k^m x)\left(\frac{g(k^n y)}{k^n}\right) \\ &= f(x)\theta(k^m y)+\lim_{n\to\infty}\phi(x)\left(\frac{g(k^n k^m x)}{k^n}\right)-k^m\lim_{n\to\infty}\phi(x)\left(\frac{g(k^n x)}{k^n}\right) \\ &= k^m f(x)\theta(y)+k^m\lim_{n\to\infty}\phi(x)\left(\frac{g(k^{n+m}x)}{k^{n+m}}\right)-k^m\lim_{n\to\infty}\phi(x)\left(\frac{g(k^n x)}{k^n}\right) \\ &= k^m f(x)\theta(y),\end{aligned} \tag{2.23}$$

for all $x, y \in A$. Then $f(x)\theta(y) = (f(k^m x)/k^m)\theta(y)$ for all $x, y \in A$ and each $m \in \mathbb{N}$, and so, by taking $m \to \infty$, we have $f(x)\theta(y) = h(x)\theta(x)$. Now we obtain $h = f$, since A is with unit. Replacing x by $k^n x$ in (2.2), we obtain

$$\|f(k^n(xy))-f(k^n x)\theta(y)-\phi(k^n x)g(y)\| \le \varphi(k^n x,y), \tag{2.24}$$

and; hence,

$$\left\| \frac{f(k^n xy)}{k^n} - \frac{f(k^n x)}{k^n}\theta(y) - \phi(x)g(y) \right\| \leq \frac{1}{|k|^n}\psi(k^n x, y) \leq K^n \psi(x,y), \tag{2.25}$$

for all $x, y \in A$ and each $n \in \mathbb{N}$. Sending n to infinite, we have

$$f(xy) = f(x)\theta(y) + \phi(x)g(y). \tag{2.26}$$

By (2.26), we get

$$\begin{aligned} \phi(z)g(xy) &= f(zxy) - f(z)\theta(xy) \\ &= f(zx)\theta(y) + \phi(zx)g(y) - f(z)\theta(xy) \\ &= \left[f(z)\theta(x) + \phi(z)g(x)\right]\theta(y) + \phi(zx)g(y) - f(z)\theta(xy) \\ &= \phi(z)\left[g(x)\theta(y) + \phi(x)g(y)\right], \end{aligned} \tag{2.27}$$

for all $x, y, z \in A$. Therefore, we have $g(xy) = g(x)\theta(y) + \phi(x)g(y)$.

Since $f(xy) = f(x)\theta(y) + \phi(x)g(y)$, f is additive, and A is with unit, g is additive. □

The proof of the following theorem is similar to that in Theorem 2.1; hence, it is omitted.

Theorem 2.2. *Let $\varphi, \psi : A \times A \to [0, \infty)$ be functions. Suppose that $f : A \to A$ and $g : A \to A$ are mappings such that*

$$\begin{aligned} \|f(x+y) - f(x) - f(y)\| &\leq \varphi(x,y), \\ \|f(xy) - xf(y) - g(x)y\| &\leq \psi(x,y), \end{aligned} \tag{2.28}$$

for all $x, y \in A$. If there exists constants $K, L < 1$ and a natural number $k \in \mathbb{K}$,

$$|k|\varphi\left(k^{-1}x, k^{-1}y\right) \leq L\varphi(x,y), |k|\psi\left(k^{-1}x, y\right), |k|\psi\left(x, k^{-1}y\right) \leq K\psi(x,y), \tag{2.29}$$

for all $x, y \in A$, then f is a generalized (θ, ϕ)-derivation and g is a (θ, ϕ)-derivation.

In the following corollaries $\mathbb{Q}_p$ is the field of p-adic numbers.

Corollary 2.3. *Let A be a non-Archimedean Banach algebra over $\mathbb{Q}_p$, $\varepsilon > 0$, and let $p_1, p_2 \in (1, \infty)$. Suppose that*

$$\begin{aligned} \|f(x+y) - f(x) - f(y)\| &\leq \varepsilon\left(\|x\|^{p_1}\|y\|^{p_2}\right), \\ \|f(xy) - xf(y) - g(x)y\| &\leq \varepsilon\left(\|x\|^{p_1}\|y\|^{p_2}\right), \end{aligned} \tag{2.30}$$

for all $x, y \in A$. Then f is a generalized (θ, ϕ)-derivation and g is a (θ, ϕ)-derivation.

Proof. Let $\varphi(x,y) = \psi(x,y) = \varepsilon(\|x\|^{p_1}\|y\|^{p_2})$ for all $x, y \in A$; then

$$\begin{aligned}
|p|^{-1}\varphi(px,py) &= |p|^{p_1+p_2-1}\varepsilon(\|x\|^{p_1}\|y\|^{p_2}),\\
|p|^{-1}\varphi(px,y) &= |p|^{p_1-1}\varepsilon(\|x\|^{p_1}\|y\|^{p_2}),\\
|p|^{-1}\varphi(x,py) &= |p|^{p_2-1}\varepsilon(\|x\|^{p_1}\|y\|^{p_2}).
\end{aligned} \tag{2.31}$$

Put

$$\begin{aligned}
L = K &= \max\left\{|p|^{p_1-1}, |p|^{p_2-1}, |p|^{p_1+p_2-1}\right\}\\
&= \max\left\{p^{1-p_1}, p^{1-p_2}, p^{1-p_1-p_2}\right\}.
\end{aligned} \tag{2.32}$$

So, by Theorem 2.1, f is a generalized (θ,ϕ)-derivation and g is a (θ,ϕ)-derivation. □

Corollary 2.4. *Let A be a non-Archimedean Banach algebra over $\mathbb{Q}_p$, $\varepsilon > 0$, and let $p_1, p_2, p_1+p_2 \in (-\infty, 1)$. Suppose that*

$$\begin{aligned}
\|f(x+y) - f(x) - f(y)\| &\le \varepsilon(\|x\|^{p_1}\|y\|^{p_2}),\\
\|f(xy) - xf(y) - g(x)y\| &\le \varepsilon(\|x\|^{p_1}\|y\|^{p_2}),
\end{aligned} \tag{2.33}$$

for all $x, y \in A$. Then f is a generalized (θ,ϕ)-derivation and g is a (θ,ϕ)-derivation.

Proof. Let $\varphi(x,y) = \psi(x,y) = \varepsilon(\|x\|^{p_1}\|y\|^{p_2})$ for all $x, y \in A$, then

$$\begin{aligned}
|p|\varphi\left(p^{-1}x, p^{-1}y\right) &= |p|^{1-p_1-p_2}\ \varepsilon(\|x\|^{p_1}\|y\|^{p_2}),\\
|p|\varphi\left(p^{-1}x, y\right) &= |p|^{1-p_1}\varepsilon(\|x\|^{p_1}\|y\|^{p_2}),\\
|p|\varphi\left(x, p^{-1}y\right) &= |p|^{1-p_2}\varepsilon(\|x\|^{p_1}\|y\|^{p_2}).
\end{aligned} \tag{2.34}$$

Put

$$\begin{aligned}
L = K &= \max\left\{|p|^{1-p_1}, |p|^{1-p_2}, |p|^{1-p_1-p_2}\right\}\\
&= \max\left\{p^{p_1-1}, p^{p_2-1}, p^{p_1+p_2-1}\right\}.
\end{aligned} \tag{2.35}$$

So, by Theorem 2.2, f is a generalized (θ,ϕ)-derivation and g is a (θ,ϕ)-derivation. □

Similarly, we can obtain the following results.

Corollary 2.5. *Let A be a non-Archimedean Banach's algebra over $\mathbb{Q}_p$, $\varepsilon > 0$, $\delta > 0$, and let $p_1, p_2 \in (1, \infty)$. Suppose that*

$$\begin{aligned} \|f(x+y) - f(x) - f(y)\| &\leq \varepsilon\left(\|x\|^{p_1} + \|y\|^{p_2}\right), \\ \|f(xy) - xf(y) - g(x)y\| &\leq \delta\left(\|x\|^{p_1}\|y\|^{p_2}\right), \end{aligned} \tag{2.36}$$

for all $x, y \in A$. Then f is a generalized (θ, ϕ)-derivation and g is a (θ, ϕ)-derivation.

Corollary 2.6. *Let A be a non-Archimedean Banach's algebra over $\mathbb{Q}_p$, $\varepsilon > 0$, $\delta > 0$, and let $p_1, p_2 \in (1, \infty)$. Suppose that*

$$\begin{aligned} &\max\{\|f(x+y) - f(x) - f(y)\|, \|f(xy) - xf(y) - g(x)y\|\} \\ &\quad \leq \varepsilon \min\{\left(\|x\|^{p_1} + \|y\|^{p_2}\right), \|x\|^{p_1}\|y\|^{p_2}\}, \end{aligned} \tag{2.37}$$

for all $x, y \in A$. Then f is a generalized (θ, ϕ)-derivation and g is a (θ, ϕ)-derivation.

Corollary 2.7. *Let A be a non-Archimedean Banach's algebra over $\mathbb{Q}_p$, $\varepsilon > 0$, $\delta > 0$, and let $p_1, p_2, p_1 + p_2 \in (-\infty, 1)$. Suppose that*

$$\begin{aligned} \|f(x+y) - f(x) - f(y)\| &\leq \varepsilon\left(\|x\|^{p_1} + \|y\|^{p_2}\right), \\ \|f(xy) - xf(y) - g(x)y\| &\leq \delta\left(\|x\|^{p_1}\|y\|^{p_2}\right), \end{aligned} \tag{2.38}$$

for all $x, y \in A$. Then f is a generalized (θ, ϕ)-derivation and g is a (θ, ϕ)-derivation.

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