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Research Article

A New Fractional Subequation Method and Its Applications for Space-Time Fractional Partial Differential Equations

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A new fractional subequation method is proposed for finding exact solutions for fractional partial differential equations (FPDEs). The fractional derivative is defined in the sense of modified Riemann-Liouville derivative. As applications, abundant exact solutions including solitary wave solutions as well as periodic wave solutions for the space-time fractional generalized Hirota-Satsuma coupled KdV equations are obtained by using this method.

1. Introduction

Fractional differential equations are generalizations of classical differential equations of integer order. Recently, fractional differential equations have been the focus of many studies due to their frequent appearance in various applications in physics, biology, engineering, signal processing, systems identification, control theory, finance, and fractional dynamics. Among the investigations for fractional differential equations, research for seeking exact solutions and approximate solutions of fractional differential equations is a hot topic. New exact solutions for fractional differential equations may help to understand better corresponding nonlinear wave phenomena they describe. Some powerful methods have been proposed so far (e.g., see [1–12]). Using these methods, a variety of fractional differential equations have been investigated.

In this paper, we propose a new fractional subequation method to establish exact solutions for fractional partial differential equations (FPDEs), which is based on the following fractional ordinary differential equation:

$$D_\xi^{2\alpha} G(\xi) + \lambda D_\xi^{\alpha} G(\xi) + \mu G(\xi) = 0, \quad 0 < \alpha \leq 1, \tag{1}$$

where $D_\xi^{\alpha} G(\xi)$ denotes the modified Riemann-Liouville derivative of order α for $G(\xi)$ with respect to ξ.

The definition and some important properties for Jumarie's modified Riemann-Liouville derivative of order α are listed as follows (see [13–16]):

$$D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dt} \displaystyle\int_0^t (t-\xi)^{-\alpha} \left(f(\xi) - f(0) \right) d\xi, & 0 < \alpha < 1, \\ \left(f^{(n)}(t) \right)^{(\alpha - n)}, & n \leq \alpha < n+1, \ n \geq 1, \end{cases} \tag{2}$$

$$D_t^{\alpha} t^r = \frac{\Gamma(1+r)}{\Gamma(1+r-\alpha)} t^{r-\alpha}, \tag{3}$$

$$D_t^{\alpha} \left(f(t) g(t) \right) = g(t) D_t^{\alpha} f(t) + f(t) D_t^{\alpha} g(t), \tag{4}$$

$$D_t^{\alpha} f[g(t)] = f_g'[g(t)] D_t^{\alpha} g(t) = D_g^{\alpha} f[g(t)] \left(g'(t) \right)^{\alpha}. \tag{5}$$

We organize this paper as follows. In Section 2, we derive the expression for $D_\xi^{\alpha} G(\xi)/G(\xi)$ related to (1). In Section 3, we give the description of the fractional subequation method for solving FPDEs. Then in Section 4 we apply this method to establish exact solutions for the space-time fractional generalized Hirota-Satsuma coupled KdV equations. Some conclusions are presented at the end of the paper.

2. The General Expression for $D_{\xi}^{\alpha}G(\xi)/G(\xi)$

In order to obtain the general solutions for (1), we suppose $G(\xi) = H(\eta)$ and a nonlinear fractional complex transformation $\eta = \xi^{\alpha}/\Gamma(1+\alpha)$. Then, by (3) and the first equality in (5), (1) can be turned into the following second ordinary differential equation:

$$H''(\eta) + \lambda H'(\eta) + \mu H(\eta) = 0. \tag{6}$$

By the general solutions of (6), we have

$$\frac{H'(\eta)}{H(\eta)} = \begin{cases} -\dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2-4\mu}}{2}\left(\dfrac{C_1\sinh\left(\sqrt{\lambda^2-4\mu}/2\right)\eta + C_2\cosh\left(\sqrt{\lambda^2-4\mu}/2\right)\eta}{C_1\cosh\left(\sqrt{\lambda^2-4\mu}/2\right)\eta + C_2\sinh\left(\sqrt{\lambda^2-4\mu}/2\right)\eta}\right), & \lambda^2-4\mu>0, \\ -\dfrac{\lambda}{2} + \dfrac{\sqrt{4\mu-\lambda^2}}{2}\left(\dfrac{-C_1\sin\left(\sqrt{4\mu-\lambda^2}/2\right)\eta + C_2\cos\left(\sqrt{4\mu-\lambda^2}/2\right)\eta}{C_1\cos\left(\sqrt{4\mu-\lambda^2}/2\right)\eta + C_2\sin\left(\sqrt{4\mu-\lambda^2}/2\right)\eta}\right), & \lambda^2-4\mu<0, \\ -\dfrac{\lambda}{2} + \dfrac{C_2}{C_1+C_2\eta}, & \lambda^2-4\mu=0, \end{cases} \tag{7}$$

where C_1 and C_2 are arbitrary constants.

Since $D_{\xi}^{\alpha}G(\xi) = D_{\xi}^{\alpha}H(\eta) = H'(\eta)D_{\xi}^{\alpha}\eta = H'(\eta)$, we obtain

$$\frac{D_{\xi}^{\alpha}G(\xi)}{G(\xi)} = \begin{cases} -\dfrac{\lambda}{2} + \dfrac{\sqrt{\lambda^2-4\mu}}{2}\left(\dfrac{C_1\sinh\left(\sqrt{\lambda^2-4\mu}/2\Gamma(1+\alpha)\right)\xi^{\alpha} + C_2\cosh\left(\sqrt{\lambda^2-4\mu}/2\Gamma(1+\alpha)\right)\xi^{\alpha}}{C_1\cosh\left(\sqrt{\lambda^2-4\mu}/2\Gamma(1+\alpha)\right)\xi^{\alpha} + C_2\sinh\left(\sqrt{\lambda^2-4\mu}/2\Gamma(1+\alpha)\right)\xi^{\alpha}}\right), & \lambda^2-4\mu>0, \\ -\dfrac{\lambda}{2} + \dfrac{\sqrt{4\mu-\lambda^2}}{2}\left(\dfrac{-C_1\sin\left(\sqrt{4\mu-\lambda^2}/2\Gamma(1+\alpha)\right)\xi^{\alpha} + C_2\cos\left(\sqrt{4\mu-\lambda^2}/2\Gamma(1+\alpha)\right)\xi^{\alpha}}{C_1\cos\left(\sqrt{4\mu-\lambda^2}/2\Gamma(1+\alpha)\right)\xi^{\alpha} + C_2\sin\left(\sqrt{4\mu-\lambda^2}/2\Gamma(1+\alpha)\right)\xi^{\alpha}}\right), & \lambda^2-4\mu<0, \\ -\dfrac{\lambda}{2} + \dfrac{C_2\Gamma(1+\alpha)}{C_1\Gamma(1+\alpha)+C_2\xi^{\alpha}}, & \lambda^2-4\mu=0. \end{cases} \tag{8}$$

3. Description of the Fractional Subequation Method

In this section, we give the main steps of the fractional subequation method for finding exact solutions for FPDEs.

Suppose that an FPDE, say in the independent variables $t, x_1, x_2, \ldots, x_n$, is given by

$$\begin{aligned} &P\left(u_1, \ldots, u_k, D_t^{\alpha}u_1, \ldots, D_t^{\alpha}u_k, D_{x_1}^{\alpha}u_1, \ldots,\right. \\ &\quad D_{x_1}^{\alpha}u_k, \ldots, D_{x_n}^{\alpha}u_1, \ldots, D_{x_n}^{\alpha}u_k, D_t^{2\alpha}u_1, \ldots, \\ &\quad \left. D_t^{2\alpha}u_k, D_{x_1}^{2\alpha}u_1, \ldots\right) = 0, \end{aligned} \tag{9}$$

where $u_i = u_i(t, x_1, x_2, \ldots, x_n)$, $i = 1, \ldots, k$, are unknown functions and P is a polynomial in u_i and their various partial derivatives including fractional derivatives.

Step 1. Suppose that

$$\begin{aligned} u_i(t, x_1, x_2, \ldots, x_n) &= U_i(\xi), \\ \xi &= ct + k_1x_1 + k_2x_2 + \cdots + k_nx_n + \xi_0. \end{aligned} \tag{10}$$

Then, by the second equality in (5), (9) can be turned into the following fractional ordinary differential equation with respect to the variable ξ:

$$\begin{aligned} &\widetilde{P}\left(U_1, \ldots, U_k, c^{\alpha}D_{\xi}^{\alpha}U_1, \ldots, c^{\alpha}D_{\xi}^{\alpha}U_k,\right. \\ &\quad k_1^{\alpha}D_{\xi}^{\alpha}U_1, \ldots, k_1^{\alpha}D_{\xi}^{\alpha}U_k, \ldots, k_n^{\alpha}D_{\xi}^{\alpha}U_1, \ldots, \\ &\quad k_n^{\alpha}D_{\xi}^{\alpha}U_k, c^{2\alpha}D_{\xi}^{2\alpha}U_1, \ldots, \\ &\quad \left. c^{2\alpha}D_{\xi}^{2\alpha}U_k, k_1^{2\alpha}D_{\xi}^{2\alpha}U_1, \ldots\right) = 0. \end{aligned} \tag{11}$$

Step 2. Suppose that the solution of (11) can be expressed by a polynomial in $(D_\xi^\alpha G/G)$ as follows:

$$U_j(\xi) = a_{j,0} + \sum_{i=1}^{m_j}\left[a_{j,i}\left(\frac{D_\xi^\alpha G}{G}\right)^i + b_{j,i}\left(\frac{D_\xi^\alpha G}{G}\right)^{i-1}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)}\right], \quad j = 1,2,\ldots,k, \tag{12}$$

where $G = G(\xi)$ satisfies (1), σ is a constant, and $a_{j,i}$, $i = 0,1,\ldots,m$, $j = 1,2,\ldots,k$, are constants to be determined later. The positive integer m can be determined by considering the homogeneous balance between the highest-order derivatives and nonlinear terms appearing in (11).

Step 3. Substituting (12) into (11), using (1), and collecting all terms with the same order of $(D_\xi^\alpha G/G)\sqrt{\sigma(1+(1/\mu)(D_\xi^\alpha G/G)^2)}$ together, the left-hand side of (11) is converted into another polynomial in $(D_\xi^\alpha G/G)$. Equating each coefficient of this polynomial to zero yields a set of algebraic equations for $a_{j0}, a_{j,i}, b_{j,i}$, $i = 1,\ldots,m$, $j = 1,2,\ldots,k$.

Step 4. Solving the equations in Step 3 and using (8), we can construct a variety of exact solutions for (9).

4. Application of the Method to Space-Time Fractional Generalized Hirota-Satsuma Coupled KdV Equations

In this section, we will apply the described method in Section 3 to solve the space-time fractional generalized Hirota-Satsuma coupled KdV equations [15, 16]:

$$\begin{gathered} D_t^\alpha u - \frac{1}{2}D_x^{3\alpha}u + 3uD_x^\alpha u - 3D_x^\alpha(vw) = 0, \\ D_t^\alpha v + D_x^{3\alpha}v - 3uD_x^\alpha v = 0, \\ D_t^\alpha w + D_x^{3\alpha}w - 3uD_x^\alpha w = 0, \\ 0 < \alpha \le 1. \end{gathered} \tag{13}$$

Equations (13) can be used to describe the interaction of two long waves with different dispersion relations [17]. In [15], the authors solved equations (13) by a proposed fractional subequation method based on the fractional Riccati equation, while in [16], (13) are solved by the known (G'/G)-expansion method. Now we apply the described method in Section 3 to solve (13). To begin with, suppose that $u(x,t) = U(\xi)$, $v(x,t) = V(\xi)$, $w(x,t) = W(\xi)$, where $\xi = kx + ct + \xi_0$, k, c, ξ_0 are all constants with $k, c \neq 0$. Then, by usinge the second equality in (4), we obtain

$$\begin{aligned} D_x^\alpha u &= D_x^\alpha U(\xi) = \left(D_\xi^\alpha U\right)\left(\xi_x'\right)^\alpha = k^\alpha D_\xi^\alpha U, \\ D_t^\alpha u &= D_t^\alpha U(\xi) = \left(D_\xi^\alpha U\right)\left(\xi_t'\right)^\alpha = c^\alpha D_\xi^\alpha U, \end{aligned} \tag{14}$$

and similarly we have

$$\begin{aligned} D_x^\alpha v &= k^\alpha D_\xi^\alpha V, \qquad D_t^\alpha v = c^\alpha D_\xi^\alpha V, \\ D_x^\alpha w &= k^\alpha D_\xi^\alpha W, \qquad D_t^\alpha w = c^\alpha D_\xi^\alpha W; \end{aligned} \tag{15}$$

then (11) can be turned into the following fractional ordinary differential equations with respect to the variable ξ:

$$\begin{gathered} c^\alpha D_\xi^\alpha U - \frac{1}{2}k^{3\alpha}D_\xi^{3\alpha}U + 3k^\alpha U D_\xi^\alpha U - 3k^\alpha D_\xi^\alpha(VW) = 0, \\ c^\alpha D_\xi^\alpha V + k^{3\alpha}D_\xi^{3\alpha}V - 3k^\alpha U D_\xi^\alpha V = 0, \\ c^\alpha D_\xi^\alpha W + k^{3\alpha}D_\xi^{3\alpha}W - 3k^\alpha U D_\xi^\alpha W = 0. \end{gathered} \tag{16}$$

Suppose that the solutions of (16) can be expressed by

$$\begin{aligned} U(\xi) &= a_0 + \sum_{i=1}^{m_1}\left[a_i\left(\frac{D_\xi^\alpha G}{G}\right)^i + b_i\left(\frac{D_\xi^\alpha G}{G}\right)^{i-1}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)}\right], \\ V(\xi) &= c_0 + \sum_{i=1}^{m_2}\left[c_i\left(\frac{D_\xi^\alpha G}{G}\right)^i + d_i\left(\frac{D_\xi^\alpha G}{G}\right)^{i-1}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)}\right], \\ W(\xi) &= e_0 + \sum_{i=1}^{m_3}\left[e_i\left(\frac{D_\xi^\alpha G}{G}\right)^i + f_i\left(\frac{D_\xi^\alpha G}{G}\right)^{i-1}\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)}\right]. \end{aligned} \tag{17}$$

Balancing the order of $D_\xi^{3\alpha}U$ and $D_\xi^\alpha(VW)$, $D_\xi^{3\alpha}V$ and $UD_\xi^\alpha V$, and $D_\xi^{3\alpha}W$ and $UD_\xi^\alpha W$ in (16), we have $m_1 = m_2 = m_3 = 2$. So

$$U(\xi) = a_0 + a_1\left(\frac{D_\xi^\alpha G}{G}\right) + a_2\left(\frac{D_\xi^\alpha G}{G}\right)^2 + b_1\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)} + b_2\left(\frac{D_\xi^\alpha G}{G}\right)\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)},$$

$$V(\xi) = c_0 + c_1\left(\frac{D_\xi^\alpha G}{G}\right) + c_2\left(\frac{D_\xi^\alpha G}{G}\right)^2 + d_1\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)} + d_2\left(\frac{D_\xi^\alpha G}{G}\right)\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)},$$

$$W(\xi) = e_0 + e_1\left(\frac{D_\xi^\alpha G}{G}\right) + e_2\left(\frac{D_\xi^\alpha G}{G}\right)^2 + f_1\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)} + f_2\left(\frac{D_\xi^\alpha G}{G}\right)\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{D_\xi^\alpha G}{G}\right)^2\right)}. \quad (18)$$

Substituting (18) into (16), using (1), and collecting all the terms with the same power of $(D_\xi^\alpha G/G)$ $\sqrt{\sigma(1+(1/\mu)(D_\xi^\alpha G/G)^2)}$ together, equating each coefficient to zero yields a set of algebraic equations. Solving these equations, with the aid of the mathematical software Maple, yields the following seven groups of values.

Case 1. One has

$$\mu = \frac{1}{4}k^{-4\alpha}\sigma b_2^2, \quad \lambda = 0, \quad a_0 = \frac{1}{3}k^{-\alpha}c^\alpha + \frac{5}{12}k^{-2\alpha}\sigma b_2^2,$$
$$a_1 = 0, \quad a_2 = 2k^{2\alpha}, \quad b_1 = 0, \quad b_2 = b_2,$$
$$c_0 = \frac{1}{24}\frac{b_2\left(16k^{-\alpha}c^\alpha f_2 + 5k^{-2\alpha}f_2\sigma b_2^2 - 6e_0 b_2\right)}{f_2^2}, \quad c_1 = 0,$$
$$c_2 = \frac{k^{2\alpha}b_2}{2f_2}, \quad d_1 = 0, \quad d_2 = \frac{b_2^2}{4f_2}, \quad e_0 = e_0,$$
$$e_1 = 0, \quad e_2 = \frac{2k^{2\alpha}f_2}{b_2}, \quad f_1 = 0, \quad f_2 = f_2. \quad (19)$$

Case 2. One has

$$\mu = \mu, \quad \lambda = \frac{1}{4}k^{-2\alpha}a_1,$$
$$a_0 = \frac{1}{48}k^{-2\alpha}a_1^2 + \frac{8}{3}k^{2\alpha}\mu + \frac{1}{3}k^{-\alpha}c^\alpha,$$
$$a_1 = a_1, \quad a_2 = 4k^{2\alpha}, \quad b_1 = 0, \quad b_2 = 0,$$
$$c_0 = \frac{1}{96}c_2\left(k^{-4\alpha}a_1^2 + 128\mu + 64c^\alpha k^{-3\alpha} - 24k^{-4\alpha}c_2 e_0\right),$$
$$c_1 = \frac{1}{4}c_2 k^{-2\alpha}a_1, \quad c_2 = c_2, \quad d_1 = 0,$$
$$d_2 = 0, \quad e_0 = e_0, \quad e_1 = \frac{k^{2\alpha}a_1}{c_2},$$
$$e_2 = \frac{4k^{4\alpha}}{c_2}, \quad f_1 = 0, \quad f_2 = 0. \quad (20)$$

Case 3. One has

$$\mu = \frac{1}{16}k^{-4\alpha}a_1^2 + c^\alpha k^{-3\alpha} - \frac{3}{4}k^{-4\alpha}c_1 e_1,$$
$$\lambda = \frac{1}{2}k^{-2\alpha}a_1,$$
$$a_0 = \frac{1}{8}k^{-2\alpha}a_1^2 + k^{-\alpha}c^\alpha - \frac{1}{2}k^{-2\alpha}c_1 e_1,$$
$$a_1 = a_1, \quad a_2 = 2k^{2\alpha}, \quad b_1 = 0,$$
$$b_2 = 0, \quad c_0 = c_0, \quad c_1 = c_1, \quad (21)$$
$$c_2 = 0, \quad d_1 = 0, \quad d_2 = 0,$$
$$e_0 = \frac{e_1\left(k^{-2\alpha}a_1 c_1 - 2c_0\right)}{2c_1}, \quad e_1 = e_1,$$
$$e_2 = 0, \quad f_1 = 0, \quad f_2 = 0.$$

Case 4. One has

$$\mu = k^{-4\alpha}\sigma b_2^2, \quad \lambda = 0,$$
$$a_0 = \frac{2}{3}k^{-2\alpha}\sigma b_2^2 + \frac{1}{3}k^{-\alpha}c^\alpha, \quad a_1 = 0,$$
$$a_2 = k^{2\alpha}, \quad b_1 = 0, \quad b_2 = b_2,$$
$$c_0 = c_0, \quad c_1 = \frac{k^{2\alpha}d_1}{b_2}, \quad c_2 = 0,$$
$$d_1 = d_1, \quad d_2 = 0, \quad (22)$$
$$e_0 = \frac{c_0 b_2^2\left(k^{-4\alpha}\sigma b_2^2 - 4c^\alpha k^{-3\alpha}\right)}{12d_1^2},$$
$$e_1 = \frac{b_2\left(-k^{-2\alpha}\sigma b_2^2 + 4k^{-\alpha}c^\alpha\right)}{12d_1}, \quad e_2 = 0,$$
$$f_1 = -\frac{b_2^2\left(k^{-4\alpha}\sigma b_2^2 - 4c^\alpha k^{-3\alpha}\right)}{12d_1}, \quad f_2 = 0.$$

Case 5. One has

$$\mu = 4c^\alpha k^{-3\alpha}, \quad \lambda = 0, \quad a_0 = 3k^{-\alpha}c^\alpha,$$
$$a_1 = 0, \quad a_2 = k^{2\alpha}, \quad b_1 = 0,$$

$$b_2 = \pm k^{-\alpha}\sqrt{\frac{c^{\alpha}k^{3\alpha}}{\sigma}}, \quad c_0 = 0, \quad c_1 = 0,$$
$$c_2 = 0, \quad d_1 = 0, \quad d_2 = 0, \quad e_0 = e_0,$$
$$e_1 = \pm\frac{k^{3\alpha} f_1}{2}\sqrt{\frac{\sigma}{c^{\alpha}k^{3\alpha}}}, \quad e_2 = 0,$$
$$f_1 = f_1, \quad f_2 = 0. \tag{23}$$

Case 6. One has

$$\mu = \mu, \quad \lambda = 0, \quad a_0 = \frac{2}{3}k^{2\alpha}\mu + \frac{1}{3}k^{-\alpha}c^{\alpha},$$
$$a_1 = 0, \quad a_2 = 2k^{2\alpha}, \quad b_1 = 0,$$
$$b_2 = 0, \quad c_0 = c_0, \quad c_1 = c_1,$$
$$c_2 = 0, \quad d_1 = 0, \quad d_2 = 0,$$
$$e_0 = -\frac{4c_0\left(-k^{4\alpha}\mu + k^{\alpha}c^{\alpha}\right)}{3c_1^2},$$
$$e_1 = \frac{4\left(-k^{4\alpha}\mu + k^{\alpha}c^{\alpha}\right)}{3c_1},$$
$$e_2 = 0, \quad f_1 = 0, \quad f_2 = 0. \tag{24}$$

Case 7. One has

$$\mu = \mu, \quad \lambda = 0, \quad a_0 = \frac{5}{3}k^{2\alpha}\mu + \frac{1}{3}k^{-\alpha}c^{\alpha},$$
$$a_1 = 0, \quad a_2 = 2k^{2\alpha}, \quad b_1 = 0,$$
$$b_2 = 0, \quad c_0 = c_0, \quad c_1 = 0,$$
$$c_2 = 0, \quad d_1 = d_1, \quad d_2 = 0,$$
$$e_0 = -\frac{2c_0\mu\left(k^{4\alpha}\mu + 2k^{\alpha}c^{\alpha}\right)}{3\sigma d_1^2},$$
$$e_1 = 0, \quad e_2 = 0,$$
$$f_1 = \frac{2\mu\left(k^{4\alpha}\mu + 2k^{\alpha}c^{\alpha}\right)}{3\left(\sigma d_1\right)}, \quad f_2 = 0. \tag{25}$$

Substituting the previous results into (18) and combining with (8), we can obtain a series of exact solutions for (13).

From Case 1, we obtain the following exact solutions. When $\mu < 0$,

$$U_1(\xi) = \frac{1}{3}k^{-\alpha}c^{\alpha} + \frac{5}{12}k^{-2\alpha}\sigma b_2^2 - 2k^{2\alpha}\mu \times \left[\left(C_1\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right)^{-1}\right]^2$$
$$+ b_2\sqrt{-\mu}\left[\left(C_1\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right)^{-1}\right]$$
$$\times\left(\sigma\left\{1 - \left[\left(C_1\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right)^{-1}\right]^2\right\}\right)^{1/2}, \tag{26}$$

$$V_1(\xi) = \frac{1}{24}\frac{b_2\left(16k^{-\alpha}c^{\alpha}f_2 + 5k^{-2\alpha}f_2\sigma b_2^2 - 6e_0b_2\right)}{f_2^2} - \frac{k^{2\alpha}b_2}{2f_2}\mu\left[\left(C_1\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right)^{-1}\right]^2$$
$$+ \frac{b_2^2}{4f_2}\sqrt{-\mu}\left[\left(C_1\sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1\cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right.\right.$$

$$+ C_2 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\Bigg)^{-1}\Bigg] \times \left(\sigma\left\{1 - \left[\left(C_1 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2 \cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1 \cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right)^{-1}\right]^2\right\}\right)^{1/2}, \quad (27)$$

$$W_1(\xi) = e_0 - \frac{2k^{2\alpha} f_2}{b_2}\mu\left[\left(C_1 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2 \cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1 \cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right)^{-1}\right]^2$$
$$+ f_2\sqrt{-\mu}\left[\left(C_1 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2 \cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1 \cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right)^{-1}\right]$$
$$\times \left(\sigma\left\{1 - \left[\left(C_1 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2 \cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right) \times \left(C_1 \cosh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right) + C_2 \sinh\left(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\right)\right)^{-1}\right]^2\right\}\right)^{1/2}, \quad (28)$$

where $\xi = kx + ct + \xi_0$, $\mu = (1/4)k^{-4\alpha}\sigma b_2^2$.

When $\mu > 0$,

$$U_2(\xi) = \frac{1}{3}k^{-\alpha}c^{\alpha} + \frac{5}{12}k^{-2\alpha}\sigma b_2^2 + 2k^{2\alpha}\mu \times \left[\left(-C_1 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha} + C_2 \cos\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right) \times \left(C_1 \cos\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha} + C_2 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right)^{-1}\right]^2$$
$$+ b_2\sqrt{\mu}\left[\left(-C_1 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha} + C_2 \cos\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right) \times \left(C_1 \cos\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha} + C_2 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right)^{-1}\right]$$
$$\times \left(\sigma\left\{1 + \left[\left(-C_1 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha} + C_2 \cos\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right) \times \left(C_1 \cos\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha} + C_2 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right)^{-1}\right]^2\right\}\right)^{1/2}, \quad (29)$$

$$V_2(\xi) = \frac{1}{24}\frac{b_2\left(16k^{-\alpha}c^{\alpha}f_2 + 5k^{-2\alpha}f_2\sigma b_2^2 - 6e_0 b_2\right)}{f_2^2} + \frac{k^{2\alpha}b_2}{2f_2}\mu\left[\left(-C_1 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha} + C_2 \cos\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right) \times \left(C_1 \cos\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha} + C_2 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right)^{-1}\right]^2$$
$$+ \frac{b_2^2}{4f_2}\sqrt{\mu}\left[\left(-C_1 \sin\left(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\right)\xi^{\alpha}\right.\right.$$

$$
\begin{aligned}
&+C_2 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr) \\
&\times\Bigl(C_1 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+C_2 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr)^{-1}\Bigr] \\
&\times\Bigl(\sigma\Bigl\{1+\Bigl[\Bigl(-C_1 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+C_2 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr) \\
&\times\Bigl(C_1 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+C_2 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr)^{-1}\Bigr]^{2}\Bigr\}\Bigr)^{1/2},
\end{aligned}
\tag{30}
$$

$$
\begin{aligned}
W_2(\xi) = e_0 &+ \frac{2k^{2\alpha} f_2}{b_2}\mu \\
&\times\Bigl[\Bigl(-C_1 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+C_2 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr) \\
&\times\Bigl(C_1 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+C_2 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr)^{-1}\Bigr]^{2} \\
&+ f_2\sqrt{\mu}\Bigl[\Bigl(-C_1 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+C_2 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr) \\
&\times\Bigl(C_1 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+C_2 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr)^{-1}\Bigr] \\
&\times\Bigl(\sigma\Bigl\{1+\Bigl[\Bigl(-C_1 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+ C_2 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr) \\
&\times\Bigl(C_1 \cos\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha} \\
&+C_2 \sin\Bigl(\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\Bigr)\xi^{\alpha}\Bigr)^{-1}\Bigr]^{2}\Bigr\}\Bigr)^{1/2},
\end{aligned}
\tag{31}
$$

where $\xi = kx + ct + \xi_0$ and $\mu = (1/4)k^{-4\alpha}\sigma b_2^2$.

In particular, if we let $C_2 = 0$ in (26)–(28), then we obtain the following solitary wave solutions, which are shown in Figures 1, 2, and 3:

$$
\begin{aligned}
U_3(\xi) = {}& \frac{1}{3}k^{-\alpha}c^{\alpha} + \frac{5}{12}k^{-2\alpha}\sigma b_2^2 \\
&- 2k^{2\alpha}\mu\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr]^2 \\
&+ b_2\sqrt{-\mu}\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr] \\
&\times\sqrt{\sigma\Bigl\{1-\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr]^2\Bigr\}},
\end{aligned}
\tag{32}
$$

$$
\begin{aligned}
V_3(\xi) = {}& \frac{1}{24}\frac{b_2\left(16k^{-\alpha}c^{\alpha}f_2 + 5k^{-2\alpha}f_2\sigma b_2^2 - 6e_0 b_2\right)}{f_2^2} \\
&- \frac{k^{2\alpha}b_2}{2f_2}\mu\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr]^2 \\
&+ \frac{b_2^2}{4f_2}\sqrt{-\mu}\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr] \\
&\times\sqrt{\sigma\Bigl\{1-\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr]^2\Bigr\}},
\end{aligned}
\tag{33}
$$

$$
\begin{aligned}
W_3(\xi) = e_0 &- \frac{2k^{2\alpha}f_2}{b_2}\mu\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr]^2 \\
&+ f_2\sqrt{-\mu}\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr] \\
&\times\sqrt{\sigma\Bigl\{1-\Bigl[\tanh\Bigl(\frac{\sqrt{-\mu}\xi^{\alpha}}{\Gamma(1+\alpha)}\Bigr)\Bigr]^2\Bigr\}}.
\end{aligned}
\tag{34}
$$

If we let $C_2 = 0$ in (29)–(31), then we obtain the following periodic wave solutions, which are shown in Figures 4, 5, and 6:

$$
\begin{aligned}
U_4(\xi) = {}& \frac{1}{3}k^{-\alpha}c^{\alpha} + \frac{5}{12}k^{-2\alpha}\sigma b_2^2 \\
&+ 2k^{2\alpha}\mu\Bigl[\tan\frac{\sqrt{\mu}}{\Gamma(1+\alpha)}\xi^{\alpha}\Bigr]^2
\end{aligned}
$$

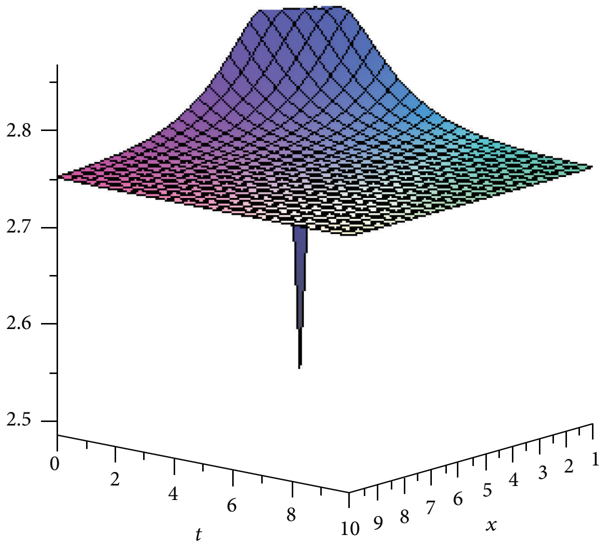

Figure 1: The solution U_3 in (32) with $\gamma = 4/5$, $k = c = \sigma = b_2 = 1$, $\mu = -1$, $\xi_0 = 0$.

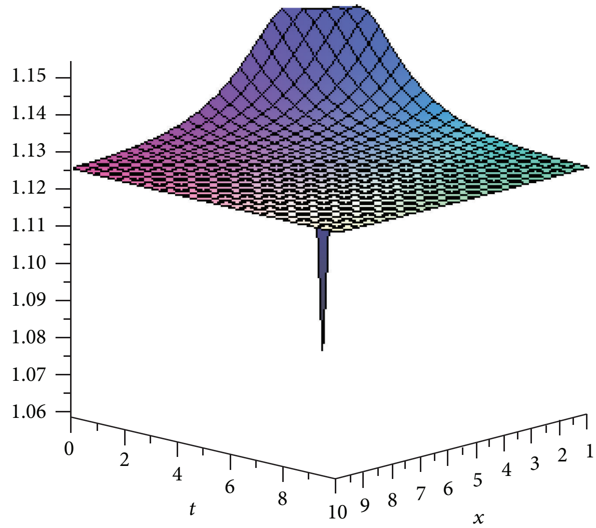

Figure 2: The solution V_3 in (33) with $\gamma = 4/5$, $k = c = \sigma = b_2 = f_2 = 1$, $\mu = -1$, $\xi_0 = 0$, $e_0 = 1$.

$$
\begin{aligned}
&- b_2 \sqrt{\mu} \left[\tan \frac{\sqrt{\mu}}{\Gamma(1+\alpha)} \xi^\alpha \right] \\
&\times \sqrt{\sigma \left\{ 1 + \left[\tan \frac{\sqrt{\mu}}{\Gamma(1+\alpha)} \xi^\alpha \right]^2 \right\}},
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
V_4(\xi) = {} & \frac{1}{24} \frac{b_2 \left(16 k^{-\alpha} c^\alpha f_2 + 5 k^{-2\alpha} f_2 \sigma b_2^2 - 6 e_0 b_2 \right)}{f_2^2} \\
&+ \frac{k^{2\alpha} b_2}{2 f_2} \mu \left[\tan \frac{\sqrt{\mu}}{\Gamma(1+\alpha)} \xi^\alpha \right]^2 \\
&- \frac{b_2^2}{4 f_2} \sqrt{\mu} \left[\tan \frac{\sqrt{\mu}}{\Gamma(1+\alpha)} \xi^\alpha \right] \\
&\times \sqrt{\sigma \left\{ 1 + \left[\tan \frac{\sqrt{\mu}}{\Gamma(1+\alpha)} \xi^\alpha \right]^2 \right\}},
\end{aligned}
\tag{36}
$$

$$
\begin{aligned}
W_4(\xi) = {} & e_0 + \frac{2 k^{2\alpha} f_2}{b_2} \mu \left[\tan \frac{\sqrt{\mu}}{\Gamma(1+\alpha)} \xi^\alpha \right]^2 \\
&- f_2 \sqrt{\mu} \left[\tan \frac{\sqrt{\mu}}{\Gamma(1+\alpha)} \xi^\alpha \right] \\
&\times \sqrt{\sigma \left\{ 1 + \left[\tan \frac{\sqrt{\mu}}{\Gamma(1+\alpha)} \xi^\alpha \right]^2 \right\}}.
\end{aligned}
\tag{37}
$$

Similar to the established solutions from Case 1, we can construct corresponding exact solutions to (13) from Cases 2–7, which are omitted here.

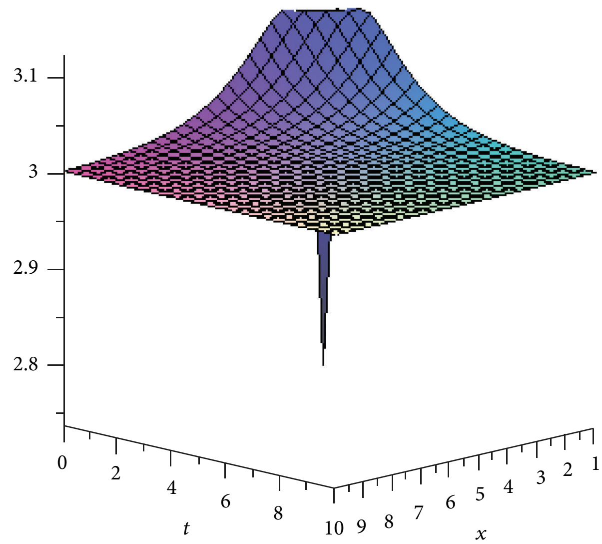

Figure 3: The solution W_3 in (34) with $\gamma = 4/5$, $k = c = \sigma = b_2 = f_2 = 1$, $\mu = -1$, $\xi_0 = 0$, $e_0 = 1$.

Remark 1. We note that the solutions obtained here are of new forms compared with the solutions obtained in [15, 16] since a fully new method is used here.

5. Conclusions

Based on the concept of the modified Riemann-Liouville derivative and a variable transformation $\xi = ct + k_1 x_1 + k_2 x_2 + \cdots + k_n x_n + \xi_0$, we have proposed a new fractional subequation method for solving fractional partial differential equations (FPDEs). By using this method, the space-time fractional generalized Hirota-Satsuma coupled KdV equations are solved successfully, and, as a result, some exact solutions are established, which may help to understand

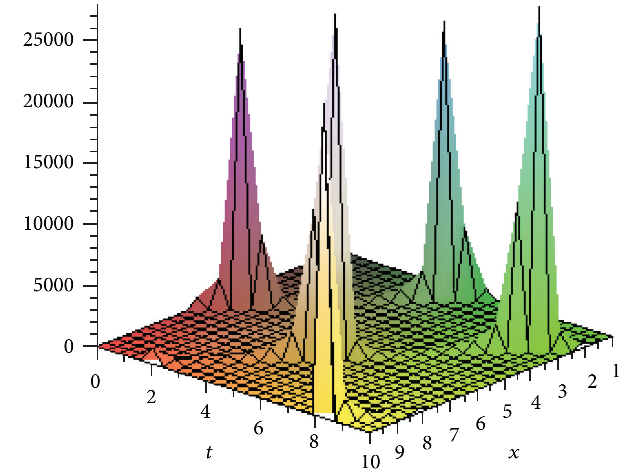

FIGURE 4: The solution U_4 in (35) with $\gamma = 4/5$, $k = c = \sigma = b_2 = 1$, $\mu = 1$, $\xi_0 = 0$.

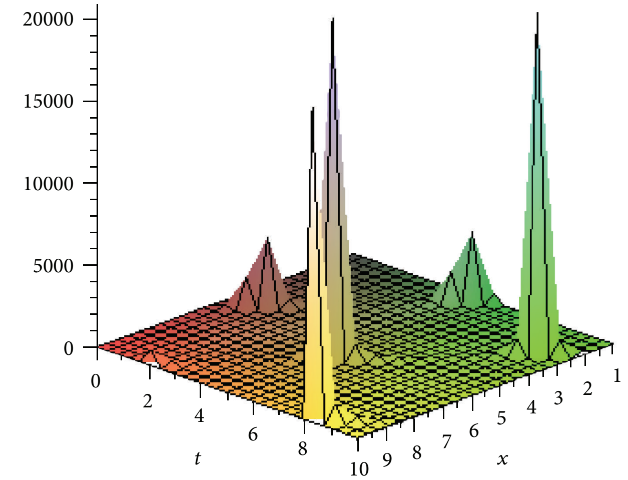

FIGURE 5: The solution V_4 in (36) with $\gamma = 4/5$, $k = c = \sigma = b_2 = f_2 = 1$, $\mu = 1$, $\xi_0 = 0$, $e_0 = 1$.

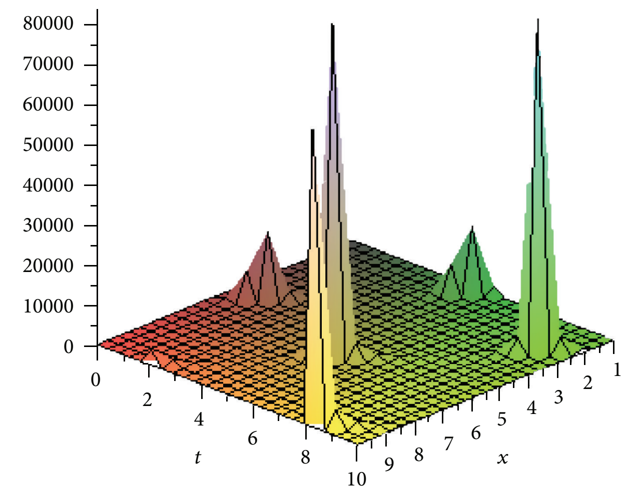

FIGURE 6: The solution W_4 in (37) with $\gamma = 4/5$, $k = c = \sigma = b_2 = f_2 = 1$, $\mu = -1$, $\xi_0 = 0$, $e_0 = 1$.

better the nonlinear wave phenomena. It is supposed that this method can be further applied to solve other FPDEs.

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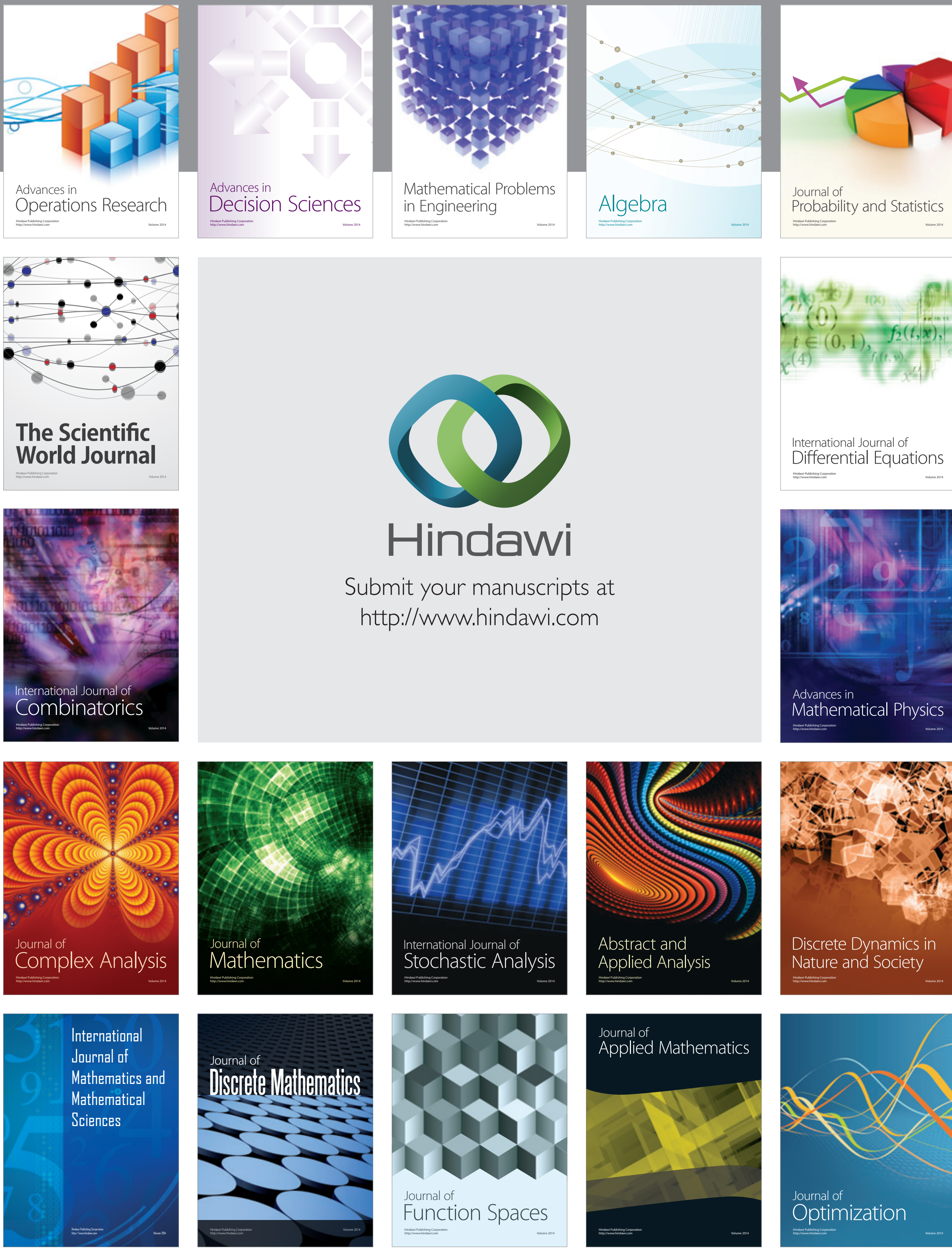
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