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Research Article

A Heuristic Approach for Determining Lot Sizes and Schedules Using Power-of-Two Policy

Esra Ekinci and Arslan M. Ornek

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We consider the problem of determining realistic and easy-to-schedule lot sizes in a multiproduct, multistage manufacturing environment. We concentrate on a specific type of production, namely, flow shop type production. The model developed consists of two parts, lot sizing problem and scheduling problem. In lot sizing problem, we employ binary integer programming and determine reorder intervals for each product using power-of-two policy. In the second part, using the results obtained of the lot sizing problem, we employ mixed integer programming to determine schedules for a multiproduct, multistage case with multiple machines in each stage. Finally, we provide a numerical example and compare the results with similar methods found in practice.

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1. Background and motivation

In practice, companies basically use single-stage models in order to find easy answers for production lot sizing and scheduling problem ignoring the multistage characteristics of their production facilities. However, multistage systems have received a considerable attention in the literature. Fundamental articles about deterministic demand condition were published early in the 1970s and the 1980s. Extensions of Harris' basic EOQ model which considered serial, assembly and distribution systems as well as more general system structures were developed for multistage systems. There are many important contributions with these extensions (see [1–7]). Another field of research in deterministic demand case is powers-of-two (PoT) policy solutions. Similar to the models mentioned in the above articles, this policy is also an extension of EOQ model (see [8–11]). Using this heuristic, reorder intervals are determined rather than lot sizes. And this policy requires replenishment frequency to be a power-of-two integer. Jackson et al. [8] also provided some interesting theoretical discussions on the optimal solution resulted from the PoT policy. That is, the optimal objective function value under PoT policy is within 6% error range.

There exist a number of extensions of PoT policy in literature. Capacitated version of the model is considered in [8] and joint replenishment problem in [12], whereas Roundy [13] and Muckstadt and Roundy [14] propose solutions for one-warehouse, multiretailer problem. Moreover, extensions on finite production rate and backlogging problems can be found in [15, 16]. Especially related with the economic lot-scheduling problem (ELSP), there exist a number of articles on the topic of cyclic schedules (see [17–20]). Cyclic schedules are a generalization of common cycle schedules in which the cycle time of a product is an integer multiple of a basic period. PoT policy on ELSP is a special type of cyclic schedules. Yao and Elmaghraby [21] studied a single-stage ELSP problem and later Lee and Yao [22] proposed a global optimum search method for the joint replenishment problem (JRP) under PoT policy. Ouenniche and Boctor [11] presented a new and efficient heuristic to solve multiproduct, multistage, economic lot-sizing problem where sequencing, lot sizing, and scheduling decisions are made for several products manufactured through several stages in a flow-shop environment so as to minimize the sum of setup and inventory holding costs while a given demand is fulfilled without backlogging. Ouenniche et al. [23], on the other hand, studied the impact of sequencing decision on total cost.

The objective of this study is to develop a heuristic approach to find reorder intervals based on PoT policy and consequently lot sizes for items to be produced, and then assign and schedule these production lots in a multistage, multimachine production environment. We consider a specific type of production, namely, flow shop type production which is a serially structured multistage system that exists in industries like paint production, food canning, or beverage bottling companies. These types of processes may have some technical restrictions. In addition, the waiting time of in-process inventories between stages should be minimal since waiting time can cause product deterioration.

We consider the problem in two parts: (i) reorder interval (lot size) determination and (ii) assignment and scheduling problem. In the first part, we solve a constrained ELSP assuming the production system as a single stage. The production rate is the rate of the bottleneck stage. Nonlinear structure of the model is modified to mixed-integer programming (MIP) form and MIP is solved to determine the reorder intervals as integer multiples of power-of-two multipliers by minimizing the objective function which consists of inventory and setup costs. Basically, the model proposed in the first part differs from the classical unconstrained ELSP model developed by Yao and Elmaghraby [21] in that it considers upper and lower limits on production lot sizes as well as lot-size-dependent setup costs in the formulation.

In the second part, the assignment and scheduling problem is solved using an MIP based on an approach developed in [11] but with the additional condition of multiple machines in each stage. An optimal schedule is achieved with the assumption of given production sequence. In practice, the production sequence is generally determined by some technical criteria.

The rest of the paper is organized as follows. Section 2 considers the model development, that is, determining optimum reorder intervals and lot sizes under PoT policy for multiproducts subject to operational constraints and then scheduling those lots in a multistage and multimachine environment. Then Section 3 presents a numerical example and compares the results obtained with similar methods found in practice. Finally, in Section 4, we present our conclusions.

2. The model development

Here, we first present the classical ELSP, then add technical constraints based on the specific production environment. Later, we develop the formulation considering the lot size dependent setup costs. In the second part, we assign and schedule these production lots to stages and machines using an MIP model. The model is designed for multistage, multiproduct problem, with multiple machines in each stage.

2.1. Determining optimum lot sizes under power-of-two policy. We define the notation as follows. Additional notation will be defined as necessary.

Indices:

(i) i = product index ($i = 1,\dots,n$),

(ii) y = container type ($y = 1,\dots,z$). A container is a receptacle for holding products.

Parameters:

(i) A_i = setup cost for product i,

(ii) A_{iy} = setup cost for product i dependent on container type y,

(iii) λ_i = demand rate for product i, constant and continuous,

(iv) h_i = holding cost per unit time for product i,

(v) p_i = finite production rate for product i,

(vi) s_i = setup time for product i,

(vii) $\rho_i = \lambda_i/p_i$,

(viii) C_y = upper-volume (lot-size) limit of container type y,

(ix) TL = technical limit showing the minimum lot amount for a machine to work effectively,

Decision variables:

(i) k_i = power-of-two multiplier for product i ($k_i = 1,2,4,\dots,2^{m_i}$; $m_i \in \{0,1,2,\dots\}$),

(ii) B = basic period,

(iii) x_{ij} = the valid power-of-two multiplier (2^j) for product i, $x_{ij} = \{0,1\}$.

The limits of the index j will be determined later.

The following assumptions are made in the first part of the model.

(1) No backlogging is allowed.

(2) The ordering decision for each product is given in every constant time interval which is a power-of-two (PoT) multiple of a basic period B, that is, k_iB.

(3) The basic period B may be specified as a shift, a day, or a week, and so forth.

(4) The finite production rate is that of the bottleneck stage.

Unconstrained economic lot-scheduling problem (ELSP) PoT problem formulated in [21] is as follows:

$$\text{Min}\, TC(B,\{k_i\}) = \sum_{i=1}^{n} \frac{A_i}{k_i B} + \frac{h_i}{2}\lambda_i(1-\rho_i)k_i B \tag{2.1a}$$

$$\text{subject to } k_i = 2^{m_i};\ m_i \in \{0,1,2,\dots\}. \tag{2.1b}$$

ELSP (PoT) is formulated as a nonlinear integer program. The objective function (2.1a) is the sum of average annual costs of setup and carrying inventory. In spite of the fact that the objective function (2.1a) is a nonlinear one, for a given value of B, the model can be restated as a linear binary integer programming (BIP) model. k_i and its reciprocal can be written in binary expansion [21]:

$$k_i = \sum_{j=0}^{v_i} 2^j x_{ij} = 2^0 x_{i0} + 2^1 x_{i1} + \cdots + 2^{v_i} x_{iv_i}, \tag{2.2}$$

$$k_i^{-1} = \sum_{j=0}^{v_i} 2^{-j} x_{ij} = 2^0 x_{i0} + 2^{-1} x_{i1} + \cdots + 2^{-v_i} x_{iv_i}, \tag{2.3}$$

$$\sum_{j=0}^{v_i} x_{ij} = 1, \quad \forall i, \tag{2.4}$$

where v_i is a nonnegative integer and $2^{v_i} * B$ is an upper bound on k_i.

We add upper and lower bound constraints on lot sizes to the above model related with the specific production type. For a single type of container, constrained ELSP (PoT) becomes

$$\text{Min}\, TC(B,\{k_i\}) = \sum_{i=1}^{n} \frac{A_i}{k_i B} + \frac{h_i}{2}\lambda_i(1-\rho_i)k_i B \tag{2.5a}$$

$$\text{subject to } k_i = 2^{m_i};\ m_i \in \{0,1,2,\dots\}, \tag{2.5b}$$

$$\lambda_i k_i B \geq TL, \quad \forall i, \tag{2.5c}$$

$$\lambda_i k_i B \leq C, \quad \forall i. \tag{2.5d}$$

In the above formulation, constraint (2.5c) indicates the minimum technical limit for effective working of the machine. In addition, constraint (2.5d) sets the upper limit for lot size according to the container restrictions. Using (2.2)–(2.4), the BIP model can be written in the following form:

$$\begin{aligned}
\operatorname{Min} TC &= \sum_{i=1}^{n} \frac{A_i}{B}\left(\sum_{j=0}^{v_i} 2^{-j} x_{ij}\right) + \sum_{i=1}^{n} \frac{h_i}{2}\lambda_i(1-\rho_i)\left(\sum_{j=0}^{v_i} 2^{j} x_{ij}\right)B && (2.6a)\\
&= \sum_{i=1}^{n}\sum_{j=0}^{v_i}\left\{\frac{A_i}{B}2^{-j}x_{ij} + \frac{h_i}{2}\lambda_i(1-\rho_i)2^{j}x_{ij}B\right\} && (2.6b)\\
&\text{subject to } \sum_{j=0}^{v_i} x_{ij} = 1 \quad \text{for } i = 0,1,\ldots,n, && (2.6c)\\
&\sum_{j=0}^{v_i} \lambda_i 2^{j} x_{ij} B \geq TL, \quad \forall i, && (2.6d)\\
&\sum_{j=0}^{v_i} \lambda_i 2^{j} x_{ij} B \leq C, \quad \forall i, && (2.6e)\\
&\text{where } x_{ij} \in \{0,1\}, \quad \forall i, j. && (2.6f)
\end{aligned}$$

However, the upper and lower limits for j have to be decided for each product. Simple bounds on j can be derived using constraints (2.6d) and (2.6e). Having determined the bounds on j, the constraints used can be eliminated from the formulation. Using constraint (2.6d), the lower bound can be set as follows:

$$\begin{gathered}
\text{when } x_{i,LB(i)} = 1,\\
\lambda_i 2^{LB(i)} B \geq TL,\\
2^{LB(i)} \geq \frac{TL}{\lambda_i B},
\end{gathered} \tag{2.7}$$

$$LB(i) \geq \log_2\left(\frac{TL}{\lambda_i B}\right), \tag{2.8}$$

where $LB(i)$ is the smallest integer that satisfies (2.8).

Similarly, using constraint (2.6e), the upper bound can be calculated:

$$UB(i) \leq \log_2\left(\frac{C}{\lambda_i B}\right), \tag{2.9}$$

where $UB(i)$ is the greatest integer that satisfies (2.9). Note that B values must be selected such that $UB(i) \geq 0$ and $LB(i) \geq 0$. Now the model becomes

$$\begin{gathered}
\operatorname{Min} TC = \sum_{i=1}^{n}\sum_{j=LB(i)}^{UB(i)}\left\{\frac{A_i}{B}2^{-j}x_{ij} + \frac{h_i}{2}\lambda_i(1-\rho_i)2^{j}x_{ij}B\right\}\\
\text{subject to } \sum_{j=LB(i)}^{UB(i)} x_{ij} = 1 \quad \text{for } i = 1,\ldots,n,\\
\text{where } x_{ij} \in \{0,1\}, \quad \forall i, j.
\end{gathered} \tag{2.10}$$

In case the setup costs vary with container sizes, we modify the objective function as follows. The lower limit (TL) is the same for all container sizes. But for the upper limits, each container has a different limit, that is, C_y. Hence, the upper bounds for different container sizes can be calculated using (2.9), that is, replacing C value by C_y values. Thus, the average annual setup cost term

$$\sum_{i=1}^{n} \sum_{j=LB(i)}^{UB(i)} \left\{ \frac{A_i}{B} 2^{-j} x_{ij} \right\} \tag{2.11}$$

for different containers now becomes

$$\sum_{i=1}^{n} \sum_{y=1}^{z} \left\{ \sum_{j=LB(i,y)}^{UB(i,y)} \left\{ \frac{A_{iy}}{B} 2^{-j} x_{ij} \right\} \right\}. \tag{2.12}$$

Another point to be emphasized is the minimum total cost functions which will be calculated for discrete B values. B would be multiples of a shift, a day, a week, or a month. Moreover, the search section of B—the highest and the lowest values—should be considered. Value found in common cycle approach is the upper bound on the value of B [21]:

$$T_{cc} = \max \left\{ \sqrt{\frac{2\sum_{i=1}^{n} A_i}{\sum_{i=1}^{n} h_i \lambda_i [1-\rho_i]}}, \frac{\sum_{i=1}^{n} s_i}{1-\sum_{i=1}^{n} \rho_i} \right\}. \tag{2.13}$$

A lower bound on the value of B can be obtained using rotation cycle policy [21]:

$$T_{rc} = \max_i \{(1+\rho_i) s_i\}. \tag{2.14}$$

This interval from the upper to lower bounds covers a feasible range of B. For example, feasible region can be between 1 day and 14-day length. Then the model can be run for basic periods of 1 day, 2 days and upto 14 days. Another simplification in this part is that, the basic periods that are power-of-two multiples of other feasible basic periods give the same optimal value for the model. Again in the same example of 1-day to 14-day feasible region of basic period, one can run the model for only 1-day, 3-day, 5-day, 7-day, 9-day, 11-day, 13-day basic period. The optimal value will be the same for 1, 2, 4, 8, and for 3, 6, 12, and for 5, 10 and similarly for 7, 14 days. The theorem given in [21] can be used to show that the program yields the same solution for a power-of-two multiple of a feasible basic period. However, in our case, there exist upper and lower limits for container types and when we change the basic period B to $2B$, then the lower bound decreases by 1 as follows:

$$\begin{aligned} LB(i) \geq \log_2\left(\frac{TL}{\lambda_i B}\right) &\Longrightarrow \log_2\left(\frac{TL}{\lambda_i 2B}\right) = \log_2\left(\frac{TL}{\lambda_i B}\right) - \log_2\left(\frac{1}{2}\right) = \log_2\left(\frac{TL}{\lambda_i B}\right) - 1 \\ &\Longrightarrow LB(i) - 1 \geq \log_2\left(\frac{TL}{\lambda_i B}\right) - 1. \end{aligned} \tag{2.15}$$

But, if one or more products have 0 as a lower limit for B, then the theorem cannot be used. However, none of the products can have lower limit as -1 for $2B$, since the reorder interval cannot be $2^{-1} * (2B)$. In this case, the limits of the containers should be calculated for $2B$ and the integer values greater than or equal to 0 should be chosen as j. Thus, the program should be executed for $2B$.

The results obtained in this part giving the minimum cost for a specific basic period will be used in the following section. An MIP is developed to assign and schedule the lots so found to a multistage multimachine production system. Here, we assume that the machines at each stage are identical and the production sequence is predetermined by some technical criteria.

2.2. Assigning and scheduling the lot-sizing problem in a multistage, multimachine environment. The lot sizes obtained in the first part will be assigned to basic periods, and for each basic period feasible schedules will be determined by an assignment procedure based on a method given in [11]; however, nonlinear program used in scheduling would be modified in order to support multimachine environment. Also, note that the production sequence of the jobs (products) is predetermined by the technical personnel.

Using the solutions of the first part, global cycle length becomes $T_G = K * B$, where $K = \max\{k_i\}$. Let P_t denote the set of products to be produced during the tth basic period of the global cycle. For a calculated value of K in the first part, we have to assign products to the subsets, $P_1, P_2, \ldots, P_K$. For example, for a product i, let k_i be 2 and global cycle length is 4 basic periods. Then product i has to be produced either in the first and third basic periods or in the second and fourth basic periods. Let π_i be the processing time of product i:

$$\pi_i = \frac{k_i \lambda_i B}{p_i}, \quad \forall i. \tag{2.16}$$

Given a vector of multipliers (k_i; $i = 1, \ldots, n$), the purpose of the following heuristic procedure is to try to find a feasible assignment of products to basic periods ($P_1, P_2, \ldots, P_K$). The procedure starts by sorting the products in ascending order of k_i, and within the products having the same multiplier k_i, in descending order of processing time. Comparison between processing times can be done using the term ρ_i^* below:

$$\rho_i^* = \frac{k_i \lambda_i}{p_i}. \tag{2.17}$$

As ρ_i^* gets larger, the processing time also becomes larger, because ρ_i^* times B is the real process time of the product i.

Then, each product i is assigned to the first basic period l, satisfying (2.18), within the first k_i periods of the global cycle that yet have the smallest summation of $\rho_i^* s$ that are assigned in that basic period P_t,

$$\min_{t<k_i} \left(\sum_{u \in P_t} \frac{k_u \lambda_u}{p_u} \right) + \frac{k_i \lambda_i}{p_i} < 1, \quad t = l, l + k_i, l + 2k_i, \ldots, l + K - k_i. \tag{2.18}$$

Note that sorting the products in ascending order of k_i, and within the products having the same multiplier k_i in the descending order of ρ_i^* allows us to assign first the products that require more production time, that is, those that are more difficult to assign.

The additional notation used in this part of the study is as follows.

Indices:

(i) j = stage index ($j = 1,\ldots,m$),

(ii) l = machine index ($l = 1,\ldots,r_j$).

Parameters:

(i) r_j = number of machines at stage j,

(ii) p_{ij} = production rate of product i at stage j,

(iii) s_{ij} = setup time of product i at stage j,

(iv) h_{ij} = inventory holding cost per unit of product i and per time unit between stages j and $j+1$,

(v) T_i = length of time interval between two successive runs of product i, called the cycle time of product i,

(vi) σ = predetermined production sequence,

(vii) $\sigma_{(Bi)}$ = the work sequence in each basic period is shown with σ but the B_i term denotes the sequence of products before product i's process.

Decision variables:

(i) d_{ij} = processing start time of product i at stage j (it is assumed that setup required in stage j is finished before the processing start time d_{ij} and for a given product, its start time is measured with respect to the starting time of the first basic period of the global cycle for which the product is scheduled),

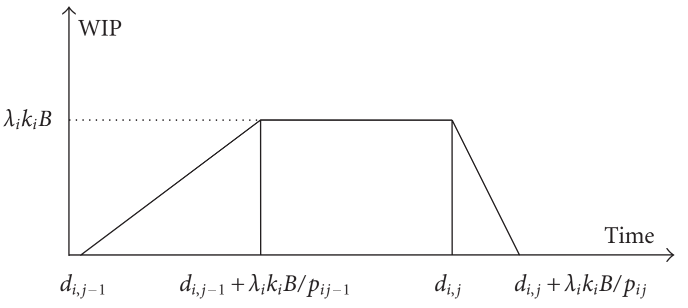
(ii) m_{ijl} = processing start time of product i at stage j on machine l (assume that necessary setup is finished before the processing start time m_{ijl}),

(iii) $a_{ijl} = 1$ if machine l at stage j is used for product i; otherwise 0,

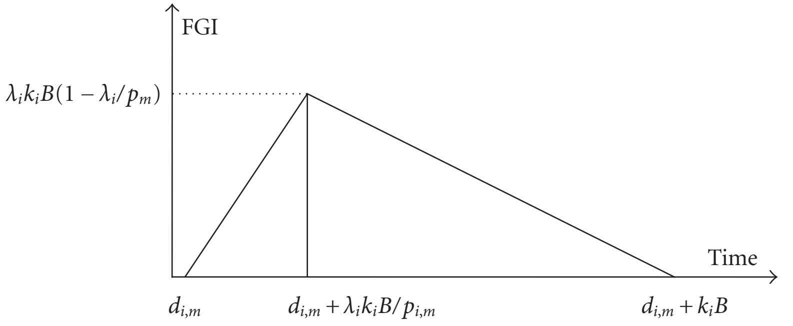
(iv) f_i = production end time of product i after the last stage.

In the scheduling model, the following assumptions are made.

(1) There are multiple stages/operations/work centers available in production and there exist multiple identical machines in each stage.
(2) Each product requires at most m stages.
(3) No machine can process more than one product at a time.
(4) Demand rates, production rates, setup times, setup costs, and inventory costs are deterministic and constant over the planning horizon.
(5) There exist external demands only for end products.
(6) Production rates can be different for different products and different stages.
(7) Different products and different stages have different setup times.
(8) Inventory holding costs are proportional to inventory levels and to holding time.
(9) Backlogging is not allowed.
(10) Preemption is not allowed, that is, at a given stage, once the processing of a lot has started it must be completed before starting the next one.
(11) Lot-splitting is not allowed, that is, a lot is not transferred to the next stage until the entire lot is processed at the current stage.
(12) In-process inventory is allowed, that is, products should wait for the next machine to be available.



(a) Work-in-process inventory between stages $j-1$ and j



(b) Finished goods inventory

Figure 2.1. Inventory of work-in-process and finished goods.

(13) Delivery of finished products is continuous.
(14) Capacity is sufficient to meet the demand.
(15) Products are manufactured in repetitive cycles and the lots of the same product are of equal size and are equally spaced through time.
(16) The same production sequence is used at all stages. (This type of production can be seen in paint industries and chemical product manufacturers.)
(17) Production may be continued indefinitely without interruption; thus, the maintenance can be done during production or during idle time (if any exists).

In the scheduling model, the problem is to determine the starting times of the operations for each product while minimizing the sum of setup and inventory holding costs. The objective function is similar to the one used in the first part but the terms related with the work-in-process inventory are added. Since the values of B and k_i's are known, the objective function is simplified. Work-in-process inventory between two stages and finished goods inventory in the system are shown in Figure 2.1.

Using Figure 2.1(a), average work-in-process inventory for product i between stages $j-1$ and j can be calculated as

$$\begin{aligned}\frac{1}{k_iB}&\left\{\frac{\lambda_ik_iB}{2}\frac{\lambda_ik_iB}{p_{i,j-1}}+\lambda_ik_iB\left(d_{ij}-d_{i,j-1}-\frac{\lambda_ik_iB}{p_{ij-1}}\right)+\frac{\lambda_ik_iB}{2}\frac{\lambda_ik_iB}{p_{ij}}\right\}\\&=\lambda_i\left(d_{ij}+\frac{\lambda_ik_iB}{2p_{ij}}-d_{i,j-1}-\frac{\lambda_ik_iB}{2p_{ij-1}}\right).\end{aligned}\tag{2.19}$$

Thus, the total work-in-process inventory holding cost per unit time is

$$\sum_{i=1}^{n}\sum_{j=2}^{m}h_{i,j-1}\lambda_i\left(d_{ij}+\frac{\lambda_ik_iB}{2p_{ij}}-d_{i,j-1}-\frac{\lambda_ik_iB}{2p_{ij-1}}\right).\tag{2.20}$$

The total finished product inventory holding cost per unit time is

$$\sum_{i=1}^{n}h_i\frac{1}{k_iB}\left(\frac{\lambda_ik_iB}{2}\left(1-\frac{\lambda_i}{p_{im}}\right)(d_{im}+k_iB-d_{im})\right)=\sum_{i=1}^{n}h_i\frac{\lambda_i}{2}\left(1-\frac{\lambda_i}{p_{im}}\right)(k_iB),\tag{2.21}$$

which is constant.

Other than holding costs, the objective function will include the setup costs. The setup cost for product i per unit time is $A_i/(k_iB)$. Therefore, the objective function of the model can be written as

$$\sum_{i=1}^{n}\frac{A_i}{k_iB}+\sum_{i=1}^{n}\sum_{j=2}^{m}h_{i,j-1}\lambda_i\left(d_{ij}+\frac{\lambda_ik_iB}{2p_{ij}}-d_{i,j-1}-\frac{\lambda_ik_iB}{2p_{ij-1}}\right)+\sum_{i=1}^{n}h_i\frac{\lambda_i}{2}\left(1-\frac{\lambda_i}{p_{im}}\right)(k_iB).\tag{2.22}$$

Objective function should be modified in order to eliminate the constant terms that do not include a decision variable. In the scheduling model, the decision variables are d_{ij}, m_{ijl}, and a_{ijl}. In part 1, the values of B and k_i are determined using the lot-sizing model, so in the objective function, only the terms including d_{ij} will remain,

$$\sum_{i=1}^{n}\sum_{j=2}^{m}h_{i,j-1}\lambda_i(d_{ij}-d_{i,j-1}).\tag{2.23}$$

Since no product can be processed before its completion at the previous stage, we add the following constraint which guarantees the processing times at each stage (2.24):

$$d_{i,j-1}+\frac{\lambda_ik_iB}{p_{i,j-1}}\le d_{ij}.\tag{2.24}$$

Also, the production end time after the last stage m can be shown using f_i,

$$d_{i,m}+\frac{\lambda_ik_iB}{p_{i,m}}\le f_i.\tag{2.25}$$

There are multiple machines at each stage. When a product enters a stage with multiple machines, it has to select the machine which is empty or will be available earliest. Hence, the model has to check for all previous jobs in sequence to find which job would leave the stage the earliest. Equation (2.27) stipulates that at each machine, no product can be processed before the completion of its predecessor. Since a_{ijl} shows that product i selects the lth machine in stage j, the term

$$\frac{\lambda_i k_i B a_{ijl}}{p_{ij}} \tag{2.26}$$

will be a positive value indicating the machining time for the chosen machine and would be zero for others,

$$m_{\sigma(Bi)jl} + \frac{\lambda_{\sigma(Bi)} k_{\sigma(Bi)} B a_{\sigma(Bi)jl}}{p_{\sigma(Bi)j}} + s_{\sigma(i)j} a_{\sigma(i)jl} \leq m_{\sigma(i)jl}. \tag{2.27}$$

Equation (2.27) has to be repeated for all predecessor products of product i in sequence (given by $\sigma(B_i)$), because in the following stages the sequence of products can be out of order. In order to choose the earliest starting time for machining, the smallest m_{ijl} should be selected. Equation (2.28) satisfies this

$$d_{ij} \leq m_{ijl}. \tag{2.28}$$

When the smallest m_{ijl} is chosen, then the selected machine should be signed. The following (2.29) implies that when process starting time for product i at stage j is equal to the smallest m_{ijl}, then machine l is used in that process and a_{ijl} is equal to 1. a_{ijl} is equal to 0 when that machine is not used. M in (2.29) can be set to basic period B.

$$m_{ijl} - d_{ij} \leq M(1 - a_{ijl}). \tag{2.29}$$

Moreover, (2.30) indicates that in each stage, for any product i, only one machine can operate:

$$\sum_l a_{ijl} = 1. \tag{2.30}$$

Finally, to prevent the production time of goods to exceed the basic period, for each product, the following constraint is added:

$$f_i + s_{1,1} - d_{1,1} \leq k_i B. \tag{2.31}$$

Other constraints are related to the setup times of first product in sequence. For the first product, the starting time of all operations will be greater than the setup time of the related operation,

$$d_{1,j} \geq s_{1,j}. \tag{2.32}$$

Also, the first product should be assigned to a specific machine, because (2.27) and (2.28) will not be appropriate for the first product at the first stage. But (2.29) should not be ignored for product 1. The following equations can be used for this purpose (it is assumed that first product is assigned to machine 1 in the first stage):

$$\begin{gathered} m_{1,1,1} = s_{1,1}, \\ m_{1,1,l} = 0, \quad \text{where } l \neq 1, \\ d_{1,1} \leq m_{1,1,1}, \\ a_{1,1,1} = 1. \end{gathered} \tag{2.33}$$

The model developed in [11] is a case for multiproduct, multistage problem with one machine in each stage.

The final formulation of the scheduling model is as follows:

$$\begin{aligned} &\operatorname{Min} z = \sum_{i=1}^{n} \sum_{j=2}^{m} h_{i,j-1} \lambda_i (d_{ij} - d_{i,j-1}) \\ &\text{subject to } d_{i,j-1} + \frac{\lambda_i k_i B}{p_{i,j-1}} \leq d_{ij}, \quad i = 1,\dots,n,\ j = 2,\dots,m, \\ &d_{i,m} + \frac{\lambda_i k_i B}{p_{im}} \leq f_i, \quad i = 1,\dots,n, \\ &m_{\sigma(Bi)jl} + \frac{\lambda_{\sigma(Bi)} k_{\sigma(Bi)} B a_{\sigma(Bi)jl}}{p_{\sigma(Bi)j}} + s_{\sigma(i)j} a_{\sigma(i)jl} \leq m_{\sigma(i)jl}, \\ &\qquad\qquad\qquad i = 2,\dots,n,\ j = 1,\dots,m,\ l = 1,\dots,r_j, \\ &d_{ij} \leq m_{ijl}, \quad \text{if } j = 1, \text{ then } i = 2,\dots,n,\ l = 1,\dots,r_j, \\ &\qquad\qquad \text{if } j = 2,\dots,m, \text{ then } i = 1,\dots,n,\ l = 1,\dots,r_j, \\ &m_{ijl} - d_{ij} \leq B(1 - a_{ijl}), \quad i = 2,\dots,n,\ j = 1,\dots,m,\ l = 1,\dots,r_j, \\ &\sum_{l} a_{ijl} = 1, \quad i = 2,\dots,n,\ j = 1,\dots,m, \\ &f_i + s_{1,1} - d_{1,1} \leq B, \quad i = 2,\dots,n, \\ &d_{1,j} \geq s_{1,j}, \quad j = 1,\dots,m, \\ &m_{1,1,1} = s_{1,1}, \\ &m_{1,1,l} = 0, \quad \text{where } l \neq 1, \\ &d_{1,1} \leq m_{1,1,1}, \\ &a_{1,1,1} = 1, \\ &d_{ij}, m_{ijl} \geq 0, \qquad a_{ijl} = 0 \text{ or } 1. \end{aligned} \tag{2.34}$$

3. A numerical example

In this section, we provide a numerical example to show the application of the proposed method, data of which has been collected from a company that produces industrial paints. There are two stages of production. In the first stage, chemicals are mixed up and in the second stage, the solution, paint, is filled into cans. At each stage, there are 2 machines

Table 3.1. Setup times and costs.

Container size (kg)	Stage 1	Stage 2	
	Setup time (min)	Setup time (min)	Setup cost ($)
800–1300	120	100	8
1300–3800	180	100	12
3800–7800	300	100	20
7800–15000	360	100	24

Table 3.2. Data for the product.

Product no.	Annual demand (kg/year)	Production rate (kg/year) in stage 1	Production rate (kg/year) in stage 2	Production costs in stage 1 ($/kg)	Total production costs ($/kg)
1	32000	2325000	2400000	2.83	3.11
2	138000	724000	750000	2.31	2.60
3	54500	766000	780000	2.31	2.66
4	27000	687000	700000	2.35	2.65
5	113000	546000	580000	2.30	2.59

Table 3.3. Total costs.

Basic period (days)	Total cost ($)	Basic period (days)	Total cost ($)
1	5102.482	7	8966.13
2	5201.231	8	8966.13
3	6108.755	9	8966.13
4	**4971.364**	10	10187.89
5	7117.474	11	10960.28
6	7805.082	12	11142.78

performing the same job with the same production rate. Depending on the size of the container used, the setup times and costs are given in Table 3.1.

Production of five different paints produced in the company is taken as an example, and the related data is given in Table 3.2.

Since the first stage of the production process is a bottleneck, we will use it as the production rate of the system to determine reorder intervals and lot sizes. To find the holding cost rate, we assume an interest rate of 30%.

Using (2.13) and (2.14), the upper and lower bounds for the basic period can be calculated. Since $T_{rc} = 0.000781$ year $= 0.28$ day and $T_{cc} = 0.033679$ year $= 12.3$ days, it is more convenient to use days from 1 to 12 as basic period candidates. For each basic period candidate, bounds for power-of-two multipliers are calculated using (2.8) and (2.9). After running the lot-size determination model, (2.10), the total costs obtained are given in Table 3.3.

Table 3.4. Multipliers for 4-day basic period.

Product no.	k_i	ρ_i^*, equation (2.17)
5	1	0.207
2	2	0.381
3	2	0.142
4	4	0.157
1	4	0.055

Table 3.5. Assignment of products to each basic period.

	Basic period			
	1	2	3	4
	5	5	5	5
Product no.	2	3	2	3
	—	4	—	1

The minimum cost is obtained for 4-day-basic period length. Since the largest k_i value is 4 in Table 3.4 (product no. 1), the global cycle length is 16 days ($k_i * B$). Production sequence is given as 1, 2, 3, 4, and 5 by the company. This sequence is important for setup purposes. Container and machine cleaning is easier when this sequence is used in production stages. In order to find schedules, products should be assigned to 4—maximum k_i value—basic periods of length 4 days. Assignment sequence is 5, 2, 3, 4, and 1. The result of assignment sorted by k_i and ρ_i^* values is given in Table 3.5.

The obtained schedules are given in Table 3.6. Starting time shows the initiation of the operation after the setup. Setup time of the initial operation starts at time 0.

In order to compare the results of the proposed method, we calculate the EOQ values for each product type. Actually, in the first part of the model, that is, during the calculations of lot sizes, the model determines the power-of-two multiplier for a given basic period candidate, so the product reorder intervals are independent of each other. Therefore, the best reorder interval can be found by individual EOQ formulas:

$$\mathrm{EOQ}_i = \sqrt{\frac{2A_i\lambda_i}{h_i(1-\lambda_i/p_i)}}. \tag{3.1}$$

However, since there are 4 different setup costs, we calculate EOQ values with each of them and analyze the feasibility of the obtained EOQ values (see Table 3.7). The last column shows the k_i values obtained in Part 1.

Since for products no.1 and no.4, EOQ values are less than min values, they are assumed to be produced at the technical lower bound of 800 kg. The total costs calculated with EOQ_i and ELSP(PoT) policy are shown in Table 3.8.

As the example indicates that the largest increases occur in products no.1 and no.4, no feasible solution can be obtained in the related regions. For products no.2 and no.3, the

Table 3.6. Schedules obtained for each basic period.

For period no.1 and no.3					
Product no.	Starting time at stage 1 (h)	Machine no. at stage 1	Starting time at stage 2 (h)	Machine no. at stage 2	End of production
2	3	1	36	1	40
5	19	2	37	2	39

For period no.2

Product no.	Starting time at stage 1 (h)	Machine no. at stage 1	Starting time at stage 2 (h)	Machine no. at stage 2	End of production
3	2	1	15	1	16
4	17	1	42	1	44
5	26	2	44	2	45

For period no.4

Product no.	Starting time at stage 1 (h)	Machine no. at stage 1	Starting time at stage 2 (h)	Machine no. at stage 2	End of production
1	3	1	6	2	8
3	2	2	15	2	16
5	8	1	27	1	28

Table 3.7. EOQ_i values.

	EOQ_i					
Product no.	$A = 8\$$ (800–1300 kg)	$A = 12\$$ (1300–3800)	$A = 20\$$ (3800–7800)	$A = 24\$$ (7800–15000)	$t_i = \text{EOQ}_i/\lambda_i$ (days)	k_i (days)
1	**746→800**	914	1180	1292	9.125	16
2	1872	**2293**	2960	3242	6.064	8
3	**1085**	1329	1715	1879	7.265	8
4	**753→800**	922	1190	1304	10.815	16
5	1715	**2100**	2711	2970	6.783	4

policy leads to insignificant increases. For product no.5, the model can capture better results with the container capacity limits which cannot be obtained by EOQ formula. Also, it should be noted that without container capacity constraints, Muckstadt and Roundy [10] state that total cost exceeds the EOQ solution no more than 6%. Using the results of this study, the most important advantage is that one can easily organize the production of lots in the manufacturing environment when the global cycle is 16 days. Nonetheless, in the above case, with the EOQ formula the scheduling process becomes cumbersome (see t_i values in Table 3.7)

Table 3.8. Total cost and percentage change.

Product no.	EOQ_i($)	ELSP(PoT) ($)	Change (%)
1	688	919	34
2	1445	1500	4
3	804	808	0
4	575	634	10
5	1291	1111	−14
Total	4803	4971	4

Finally, we note that the models were solved on P IV 3.2 MHz 512 Mb ram computer using LINGO 8 software in a matter of seconds. Hence larger scale instances of the model can easily be solved.

4. Conclusions

In this paper, we propose a heuristic decomposition approach to model the problem of determining realistic and easy-to-schedule lot sizes in a multiproduct, multistage, multimachine manufacturing environment using PoT policy. The approach is composed of two parts in order to find solutions to reorder interval determination (and consequently lot sizing), and assignment and scheduling problems separately.

In the first part of the method, we develop a binary integer programming model to find optimum reorder intervals which are power-of-two multiples of a basic period. The basic period length can be a day, a week, or a month to secure a realistic time period. As the optimum reorder intervals are determined, we then calculate the optimum lot sizes for each product. The basic contribution of the first model of the decomposition approach is that we not only consider the technical upper and lower bounds of the production process, but also develop the formulation to accommodate process-dependent setup costs. To facilitate the solution of the problem, we assume the production system as a single entity.

In the second part of the study, we first try to assign those lots to basic periods of the global cycle length (a cycle length in which production cycles of all products are completed). Then a mixed integer programming model schedules the lots already assigned to basic periods to multistages and multimachines at each stage.

Although the approach is designed for a specific flow-shop-type production, it could be easily modified to accommodate other types of production systems, for example, assembly systems.

Despite the problem is designed for deterministic demand, the results of lot sizing problem are stable since they are based on reorder intervals rather than lot sizes. They can be rescheduled again in case of small demand variations.

Finally, in our opinion, the main advantage of the approach is that we find realistic and easy-to-apply reorder intervals. As seen in Tables 3.7 and 3.8, it is rather difficult to order lots every 9.125 days for product no. 1, 6.064 days for product no. 2, and so forth, and on the overall, the cost increase is only 4%.

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Esra Ekinci: Industrial Engineering Depatment, Dokuz Eylul University, Bornova,
35100 Izmir, Turkey
Email address: esra.ekinci@tescokipa.com.tr

Arslan M. Ornek: Industrial Engineering Depatment, Dokuz Eylul University, Bornova,
35100 Izmir, Turkey
Email address: arslan.ornek@deu.edu.tr