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CONSTRUCTION OF J–VARIATE DISTRIBUTION FUNCTIONS AND APPLICATIONS TO DISCRETE DECISION MODELS

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Abstract. The construction of J-variate distribution functions, introducing dependences among J random variables and keeping fixed the J marginal distribution functions, is important in the development of theoretical and empirical statistical analysis. Here, a method for generating such distribution functions is developed. Characteristics of the resulting distribution functions are discussed. An application to discrete regression models is presented. The latter is specialized to model choice of mode od transportation by travelers.

Keywords: Generalized joint distributions functions, fixed margins, dependencies, Extreme Value distribution, discrete regression models.

1. Introduction

In the development of probabilistic model and statistical analysis, the postulation of a joint cumulative distribution function (JCDF), keeping fixed some marginal cumulative distributions (MCDF), is often required (Yadlin, 1991; Long and Krysztofowicz, 1995). Some examples follow.

In survival models, it is usually assumed that the duration D of a component has an exponential distribution. If there are J components functioning in parallel, the formulation of a JCDF for the $D_j, J = 1, \ldots, J$, with exponential MCDF, that becomes the product of these MCDF under restrictions, may be convenient (Sarkar, 1987).

Here, we want to model the survival function $Pr\left[\cap_{j=1}^{J}(D_j \geq d_j)\right]$. where the D_j are not necessarily independent.

In dosage-response models, it is assumed that there exists a random tolerance function $T_r, r = 1, \ldots, R$, underlying each of the R possible response (Berkson, 1953). Examples are the response of an individual to some stimuli or the reaction of an insect to an insecticide. If there are $J \geq 2$ stimuli, hence J tolerance functions, $T_r^{(1)}, T_r^{(2)}, \ldots, T_r^{(J)}$ associated to the rth response, a JCDF, with Logistic MCDF for the $T_r^{(j)}$, which becomes the product of these marginal Logistics under restrictions may be established. Here we want to model the probability that the response under dosages $x^{(1)}, \ldots, x^{(j)}$ is the rth response, that is,

$$Pr\left[\cap_{k=1}^{r-1}(T_k^{(1)} > x^{(1)}, \ldots, T_k^{(J)} > x^{(J)},) \cap (T_r^{(1)} \leq x^{(1)}, \ldots, (T_r^{(J)} \leq x^{(J)})\right],$$

where the $T_r^{(J)}$ may or may nor be independent.

In discrete decision models (DDM), it is assumed that there exists a radom utility functions $U_m = i = 1, \ldots, J$, underlying each of the J possible decisions, for each sujects $n = 1, \ldots, N$. Here $U_{in} = V_{in} + \epsilon_{in}$, where V_{in} is non stochastic and ϵ_{in} is a random perturbation. Examples are the decision by a high-school graduate about a career, the selection by a family of a vacation resort and the choice by a user of mode of transportation. Assuming that $\epsilon_{1n}, \ldots, \epsilon_{Jn}$ are iid Extreme Value (EV) yields the prevailing methodology, which is the Multinomial Logit Model (MLM), which exhibits analytical and computational simplicity, but disregards interrelations among the J possible choices. A Nesting MLM, that overcomes the latter drawback, can be obtained by constructing a Nesting EV JCDF that keeps the J MCDF as EV. This Nesting MLM is also consistent with the principle of utility maximization, exhibits a closed functional form manageable for theoretical and empirical investigations, and allows for the introduction of relations among alternatives through the inclusion of a dependence parameter. Moreover, the Nesting MLM constitutes a parametric generalization of the MLM, in the sense that, under restrictions, the Nesting EV distribution becomes the product of J EV distributions, while the Nesting MLM reduces to the original MLM, so that classical testing procedures can be applied to decide between them. We will return to this example is Section 3.

Thus, the purpose is to build a JDCF for $X_1, \ldots, X_J$ such that: 1. The arbitrary MCDF of the X_j are mantained. 2. Patterns of dependences are parametrized through a parameter $\alpha \in A \subset R^L$ with $0 \in A$. 3. The JCDF reduces to the product of the J marginals when $\alpha = 0$. 4. Parametric generalizations of statistical models, with analytical and computational tractability, are accomplished.

Schematically, let F_j be the MCDF of X_j, F_I be $P_{j=1}^J F_j, M_I$ be model under F_I, F be the generalized JCDF of $X_1, \ldots, X_J$, and M be the model under F Then we have $(X_1, \ldots, X_J) \sim F_I$; if and only if the true model is M_1; if and only if $\alpha = 0$ versus $(X_1, \ldots, X_J) \sim F$; if and only if the model is M; if and only if $\alpha \in A$. Thus, since $M_I \subset M$; likelihood methods can be applied to test M_I versus M.

2. Generating Method

Considering the JCDF for $X_1, \ldots, X_J$, when the X_j are grouped into subvectors $X_j : j \in B_m), m = 1, \ldots, M$, with $|B_m| \geq 2$. $B_m \subset \{1, \ldots, J\}$ and $U_{m=1}^M B_m = \{1, \ldots, J\}$, leads to the following reformulation of a method designed by Johnson and Kotz (1975)

$$F(x_1, \ldots, x_j) = \prod_{j=1}^{J} F_j(x_j) \left\{1 + \sum_{m=1}^{M} \alpha_m \prod_{k \in B_m} [1 - F_k(x_k)]\right\}, \tag{1}$$

where α_m represents dependences among the $X_j, j \in B_m$. If the marginal densities f_j exist, the joint density is given by

$$f(x_1, \ldots, x_J) = \prod_{j=1}^{J} f_j(x_j) \left\{ 1 + \sum_{m=1}^{M} \alpha_m \prod_{k \in B_m} [1 - 2F_k(x_k)] \right\}. \quad (2)$$

Whatever the values of the α_m, the MCDF and be marginal densities are F_j and f_j, respectively. Also, whatever the values of the α_m, F has all the properties of a JCDF, except that it may assign negative probabilities to some rectangles in R^J. Similarly, f integrates to 1, but it may take negative values. Thus, 2^M restrictions are required

$$1 + \sum_{m=1}^{M} C_m \alpha_m \geq 0, \quad C_m = \pm 1. \quad (3)$$

Hence, A is determined by (3).

It is clear that if $B \subset C$ is the union of some of the $B_m, B = U_{s=1}^{S} B_{ls}$, where $\{l_1, \ldots, l_S\} \subset \{1, \ldots, M\}$, then the JCDF of $\{X_j; j \in B\}$ belongs to the same family (1). Here the $X_j, j \in B$ are grouped into S subsets $\{X_j; j \in B_{l_s}\}, s = 1, \ldots, S$.

The moment generating functions of $(X_1, \ldots, X_J)$ is derived as

$M(s_1, \ldots, s_J) = E(\prod_{j=1}^{J} e^{s_j X_j})$

$= \int \ldots \int \left\{ \prod_{j=1}^{J} e^{s_j t_j} f_j(t_j) \left\{ 1 + \sum_{m=1}^{M} \alpha_m \prod_{k \in B_m} [1 - 2F_k(s_k)] \right\} dt_1, \ldots, dt_J \right\}$

$= \prod_{j=1}^{J} M_j(S_j) +$

$+ \sum_{m=1}^{M} \alpha_m \prod_{k \in B_m} \left[M_k(s_k) - 2 \int e^{s_k t_k} F_k(t_k) f_k(t_k) dt_k \right] \prod_{j \notin B_m} M_j(s_j).$

Thus,

$$M(s_1, \ldots, s_J) = \prod_{j=1}^{J} M_j(s_j) + \sum_{m=1}^{M} \alpha_m \prod_{k \in B_m} \left\{ M_k(s_k) - 2E\left[e^{s_k X_k} F_k(X_k) \right] \right\} \times$$

$$\times \prod_{j \in B_m} M_j(s_j), \quad (4)$$

where M_j is the moment generating function of X_J.

Formulae (1), (2) and (4) reduce to the formulae corresponding to indepedence when $\alpha_m = 0, m = 1, \ldots, M$.

From (4), we can compute correlation $(X_i, , X_l)$ for any $\{i, l\} \subset \{1, \ldots, J\}$. Setting α_{il} be the dependence parameter associated to $\{X_i, X_l\}$ we have

$E(X_i X_l) = \frac{\partial^2}{\partial s_i \partial s_l} M(s_1, \ldots, s_J) | s_1 = \ldots = s_J = 0$

$$= \frac{\partial^2}{\partial s_i \partial s_l} \left\{ M_i(s_i) M_l(s_l) + \alpha_{il} \left\{ M_i(s_i) - 2E\left[e^{s_i X_i} F(X_i) \right] \right\} \times \right.$$

$$\left. \times \; \left\{ M_l(s_l) - 2E\left[e^{s_l X_l} F_l(X_l) \right] \right\} |s_i = s_l = 0$$

$$= \; E(X_i)E(X_l) + \alpha_{il} \left\{ E(X_i) - 2E\left[X_i F_i(X_i) \right] \right\} \left\{ E(X_l) - 2E\left[X_l F_l(X_l) \right] \right\}$$

$= E(X_i)E(X_l) + \alpha_{il}$ 4 Covariance $(X_i, F_i(X_i))$ Covariance $(X_l, F_l(X_l))$.
Since $F_l(X_j)$ follows a Uniform (0,1) distribution, we get

$$\text{Correlation}(X_i, X_l) = \alpha_{il} \text{ Correlation } (X_i, F_i(X_i)) \text{ Correlation } (X_l, F_l(X_l)). \tag{5}$$

Hence, the correlation between X_i and X_l is zero whenever the dependence parameter α_{il} is zero.
As an axample, suppose X_j has an EV distribution. Then

$$F_j(x) = exp\{-e^{-x}\}, x \in R, \tag{6}$$

$$f_j(x) = exp\{-x - e^{-x}\}, x \in R, \tag{7}$$

$$M_j(s) = \Gamma(1 - s), s < 1, \tag{8}$$

$$E(X_j) = 0.577, \tag{9}$$

$$Var(X_j) = 1.645. \tag{10}$$

Hence,

$$F(x_1, \ldots, x_J) = \prod_{j=1}^{J} \text{ exp } \left\{ -e^{-x_j} \right\} \left\{ 1 + \sum_{m=1}^{M} \alpha_m \prod_{k \in B_m} \left[1 - \text{ exp } \left\{ -e^{-x_k} \right\} \right] \right\}, \tag{11}$$

where α_m must satisfy (3),

$$f(x_1, \ldots, x_J) = \prod_{j=1}^{J} \text{ exp } \left\{ -x_j - e^{-x_j} \right\} \left\{ 1 + \sum_{m=1}^{M} \alpha_m \prod_{k \in B_m} \left[1 - 2 \text{ exp } \left\{ -e^{-x_k} \right\} \right] \right\} \tag{12}$$

We call (11) and (12) Nesting EV distribution and density, respectively.

Also, substituting (8) in (4)

$$M(s_1,\dots,S_J)=\prod_{j=1}^{J}\Gamma(1-s_j)\left\{1+\sum_{m=1}^{M}\alpha_m\prod_{k\in B_m}[1-2^{s_k}]\right\}, s_j<1, \tag{13}$$

and, from (5)
Correlation $(X_i,X_l)=\alpha_{il}$ Correlation $(X_i,\exp\{e^{-X_i}\})$ Correlation $(X_l,\exp\{e^{-X_l}\})=\alpha_{il}(\log 2)^2/1.645$,

$$\text{Correlation }(X_i,X_l)=0.292\ \alpha_{il}. \tag{14}$$

3. An Application: Generalization of Discrete Decision Models

A DDM serves to explain the probabilities with which a subject will choose one and only one of J objects (Rust 1988; Yadlin 1991 and 1993). Letting $C=\{1,\dots,J\}$ be the choice set, a DDM specifies $P_B^n(i)$, the probability that subject n will choose i from B, for all choice subsets $B\subset C$ and all $i\in B$, on the basis of the principle of utility maximization. In this way, a DDM postulates that individuals' preferences can be expressed by means of a vector valued random utility function on C. It is assumed that subject n associates to alternative a random value U_{in}, and them he chooses so as to maximize this value.

Thus,

$$P_B^n(i)=Pr(U_{in}>U_{jn};\forall j\in B-\{i\}).$$

U_{in} can be decomposed into a deterministic part V_{in} and a random perturbation ε_{in},

$$U_{in}=V_{in}+\epsilon_{in},$$

Hence,

$$P_B^n(i)=Pr(\epsilon_{jn}-\epsilon_{in}<V_{in}-V_{jn};\forall_j\in B-\{i\}).$$

Conditioning on ϵ_{in}, we get

$$P_B^n(i)=\int\left[\frac{\partial Pr(\epsilon_{jn}<t_j;\forall j\in B)}{\partial t_i}|_{t_j=V_{in}-V_{jn}+t}\right]dt. \tag{15}$$

Thus, the functional form of a DDM is determined by F, the JCDF of $\epsilon_{1n},\dots,\epsilon_{jn}$. In particular, under independence

$$P_B^n(i)=\int f_i(t)\prod_{j\in B-\{i\}}F_j(V_{in}-V_{jn}+t)dt$$

It is usually assumed that

$$V_{in}=\beta^T Z_{in},$$

where β is a vector of unknown parameters and Z_{in} is a vector of observable exogeneous variables.

In what follows, we omit n and concentrate on $P_C(i)$, denoted by $P(i)$, for shortness. No loss of generality is induced, because of the comment under (16) below.

It is clear that we can generalize any DDM based on independence of the ϵ_j using (1). We obtain a model that embeds the independence model in a simple way, and whose decision probabilities are derived via (15) and are formulated in terms of the F_j as

$$P(i) = \int \left\{ f_i(x_i) \prod_{j \in C-\{i\}} F_i(x_i) \left\{ 1 + \sum_{m=1}^{M} \alpha_m \prod_{k \in B_m} [1 - F_k(x_k)] - F_i(x_i) \times \right.\right.$$

$$\left.\left. \times \sum_{\{m: i \in B_m\}} \alpha_m \prod_{k \in B_m} [1 - F_k(x_k)] \right\} |x_j = V_i - V_j + t \right\} dt.$$

Thus,

$$\begin{aligned} P(i) \;&=\; \int \left\{ f_i(x_i) \prod_{j \in C-\{i\}} F_j(V_i - V_j + t) \left\{ 1 + \sum_{m=1}^{M} \alpha_m \right\} \right. \\ &- \; 2 \sum_{\{m: i \in B_m\}} \alpha_m F_i(t) + \sum_{\{m: i \in B_m\}} \alpha_m \prod_{k=1}^{B_m - 1} (-1)^k \\ &\sum_{j_1 < \ldots < j_k, j_l \neq i} F_{j_1}(V_i - V_{j_1} + t) \ldots F_{j_k}(V_i - V_{j_k} + t) \qquad (16) \\ &- \; 2F_i(t) \sum_{\{m: i \in B_m\}} \alpha_m \sum_{k=1}^{|B_m|-1} (-1)^k \sum_{j_1 < \ldots < j_k, j_l \neq i} F_{j_1}(V_i - V_{j_1} + t) \ldots F_{j_k}(V_i - V_{jk} + t) \\ &+ \; \left. \sum_{\{m: i \notin B_m\}} \alpha_m \sum_{k=1}^{B_m|-1} (-1)^k \sum_{j_1 < \ldots < j_k, j_l \neq i} F_{j_1}(V_i - V_{j_1} + t) \ldots F_{j_k}(V_i - V_{j_k} + t) \right\} dt. \end{aligned}$$

If $B \subset C$ is the union of some the B_m, the probabilities $P_B(i), i \in B$, belong to the same family (16).

The $P(i)$ in (16) reduce to the model under independence, when $\alpha_m = 0, m = 1, \ldots, M$.

The MLM is the prevailing model in applications of DDM. Denoting $P(i)$ by $Q(i)$, this model is given by

$$Q(i) = \frac{e^{Vi}}{\sum_{k=1}^{J} e^{V_k}}, \qquad (17)$$

Besides its great tractability, the MLM presents flexibility for interpreting the decision probabilities in terms of the relative mean utilities of the alternatives, and for evaluating the impact of manipulating the decision set C or the exogeneous variables Z_{in}.

However, the likelihood odds for deciding alternative i over alternative j are constant whatever the set C is, so that these odds are independent of third alternatives available to the decision maker. In fact writing

$$Q(i) = \frac{e^{V_i - V_j}}{\sum_{k=1}^{J} e^{V_k - V_j}}, \tag{18}$$

it can be seen that the evaluation of an alternative is based on binary comparisons only.

To derive the MLM from the principle of utility maximization, it is sufficient to assume that $\epsilon_1, \ldots, \epsilon_J$ are iid with EV distributions. Thus, replacing in (15) F_j and f_j, by (6) and (7), the following model is obtained

$$P(i) = \left\{1 + \sum_{m=1}^{M} \alpha_m\right\} \frac{e^{V_i}}{\sum_{j\in C} e^{V_j}} - 2\sum_{\{m:i\in B_m\}} \alpha_m \frac{e^{V_i}}{e^{V_i} + \sum_{j\in C} e^{V_i}} +$$
$$+ \sum_{\{m:i\in B_m\}} \alpha_m \sum_{k=1}^{|B_m|-1} (-1)^k \sum_{j_1<\ldots<j_k, j_l\neq i} \frac{e^{V_i}}{\sum_{l=1}^{k} e^{V_{j_l}} + \sum_{j\in C} e^{V_j}} +$$
$$-2\sum_{\{m:i\in B_m\}} \alpha_m \sum_{k=1}^{|B_m|-1} (-1)^k \sum_{j_1<\ldots<j_k, j_l\neq i} \frac{e^{V_i}}{e^{V_i} + \sum_{l=1}^{k} e^{V_{j_l}} + \sum_{j\in C} e^{V_j}} +$$
$$+ \sum_{\{m:i\in B_m\}} \alpha_m \sum_{k=1}^{|B_m|} (-1)^k \sum_{j_1<\ldots<j_k, j_l\neq i} \frac{e^{V_i}}{\sum_{l=1}^{K} e^{V_{j_l}} + \sum_{j\in C} e^{V_j}}.$$

Upon deviding and multiplying by $\sum_{k=1}^{J} e^{V_k}$, we obtain

$$\begin{aligned} P(i) \;=\; & \left\{1 + \sum_{m=1}^{M} \alpha_m\right\} Q(i) - 2 \sum_{\{m:i\in B_m\}} \alpha_m \frac{Q(i)}{1+Q(i)} + \\ & + \sum_{\{m:i\in B_m\}} \alpha_m \sum_{k=1}^{|B_m|-1} (-1)^k \sum_{j_1<\ldots<j_k, j_l\neq i} \frac{Q(i)}{1 + \sum_{l=1}^{k} Q(j_l)} + \\ & -2 \sum_{\{m:i\in B_m\}} \alpha_m \sum_{k=1}^{|B_m|-1} (-1)^k \sum_{j_1<\ldots<j_k, j_l\neq i} \frac{Q(i)}{1 + Q(i) + \sum_{l=1}^{k} Q(j_l)} + \\ & + \sum_{\{m:i\in B_m\}} \alpha_m \sum_{k=1}^{|B_m|-1} (-1)^k \sum_{j_1<\ldots<j_k, j_l\neq i} \frac{Q(i)}{1 + \sum_{l=1}^{k} Q(j_l)} \end{aligned} \tag{19}$$

The terms in the $P(i)$ involve the MLM probabilities $Q(i)$ and the dependence parameters α_j, so that they consider interrelations among the elements inside each subser B_m of the choice set C. The generalized probabilities in (19) constitute the Nesting EV Model. Unlike other generalizations of the MLM (Hausman and Wise, 1978; Small, 1994), the probabilities of the Nesting EV Model constitute simple functions of the probabilities of the MLM.

When the dependence parameters α_m are zero for $m = 1, \ldots, M$, the $P(i)$ reduce to the $Q(i)$. In this way, we have constructed a parametric generalization of the MLM, allowing us to test whether we can maintain the advantages of the MLM in any practical situation. In the event that the null hypothesis that all α_m are zero, that is, the MLM is the true model, is rejected, we are still left with a manageable model, that takes into account relationship among the J possible decisions.

In the remainder of this section, an empirical illustration (Yadlin, 1991) is described. This illustration deals with the choice of mode of transportation by users who travel between two places. This problem has been usually approach with discrete regression models. Thus, the analysis of these models play an important role in the evaluation of transportation models, becoming fundamental to detect whether the simplest model provides an adequate fit to the available data.

Here we consider individuals who move from their homes to their working or studying place in Santiago.

The choice set consists of $C = \{1 = \text{Metro}, 2 = \text{Microbus}, 3 = \text{Minibus}\}$. Since the Microbus andthe Minibus are close substitutes and the Metro is regarded as unrelated with the former, we let $B_1 = \{1\}, B_2 = \{2, 3\}, \alpha_2 = \alpha$ in (19), getting $|\alpha| \leq 1$ by (3), and Correlation $(\epsilon_2, \epsilon_3) = 0.292\alpha$ by (14).

In order to assess the performance of the MLM, completely known values of the true probabilities are required. Then, the information about the choices is simulated. 160 strata of 100 travellers echa are created. The subjects in each strata are homogeneous with respect to the value of the following exogeneous variables:

Z_{in1} = Cost of mode i in Chilean money / Per-hour income in Chilean money of subjects in stratum $n(C_0/I)$

Z_{in2} = Out-of-vehicle travel (walking plus waiting) time in hours for mode i and individuals in stratum n (OVT)

Z_{in3} = In-vehicle travel time for mode i and individuals in stratum n (IVT)

Z_{in4} = 1 if $i = 1$ and 0 if $i \neq 1$. (Relates to the level of comfort of the Metro).

Others variables may have been considered. It was decided to have a very simple utility function because, the purpose is to evaluate the performance of probabilistic choice models under misspecification of the CDF F and not of the utility function, whose form is kept fixed. Thus the objetive is the comparison of the performance of models MLM and Nesting EV Model in the three-alternative, two-substitute situation, under misspecification of the probabilistic mechanism (CDF) underlyig the models, keeping the formulation of the utility function fixed, in the context of demand for transportation. The main steps of the methodology are the following:

1. Defining the choice set C described above and the target population, whose members are noncar owners transportation users, who travel from their resi-

dence to their working or studying place by either of the modes in C at the morning peak hours.

2. The mean utility function is specified:

$$\begin{aligned} V_{in} &= \sum_{k=1}^{n} Z_{ink}\beta_k \\ &= (C_0/I)_{in}\beta_1 + (OVT)_{in}\beta_2 + (IVT)_{in}\beta_3 + \text{Dummy Metro}\beta_4, \quad (20) \\ & \quad i = 1,2,3,\ n = 1,\ldots,160. \end{aligned}$$

3. The values for the $Z_{ink}, i = 1,2,3, n = 1,\ldots,160, k = 1,2,3,4$, are determined, so that Z_{2nk} is close to Z_{3nk}, while Z_{1nk} differ from the former. In this way, the attributes of mode 1 are different from those of modes 2 and 3, which are similar.

4. For each $n = 1,\ldots,160,\ Z_{ink}$ are replicated 100 times, so as to obtain a population of 16,000 travellers, composed by 160 strata of size 100 each.

5. A value is assigned to $(\beta_1,\beta_2,\beta_3,\beta_4,\alpha)$: $\beta_1 = 4.4$, $\beta_2 = 5.6$, $\beta_3 = 4.8$, $\beta_4 = 2.0$, and $\alpha = 0$ for the MLM and 1 for the Nesting EV Model.

6. The probabilities $Q^n(i)$ of the MLM and $P^n(i)$ of the Nesting EV Model, which are completely specified, are calculated.

7. Two populations of 16,000 modal decisions each are constructed. This is achieved by generating one modal selection for each of the 100 travellers in each of the 160 strata, by means of Trinomial simulation, based either on the $Q^n(i)$ (MLM population) or on the$P^n(i)$ (Nesting EV population).

8. 25 samples of size 1,600 each are extracted from each of the two populations. This is accomplished by drawing 25 stratified random samples with 10 observations per stratum from either of the two populations.

9. The parameters corresponding to both models are estimated by means of Maximum Likelihood Methods. The asymptotic Likelihood Ratio Test (LRT) statistics is calculated with the data of each of the 25 samples drawn from either the MLM or the Nesting EV Model populations. The LRT statistics is used to test the null hypothesis that the MLM is the true model ($H_0 : \alpha = 0$) versus the alternative hypothesis that the Nesting EV Model is the true model ($H_1 : \alpha \neq 0$).

   When the population of decisions is derived from the $Q^n(i)$ (MLM) the null hypothesis, is accepted in the 25 samples, and when the population of decisions is derived from the $P^n(i)$ (Nesting EV Model), the null hypothesis is rejected in the 25 samples.

Thus, the Likelihhod Ratio Test rejects the false model always.

4. Conclusion

A straightforward method to build parametric generalizations of statistical models, that hold under independence of certain random variables have been presented.

In this way, a nesting statistical framework for non-separate models, compatible with likelihood methods is provided.

The importance of these developments becomes apparent in their applications in DDM.

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