Hindawi Publishing Corporation
International Journal of Stochastic Analysis
Volume 2012, Article ID 249201, 14 pages
doi:10.1155/2012/249201

Research Article

Application of Stochastic Sensitivity Analysis to Integrated Force Method

X. F. Wei[1] and S. N. Patnaik[2]

[1] *Mechanical Engineering Department, The University of Akron, OH 44325, USA*
[2] *Ohio Aerospace Institute, Cleveland, OH 44135, USA*

Correspondence should be addressed to X. F. Wei, weixiaofanhome@yahoo.com

Received 18 February 2012; Accepted 18 April 2012

Academic Editor: Josefa Linares-Perez

As a new formulation in structural analysis, Integrated Force Method has been successfully applied to many structures for civil, mechanical, and aerospace engineering due to the accurate estimate of forces in computation. Right now, it is being further extended to the probabilistic domain. For the assessment of uncertainty effect in system optimization and identification, the probabilistic sensitivity analysis of IFM was further investigated in this study. A set of stochastic sensitivity analysis formulation of Integrated Force Method was developed using the perturbation method. Numerical examples are presented to illustrate its application. Its efficiency and accuracy were also substantiated with direct Monte Carlo simulations and the reliability-based sensitivity method. The numerical algorithm was shown to be readily adaptable to the existing program since the models of stochastic finite element and stochastic design sensitivity are almost identical.

1. Introduction

As an alternative of the classical stiffness method, the force method [1–3] was also popular in structural analysis of civil, mechanical, and aerospace engineering because of its accurate estimates of forces in structural analysis. A new formulation in the force method, termed the Integrated Force Method [4–7] (IFM), was proposed by Patnaik for the analysis of discrete and continuous systems. IFM is a force method [8], which integrates both the system equilibrium equations and the global compatibility conditions together. This method has been successfully applied to many structures for their deterministic models with well-defined parameters. Realizing its potential in structural analysis, it is being further extended to consider investigations on the probabilistic analysis. Meanwhile, the probabilistic sensitivity formulation for IFM is also desirable to be investigated for assessing uncertainty effect in system optimization and identification.

In general, sensitivity analysis of structural systems involves the computation of the partial derivatives of some response function with respect to the design parameters. The sensitivity analysis of structural systems to variations in their parameters is one of the ways to evaluate the performance of structures. It is very important for system optimization, parameter identification, reliability assessment, and so forth in engineering analysis. However, the conventional sensitivity analysis is based on the assumptions of complete determinacy of structural parameters. In reality, the occurrence of uncertainty associated with the system parameters is inevitable and intrinsic. Hence, there is a necessity to estimate the effect of uncertainty in stochastic sensitivity derivatives with respect to random design variables on structural responses.

There has been a great interest in developing various methods for computing response sensitivity of structure [9–11]. Hien and Kleiber [12] formulated the stochastic design sensitivity problems of structural static in an effective way by using the perturbation approach and adjoint variables method. Numerical algorithms are developed and turn out to be readily adaptable to existing finite element codes. The structural design sensitivity and stochastic finite elements are similar in terms of the methodology and computer implementation, which greatly facilitate the combined analysis. Based on the first-order perturbation method, Ghosh et al. [13] illustrated that the stochastic structural response sensitivity is quite satisfactory compared to Monte Carlo simulation results under small variation of input random parameters.

It was noted that literatures about the stochastic sensitivity analysis are limited and almost all of them focused on the common stiffness method. Therefore, this paper was devoted to develop a set of formulations of the probabilistic sensitivity analysis for Integrated Force Method using the perturbation method. Under the assumption of the spatial homogonous random fields, two approximate formulas of the stochastic sensitivity analysis were implemented for IFM. Another stochastic sensitivity analysis from the reliability-based method was also evaluated for comparisons according to the inverse cumulative distribution function of design variables. Numerical examples are presented in this paper to illustrate the application and differences of these methods. The efficiency and accuracy of these analytical formulas were also compared with results from the direct Monte Carlo simulations and the reliability-based sensitivity method. The numerical algorithm was shown to be readily adaptable to the existing program since the models of stochastic finite element and stochastic design sensitivity are almost identical.

2. Integrated Force Method and Deterministic Sensitivity Formulas

The governing equation of IFM [4] for a continuum is discretized by a finite element model with n force and m displacement unknowns. The m equilibrium equations and the $r(r = n - m)$ compatibility conditions are coupled to obtain the IFM governing equation in static analysis as

$$\left[\frac{[B]}{[C][G]}\right]\{F\} = \left\{\frac{\{P^*\}}{\{\delta R\}}\right\} \quad \text{or} \quad [S]\{F\} = \{P\}, \tag{2.1}$$

where $[B]$ is the $(m \times n)$ equilibrium matrix, $[C]$ is the $(r \times n)$ compatibility matrix, $[G]$ is the $(n \times n)$ concatenated flexibility matrix, and $\{P^*\}$ is the m components load vector. $\{P\}$ is the n components vector of combination of $\{P^*\}$ and $\{\delta R\}$. The r-component initial deformation vector $\{\delta R\}$ is $\{\delta R\} = -[C]\{\beta^0\}$. Here, $\{\beta^0\}$ is the n-component initial deformation vector and $[S]$ is the $(n \times n)$ governing matrix.

The solution of (2.1) yields the n forces $\{F\}$. The m displacements $\{X\}$ are obtained from the force $\{F\}$ by back-substitution

$$\{X\} = [J]\left([G]\{F\} + \left\{\beta^0\right\}\right), \tag{2.2}$$

where $[J]$ is the $(m \times n)$ deterministic deformation coefficient matrix defined as $[J] = m$ rows of $[S]^{-1}$. It must be stressed here that (2.1) and (2.2) in IFM represent two key relationships in structural analysis.

Based on the above governing equation (2.1), the differentiating equation (2.1) with respect to the "kth" design variable h_k results in

$$[S]\frac{\partial\{F\}}{\partial h_k} + \frac{\partial[S]}{\partial h_k}\{F\} = \frac{\partial\{P\}}{\partial h_k}, \tag{2.3}$$

that is,

$$[S]\{F_k\} = \{P_k\} - [S_k]\{F\}, \tag{2.4}$$

where $\{F_k\}$ is the internal force sensitivity vector with respect to h_k, that is, $\{F_k\} = \partial\{F\}/\partial h_k$. $\{P_k\}$ is the load derivative vector with respect to h_k, that is, $\{P_k\} = \partial\{P\}/\partial h_k$. $[S_k]$ is the governing derivative matrix with respect to h_k, that is, $[S_k] = \partial[S]/\partial h_k$. k is the number of design variables, $k = 1, 2, \ldots, M$.

Similarly, the deterministic sensitivity formula of displacement with respect to the "kth" design variable h_k in IFM can be expressed by

$$\{X_k\} = [J_k]\left([G]\{F\} + \left\{\beta^0\right\}\right) + [J]\left([G_k]\{F\} + [G]\{F_k\} + \left\{\beta_k^0\right\}\right), \tag{2.5}$$

where $\{X_k\}$ is the displacement sensitivity vector with respect to h_k, that is, $\{X_k\} = \partial\{X\}/\partial h_k$. $\{\beta_k^0\}$ is the initial deformation derivative vector with respect to h_k, that is, $\{\beta_k^0\} = \partial\{\beta^0\}/\partial h_k$. $[J_k]$ is the deformation coefficient derivative matrix with respect to h_k, that is, $[J_k] = \partial[J]/\partial h_k$. Note that the deterministic sensitivity formulas are more complicated than the deterministic analysis ones.

3. Stochastic Sensitivity Analysis of IFM

The stochastic structural parameters are assumed to vary spatially as a homogeneous stochastic field. For example, the Young's modulus E_i corresponding to the ith element with a mean value μ_{E_i} may be written as

$$E_i(x) = \mu_{E_i}\left(1 + q_{E_i}(x)\right), \tag{3.1}$$

where $q_{E_i}(x)$ is the fluctuating component of the Young's modulus, called the normalized primitive random variable of the Young's modulus. Note that the defined random Young's modulus has zero mean and standard deviation $q_{E_i}(x)$. Similarly, other random structural parameters can also be defined by the normalized primitive random variables.

It is well known that the perturbation technique had been applied to the stochastic analysis of finite element method for structures. The uncertain parameters in structures can cause the response to be uncertain as well as make the response sensitivity uncertain. Assuming the random variables have small fluctuations, a second-order Taylor's expansion in relation to the random variables is employed for stochastic response sensitivity calculation in IFM. For example, the Taylor series expansion of the internal force sensitivity vector $\{F_k\}$ has the following form:

$$\{F_k\} = \left\{\overline{F_k}\right\} + \sum_{i=1}^{N}\{F_{k,i}\}q_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{F_{k,ij}\right\}q_iq_j + \cdots \quad i,j = 1,2,\ldots,N, \tag{3.2}$$

where $\{\overline{F_k}\}$ denote the corresponding deterministic part of the vector at $q_i = 0$ $(i = 1,2,\ldots,N)$ and $\{F_{k,i}\}$ and $\{F_{k,ij}\}$ are partial derivatives of $\{F_k\}$ defined as follows:

$$\{F_{k,i}\} = \left.\frac{\partial\{F_k\}}{\partial q_i}\right|_{\{q\}=\{0\}} \qquad \left\{F_{k,ij}\right\} = \left.\frac{\partial^2\{F_k\}}{\partial q_i\partial q_j}\right|_{\{q\}=\{0\}} \qquad i,j = 1,2,\ldots,N. \tag{3.3}$$

Similarly, the Taylor series expansions of $[S]$, $[S_k]$, $\{F\}$, and $\{P_k\}$ in the deterministic sensitivity (2.4) can be also obtained. Substituting these expressions into (2.4), retaining terms up to the second-order terms, and by equating terms of equal orders, the zeroth-, first- and second-order equations on both sides corresponding to (2.4) can be written by the following set of recursive equations:

$$\begin{aligned}
\text{zeroth-order}: \left\{\overline{F_k}\right\} &= \left[\overline{S}\right]^{-1}\left(\left\{\overline{P_k}\right\} - \left[\overline{S_k}\right]\left\{\overline{F}\right\}\right),\\
\text{first-order}: \{F_{k,i}\} &= \left[\overline{S}\right]^{-1}\left(\{P_{k,i}\} - \left[\overline{S_k}\right]\{F_{,i}\} - [S_{k,i}]\left\{\overline{F}\right\} - [S_{,i}]\left\{\overline{F}_k\right\}\right),\\
\text{second-order}: \{F_{k,ij}\} &= \left[\overline{S}\right]^{-1}\left(\{P_{k,ij}\} - \left[\overline{S_k}\right]\{F_{,ij}\} - 2[S_{k,i}]\{F_{,j}\}\right.\\
&\quad \left. - [S_{k,ij}]\left\{\overline{F}\right\} - 2[S_{,i}]\{F_{k,j}\} - [S_{,ij}]\left\{\overline{F_k}\right\}\right).
\end{aligned} \tag{3.4}$$

Therefore, the first-order approximation for the internal force sensitivity is obtained by truncating the right-hand side of (3.2) after the second term as

$$\begin{aligned}
\{F_k\} &= \left\{\overline{F_k}\right\} + \sum_{i=1}^{N}\{F_{k,i}\}q_i,\\
\mu_{F_k}^{\mathrm{I}} &= E^{\mathrm{I}}[\{F_k\}] = \left\{\overline{F_k}\right\},\\
\mathrm{Cov}^{\mathrm{I}}\left(\{F_k\},\{F_k\}^T\right) &= \sum_{i=1}^{N}\sum_{j=1}^{N}\{F_{k,i}\}\left\{F_{k,j}\right\}^T E\left[q_iq_j\right]
\end{aligned} \tag{3.5}$$

and the second-order approximation

$$\{F_k\} = \left\{\overline{F_k}\right\} + \sum_{i=1}^{N}\{F_{k,i}\}q_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{F_{k,ij}\right\}q_i q_j,$$

$$\mu_{F_k}^{\mathrm{II}} = E^{\mathrm{II}}[\{F_k\}] = \left\{\overline{F_k}\right\} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{F_{k,ij}\right\}E\left[q_i q_j\right], \tag{3.6}$$

$$\mathrm{Cov}^{\mathrm{II}}\left(\{F_k\}, \{F_k\}^T\right) = \sum_{i=1}^{N}\sum_{j=1}^{N}\{F_{k,i}\}\left\{F_{k,j}\right\}^T E\left[q_i q_j\right],$$

where $\mu_{F_k}^{I}$ and $\mu_{F_k}^{II}$ are the mean values of the first- and second-order approximate formulations of the internal force sensitivity vector with respect to h_k, respectively. $\mathrm{Cov}^{\mathrm{I}}(\{F_k\}, \{F_k\}^T)$ and $\mathrm{Cov}^{\mathrm{II}}(\{F_k\}, \{F_k\}^T)$ are the covariance matrices of the first- and second-order approximate formulations of the internal force sensitivity matrices with respect to h_k, respectively. In the same way, the displacement sensitivity formulas of IFM can be also obtained by.

The recursive equations:

$$\begin{aligned}
\left\{\overline{X_k}\right\} &= [J_k]\left(\left[\overline{G}\right]\left\{\overline{F}\right\} + \left\{\overline{\beta^0}\right\}\right) + [J]\left(\left[\overline{G_k}\right]\left\{\overline{F}\right\} + \left[\overline{G}\right]\left\{\overline{F_k}\right\} + \left\{\overline{\beta_k^0}\right\}\right),\\
\{X_{k,i}\} &= [J_k]\left(\left[\overline{G}\right]\{F_{,i}\} + [G_{,i}]\left\{\overline{F}\right\} + \left\{\beta_{,i}^0\right\}\right)\\
&\quad + [J]\left(\left[\overline{G_k}\right]\{F_{,i}\} + [G_{k,i}]\left\{\overline{F}\right\} + \left[\overline{G}\right]\{F_{k,i}\} + [G_{,i}]\left\{\overline{F_k}\right\} + \left\{\beta_{k,i}^0\right\}\right),\\
\{X_{k,ij}\} &= [J_k]\left(\left[\overline{G}\right]\{F_{,ij}\} + 2[G_{,i}]\{F_{,j}\} + [G_{,ij}]\left\{\overline{F}\right\} + \left\{\beta_{,ij}^0\right\}\right) + [J]\\
&\quad \times \left(\left[\overline{G_k}\right]\{F_{,ij}\} + 2[G_{,i}]\{F_{,j}\} + [G_{k,ij}]\left\{\overline{F}\right\} + \left[\overline{G}\right]\{F_{k,ij}\}\right.\\
&\qquad \left. +2[G_{,i}]\{F_{k,j}\} + [G_{,ij}]\left\{\overline{F_k}\right\} + \left\{\beta_{k,ij}^0\right\}\right).
\end{aligned} \tag{3.7}$$

The first-order approximation:

$$\{X_k\} = \left\{\overline{X_k}\right\} + \sum_{i=1}^{N}\{X_{k,i}\}q_i,$$

$$\mu_{X_k}^{\mathrm{I}} = E^{\mathrm{I}}[\{X_k\}] = \left\{\overline{X_k}\right\}, \tag{3.8}$$

$$\mathrm{Cov}^{\mathrm{I}}\left(\{X_k\}, \{X_k\}^T\right) = \sum_{i=1}^{N}\sum_{j=1}^{N}\{X_{k,i}\}\left\{X_{k,j}\right\}^T E\left[q_i q_j\right].$$

The second-order approximation:

$$\{X_k\} = \left\{\overline{X_k}\right\} + \sum_{i=1}^{N}\{X_{k,i}\}q_i + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{X_{k,ij}\right\}q_i q_j,$$

$$\mu_{X_k}^{\mathrm{II}} = E^{\mathrm{II}}[\{X_k\}] = \left\{\overline{X_k}\right\} + \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left\{X_{k,ij}\right\}E[q_i q_j], \tag{3.9}$$

$$\mathrm{Cov}^{\mathrm{II}}\left(\{X_k\},\{X_k\}^T\right) = \sum_{i=1}^{N}\sum_{j=1}^{N}\{X_{k,i}\}\left\{X_{k,j}\right\}^T E[q_i q_j],$$

where $\mu_{X_k}^{\mathrm{I}}$ and $\mu_{X_k}^{\mathrm{II}}$ are the mean values of the first- and second-order approximate formulations of the displacement sensitivity vector with respect to h_k, respectively. $\mathrm{Cov}^{\mathrm{I}}(\{X_k\},\{X_k\}^T)$ and $\mathrm{Cov}^{\mathrm{II}}(\{X_k\},\{X_k\}^T)$ are the covariance matrices of the first- and second-order approximate formulations of the displacement sensitivity matrices with respect to h_k, respectively.

It is noted that the stochastic response sensitivity formulas of IFM have similar features of the stochastic response formulas [14]. The stochastic response sensitivity analysis formulas consist of the deterministic and random parts. The zeroth-order sensitivity equations are identical to the deterministic sensitivity equations evaluated at the mean value. The first- and second-order sensitivity equations consist of the recursive items. Moreover, covariance matrices in the first- and second-order approximate formulas have the same forms in expressions with the assumption of $E[q_i q_j q_k] = 0$ and $\mathrm{Cov}(q_i, q_j, q_k, q_l) = 0$. However, to obtain the acceptable accuracy, the variance of uncertainties must be small enough because only finite terms in the above series were used. In this study, the algorithms of stochastic sensitivity analysis for IFM had been programmed in a closed form to evaluate the mean values and covariance matrices using the symbolic software tools, Maple V.

4. Reliability Sensitivity Analysis of IFM

Since the first two probabilistic moments of responses have been obtained in the stochastic response formulation of IFM, the response value at p probability of occurrence can be calculated by the inverse of the cumulative distribution function. By using the reliability-based method, stochastic response sensitivity formulation may be expressed by another form.

Let ν, μ_ν, and σ_ν denote the response value at p probability of occurrence, the mean value, and standard deviation, respectively. By means of the transformation in the probability theory, the inverse of the cumulative distribution function can be written as

$$\Phi^{-1}(p) = \frac{\nu - \mu_\nu}{\sigma_\nu}, \quad \text{that is,} \quad \nu = \mu_\nu + \Phi^{-1}(p)\sigma_\nu, \tag{4.1}$$

where $\Phi^{-1}(p)$ is the cumulative distribution function for the standard normal distribution.

The sensitivity of ν with respect to the mean of a primitive random variable, μ_R, can be given by

$$\frac{\partial \nu}{\partial \mu_R} = \frac{\partial \mu_\nu}{\partial \mu_R} + \Phi^{-1}(p)\frac{\partial \sigma_\nu}{\partial \mu_R}. \tag{4.2}$$

It should be noted that the response sensitivity is the sum of the derivatives of the mean response and the standard deviation that is prorated by the inverse standard normal cumulative distribution function $\Phi^{-1}(p)$. Obviously, the sensitivity formula is a complicated expression, even for the simple structure.

5. Illustration Examples

To clearly demonstrate the methodology application, an example, the six-bar truss with mechanical and thermal loads, is evaluated by the following two cases.

(1) Only one random variable, Young's modulus E, is considered in the structure; other variables take their mean values as deterministic variables in the computation.

(2) Multirandom variables are considered in the computation.

The six-bar truss shown in Figure 1 has a Young's modulus E with a mean of 68.947 GPa and standard deviation of 3.776 GPa and a coefficient of thermal expansion α with a mean of 1.08×10^{-5}/°C and standard deviation of 4.183×10^{-7}/°C. It is subjected to a mechanical load P with a mean of 4.448 kN and standard deviation of 0.222 kN at node 1 along the y-direction. The temperature on member 3 has an increasing mean of 37.778°C and standard deviation of 13.299°C. The sizing variables, six cross-sectional areas, have the mean, standard deviation, and correlation coefficient matrix as follows:

$$\begin{Bmatrix} \mu_{A_1} \\ \mu_{A_2} \\ \mu_{A_3} \\ \mu_{A_4} \\ \mu_{A_5} \\ \mu_{A_6} \end{Bmatrix} = \begin{Bmatrix} 6.452 \\ 4.562 \\ 6.452 \\ 4.562 \\ 6.452 \\ 6.452 \end{Bmatrix} \text{cm}^2, \qquad \begin{Bmatrix} \sigma_{A_1} \\ \sigma_{A_2} \\ \sigma_{A_3} \\ \sigma_{A_4} \\ \sigma_{A_5} \\ \sigma_{A_6} \end{Bmatrix} = \begin{Bmatrix} 0.6452 \\ 0.4446 \\ 0.5770 \\ 0.4206 \\ 0.5398 \\ 0.4785 \end{Bmatrix} \text{cm}^2,$$
$$[\rho_A] = \begin{bmatrix} 1.000 & 0.641 & 0.615 & 0.461 & 0.598 & 0.607 \\ 0.641 & 1.000 & 0.516 & 0.557 & 0.521 & 0.553 \\ 0.615 & 0.516 & 1.000 & 0.546 & 0.535 & 0.641 \\ 0.461 & 0.556 & 0.546 & 1.000 & 0.778 & 0.804 \\ 0.598 & 0.521 & 0.535 & 0.778 & 1.000 & 0.806 \\ 0.607 & 0.553 & 0.641 & 0.804 & 0.806 & 1.000 \end{bmatrix}. \tag{5.1}$$

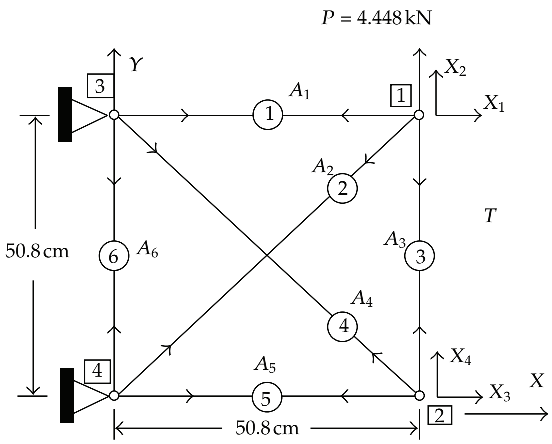

Figure 1: Six-bar truss.

It should be noted that the problem has ten random variables defined as follows:

$$
\begin{aligned}
&A_1 = \mu_{A_1}(1 + q_{A_1}), \qquad A_2 = \mu_{A_2}(1 + q_{A_2}), \qquad A_3 = \mu_{A_3}(1 + q_{A_3}), \\
&A_4 = \mu_{A_4}(1 + q_{A_4}), \qquad A_5 = \mu_{A_5}(1 + q_{A_5}), \qquad A_6 = \mu_{A_6}(1 + q_{A_6}), \\
&E = \mu_E(1 + q_E), \qquad \alpha = \mu_\alpha(1 + q_\alpha), \qquad P = \mu_P(1 + q_P), \qquad T = \mu_T(1 + q_T),
\end{aligned}
\tag{5.2}
$$

where q_{R_i}, is the normalized primitive random variable with zero mean and standard deviation q_{R_i} and the subscript R represents any of random variables. Note that the sixth bar was omitted in the results since it is not subjected to any force.

In Case (1), as it is observed from Figures 2 and 3, after 18,000 simulations the fluctuations of mean value and standard deviation of response sensitivities (here, we just calculated force F_1 in the first bar and displacement x_1 in the horizontal direction at node 1) with respect to the first bar area A_1 are small enough and gradually trend to be smooth with increasing simulations. In fact, the number in simulations has been taken as 25,000.

The comparisons of the mean value and standard deviation of response sensitivities with varying c.o.v. of E between the perturbation methods and direct Monte Carlo Simulation (DMCS) of IFM are shown in Figures 4 and 5, respectively. It is noted in Figure 4 that in the range of small variance of the Young's modulus the mean force sensitivity curve of DMCS increases linearly, whereas the first- and second-order perturbation curves overlap each other and remain constant. And the standard deviation force sensitivity curves of three methods are close to each other and linearly increase. In Figure 5, the mean displacement sensitivity curves of both DMCS and the second-order perturbation have an accelerated rate of increase and are close to each other prior to c.o.v. of $E = 0.125$, whereas the first-order perturbation curve remains constant. The standard deviation displacement sensitivity curve of both the first- and second-order perturbation linearly increases with the coefficient of variation E and is close to the DMCS curve before c.o.v. of $E = 0.15$.

It is important to note that the perturbation methods underestimate the response variability after the coefficient of variation $E = 0.1$. It is consistent with the earlier results as demonstrated in some papers [15, 16].

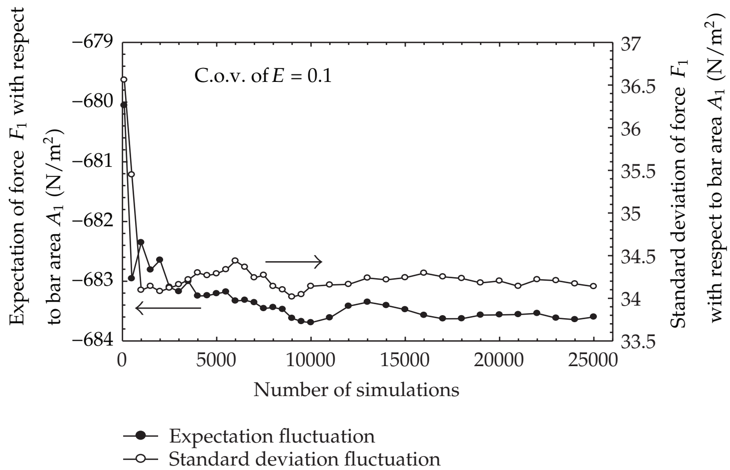

Figure 2: Fluctuation of mean and standard deviation of force sensitivity in the first bar with respect to A_1.

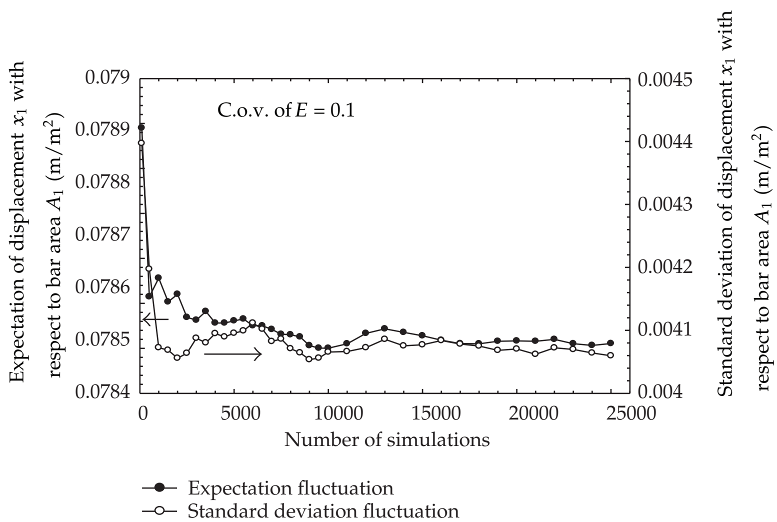

Figure 3: Fluctuation of mean and standard deviation of displacement sensitivity, x_1, with respect to A_1.

For easy comparisons, assuming the probability of occurrence as 75%, the force sensitivity in the first bar and displacement sensitivity in the horizontal direction at 1 node with respect to bar areas are shown in Figures 6 and 7, respectively. In Figure 6, the force sensitivity values obtained from the reliability method, the perturbation method, and direct Monte Carlo simulation are almost equal to each other. The first bar force is more sensitive to the second bar area A_2, not sensitive to bar areas A_3 and A_5. The deterministic force sensitivity values are different from results of other methods. In Figure 7, the horizontal displacement sensitivity values from both the perturbation method and Monte Carlo simulation are close to each other, but the reliability method and deterministic sensitivity have different values with results of other methods. The horizontal displacement is very sensitive to the first bar area A_1, secondly to A_2, and insensitive to other bar areas.

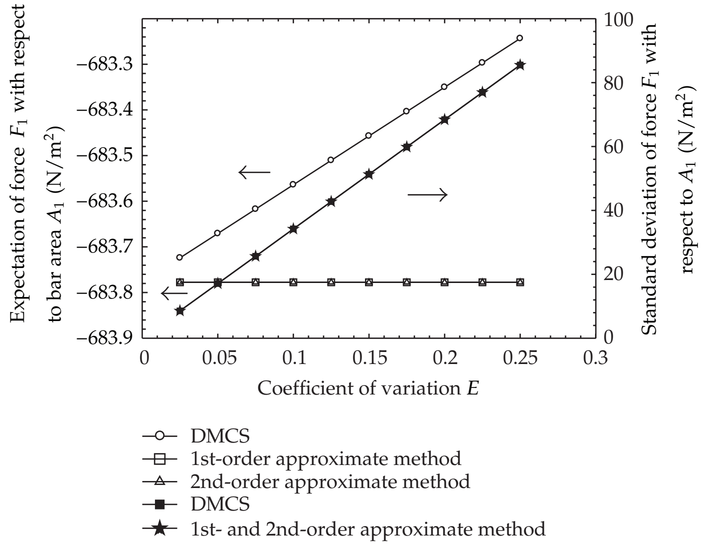

Figure 4: Comparison of mean and standard deviation of force sensitivity in the first bar with varying c.o.v. of uncertain variable *E*.

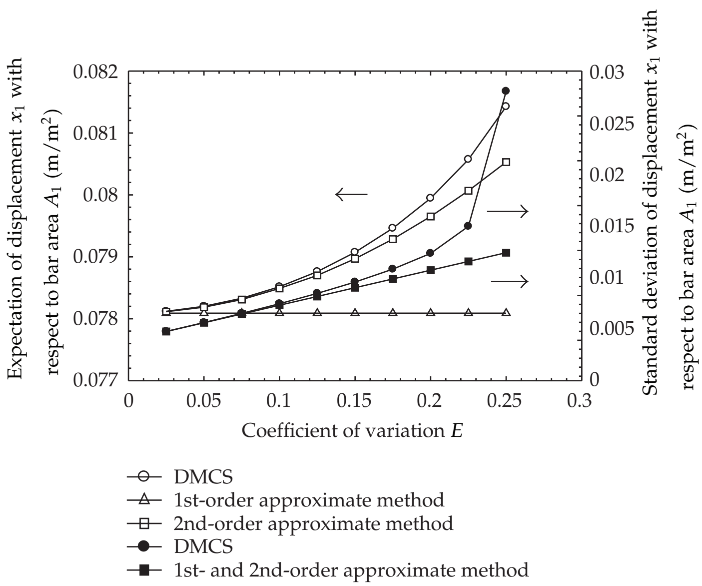

Figure 5: Comparison of mean and standard deviation of displacement sensitivity, x_1, with varying c.o.v. of uncertain variable *E*.

In Case (2), also assuming the probability of occurrence as 75%, the comparisons of the force sensitivity in the first bar and displacement sensitivity in the horizontal direction at 1 node with respect to bar areas are shown in Figures 8 and 9, respectively. In Figure 8, note that there are small differences between the perturbation methods and direct Monte Carlo simulation; moreover, the second-order approximation perturbation method has better results with Monte Carlo simulation. However, the reliability method and deterministic

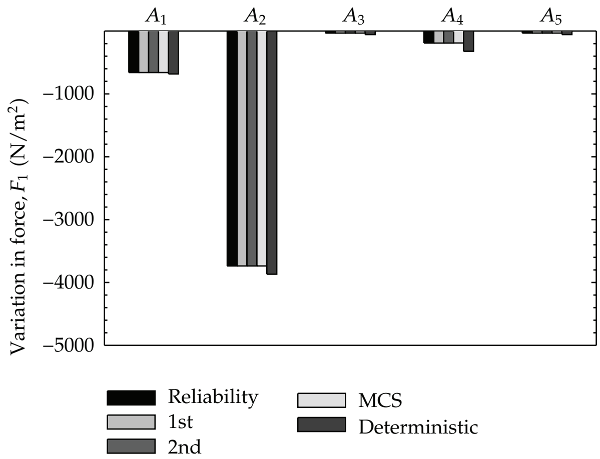

Figure 6: Comparison of force sensitivity in the first bar with one random variable E at 75% probability of occurrence.

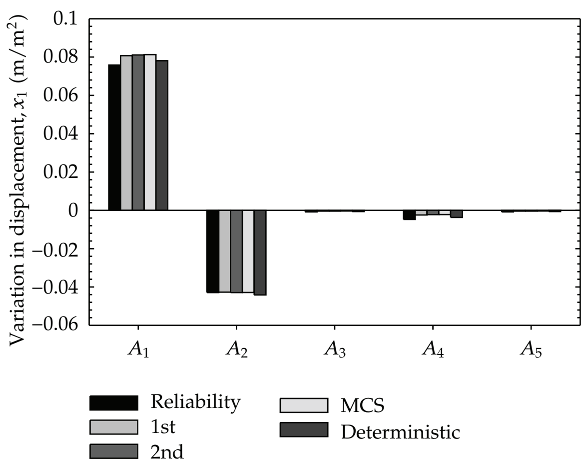

Figure 7: Comparison of displacement sensitivity at the bar with one random variable E at 75% probability of occurrence.

sensitivity are not correlated to DMCS. In Figure 9, the reliability method and deterministic sensitivity have big differences with DMCS, whereas the perturbation methods have a good agreement with DMCS. The first bar force and horizontal displacement in Case (2) have the same sensitive and insensitive design variables in Case (1).

The comparisons of the first bar force and horizontal displacement sensitivities with respect to bar areas between the perturbation methods and direct Monte Carlo simulation are shown in Table 1. The number of simulations in DMCS was taken as 20,000. It was noted that the second-order approximation has a better agreement with DMCS than the first-order approximation on the mean values and standard deviations of response sensitivities.

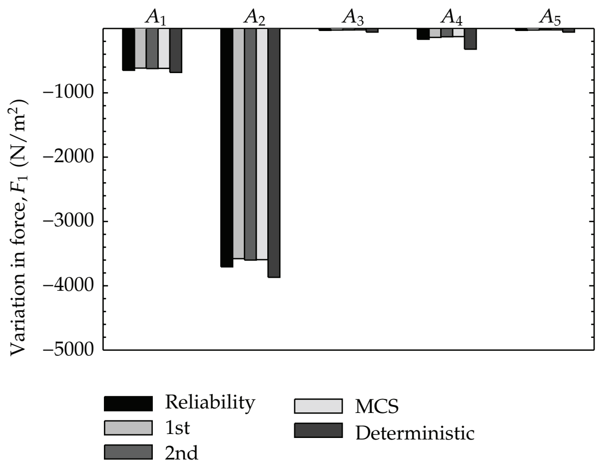

Figure 8: Comparison of force sensitivity in the first bar with Multirandom variables at 75% probability of occurrence.

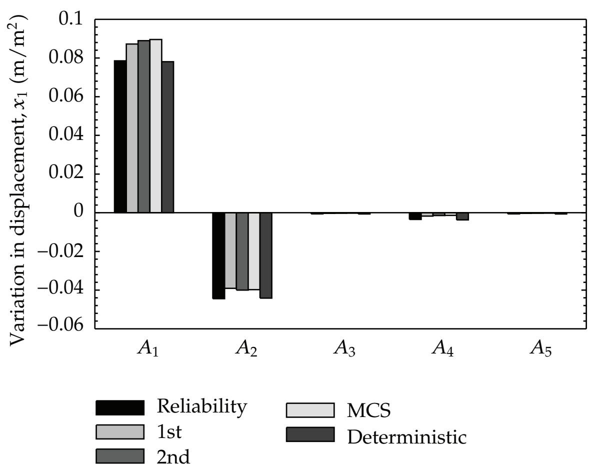

Figure 9: Comparison of displacement sensitivity at the bar with Multirandom variables at 75% probability of occurrence.

6. Conclusions

Using the perturbation method, stochastic sensitivity analysis for Integrated Force Method (IFM) had been formulated with the first- and second-order approximations in this study. Furthermore, the developed algorithm can be easily adopted to fit into the existing deterministic sensitivity analysis program of IFM, only incorporating the random variable part subroutine. Without question, the stochastic analysis should enhance IFM available in the probability field for advanced structure analysis. It was observed that the sensitivity analytical results obtained through the second-order approximation have better accuracy than that through the first-order one compared to Monte Carlo results for either single- or Multirandom variables if the coefficients of variation of input random parameter are small.

Table 1: Comparison of stochastic sensitivities between PM and MCS.

Response sensitivities		Methods				
		Perturbation Method			DMCS	
		μ^{I}	μ^{II}	σ_p	μ	σ_M
F_1 (N/m^2)	A$_1$	−683.95	−691.34	100.42	−690.74	104.30
	A$_2$	−3868.99	−3891.04	429.72	−3892.00	440.63
	A$_3$	−57.00	−55.55	48.46	−55.47	49.28
	A$_4$	−322.42	−312.47	271.84	−312.07	275.60
	A$_5$	−57.00	−55.48	48.44	−55.42	49.09
x_1 (m/m^2)	A$_1$	0.0781	0.0798	0.0136	0.0798	0.0144
	A$_2$	−0.0442	−0.0451	0.00755	−0.0451	0.00798
	A$_3$	−0.00065	−0.00061	0.000516	−0.00060	0.000541
	A$_4$	−0.00368	−0.00341	0.00288	−0.00339	0.00300
	A$_5$	−0.00065	−0.00061	0.000515	−0.00060	0.000537

Note. μ^{I}, μ^{II} are the mean value of the first- and second-order approximation of responses, respectively. σ_p is the standard deviation of the perturbation method of responses. μ is the mean value of direct Monte Carlo simulation of responses. σ_M is the standard deviation of direct Monte Carlo simulation of responses.

The reliability-based sensitivity analysis shows a big difference with Monte Carlo results and has a computational complexity, even for the simplest structures.

However, for larger coefficients of variation of random parameters, the perturbation method with the first- and second-order approximations is not effective and the accuracy of their calculations also decreases due to the limitation of truncating error and neglect of the higher-order covariance matrices in the approximation formulas. Thus, the further development of the methodology for providing more accurate calculations in stochastic sensitivity analysis of IFM would be desirable for larger coefficient of variation of randomness.

Acknowledgments

The support of Glenn Research Center and help from Dr. James D. Guptill and Mrs. Rula M. Coroneos for this research are gratefully acknowledged.

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