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Research Article

$M^{[X]}/G_1, G_2/1$ with Setup Time, Bernoulli Vacation, Break Down, and Delayed Repair

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We present a single server in which customers arrive in batches and the server provides service one by one. The server provides two heterogeneous service stages such that service time of both stages is different and mandatory to all arriving customers in such a way that, after the completion of first stage, the second stage should also be provided to the customers. The server may subject to random breakdowns with brake down rate λ and, after break down, it should be repaired but it has to wait for being repaired and such waiting time is called delay time. Both the delay time and repair time follow exponential distribution. Upon the completion of the second stage service, the server will go for vacation with probability p or stay back in the system probability $1 - p$, if any. The vacation time follows general (arbitrary) distribution. Before providing service to a new customer or a batch of customers that joins the system in the renewed busy period, the server enters into a random setup time process such that setup time follows exponential distribution. We discuss the transient behavior and the corresponding steady state results with the performance measures of the model.

1. Introduction

Due to the improvement and advancement of science and technology, performance in modeling is one of the vital parts that affects the design, configuration, and implementation of any real time system. Queueing modeling is being used tremendously and effectively in congestion problems encountered in day to day life as well as industrial scenario including computer systems, web services and communication networks, waiting lines at airports, railway stations, and banks.

Many authors have put their contributions on queueing systems with random setup time. Setup time plays a significant role in the study of queueing systems and which is defined as follows: at every beginning of new busy period, the server enters into a random setup time process before actually providing service to a new customer. Levy and Kleinrock [1] studied such types of models. Choudhury [2] studied a batch arrival queueing system having a setup period and a vacation period. Ke [3] examined the steady-state results of the unreliable system and startup time with also modified T vacation policy. The same author [4] extended his work with also NT policies and close down period. Wang et al. [5] found important performance measures for such systems with both N-policy and T-policy. Yang et al. [6] studied optimal control policy for an unreliable system with second optional service and startup period.

Baba [7] studied about batch arrival single server with vacation. A comprehensive survey can be found in Doshi [8]. Keilson and Servi [9] studied the dynamics of non-Markovian vacation. Maraghi et al. [10] have obtained steady state solution of batch arrival queueing system with random breakdowns and Bernoulli schedule server vacations having general vacation time. In most of the research study of queueing models, the server is assumed to be reliable such that the server works forever, but this is not the case in most of the real scenarios that the servers are reliable such that the servers may meet breakdowns. Also there are numerous papers on queueing models with vacations and breakdowns.

Many researchers have paid their attention and efforts in queueing theory by considering various aspects like two phases queue system with random break downs and Bernoulli vacation. Anabosi and Madan [11] studied a single server queue with two types of service, Bernoulli schedule server vacations, and a single vacation policy. Artalejo and Choudhury [12] discussed about the steady state analysis of an $M/G/1$ queue with repeated attempts and two-phase service. Zadeh and Shahkar [13] studied two phases queue system with Bernoulli feedback and Bernoulli schedule server vacation. Choudhury and Madan [14] analyzed a two-phase batch arrival queueing system with a vacation time under Bernoulli schedule. Choudhury and Paul [15] studied a two-phase queueing system with Bernoulli feedback. Madan [16, 17] discussed a single server with two types of service and deterministic server vacations as well as Bernoulli vacation.

Whenever the server encounters a break down, it would not be able to serve unless it should be repaired. Therefore the server should undergo a repair process, but sometimes the repair process will not be started immediately due to the nonavailability of the repairing equipment or repairmen. Such situations can also be modeled as queueing model and which has been studied by many authors. Burke [18] studied delays in single-server queues with batch input. Madan [19] studied queueing system with random failures and delayed repairs. Choudhury et al. [20] discussed a batch arrival, single server queue with two phases of service subject to the server breakdown, and delay time. Khalaf et al. [21] have obtained the steady state solution of an $M^{[X]}/G/1$ queue with Bernoulli schedule, general vacation times, random breakdowns, general delay times, and general repair times.

Most recently, the studies of transient behavior in queueing systems have been growing extensively due to their potential applications in which a practitioner needs to know how the system will function up to a time horizon. Takagi [22] analyzed time-dependent analysis of $M/G/1$ vacation models with exhaustive service. Thangaraj and Vanitha [23] have obtained transient solution of two-phase heterogeneous services with compulsory vacation and random break downs.

In this paper, we consider a queueing system where the customers arrive in batches and the server provides service one by one in FCFS basis. Each arriving batch has to undergo two stages of service provided by a single server and the service time for two stages is assumed to follow general distribution. As soon as the second stage of a customer's service is completed, the server may go for a vacation with probability p or continue staying in the system to provide service to the next customer, if any, with probability $1-p$. Further, assuming that, after returning from a vacation, if the server does not find any customers in the system, even then he joins the system without taking any further vacations and this policy is termed as single vacation with Bernoulli schedule. Further, we assume that whenever the system becomes empty, the server is turned off each time and which is called turned off period. During this period, the server may be either available but turned off in the system or else it may be on vacation. After service completion, the server goes on vacation with probability p or the server stays back in the system with probability $1-p$. If the server is ready for service in the system, then the system becomes operative only when a new customer or a batch of customers arrives to the system. The server startup corresponds to the preparatory work of the server before starting the service. In some actual situations, the server often needs a startup time before providing service. In this case, it will take a random setup time before it actually starts serving a new customer. This random setup time is usually termed as SET (during which no proper work is done) in order to set the system into operative mode before actual service begins (setup period). On account of that, the system may be subject to breakdowns; the breakdowns occur according to Poisson process. Once the system breaks down, the repair process will not be started immediately so that the system has to wait before it could be repaired; such a waiting time is known as "delay time" which follows exponential distribution. The repair time follows exponential distribution. After the repair process is complete, the server resumes its work immediately. Also, whenever the system meets a break down, the customer whose service is interrupted goes back to the head of the queue and the interrupted customer restarts its service from the beginning again.

The rest of the paper is organized as follows. The mathematical description of our model is in Section 2 and equations governing the model are given in Section 3. The corresponding steady state results have been derived explicitly in Section 4, followed by the particular cases of the prescribed model which have been discussed in Section 5, and in Section 6, the concluding marks have been given.

2. Mathematical Description of the Model

We assume the following to describe the queueing model of our study.

(i) Customers arrive at the system in batches of variable size in a compound Poisson process. Let $\lambda c_i \Delta t$ ($i = 1, 2, 3, \ldots$) be the first order probability that a batch of i customers arrives at the system during a short interval of time $(t, t+\Delta t)$, where $0 \le c_i \le 1$, $\sum_{i=1}^{\infty} c_i = 1$, and $\lambda > 0$ are the mean arrival rates of batches. The customers are served one-by-one on a "first come-first served" basis.

(ii) The random setup time is a random variable called SET variable following exponential distribution with mean setup time being ν.

(iii) Each customer undergoes two stages of heterogeneous service provided by a single server on a first come-first served basis. The service time of the two stages follows different general (arbitrary) distributions with distribution function $B_j(\nu)$ and the density function $b_j(\nu)$, $j = 1, 2$.

(iv) Let $\mu_i(x)dx$ be the conditional probability of completion of the ith stage of service during the interval $(x, x+dx]$ given that elapsed service time is x, so that

$$\mu_i(x) = \frac{b_i(x)}{1 - B_i(x)}, \quad i = 1, 2. \tag{1}$$

And therefore,

$$b_i(v) = \mu_i(v)\, e^{-\int_0^v \mu_i(x)dx}, \quad i = 1, 2. \tag{2}$$

(v) As soon as the second stage service of a customer is completed, the server may go for a vacation of random length V with probability p $(0 \le p \le 1)$ or it may continue to serve the next customer $(1 - p)$.

(vi) The vacation time also follows general (arbitrary) distribution with distribution function $V(s)$ and the density function $v(s)$. Let $\gamma(x)dx$ be the conditional probability of a completion of a vacation during the interval $(x, x + dx]$ given that the elapsed vacation time is x, so that

$$\gamma(x) = \frac{v(x)}{1 - V(x)}, \tag{3}$$

and therefore,

$$v(s) = \gamma(s)\, e^{-\int_0^s \gamma(x)dx}. \tag{4}$$

(vii) On returning from vacation, the server instantly starts serving the customer at the head of the queue, if any. The server stays in the system for being available if there are no customers.

(viii) The system may break down at random and breakdowns are assumed to occur according to a Poisson stream with mean breakdown rate $\alpha > 0$.

(ix) Whenever the system breaks down, its repairs do not start immediately and there is a delay time. The delay time follows exponential distribution with mean $1/\eta$.

(x) The repair time of the server is exponentially distributed with mean $1/\beta$.

(xi) The server's breakdown does not occur during setup time.

(xii) Various stochastic processes involved in the system are assumed to be independent of each other.

3. Definitions and Equations Governing the System

We let

(i) $S_n(t)$ = Probability that at time t, the server is in setup time while there are "n" $(n \ge 1)$ customers in the queue;

(ii) $P_n^{(1)}(x, t)$ = Probability that at time "t" the server is active providing first stage service and there are "n" $(n \ge 1)$ customers in the queue excluding the one being served and the elapsed service time for this customer is x. Consequently, $P_n^{(1)}(t)$ denotes the probability that at time "t," there are "n" customers in the queue excluding the one customer in the first stage service irrespective of the value of x;

(iii) $P_n^{(2)}(x, t)$ = Probability that at time "t," the server is active providing second stage service and there are "n" $(n \ge 1)$ customers in the queue excluding the one being served and the elapsed service time for this customer is x. Consequently, $P_n^{(2)}(t)$ denotes the probability that at time "t," there are "n" customers in the queue excluding the one customer in the second stage service irrespective of the value of x;

(iv) $V_n(x, t)$ = probability that at time "t," the server is on vacation with elapsed vacation time x, and there are "n" $(n \ge 0)$ customers waiting in the queue for service. Consequently $V_n(t)$ denotes the probability that at time "t," there are "n" customers in the queue and the server is on vacation irrespective of the value of x;

(v) $D_n(t)$ = Probability that at time t, the server is inactive due to breakdown and the system is under waiting time before the server getting repaired while there are "n" $(n \ge 1)$ customers in the queue;

(vi) $R_n(t)$ = Probability that at time t, the server is inactive due to breakdown and the system is under repair while there are "n" $(n \ge 1)$ customers in the queue;

(vii) $Q(t)$ = probability that at time "t," there are no customers in the system and the server is idle but available in the system.

The queueing model is then governed by the following set of differential-difference equations:

$$\frac{d}{dt}S_n(t) = -(\lambda + \nu) S_n(t) + \lambda \sum_{i=1}^{n} c_i S_{n-i}(t) + \lambda c_n Q(t), \quad n \ge 1,$$

$$\frac{\partial}{\partial t}P_n^{(1)}(x,t) + \frac{\partial}{\partial x}P_n^{(1)}(x,t) + \left(\lambda + \mu_1(x) + \alpha\right) P_n^{(1)}(x,t) = \lambda \sum_{i=1}^{n-1} c_i P_{n-i}^{(1)}(x,t), \quad n \ge 1,$$

$$\frac{\partial}{\partial t}P_n^{(2)}(x,t) + \frac{\partial}{\partial x}P_n^{(2)}(x,t) + \left(\lambda + \mu_2(x) + \alpha\right) P_n^{(2)}(x,t) = \lambda \sum_{i=1}^{n-1} c_i P_{n-i}^{(2)}(x,t), \quad n \ge 1,$$

$$\frac{\partial}{\partial t}V_n(x,t) + \frac{\partial}{\partial x}V_n(x,t) + \left(\lambda + \gamma(x)\right) V_n(x,t) = \lambda \sum_{i=1}^{n-1} c_i V_{n-i}(x,t), \quad n \ge 1,$$

$$\frac{\partial}{\partial t}V_0(x,t) + \frac{\partial}{\partial x}V_0(x,t) + \left(\lambda + \gamma(x)\right) V_0(x,t) = 0,$$

$$\frac{d}{dt}D_n(t) = -\left(\lambda + \eta\right) D_n(t) + \lambda \sum_{i=1}^{n} c_i D_{n-i}(t) + \alpha \int_0^\infty P_{n-1}^{(1)}(x,t)\,dx + \alpha \int_0^\infty P_{n-1}^{(2)}(x,t)\,dx,$$

$$\frac{d}{dt}R_n(t) = -(\lambda+\beta)R_n(t) + \lambda\sum_{i=1}^{n-1}c_iR_{n-i}(t) + \eta D_n(t),$$
$$\frac{d}{dt}Q(t) = -\lambda Q(t) + \int_0^\infty V_0(x,t)\gamma(x)\,dx + (1-p)\int_0^\infty P_1^{(2)}(x,t)\mu_2(x)\,dx. \quad (5)$$

Equation (5) is to be solved subject to the following boundary conditions:

$$P_n^{(1)}(0,t) = \beta R_n(t) + \int_0^\infty V_n(x,t)\gamma(x)\,dx + (1-p)\int_0^\infty P_{n+1}^{(2)}(x,t)\mu_2(x)\,dx + \nu S_n(t), \quad n\ge 1, \quad (6)$$
$$P_n^{(2)}(0,t) = \int_0^\infty P_n^{(1)}(x,t)\mu_1(x)\,dx, \quad n\ge 1,$$
$$V_n(0,t) = p\int_0^\infty P_n^{(2)}(x,t)\mu_2(x)\,dx, \quad n\ge 0.$$

We assume that initially there are no customers in the system and the server is idle. So the initial conditions are

$$P_n^{(j)}(0) = 0; \quad n = 0,1,2\ldots, \; j=1,2;$$
$$V_0(0) = V_n(0) = 0; \qquad Q(0) = 1. \quad (7)$$

We define the probability generating function as follows:

$$P_q^{(i)}(x,z,t) = \sum_{n=0}^\infty z^n P_n^{(i)}(x,t);$$
$$P_q^{(i)}(z,t) = \sum_{n=0}^\infty z^n P_n^{(i)}(t);$$
$$i = 1,2;$$
$$V_q(x,z,t) = \sum_{n=0}^\infty z^n V_n(x,t); \qquad V_q(z,t) = \sum_{n=0}^\infty z^n V_n(t);$$
$$S_q(z,t) = \sum_{n=0}^\infty z^n S_n(t);$$
$$D_q(z,t) = \sum_{n=0}^\infty z^n D_n(t);$$
$$R_q(z,t) = \sum_{n=0}^\infty z^n R_n(t);$$
$$C(z) = \sum_{n=1}^\infty c_n z^n, \quad (8)$$

which are convergent inside the circle given by $|z|\le 1$, and define the Laplace transform of a function $f(t)$ as

$$\overline{f}(s) = \int_0^\infty f(t)e^{-st}dt. \quad (9)$$

Taking Laplace transforms of (5),

$$(s+\lambda+\nu)\overline{S}_n(s) = \lambda\sum_{i=1}^{n}c_i\overline{S}_{n-i}(s) + \lambda c_n\overline{Q}(s), \quad n\ge 1, \quad (10)$$
$$\frac{\partial}{\partial x}\overline{P}_n^{(1)}(x,s) + (s+\lambda+\mu_1(x)+\alpha)\overline{P}_n^{(1)}(x,s), = \lambda\sum_{i=1}^{n-1}c_i\overline{P}_{n-i}^{(1)}(x,s), \quad n\ge 1, \quad (11)$$
$$\frac{\partial}{\partial x}\overline{P}_n^{(2)}(x,s) + (s+\lambda+\mu_2(x)+\alpha)\overline{P}_n^{(2)}(x,s) = \lambda\sum_{i=1}^{n-1}c_i\overline{P}_{n-i}^{(2)}(x,s), \quad n\ge 1, \quad (12)$$
$$\frac{\partial}{\partial x}\overline{V}_n(x,s) + (s+\lambda+\gamma(x))\overline{V}_n(x,s) = \lambda\sum_{i=1}^{n-1}c_i\overline{V}_{n-i}(x,s), \quad (13)$$
$$\frac{\partial}{\partial x}\overline{V}_0(x,s) + (s+\lambda+\gamma(x))\overline{V}_0(x,s) = 0, \quad (14)$$
$$(s+\lambda+\eta)\overline{D}_n(s) = \lambda\sum_{i=1}^{n-1}c_i\overline{D}_{n-i}(s) + \alpha\int_0^\infty \overline{P}_{n-1}^{(1)}(x,s)\,dx + \alpha\int_0^\infty \overline{P}_{n-1}^{(2)}(x,s)\,dx, \quad (15)$$
$$(s+\lambda+\beta)\overline{R}_n(s) = \lambda\sum_{i=1}^{n-1}c_i\overline{R}_{n-i}(s) + \eta\overline{D}_n(s), \quad (16)$$
$$(s+\lambda)\overline{Q}(s) = 1 + \int_0^\infty \overline{V}_0(x,s)\gamma(x)\,dx + (1-p)\int_0^\infty \overline{P}_0^{(2)}(x,s)\mu_2(x)\,dx, \quad (17)$$

for the following boundary conditions:

$$\overline{P}_n^{(1)}(0,s) = (1-p)\int_0^\infty \overline{P}_{n+1}^{(2)}(x,s)\mu_2(x)\,dx + \int_0^\infty \overline{V}_n(x,s)\gamma(x)\,dx + \beta\overline{R}_n(s) + \nu\overline{S}_n(s); \quad n\ge 1; \quad (18)$$

$$\overline{P}_n^{(2)}(0,s) = \int_0^\infty \overline{P}_n^{(1)}(x,s)\,\mu_1(x)\,dx; \quad n = 0,1,2,\ldots; \tag{19}$$

$$\overline{V}_n(0,s) = p\int_0^\infty \overline{P}_n^{(2)}(x,s)\,\mu_2(x)\,dx; \quad n = 0,1,2,\ldots. \tag{20}$$

Now multiplying (10) by z^n, summing over n from 1 to ∞, and using the definition of probability generating function, we obtain

$$(s+\lambda-\lambda C(z)+\nu)\,\overline{S}_q(z,s) = \lambda C(z)\,\overline{Q}(s), \quad n \geq 1. \tag{21}$$

Performing similar operations on (11) to (16), we have

$$\frac{\partial}{\partial x}\overline{P}_q^{(1)}(x,z,s) + (s+\lambda-\lambda C(z)+\mu_1(x)+\alpha)\,\overline{P}_q^{(1)}(x,z,s) = 0 \tag{22}$$

$$\frac{\partial}{\partial x}\overline{P}_q^{(2)}(x,z,s) + (s+\lambda-\lambda C(z)+\mu_2(x)+\alpha)\,\overline{P}_q^{(2)}(x,z,s) = 0 \tag{23}$$

$$\frac{\partial}{\partial x}\overline{V}_q(x,z,s) + (s+\lambda-\lambda C(z)+\gamma(x))\,\overline{V}_q(x,z,s) = 0 \tag{24}$$

$$(s+\lambda-\lambda C(z)+\eta)\,\overline{D}_q(z,s) = \alpha z\left[\int_0^\infty \overline{P}_q^{(1)}(x,z,s)\,dx + \int_0^\infty \overline{P}_q^{(2)}(x,z,s)\,dx\right] \tag{25}$$

$$(s+\lambda-\lambda C(z)+\beta)\,\overline{R}_q(z,s) = \eta\overline{D}_q(z,s). \tag{26}$$

Multiplying both sides of (18) by z^{n+1}, summing over 1 to ∞, and using the definition of probability generating function, we get

$$\begin{aligned} z\overline{P}_q^{(1)}(0,z,s) &= (1-p)\int_0^\infty \overline{P}_q^{(2)}(x,z,s)\,\mu_2(x)\,dx \\ &\quad + z\int_0^\infty \overline{V}_q(x,z,s)\,\gamma(x)\,dx + z\nu\overline{S}_q(z,s) \\ &\quad + z\beta\overline{R}_q(z,s) + z\left[1-s\overline{Q}(s)\right] \\ &\quad + \lambda z\,(C(z)-1)\,\overline{Q}(s). \end{aligned} \tag{27}$$

Performing similar operations on (19) and (20), we obtain

$$\overline{P}_q^{(2)}(0,z,s) = \int_0^\infty \overline{P}_q^{(1)}(x,z,s)\,\mu_1(x)\,dx \tag{28}$$

$$\overline{V}_q(0,z,s) = p\int_0^\infty \overline{P}_q^{(2)}(x,z,s)\,\mu_2(x)\,dx. \tag{29}$$

Integrating (22) from 0 to x yields

$$\overline{P}_q^{(1)}(x,z,s) = \overline{P}_q^{(1)}(0,z,s)\,e^{-(s+\lambda-\lambda C(z)+\alpha)x-\int_0^x \mu_1(t)dt}, \tag{30}$$

where $\overline{P}_q^{(1)}(0,z,s)$ is given by (27).

Again integrating (30) by parts with respect to x yields

$$\overline{P}_q^{(1)}(z,s) = \overline{P}_q^{(1)}(0,z,s)\left[\frac{1-\overline{B}_1(s+\lambda-\lambda C(z)+\alpha)}{(s+\lambda-\lambda C(z)+\alpha)}\right], \tag{31}$$

where

$$\overline{B}_1(s+\lambda-\lambda C(z)+\alpha) = \int_0^\infty e^{-(s+\lambda-\lambda C(z)+\alpha)x}\,dB_1(x) \tag{32}$$

is a Laplace-Stieltjes transform of the first stage service time $B_1(x)$. Now multiplying both sides of (30) by $\mu_1(x)$ and integrating over x, we get

$$\int_0^\infty \overline{P}_q^{(1)}(x,z,s)\,\mu_1(x)\,dx = \overline{P}_q^{(1)}(0,z,s)\,\overline{B}_1(s+\lambda-\lambda C(z)+\alpha). \tag{33}$$

Similarly, on integrating (23) and (24) from 0 to x, respectively, we get

$$\overline{P}_q^{(2)}(x,z,s) = \overline{P}_q^{(2)}(0,z,s)\,e^{-(s+\lambda-\lambda C(z)+\alpha)x-\int_0^x \mu_2(t)dt} \tag{34}$$

$$\overline{V}_q(x,z,s) = \overline{V}_q(0,z,s)\,e^{-(s+\lambda-\lambda C(z))x-\int_0^x \gamma(t)dt}, \tag{35}$$

where $\overline{P}_q^{(2)}(0,z,s)$ and $\overline{V}_q(0,z,s)$ are given by (28) and (29), respectively.

Again integrating (34) and (35) by parts with respect to x, respectively, yields

$$\overline{P}_q^{(2)}(z,s) = \overline{P}_q^{(2)}(0,z,s)\left[\frac{1-\overline{B}_2(s+\lambda-\lambda C(z)+\alpha)}{(s+\lambda-\lambda C(z)+\alpha)}\right] \tag{36}$$

$$\overline{V}_q(z,s) = \overline{V}_q(0,z,s)\left[\frac{1-\overline{V}(s+\lambda-\lambda C(z))}{(s+\lambda-\lambda C(z))}\right], \tag{37}$$

where

$$\overline{B}_2(s+\lambda-\lambda C(z)+\alpha) = \int_0^\infty e^{-(s+\lambda-\lambda C(z)+\alpha)x}\,dB_2(x) \tag{38}$$

is a Laplace-Stieltjes transform of the second stage service time $B_2(x)$, and

$$\overline{V}(s+\lambda-\lambda C(z)) = \int_0^\infty e^{-(s+\lambda-\lambda C(z))x}\,dV(x) \tag{39}$$

is Laplace-Stieltjes transform of the vacation time $V(x)$. Now multiplying both sides of (34) by $\mu_2(x)$ and integrating over x, we get

$$\int_0^\infty \overline{P}_q^{(2)}(x,z,s)\,\mu_2(x)\,dx = \overline{P}_q^{(2)}(0,z,s)\,\overline{B}_2(s+\lambda-\lambda C(z)+\alpha). \tag{40}$$

Now, using (33), (28) is reduced to

$$\overline{P}_q^{(2)}(0,z,s)=\overline{P}_q^{(1)}(0,z,s)\overline{B}_1(s+\lambda-\lambda C(z)+\alpha). \quad (41)$$

Now multiplying both sides of (35) by $\gamma(x)$ and integrating over x, we get

$$\int_0^\infty \overline{V}_q(x,z,s)\gamma(x)\,dx=\overline{V}_q(0,z,s)\overline{V}(s+\lambda-\lambda C(z)). \quad (42)$$

Now using (40) and (41), (29) can be written as

$$\overline{V}_q(0,z,s)=p\overline{P}_q^{(1)}(0,z,s)\overline{B}_1(s+\lambda-\lambda C(z)+\alpha)\times\overline{B}_2(s+\lambda-\lambda C(z)+\alpha). \quad (43)$$

From (21), we get

$$\overline{S}_q(z,s)=\frac{\lambda C(z)\overline{Q}(s)}{s+\lambda-\lambda C(z)+\nu}. \quad (44)$$

Using (43), (37) becomes

$$\overline{V}_q(z,s)=p\overline{P}_q^{(1)}(0,z,s)\overline{B}_1(s+\lambda-\lambda C(z)+\alpha)\overline{B}_2\times(s+\lambda-\lambda C(z)+\alpha)\left[\frac{1-\overline{V}(s+\lambda-\lambda C(z))}{(s+\lambda-\lambda C(z))}\right]. \quad (45)$$

Using (31) and (36), (25) becomes

$$\overline{D}_q(z,s)=\alpha z\overline{P}_q^{(1)}(0,z,s)\times\left[\frac{1-\overline{B}_1(s+\lambda-\lambda C(z)+\alpha)\overline{B}_2(s+\lambda-\lambda C(z)+\alpha)}{(s+\lambda-\lambda C(z)+\alpha)(s+\lambda-\lambda C(z)+\eta)}\right]. \quad (46)$$

Using (46), (26) becomes

$$\begin{aligned}\overline{R}_q(z,s)&=\alpha\eta z\overline{P}_q^{(1)}(0,z,s)\\&\times\Big[\big(1-\overline{B}_1(s+\lambda-\lambda C(z)+\alpha)\\&\times\overline{B}_2(s+\lambda-\lambda C(z)+\alpha)\big)\\&\times\big((s+\lambda-\lambda C(z)+\alpha)\\&\times(s+\lambda-\lambda C(z)+\beta)\\&\times(s+\lambda-\lambda C(z)+\eta)\big)^{-1}\Big].\end{aligned} \quad (47)$$

Now using (44), (45), (46), and (47) in (27) and solving for $\overline{P}_q^{(1)}(0,z,s)$, we get

$$\begin{aligned}\overline{P}_q^{(1)}(0,z,s)&=\Big(f_1(z,s)f_2(z,s)f_3(z,s)\\&\times\Big[z\big(1-s\overline{Q}(s)\big)+\lambda z\overline{Q}(s)\frac{\nu C(z)}{(s+\lambda-\lambda C(z)+\nu)}-1\Big]\Big)\\&\times(D(z,s))^{-1},\end{aligned} \quad (48)$$

where

$$\begin{aligned}D(z,s)&=f_1(z,s)f_2(z,s)f_3(z,s)\\&\times\{z-[(1-p)-p\overline{V}(s+\lambda-\lambda C(z))]\\&\times\overline{B}_1[f_1(z,s)]\overline{B}_2[f_1(z,s)]\}\\&-\alpha\beta\eta z\big(1-\overline{B}_1[f_1(z,s)]\overline{B}_2[f_1(z,s)]\big)\end{aligned} \quad (49)$$

$$\begin{aligned}f_1(z,s)&=s+\lambda-\lambda C(z)+\alpha\\f_2(z,s)&=s+\lambda-\lambda C(z)+\beta\\f_3(z,s)&=s+\lambda-\lambda C(z)+\eta.\end{aligned} \quad (50)$$

Substituting the value of $\overline{P}_q^{(1)}(0,z,s)$ from (48) into (31), (36), (45), (46), and (47), we get

$$\begin{aligned}\overline{P}_q^{(1)}(z,s)&=\Big(f_2(z,s)f_3(z,s)\Big\{z\big(1-s\overline{Q}(s)\big)+\lambda z\overline{Q}(s)\\&\times\Big[\frac{\nu C(z)}{(s+\lambda-\lambda C(z)+\nu)}-1\Big]\Big\}\\&\times\big[1-\overline{B}_1[f_1(z,s)]\big]\Big)\times(D(z,s))^{-1}\end{aligned} \quad (51)$$

$$\begin{aligned}\overline{P}_q^{(2)}(z,s)&=\Big(f_2(z,s)f_3(z,s)\Big\{z\big(1-s\overline{Q}(s)\big)+\lambda z\overline{Q}(s)\\&\times\Big[\frac{\nu C(z)}{(s+\lambda-\lambda C(z)+\nu)}-1\Big]\Big\}\\&\times\overline{B}_1[f_1(z,s)]\big[1-\overline{B}_2[f_1(z,s)]\big]\Big)\\&\times(D(z,s))^{-1}\end{aligned} \quad (52)$$

$$\overline{V_q}(z,s) = \left(p f_1(z,s) f_2(z,s) f_3(z,s) \times \overline{B}_1[f_1(z,s)]\overline{B}_2[f_1(z,s)] \times \left\{ z\left(1 - s\overline{Q}(s)\right) + \lambda z \overline{Q}(s) \times \left[\frac{\nu C(z)}{(s+\lambda-\lambda C(z)+\nu)} - 1 \right] \right\} \times \left[\frac{1-\overline{V}(s+\lambda-\lambda C(z))}{(s+\lambda-\lambda C(z))} \right] \right) \times (D(z,s))^{-1} \tag{53}$$

$$\overline{D}_q(z,s) = f_2(z,s) \times \left(\alpha z \left[1 - \overline{B}_1[f_1(z,s)]\overline{B}_2[f_1(z,s)]\right] \times \left\{ z\left(1 - s\overline{Q}(s)\right) + \lambda z \overline{Q}(s) \times \left[\frac{\nu C(z)}{(s+\lambda-\lambda C(z)+\nu)} - 1 \right] \right\} \right) \times (D(z,s))^{-1} \tag{54}$$

$$\overline{R}_q(z,s) = \alpha\eta z \left(\left[1 - \overline{B}_1[f_1(z,s)]\overline{B}_2[f_1(z,s)]\right] \times \left\{ z\left(1 - s\overline{Q}(s)\right) + \lambda z \overline{Q}(s) \times \left[\frac{\nu C(z)}{(s+\lambda-\lambda C(z)+\nu)} - 1 \right] \right\} \right) \times (D(z,s))^{-1}, \tag{55}$$

where $D(z,s)$ is given by (49).

4. The Steady State Analysis

In this section, we will derive the steady state probability distribution for our queueing model. To define the steady state probabilities, suppress the argument "t" where ever it appears in the time dependent analysis.

By using well known Tauberian property,

$$Lt_{s\to 0} s\overline{f}(s) = Lt_{t\to\infty} f(t). \tag{56}$$

Multiplying both sides of (44), (51), (52), (53), (54), and (55) by s and applying property (56) and simplifying, we get

$$S_q(z) = \frac{\lambda Q C(z)}{\lambda - \lambda C(z) + \nu}$$

$$P_q^{(1)}(z) = \frac{f_2(z) f_3(z) z(\lambda+\nu)[\lambda(C(z)-1)]\left[1-\overline{B}_1[f_1(z)]\right]Q}{Drf_4(z)}$$

$$P_q^{(2)}(z) = \left(f_2(z) f_3(z) z(\lambda+\nu) \times [\lambda((C(z))-1)]\left[\overline{B}_1[f_1(z)]\right] \times \left[1-\overline{B}_2[f_1(z)]\right]Q\right) \times \left(Drf_4(z)\right)^{-1}$$

$$V_q(z) = p\left(\left[f_1(z) f_2(z) f_3(z)(\lambda+\nu)\overline{B}_1[f_1(z)]\overline{B}_2 \times [f_1(z)]\right]\left[\overline{V}(\lambda-\lambda C(z)) - 1\right]Q\right) \times \left(Drf_4(z)\right)^{-1}$$

$$D_q(z) = \alpha z\left(f_2(z)(\lambda+\nu)\lambda(C(z)-1) \times \left[1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right]Q\right) \times \left(Drf_4(z)\right)^{-1}$$

$$R_q(z) = \alpha\eta z\left((\lambda+\nu)\lambda(C(z)-1) \times \left[1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right]Q\right) \times \left(Drf_4(z)\right)^{-1}, \tag{57}$$

where

$$Dr = f_1(z) f_2(z) f_3(z)\left\{z - \left[(1-p) - p\overline{V}(\lambda-\lambda C(z))\right] \times \overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right\} - \alpha\beta\eta z\left(1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right), \tag{58}$$

$$f_1(z) = \lambda - \lambda C(z) + \alpha$$
$$f_2(z) = \lambda - \lambda C(z) + \beta$$
$$f_3(z) = \lambda - \lambda C(z) + \eta \tag{59}$$
$$f_4(z) = \lambda - \lambda C(z) + \nu.$$

Let $P(z)$ denote the probability generating function of queue size irrespective of the state of the system. Then, adding (57), we get

$$P(z) = S_q(z) + P_q^{(1)}(z) + P_q^{(2)}(z) + V_q(z) + D_q(z) + R_q(z). \tag{60}$$

Let $W_q(z)$ be defined as

$$W_q(z) = P_q^{(1)}(z) + P_q^{(2)}(z) + V_q(z) + D_q(z) + R_q(z) \quad (61)$$

$$\begin{aligned}
W_q(z) = {} & \big(f_2(z) f_3(z) z (\lambda+\nu) [\lambda (C(z)-1)] \\
& \times \left[1-\overline{B}_1[f_1(z)]\right] Q\big) \times \left(Drf_4(z)\right)^{-1} \\
& + \big(f_2(z) f_3(z) z (\lambda+\nu) [\lambda ((C(z))-1)] \\
& \times \left[\overline{B}_1[f_1(z)]\right]\left[1-\overline{B}_2[f_1(z)]\right] Q\big) \\
& \times \left(Drf_4(z)\right)^{-1} \\
& + p\big(\big[f_1(z) f_2(z) f_3(z) (\lambda+\nu) \overline{B}_1[f_1(z)] \\
& \times \overline{B}_2[f_1(z)]\big]\left[\overline{V}(\lambda-\lambda C(z))-1\right] Q\big) \\
& \times \left(Drf_4(z)\right)^{-1} \\
& + \lambda\alpha z \big(f_2(z)(\lambda+\nu)(C(z)-1) \\
& \times \left[1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right] Q\big) \\
& \times \left(Drf_4(z)\right)^{-1} \\
& + \lambda\alpha\eta z \big((\lambda+\nu)(C(z)-1) \\
& \times \left[1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right] Q\big) \\
& \times \left(Drf_4(z)\right)^{-1}.
\end{aligned} \quad (62)$$

In order to obtain Q, we use the normalization condition as follows:

$$P(1) + Q = 1, \quad (63)$$

where $\overline{B}_i(0) = 1$, $i = 1, 2$; $\overline{V}(0) = 1$, $-V'(0) = E[V]$ is the mean vacation time.

Now,

$$S_q(1) = \frac{\lambda Q}{\nu}$$

$$P_q^{(1)}(1) = \frac{(\lambda+\nu)\lambda\beta\eta Q\left[1-\overline{B}_1(\alpha)\right]E(I)}{dr}$$

$$P_q^{(2)}(1) = \frac{(\lambda+\nu)\lambda\beta\eta Q\left[1-\overline{B}_2(\alpha)\right]\overline{B}_1(\alpha)E(I)}{dr}$$

$$V_q(1) = p\frac{(\lambda+\nu)\lambda\alpha\beta\eta Q\overline{B}_1(\alpha)\overline{B}_2(\alpha)E(I)E(V)}{dr}$$

$$D_q(1) = \frac{(\lambda+\nu)\lambda\alpha\beta Q E(I)\left(1-\overline{B}_1(\alpha)\overline{B}_2(\alpha)\right)}{dr}$$

$$R_q(1) = \frac{(\lambda+\nu)\lambda\alpha\eta Q E(I)\left(1-\overline{B}_1(\alpha)\overline{B}_2(\alpha)\right)}{dr}$$

$$\begin{aligned}
W_q(1) & \\
= {} & \big(\lambda Q E(I)(\lambda+\nu)\big\{(\alpha\beta+\beta\eta+\eta\alpha) \\
& \times\left[1-\overline{B}_1(\alpha)\overline{B}_2(\alpha)\right] \\
& + \alpha\beta\eta p\overline{B}_1(\alpha) \\
& \times \overline{B}_2(\alpha)E(V)\big\}\big) \\
& \times (dr)^{-1},
\end{aligned} \quad (64)$$

where

$$\begin{aligned}
dr = \nu\big\{ & \alpha\beta\eta\overline{B}_1(\alpha)\overline{B}_2(\alpha) - \lambda E(I)\big[(\alpha\beta+\beta\eta+\eta\alpha) \\
& \times\left[1-\overline{B}_1(\alpha)\overline{B}_2(\alpha)\right]\big] \\
& - p\alpha\beta\eta E(I)E(V)\overline{B}_1(\alpha)\overline{B}_2(\alpha)\big\}
\end{aligned} \quad (65)$$

$$\begin{aligned}
Q = \frac{1}{1+\lambda/\nu}\Bigg\{1-\lambda E(I)\Bigg[& \frac{1}{\eta\overline{B}_1(\alpha)\overline{B}_2(\alpha)} + \frac{1}{\beta\overline{B}_1(\alpha)\overline{B}_2(\alpha)} \\
& + \frac{1}{\alpha\overline{B}_1(\alpha)\overline{B}_2(\alpha)} - \frac{1}{\eta} - \frac{1}{\beta} - \frac{1}{\alpha} \\
& + pE(V)\Bigg]\Bigg\},
\end{aligned} \quad (66)$$

and the utilization factor ρ of the system is given by

$$\begin{aligned}
\rho = \lambda E(I)\Bigg[& \frac{1}{\eta\overline{B}_1(\alpha)\overline{B}_2(\alpha)} + \frac{1}{\beta\overline{B}_1(\alpha)\overline{B}_2(\alpha)} \\
& + \frac{1}{\alpha\overline{B}_1(\alpha)\overline{B}_2(\alpha)} - \frac{1}{\eta} - \frac{1}{\beta} - \frac{1}{\alpha} + pE(V)\Bigg],
\end{aligned} \quad (67)$$

where $\rho < 1$ is the stability condition under which the steady state exists; (66) gives the probability that the server is idle. Substitute Q from (66) into (60), $P(z)$ have been completely and explicitly determined which is the probability generating function of the queue size.

Let L_q denote the mean number of customers in the queue under the following steady state:

$$L_q = \frac{d}{dz}P(z)\big|_{z=1}, \quad (68)$$

that is,

$$L_q = \frac{d}{dz}W_q(z)\big|_{z=1} + \frac{d}{dz}D_q(z)\big|_{z=1}. \quad (69)$$

Since the formula for $W_q(z)$ gives 0/0 form, we write $W_q(z) = N(z)/D(z)$, where $N(z)$ and $D(z)$ are the numerator and denominator of the right hand side of (61), respectively; then (69) becomes

$$L_q = \frac{D'(1)N''(1) - N'(1)D''(1)}{2[D'(1)]^2} + \lambda E(I)Q\left[\frac{\lambda+\nu}{\nu^2}\right], \quad (70)$$

where primes and double primes in (70) denote first and second derivation at $z = 1$, respectively. Carrying out the derivatives at $z = 1$, we have

$$\begin{aligned} N'(1) = (\lambda+\nu)\{&\lambda E(I)Q\{(\alpha\beta+\beta\eta+\eta\alpha) \\ &+ (\overline{B}_1(\alpha)\overline{B}_2(\alpha))[p\alpha\beta\eta E(V) \\ &- (\alpha\beta+\beta\eta+\eta\alpha)]\}\} \end{aligned} \quad (71)$$

$$\begin{aligned} N''(1) &= (\lambda+\nu)\Big\{2Q[\lambda E(I)]^2\Big\{\Big(\frac{\alpha}{\lambda E(I)} - 1\Big) + \overline{B}_1(\alpha)\overline{B}_2(\alpha) \\ &\times\Big[1 - \frac{\alpha}{\lambda E(I)} - p\alpha E(V) - p\beta E(V) - p\eta E(V) + \frac{1}{2}p\alpha\beta\eta E(V^2)\Big] \\ &+ \overline{B}_1'(\alpha)[(\alpha\beta+\beta\eta+\eta\alpha) - p\alpha\beta\eta E(V)] \\ &+ \overline{B}_2'(\alpha)[(\alpha\beta+\beta\eta+\eta\alpha) - p\alpha\beta\eta E(V)]\Big\} \\ &+ \lambda Q E(I(I-1))\{(\alpha\beta+\beta\eta+\eta\alpha) + \overline{B}_1(\alpha)\overline{B}_2(\alpha) \\ &\times[p\alpha\beta\eta E(V) - (\alpha\beta+\beta\eta+\eta\alpha)]\}\Big\} \end{aligned} \quad (72)$$

$$\begin{aligned} D'(1) = \nu\{&-\lambda E(I)(\alpha\beta+\beta\eta+\eta\alpha) + \overline{B}_1(\alpha)\overline{B}_2(\alpha) \\ &\times\{\alpha\beta\eta + \lambda E(I)(\alpha\beta+\beta\eta+\eta\alpha) \\ &- p\alpha\beta\eta E(V)\}\} \end{aligned} \quad (73)$$

$$\begin{aligned} D''(1) &= \nu\Big\{2[\lambda E(I)]^2\Big\{\Big(1 - \frac{\alpha\beta+\beta\eta+\eta\alpha}{\lambda E(I)}\Big) + \overline{B}_1(\alpha)\overline{B}_2(\alpha) \\ &\times\Big[(-1 - pE(V))(\alpha\beta+\beta\eta+\eta\alpha) - \frac{1}{2}\alpha\beta\eta E(V^2)\Big] + \overline{B}_1'(\alpha) \\ &\times\Big[-\frac{\alpha\beta\eta}{\lambda E(I)} - (\alpha\beta+\beta\eta+\eta\alpha) + \alpha\beta\eta p E(V)\Big] + \overline{B}_2'(\alpha) \\ &\times\Big[-\frac{\alpha\beta\eta}{\lambda E(I)} - (\alpha\beta+\beta\eta+\eta\alpha) + \alpha\beta\eta p E(V)\Big]\Big\} + \lambda E(I(I-1)) \\ &\times\{-(\alpha\beta+\beta\eta+\eta\alpha) + \overline{B}_1(\alpha)\overline{B}_2(\alpha) \\ &\times[(\alpha\beta+\beta\eta+\eta\alpha) - \alpha\beta\eta p E(V)]\}\Big\} \\ &- 2\lambda E(I)\{-\lambda E(I)(\alpha\beta+\beta\eta+\eta\alpha) + \overline{B}_1(\alpha)\overline{B}_2(\alpha) \\ &\times\{\alpha\beta\eta + \lambda E(I)(\alpha\beta+\beta\eta+\eta\alpha) \\ &- p\alpha\beta\eta E(V)\}\}, \end{aligned} \quad (74)$$

where $E(V^2)$ is the second moment of the vacation time and Q has been found in (65). Then, if we substitute the values of $N'(1)$, $N''(1)$, $D'(1)$ and $D''(1)$ from (70), (71), (72) and (73) into (69), we obtain L_q in a closed form.

Mean waiting time of a customer could be found

$$W_q = \frac{L_q}{\lambda} \quad (75)$$

by using Little's formula.

5. Particular Cases

Case 1 (no setup time). When the server has no option to take setup time, we let the mean setup time $1/\nu = 0$; we have

$$\begin{aligned} W_q(z) &= \frac{f_2(z)f_3(z)z[\lambda(C(z)-1)][1-\overline{B}_1[f_1(z)]]Q}{Dr} \\ &+ (f_2(z)f_3(z)z[\lambda((C(z))-1)][\overline{B}_1[f_1(z)]] \\ &\times[1-\overline{B}_2[f_1(z)]]Q) \times (Dr)^{-1} \\ &+ p([f_1(z)f_2(z)f_3(z)\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]] \\ &\times[\overline{V}(\lambda-\lambda C(z))-1]Q) \times (Dr)^{-1} \end{aligned}$$

$$+\lambda\alpha z\frac{f_2(z)(C(z)-1)\left[1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right]Q}{Dr}$$
$$+\lambda\alpha\eta z\frac{(C(z)-1)\left[1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right]Q}{Dr} \tag{76}$$

$$Q=\left\{1-\lambda E(I)\left[\frac{1}{\eta\overline{B}_1(\alpha)\overline{B}_2(\alpha)}+\frac{1}{\beta\overline{B}_1(\alpha)\overline{B}_2(\alpha)}+\frac{1}{\alpha\overline{B}_1(\alpha)\overline{B}_2(\alpha)}-\frac{1}{\eta}-\frac{1}{\beta}-\frac{1}{\alpha}+pE(V)\right]\right\}, \tag{77}$$

and the utilization factor ρ of the system is given by

$$\rho=\lambda E(I)\left[\frac{1}{\eta\overline{B}_1(\alpha)\overline{B}_2(\alpha)}+\frac{1}{\beta\overline{B}_1(\alpha)\overline{B}_2(\alpha)}+\frac{1}{\alpha\overline{B}_1(\alpha)\overline{B}_2(\alpha)}-\frac{1}{\eta}-\frac{1}{\beta}-\frac{1}{\alpha}+pE(V)\right]. \tag{78}$$

These results agree with the results obtained in [21] in which the model repair and delay time are assumed to follow general distribution.

Case 2 (no delay time and no setup time). In this case, $1/\eta=0$ and $1/\nu=0$; we have

$$\begin{aligned}W_q(z)={}&\left(f_2(z)z[\lambda(C(z)-1)]\left[1-\overline{B}_1[f_1(z)]\right]Q\right)\\&\times\left(f_1(z)f_2(z)\left\{z-\left[(1-p)-p\overline{V}(\lambda-\lambda C(z))\right]\times\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right\}\right.\\&\left.-\alpha\beta z\left(1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right)\right)^{-1}\\&+\left(f_2(z)z[\lambda((C(z))-1)]\left[\overline{B}_1[f_1(z)]\right]\times\left[1-\overline{B}_2[f_1(z)]\right]Q\right)\\&\times\left(f_1(z)f_2(z)\left\{z-\left[(1-p)-p\overline{V}(\lambda-\lambda C(z))\right]\times\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right\}\right.\\&\left.-\alpha\beta z\left(1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right)\right)^{-1}\\&+p\left(\left[f_1(z)f_2(z)\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right]\times\left[\overline{V}(\lambda-\lambda C(z))-1\right]Q\right)\\&\times\left(f_1(z)f_2(z)\left\{z-\left[(1-p)-p\overline{V}(\lambda-\lambda C(z))\right]\times\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right\}\right.\\&\left.-\alpha\beta z\left(1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right)\right)^{-1}\\&+\left(\alpha\lambda z(C(z)-1)\left[1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right]Q\right)\\&\times\left(f_1(z)f_2(z)\left\{z-\left[(1-p)-p\overline{V}(\lambda-\lambda C(z))\right]\times\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right\}\right.\\&\left.-\alpha\beta z\left(1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right)\right)^{-1}\end{aligned} \tag{79}$$

$$Q=\left\{1-\lambda E(I)\left[\frac{1}{\beta\overline{B}_1(\alpha)\overline{B}_2(\alpha)}+\frac{1}{\alpha\overline{B}_1(\alpha)\overline{B}_2(\alpha)}-\frac{1}{\beta}-\frac{1}{\alpha}+pE(V)\right]\right\}, \tag{80}$$

and the utilization factor ρ of the system is given by

$$\rho=\lambda E(I)\left[\frac{1}{\beta\overline{B}_1(\alpha)\overline{B}_2(\alpha)}+\frac{1}{\alpha\overline{B}_1(\alpha)\overline{B}_2(\alpha)}-\frac{1}{\beta}-\frac{1}{\alpha}+pE(V)\right], \tag{81}$$

and this model coincides with the model discussed in [23] in which model they have considered compulsory vacation with $p=1$.

Case 3 (no delay time, no setup time, and no vacation). In this case, $1/\eta=0$, $1/\nu=0$, and $p=0$; we have

$$\begin{aligned}W_q(z)={}&\left(f_2(z)z[\lambda(C(z)-1)]\left[1-\overline{B}_1[f_1(z)]\right]Q\right)\\&\times\left(f_1(z)f_2(z)\left\{z-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right\}-\alpha\beta z\left(1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right)\right)^{-1}\\&+\left(f_2(z)z[\lambda((C(z))-1)]\left[\overline{B}_1[f_1(z)]\right]\times\left[1-\overline{B}_2[f_1(z)]\right]Q\right)\\&\times\left(f_1(z)f_2(z)\times\left\{z-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right\}-\alpha\beta z\left(1-\overline{B}_1[f_1(z)]\overline{B}_2[f_1(z)]\right)\right)^{-1}\end{aligned}$$

$$+\left(\alpha\lambda z\left(C\left(z\right)-1\right)\left[1-\overline{B}_1\left[f_1\left(z\right)\right]\overline{B}_2\left[f_1\left(z\right)\right]\right]Q\right)$$
$$\times\left(f_1\left(z\right)f_2\left(z\right)\left\{z-\overline{B}_1\left[f_1\left(z\right)\right]\overline{B}_2\left[f_1\left(z\right)\right]\right\}\right.$$
$$\left.-\alpha\beta z\left(1-\overline{B}_1\left[f_1\left(z\right)\right]\overline{B}_2\left[f_1\left(z\right)\right]\right)\right)^{-1}$$
$$Q=\left\{1-\lambda E\left(I\right)\left[\frac{1}{\beta\overline{B}_1\left(\alpha\right)\overline{B}_2\left(\alpha\right)}+\frac{1}{\alpha\overline{B}_1\left(\alpha\right)\overline{B}_2\left(\alpha\right)}\right.\right.$$
$$\left.\left.-\frac{1}{\beta}-\frac{1}{\alpha}\right]\right\}, \tag{82}$$

and the utilization factor ρ of the system is given by

$$\rho=\lambda E\left(I\right)\left[\frac{1}{\beta\overline{B}_1\left(\alpha\right)\overline{B}_2\left(\alpha\right)}+\frac{1}{\alpha\overline{B}_1\left(\alpha\right)\overline{B}_2\left(\alpha\right)}-\frac{1}{\beta}-\frac{1}{\alpha}\right]. \tag{83}$$

Case 4 (no break down and no setup time). In this case, $\alpha=0$ and $1/\nu=0$; we have

$$Q=1-\lambda E\left(I\right)\left[E\left(S_1\right)+E\left(S_2\right)+pE\left(V\right)\right], \tag{84}$$

and the utilization factor ρ of the system is given by

$$\rho=\lambda E\left(I\right)\left[E\left(S_1\right)+E\left(S_2\right)+pE\left(V\right)\right]. \tag{85}$$

Case 5 (no break down, no setup time, and no vacation). In this case, $\alpha=0$, $1/\nu=0$, and $p=0$; we have

$$Q=1-\lambda E\left(I\right)\left[E\left(S_1\right)+E\left(S_2\right)\right], \tag{86}$$

and the utilization factor ρ of the system is given by

$$\rho=\lambda E\left(I\right)\left[E\left(S_1\right)+E\left(S_2\right)\right]. \tag{87}$$

6. Concluding Remarks

We have proposed a single server with a two-stage heterogeneous service unreliable server with setup time, delayed repair, and Bernoulli scheduled vacation. At the end of each busy period, the server takes a setup time before giving proper service to the customers. There is a delay before the server gets repaired which has been incorporated whenever the server meets breakdown. The probability generating function of transient solutions is obtained explicitly, and along with this, the steady state has also been analyzed. Further performance measures like average number of customers in the queue and the average waiting time of a customer in the queue are obtained.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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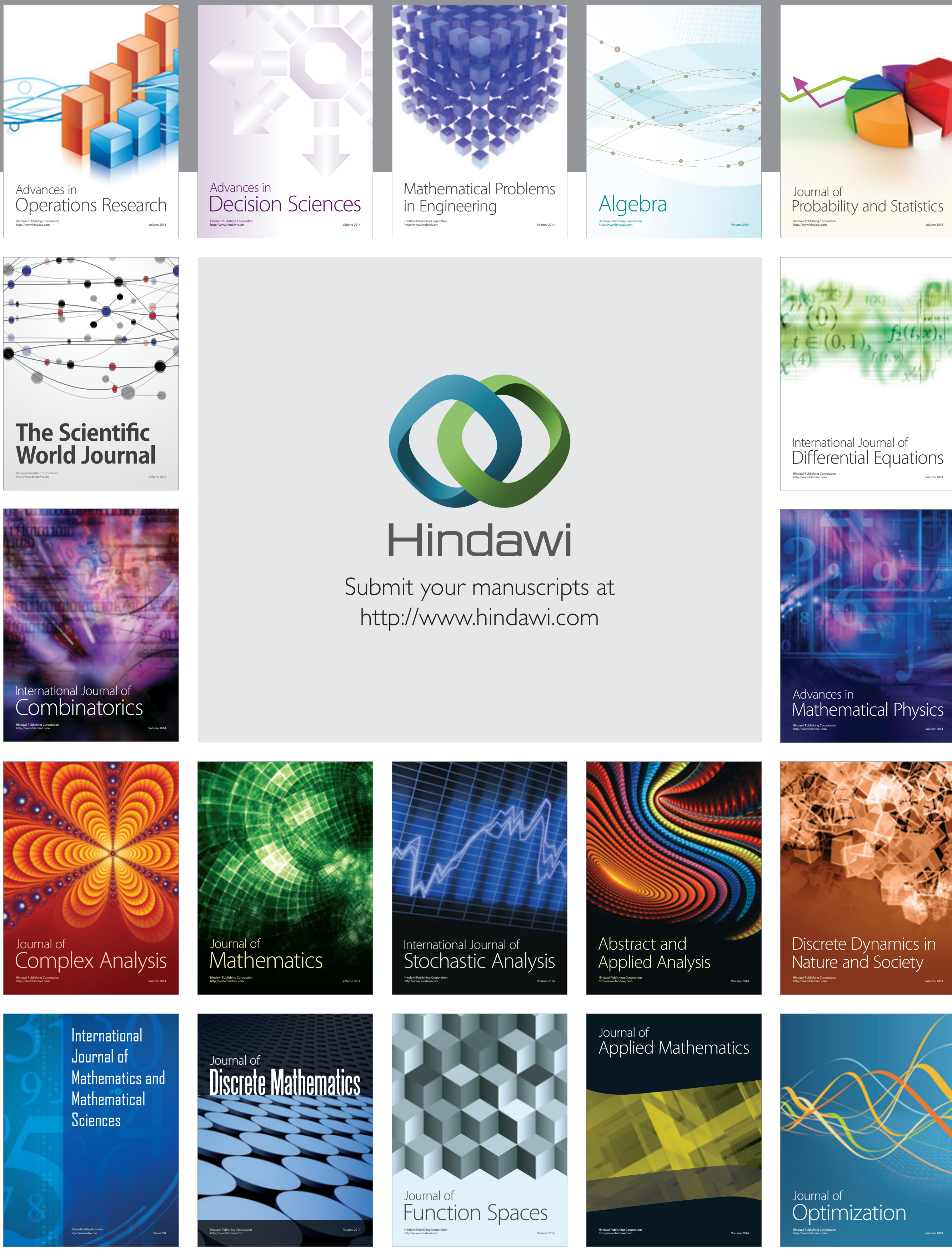

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