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Research Article

System of Generalized Implicit Vector Quasivariational Inequalities

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We will introduce a system of generalized implicit vector quasivariational inequalities (in short, SGIVQVI) which generalizes and unifies the system of generalized implicit variational inequalities, the system of generalized vector quasivariational-like inequalities, the system of generalized vector variational inequalities, the system of variational inequalities, the generalized implicit vector quasivariational inequality, as well as various extensions of the classic variational inequalities in the literature, and we present some existence results of a solution for the SGIVQVI without any monotonicity conditions.

1. Introduction

The vector variational inequality (in short, VVI) in a finite-dimensional Euclidean space has been introduced in [1] and applications have been given. Chen and Cheng [2] studied the VVI in infinite-dimensional space and applied it to vector optimization problem (in short, VOP). Since then, many authors [3–11] have intensively studied the VVI on different assumptions in infinite-dimensional spaces. Lee et al. [12, 13], Lin et al. [14], Konnov and Yao [15], Daniilidis and Hadjisavvas [16], Yang and Yao [17], and Oettli and Schläger [18] studied the generalized vector variational inequality and obtained some existence results. Chen and Li [19] and Lee et al. [20] introduced and studied the generalized vector quasivariational inequality and established some existence theorems. Ansari [21, 22] and Ding and Tarafdar [23] studied the generalized vector variational-like inequalities. Ding [24] studied the generalized vector quasivariational-like inequality. Ansari et al. [25] studied the generalized implicit vector variational inequality and Chiang et al. [26] studied the implicit vector quasivariational inequality. Pang [27], Cohen and Chaplais [28], Bianchi [29], and Ansari and Yao [30] considered the system of scalar variational inequalities and Pang showed that the traffic equilibrium problem, the spatial equilibrium problem, the Nash equilibrium problem, and the general equilibrium programming problem can be modeled as a system of variational inequalities. Ansari and Yao [31] introduced and studied the system of generalized implicit variational inequalities and the system of generalized variational-like inequalities. Ansari et al. [32] introduced and studied the system of vector variational inequalities. Allevi et al. [33] introduced the system of generalized vector variational inequalities with set-valued mappings and got its several existence results which are based on some monotonicity-type conditions. Peng [34] introduced the system of generalized vector quasivariational-like inequalities with set-valued mappings and got its several existence results without any monotonicity conditions.

In this paper, a system of generalized implicit vector quasivariational inequalities (in short, SGIVQVI) which generalizes and unifies the system of generalized implicit variational inequalities, the system of variational-like inequalities, the system of vector variational inequalities, the system of vector quasivariational-like inequalities, the system of variational inequalities, the generalized implicit vector quasivariational inequality, as well as various extensions of the classic variational inequalities in the literature will be introduced, and some existence results of a solution for the SGIVQVI without any monotonicity conditions will be shown.

2. Problem statement and preliminaries

Let $\operatorname{int} A$ denote the interior of a set A and let I be an index set, for each $i \in I$. Let Z_i be a Hausdorff topological vector space, and let E_i and F_i be two locally convex Hausdorff topological vector spaces. Let $L(E_i,F_i)$ denote the space of the continuous linear operators from E_i to F_i and let D_i be a nonempty subset of $L(E_i,F_i)$. Consider a family of nonempty convex subsets $\{X_i\}_{i\in I}$ with $X_i \subset E_i$. Let $X = \prod_{i\in I} X_i$, and let $E = \prod_{i\in I} E_i$. An element of the set $X^i = \prod_{j\in I\setminus i} X_i$ will be denoted by x^i; therefore, $x \in X$ will be written as $x = (x^i,x_i) \in X^i \times X_i$. For each $i \in I$, let $f_i : D_i \times X_i \times X_i \to Z_i$ be a single-valued mapping and let $C_i : X \to 2^{Z_i}$ be a set-valued mapping such that $C_i(x)$ is a closed, pointed, and convex cone with $\operatorname{int} C_i(x) \neq \varnothing$ for each $x \in X$. Let $S_i : X \to 2^{X_i}$ and $T_i : X \to 2^{D_i}$ be two set-valued mappings. Then, we introduce a system of generalized implicit vector quasivariational inequalities (in short, SGIVQVI) which is to find $\overline{x} = (\overline{x}^i,\overline{x}_i)$ in X such that, for each $i \in I$, $\overline{x}_i \in S_i(\overline{x})$:

$$\forall y_i \in S_i(\overline{x}), \quad \exists \overline{v}_i \in T_i(\overline{x}) : f_i(\overline{v}_i,\overline{x}_i,y_i) \notin -\operatorname{int} C_i(\overline{x}). \tag{2.1}$$

Then, the point $\overline{x}$ is said to be a solution of the SGIVQVI.

It is easy to see that $\overline{x}$ is a solution of the SGIVQVI which, for each $i \in I$, is equivalent to

$$\overline{x}_i \in S_i(\overline{x}), \quad \forall y_i \in S_i(\overline{x}) : f_i(T_i(\overline{x}),\overline{x}_i,y_i) \nsubseteq -\operatorname{int} C_i(\overline{x}), \tag{2.2}$$

where

$$f_i(T_i(\overline{x}),\overline{x}_i,y_i) = \bigcup_{v_i\in T_i(\overline{x})} f_i(v_i,\overline{x}_i,y_i). \tag{2.3}$$

The following problems are some special cases of the SGIVQVI.

(i) For each $i \in I$, if $S_i(x) = X_i$ for every $x \in X$, then the SGIVQVI reduces to the system of generalized implicit vector variational inequalities (in short, SGIVVI) which is to find $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that, for each $i \in I$, $\overline{x}_i \in X_i$:

$$\forall y_i \in X_i, \quad \exists \overline{v}_i \in T_i(\overline{x}) : f_i(\overline{v}_i, \overline{x}_i, y_i) \notin -\operatorname{int} C_i(\overline{x}). \tag{2.4}$$

For each $i \in I$, let $Z_i = R$ and let $C_i(x) = R^+ = \{r \in R \mid r \geq 0\}$. Then, the SGIVVI reduces to the system of generalized implicit variational inequalities (in short, SGIVI) which is to find $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that, for each $i \in I$, $\overline{x}_i \in X_i$:

$$\forall y_i \in X_i, \quad \exists \overline{v}_i \in T_i(\overline{x}) : f_i(\overline{v}_i, \overline{x}_i, y_i) \geq 0. \tag{2.5}$$

This problem was studied by Ansari and Yao [31].

(ii) For each $i \in I$, let $\eta_i : X_i \times X_i \to E_i$ be a function and let $f_i(T_i(x), x_i, y_i) = \langle v_i, \eta_i(y_i, x_i)\rangle : v_i \in T_i(x)\}$. Then, the SGIVQVI reduces to the system of generalized vector quasivariational-like inequalities (in short, SGVQVLI) which is to find $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that, for each $i \in I$, $\overline{x}_i \in S_i(\overline{x})$:

$$\forall y_i \in S_i(\overline{x}), \quad \exists \overline{v}_i \in T_i(\overline{x}) : \langle \overline{v}_i, \eta_i(y_i, \overline{x}_i)\rangle \notin -\operatorname{int} C_i(\overline{x}), \tag{2.6}$$

where $\langle s_i, x_i\rangle$ denotes the evaluation of $s_i \in L(E_i, F_i)$ at $x_i \in E_i$.

The SGVQVLI was introduced and studied by Peng [34], and it contains many mathematical models as special cases, for example, consider the following cases.

For each $i \in I$, let $S_i(x) = X_i$, then the SGVQVLI reduces to a system of generalized vector variational-like inequalities (in short, SGVVLI) which is to find $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that, for each $i \in I$,

$$\forall y_i \in X_i, \quad \exists \overline{v}_i \in T_i(\overline{x}) : \langle \overline{v}_i, \eta_i(y_i, \overline{x}_i)\rangle \notin -\operatorname{int} C_i(\overline{x}). \tag{2.7}$$

For each $i \in I$, let $Z_i = R$ and let $C_i(x) = R^+ = \{r \in R \mid r \geq 0\}$ for all $x \in X$, then the SGVVLI reduces to the system of generalized variational-like inequalities studied by Ansari and Yao [31].

For each $i \in I$, let $\eta_i(y_i, \overline{x}_i) = y_i - x_i$. Then, the SGVQVLI reduces to a system of generalized vector quasivariational inequalities (in short, SGVQVI) which is to find $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that, for each $i \in I$, $\overline{x}_i \in S_i(\overline{x})$:

$$\forall y_i \in S_i(\overline{x}), \quad \exists \overline{v}_i \in T_i(\overline{x}) : \langle \overline{v}_i, y_i - \overline{x}_i\rangle \notin -\operatorname{int} C_i(\overline{x}). \tag{2.8}$$

For each $i \in I$, let $S_i(x) = X_i$, then the SGVQVI reduces to the system of generalized vector variational inequalities (for short, SGVVI) which is to find $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that, for each $i \in I$,

$$\forall y_i \in X_i, \quad \exists \overline{v}_i \in T_i(\overline{x}) : \langle \overline{v}_i, y_i - \overline{x}_i\rangle \notin -\operatorname{int} C_i(\overline{x}). \tag{2.9}$$

For each $i \in I$, for all $x_i \in X_i$, if $Y_i \equiv Y$ and $C_i(x) \equiv C$, where C is a convex, closed, and pointed cone in Y with $\operatorname{int} C \neq \varnothing$, then the SGVVI reduces to the system of set-valued variational inequalities (in short, SSVI) which is to find $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that

$$\forall y_i \in X_i, \quad \exists \overline{v}_i \in T_i(\overline{x}) : \langle \overline{v}_i, y_i - \overline{x}_i \rangle \notin -\operatorname{int} C. \tag{2.10}$$

This was introduced and studied by Allevi et al. [33].

If T_i is single-valued function, then the SSVI reduces to the system of vector variational inequalities (in short, SVVI) which is to find $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that

$$\langle T_i(\overline{x}), y_i - \overline{x}_i \rangle \notin -\operatorname{int} C, \quad \forall y_i \in X_i. \tag{2.11}$$

This was considered by Ansari et al. [32].

For each $i \in I$, for all $x_i \in X_i$, let $Z_i = R$ and let $C_i(x) = R^+ = \{r \in R : r \geq 0\}$. Let T_i be replaced by $f_i : X \to R$, then the SVVI reduces to the system of scalar variational inequalities which is finding $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that

$$\langle f_i(\overline{x}), y_i - \overline{x}_i \rangle \geq 0, \quad \forall y_i \in X_i. \tag{2.12}$$

This problem was considered by several authors in [27–30].

(iii) If I is a singleton, then the SGIVQVI reduces to the generalized implicit vector quasivariational inequality (in short, GIVQVI) which is to find $\overline{x}$ in X such that $\overline{x} \in S(\overline{x})$:

$$\forall y \in S(\overline{x}), \quad \exists \overline{v} \in T(\overline{x}) : f(\overline{v}, \overline{x}, y) \notin -\operatorname{int} C(\overline{x}). \tag{2.13}$$

This new problem contains the generalized implicit vector variational inequality in [25], the implicit vector quasivariational inequality in [26], the generalized set-valued quasivariational-like inequality in [24], the generalized vector variational-like inequality in [21–23], the set-valued quasivariational inequality in [19, 20], the generalized vector variational inequality in [12–18], and the vector variational inequality in [1–11] as special cases.

In order to prove the main results, we need the following definitions and lemmas.

Definition 2.1 [35]. Let X and Y be two topological spaces and let $T : X \to 2^Y$ be a set-valued mapping. Then,

(1) T is said to be upper semicontinuous if, for any $x_0 \in X$ and for each open set U in Y containing $T(x_0)$, there is a neighborhood V of x_0 in X such that $T(x) \subset U$ for all $x \in V$;

(2) T is said to have open lower sections if the set $T^{-1}(y) = \{x \in X : y \in T(x)\}$ is open in X for each $y \in Y$;

(3) T is said to be closed, if the set $\{(x, y) \in X \times Y : y \in T(x)\}$ is closed in $X \times Y$.

Lemma 2.2 [36]. *Let X be a paracompact Hausdorff space and let Y be a linear topological space. Suppose that $T : X \to 2^Y$ is a set-valued mapping such that*

(i) *for each $x \in X$, $T(x)$ is nonempty,*

(ii) *for each $x \in X$, $T(x)$ is convex,*

(iii) *T has open lower sections.*

Then, there exists a continuous function $f : X \to Y$ such that $f(x) \in T(x)$ for all $x \in X$.

Lemma 2.3 [35]. *Let X and Y be topological spaces. If $T: X \to 2^Y$ is an upper semicontinuous set-valued mapping with closed values, then T is closed.*

Lemma 2.4 [37]. *Let X and Y be topological spaces and let $T: X \to 2^Y$ be an upper semicontinuous set-valued mapping with compact values. Suppose that $\{x_\alpha\}$ is a net in X such that $x_\alpha \to x_0$. If $y_\alpha \in T(x_\alpha)$ for each α, then there are a $y_0 \in T(x_0)$ and a subset $\{y_\beta\}$ of $\{y_\alpha\}$ such that $y_\beta \to y_0$.*

Lemma 2.5 [36]. *Let X and Y be two topological spaces. Suppose that $T: X \to 2^Y$ and $K: X \to 2^Y$ are set-valued mappings having open lower sections, then (i) the set-valued mapping $F: X \to 2^Y$ defined by $F(x) = \mathrm{Co}(T(x))$, for each $x \in X$, has open lower sections. (ii) the set-valued mapping $\theta: X \to 2^Y$ defined by $\theta(x) = T(x) \cap K(x)$, for each $x \in X$, has open lower sections.*

Lemma 2.6 [38]. *Let E be a locally convex topological linear space and let X be a compact convex subset in E. Suppose that $T: X \to 2^X$ is a set-valued mapping such that*

(i) *for each $x \in X$, $T(x)$ is nonempty,*
(ii) *for each $x \in X$, $T(x)$ is convex and closed,*
(iii) *T is upper semicontinuous.*

Then, there exists a $\overline{x} \in X$ such that $\overline{x} \in T(\overline{x})$.

3. Existence results

In this section, we will present some existence results of a solution for the SGIVQVI without any monotonicity conditions.

Theorem 3.1. *Let I be an index set and let I be countable. For each $i \in I$, let Z_i be a Hausdorff topological vector space, let E_i and F_i be two locally convex Hausdorff topological vector spaces, let D_i be a nonempty subset of $L(E_i, F_i)$, let X_i be a nonempty, compact, convex, and metrizable set in E_i, let $f_i: D_i \times X_i \times X_i \to Z_i$ be a single-valued mapping, and let $C_i: X \to 2^{Z_i}$ be a set-valued mapping such that $C_i(x)$ is a closed, pointed, and convex cone with $\operatorname{int} C_i(x) \neq \varnothing$ for each $x \in X$. Let $S_i: X \to 2^{X_i}$ and $T_i: X \to 2^{D_i}$ be two set-valued mappings. For each $i \in I$, assume that*

(i) *$S_i: X \to 2^{X_i}$ is an upper semicontinuous set-valued mapping with nonempty convex closed values and open lower sections;*
(ii) *the set-valued mapping $M_i = Y_i \setminus (-\operatorname{int} C_i): X_i \to 2^{Z_i}$ is upper semicontinuous;*
(iii) *$T_i: X \to 2^{D_i}$ is an upper semicontinuous set-valued mapping with nonempty compact values;*
(iv) *for all $x \in X$, $\exists\, v_i \in T_i(x)$, $f_i(v_i, x_i, x_i) \notin -\operatorname{int} C_i(x)$;*
(v) *for each $x \in X$, $P_i(x) = \{y_i \in X_i : f_i(v_i, x_i, y_i) \in -\operatorname{int} C_i(x), \forall\, v_i \in T_i(x)\}$ is a convex set;*
(vi) *for all $y_i \in X_i$, the map $(v_i, x_i) \mapsto f_i(v_i, x_i, y_i)$ is continuous on $D_i \times X_i$.*

Then, there exists $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that, for each $i \in I$,

$$\begin{gathered} \overline{x}_i \in S_i(\overline{x}), \quad \forall\, y_i \in S_i(\overline{x}), \\ \exists\, \overline{v}_i \in T_i(\overline{x}) : f_i(\overline{v}_i, \overline{x}_i, y_i) \notin -\operatorname{int} C_i(\overline{x}). \end{gathered} \tag{3.1}$$

That is, the SGIVQVI has a solution $\overline{x} \in X$.

Proof. We first prove that $x_i \notin \mathrm{Co}(P_i(x))$ for all $x = (x^i, x_i) \in X$. To see this, suppose, by way of contradiction, that there exist some $i \in I$ and some point $\bar{x} = (\bar{x}^i, \bar{x}_i) \in X$ such that $\bar{x}_i \in \mathrm{Co}(P_i(\bar{x}))$. Then, there exist finite points $y_{i_1}, y_{i_2}, \ldots, y_{i_n}$ in X_i, and $\alpha_j \geq 0$ with $\sum_{j=1}^{n} \alpha_j = 1$ such that $\bar{x}_i = \sum_{j=1}^{n} \alpha_j y_{i_j}$ and $y_{i_j} \in P_i(\bar{x})$ for all $v_i \in T_i(\bar{x})$ and for all $j = 1, 2, \ldots, n$. Since $P_i(x) = \{y_i \in X_i : f_i(v_i, x_i, y_i) \in -\mathrm{int}\, C_i(x), \forall v_i \in T_i(x)\}$ is a convex set, $x_i \in P_i(x)$. That is, for all $v_i \in T_i(x)$, $f_i(v_i, x_i, x_i) \in -\mathrm{int}\, C_i(x)$ which contradicts the condition (iv). □

Now, we prove that the set

$$P_i^{-1}(y_i) = \{x \in X : f_i(v_i, x_i, y_i) \in -\mathrm{int}\, C_i(x), \forall v_i \in T(x)\} \tag{3.2}$$

is open for each $i \in I$ and for each $y_i \in X_i$. That is, P_i has open lower sections in X. We only need to prove that $Q_i(y_i) = \{x \in X : \exists\, v_i \in T_i(x) \text{ such that } f_i(v_i, x_i, y_i) \notin -\mathrm{int}\, C_i(x)\}$ is closed for all $y_i \in X_i$.

In fact, consider a net $x_t \in Q_i(y_i)$ such that $x_t \to x \in X$, then $x_{i_t} \to x_i \in X_i$ for each $i \in I$. Since $x_t \in Q_i(y_i)$, there exists $v_t \in T_i(x_t)$ such that

$$f_i(v_t, x_{i_t}, y_i) \notin -\mathrm{int}\, C_i(x_t). \tag{3.3}$$

From the upper semicontinuous and compact values of T_i and Lemma 2.4, it suffices to find a subset $\{v_{t_j}\}$ which converges to some $v \in T_i(x)$. By assumption (iv), the map $(v_i, x_i) \mapsto f_i(v_i, x_i, y_i)$ is continuous on $D_i \times X_i$:

$$f_i(v_{t_j}, x_{i_{t_j}}, y_i) \longrightarrow f_i(v, x_i, y_i). \tag{3.4}$$

By Lemma 2.3 and upper semicontinuity of M_i, we have $f_i(v, x_i, y_i) \notin -\mathrm{int}\, C_i(x)$, and hence $x \in Q_i(y_i)$ and $Q_i(y_i)$ is closed.

For each $i \in I$, also define another set-valued mapping, $G_i : X \to 2^{X_i}$, by $G_i(x) = S_i(x) \cap \mathrm{Co}(P_i(x))$, for all $x \in X$. Let the set $W_i = \{x \in X : G_i(x) \neq \varnothing\}$. Since S_i and P_i have open lower sections in X, and by Lemma 2.5, we know that $\mathrm{Co}(P_i)$ and G_i also have open lower sections in X. Hence, $W_i = \cup_{y_i \in X_i} G_i^{-1}(y_i)$ is an open set in X. Then, the set-valued mapping $G_i|_{W_i} : W_i \to 2^{X_i}$ has open lower sections in W_i, and for all $x \in W_i$, $G_i(x)$ is nonempty and convex. Also, since X is a metrizable space [39, page 50], W_i is paracompact [40, page 831]. Hence, by Lemma 2.2, there is a continuous function $f_i : W_i \to X_i$ such that $f_i(x) \in G_i(x) \subset S_i(x)$ for all $x \in W_i$. Define $H_i : X \to 2^{X_i}$ by

$$H_i(x) = \begin{cases} f_i(x) & \text{if } x \in W_i, \\ S_i(x) & \text{if } x \notin W_i. \end{cases} \tag{3.5}$$

Now, we prove that H_i is upper semicontinuous. In fact, for each open set V_i in X_i, the set

$$\begin{aligned} \{x \in X : H_i(x) \subset V_i\} &= \{x \in W_i : f_i(x) \in V_i\} \cup \{x \in X \setminus W_i : S_i(x) \subset V_i\} \\ &\subset \{x \in W_i : f_i(x) \in V_i\} \cup \{x \in X : S_i(x) \subset V_i\}. \end{aligned} \tag{3.6}$$

On the other hand, when $x \in W_i$, and $f_i(x) \in V_i$, we have $H_i(x) = f_i(x) \in V_i$. When $x \in X$ and $S_i(x) \subset V_i$, since $f_i(x) \in S_i(x)$ if $x \in W_i$, we know that $H_i(x) \subset V_i$ and so

$$\{x \in W_i : f_i(x) \in V_i\} \cup \{x \in X : S_i(x) \subset V_i\} \subset \{x \in X : H_i(x) \subset V_i\}. \tag{3.7}$$

Therefore,

$$\{x \in X : H_i(x) \subset V_i\} = \{x \in W_i : f_i(x) \in V_i\} \cup \{x \in X : S_i(x) \subset V_i\}. \tag{3.8}$$

Since f_i is continuous and S_i is upper semicontinuous, the sets $\{x \in W_i : f_i(x) \in V_i\}$ and $\{x \in X : S_i(x) \subset V_i\}$ are open. It follows that $\{x \in X : H_i(x) \subset V_i\}$ is open and so the mapping $H_i : X \to 2^{X_i}$ is upper semicontinuous. Now, define $H : X \to 2^X$ by $H(x) = \prod_{i \in I} H_i(x)$ for each $x \in X$. By [38, Lemma 3, page 124], H is upper semicontinuous. Since for each $x \in X$, $H(x)$ is convex, closed, and nonempty, by Lemma 2.6, there is $\overline{x} \in X$ such that $\overline{x} \in H(\overline{x})$. Note that for each $i \in I$, $\overline{x} \notin W_i$. Otherwise, there is some $i \in I$ such that $\overline{x} \in W_i$. Then, $\overline{x}_i = f_i(\overline{x}) \in \mathrm{Co}(P_i(\overline{x}))$ which contradicts $x_i \in \mathrm{Co}(P_i(x))$ for all $x = (x^i, x_i) \in X$.

Thus, $\overline{x}_i \in S_i(\overline{x})$ and $G_i(\overline{x}) = \emptyset$ for each $i \in I$. That is, $\overline{x}_i \in S_i(\overline{x})$ and $S_i(\overline{x}) \cap \mathrm{Co}(P_i(\overline{x})) = \emptyset$ for each $i \in I$, which implies $\overline{x}_i \in S_i(\overline{x})$ and $S_i(\overline{x}) \cap P_i(\overline{x}) = \emptyset$ for each $i \in I$. Consequently, there exists $\overline{x} = (\overline{x}^i, \overline{x}_i)$ in X such that, for each $i \in I$,

$$\begin{gathered} \overline{x}_i \in S_i(\overline{x}), \quad \forall y_i \in S_i(\overline{x}), \\ \exists \overline{v}_i \in T_i(\overline{x}) : f_i(\overline{v}_i, \overline{x}_i, y_i) \notin -\mathrm{int}\, C_i(\overline{x}_i). \end{gathered} \tag{3.9}$$

Hence, the solution set of the SGIVQVI is nonempty.

Remark 3.2. By Theorem 3.1, it is easy to obtain the existence results for all of the special models of the SGIVQVI mentioned in Section 2. Hence, Theorem 3.1 is a generalization of the main results in [24–26, 32, 34].

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