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Research Article

Fractional Calculus and p-Valently Starlike Functions

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In this investigation, the authors prove coefficient bounds, distortion inequalities for fractional calculus of a family of multivalent functions with negative coefficients, which is defined by means of a certain nonhomogenous Cauchy-Euler differential equation.

1. Introduction and Definitions

Let $\mathcal{T}_n(p)$ denote the class of functions $f(z)$ of the form

$$f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k \quad (a_k \geq 0;\ n, p \in \mathbb{N} = \{1, 2, 3, \ldots\}), \tag{1.1}$$

which are *analytic* and *multivalent* in the unit disk $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$.

The fractional calculus are defined as follows (e.g., [1, 2]).

Definition 1.1. The fractional integral of order δ is defined by

$$\mathfrak{D}_z^{-\delta} f(z) = \frac{1}{\Gamma(\delta)} \int_0^z \frac{f(\xi)}{(z-\xi)^{1-\delta}}\, d\xi \quad (\delta > 0), \tag{1.2}$$

where $f(z)$ is an analytic function in a simply-connected region of the z-plane containing the origin and the multiplicity of $(z-\xi)^{\delta-1}$ is removed by requiring $\log(z-\xi)$ to be real when $z - \xi > 0$.

Definition 1.2. The fractional derivative of order δ is defined by

$$\mathfrak{D}_z^{\delta} f(z) = \frac{1}{\Gamma(1-\delta)} \frac{d}{dz} \int_0^z \frac{f(\xi)}{(z-\xi)^{\delta}}\, d\xi \quad (0 \le \delta < 1), \tag{1.3}$$

where $f(z)$ is constrained and multiplicity of $(z-\xi)^{-\delta}$ is removed as in Definition 1.1.

Definition 1.3. Under the hypotheses of Definition 1.1, the fractional derivative of order $(n+\delta)$ is defined by

$$\mathfrak{D}_z^{n+\delta} f(z) = \frac{d^n}{dz^n} \mathfrak{D}_z^{\delta} f(z) \quad \left(0 \le \delta < 1,\ n \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}\right). \tag{1.4}$$

$(a)_v$ denotes the *Pochhammer* symbol *(or the shifted factorial)*, since

$$(1)_n = n! \quad \text{for } n \in \mathbb{N}_0 := \mathbb{N} \cup \{0\}, \tag{1.5}$$

defined (for $a, v \in \mathbb{C}$ and in terms of the Gamma function) by

$$(a)_v := \frac{\Gamma(a+v)}{\Gamma(a)} = \begin{cases} 1, & (v = 0,\ a \in \mathbb{C} \setminus \{0\}), \\ a(a+1)\cdots(a+n-1), & (v = n \in \mathbb{N};\ a \in \mathbb{C}). \end{cases} \tag{1.6}$$

The earlier investigations by Goodman [3, 4] and Ruscheweyh [5], we define the (n, p, ε)-neighborhood of a function $f \in \mathcal{T}_n(p)$ by

$$\mathcal{N}_{n,p}^{\varepsilon}\left(\mathfrak{D}_z^{\delta} f, \mathfrak{D}_z^{\delta} g\right) := \left\{ g \in \mathcal{T}_n(p) : g(z) = z^p - \sum_{k=n+p}^{\infty} b_k z^k,\ \sum_{k=n+p}^{\infty} (k+1-\delta)_{\delta} k \left| a_k - b_k \right| \le \varepsilon \right\}, \tag{1.7}$$

so that, obviously,

$$\mathcal{N}_{n,p}^{\varepsilon}\left(\mathfrak{D}_z^{\delta} h, \mathfrak{D}_z^{\delta} g\right) := \left\{ g \in \mathcal{T}_n(p) : g(z) = z^p - \sum_{k=n+p}^{\infty} b_k z^k,\ \sum_{k=n+p}^{\infty} (k+1-\delta)_{\delta} k \left| b_k \right| \le \varepsilon \right\}, \tag{1.8}$$

where

$$h(z) := z^p. \tag{1.9}$$

The class $\mathcal{S}_{n,p}^{\delta}(\lambda,\alpha)$ denote the subclass of $\mathfrak{T}_n(p)$ consisting of functions $f(z)$ which satisfy

$$\Re e \frac{z\mathcal{F}'(z)}{\mathcal{F}(z)} > \alpha \quad (0 \le \alpha < p,\ p \in \mathbb{N}), \tag{1.10}$$

where

$$\mathcal{F}(z) = \lambda z \mathfrak{D}_z^{1+\delta} f(z) + (1-\lambda)\mathfrak{D}_z^{\delta} f(z) \quad (0 \le \lambda \le 1,\ 0 \le \delta < 1). \tag{1.11}$$

We note that the class $\mathcal{S}_{1,1}^{0}(\lambda,\alpha)$ was investigated by Altıntaş [6] and the class $\mathcal{S}_{n,p}^{0}(\lambda,\alpha)$ was studied by Altıntaş et al. [7, 8] .

We donote by

$$\mathcal{S}_{n,p}^{0}(0,\alpha) = \mathcal{S}_n^{*}(p,\alpha), \qquad \mathcal{S}_{n,p}^{0}(1,\alpha) = \mathcal{C}_n(p,\alpha) \tag{1.12}$$

the classes of p-valently starlike functions of order α in $\mathbb{U}$ $(0 \le \alpha < p)$ and p-valently convex functions of order α in $\mathbb{U}$ $(0 \le \alpha < p)$, respectively [see, [2, 9]].

Finally $\mathcal{K}_{n,p}^{\delta}(\lambda,\alpha,\mu)$ denote the subclass of the general class $\mathfrak{T}_n(p)$ consisting of functions $f(z) \in \mathfrak{T}_n(p)$ satisfying the following nonhomogeneous Cauchy-Euler differential equation:

$$z^2\mathfrak{D}_z^{2+\delta}\omega + 2(1+\mu)z\mathfrak{D}_z^{1+\delta}\omega + \mu(1+\mu)\mathfrak{D}_z^{\delta}\omega = (p-\delta+\mu)(p-\delta+\mu+1)\mathfrak{D}_z^{\delta}g, \tag{1.13}$$

where $\omega = f(z), f(z) \in \mathfrak{T}_n(p), g = g(z) \in \mathcal{S}_{n,p}^{\delta}(\lambda,\alpha)$ and $\mu > \delta - p$.

The main object of the present paper is to give coefficients bounds and distortion inequalities for functions in the classes $\mathcal{S}_{n,p}^{\delta}(\lambda,\alpha)$ and $\mathcal{K}_{n,p}^{\delta}(\lambda,\alpha,\mu)$.

2. Coefficient Bounds and Distortion Inequalities

We begin by proving the following result.

Lemma 2.1. *Let the function $f(z) \in \mathfrak{T}_n(p)$ be defined by* (1.1). *Then $f(z)$ is in the class $\mathcal{S}_{n,p}^{\delta}(\lambda,\alpha)$ if and only if*

$$\sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}(k-\alpha-\delta)[1+\lambda(k-1-\delta)]a_k \le (p+1-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)]$$
$$(0 \le \lambda \le 1;\ 0 \le \alpha < p-\delta;\ 0 \le \delta < 1;\ p \in \mathbb{N}). \tag{2.1}$$

The result is sharp for the function $f(z)$ given by

$$f(z) = z^p - \frac{(p+1-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)]}{(n+p+1-\delta)_{\delta}(n+p-\alpha-\delta)[1+\lambda(n+p-1-\delta)]}z^{n+p}. \tag{2.2}$$

Proof. Let $f(z) \in \mathcal{T}_n(p)$ and $\mathcal{F}(z)$ be defined by (1.11). Suppose that $f(z) \in \mathcal{S}_{n,p}^{\delta}(\lambda,\alpha)$. Then, in conjunction with (1.10) and (1.11) yields

$$\Re e \frac{(p+1-\delta)_{\delta}(p-\delta)[1+\lambda(p-1-\delta)]z^{p-\delta}-\sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}(k-\delta)[1+\lambda(k-1-\delta)]a_k z^{k-\delta}}{(p+1-\delta)_{\delta}[1+\lambda(p-1-\delta)]z^{p-\delta}-\sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}[1+\lambda(k-1-\delta)]a_k z^{k-\delta}} > \alpha. \tag{2.3}$$

By letting $z \to 1^-$ along the real axis, we arrive easily at the inequality in (2.1). □

Lemma 2.2. *Let the function $f(z)$ given by* (1.1) *be in the class $\mathcal{S}_{n,p}^{\delta}(\lambda,\alpha)$. Then*

$$\sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}\, a_k \leq \frac{(p+1-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)]}{(n+p-\alpha-\delta)[1+\lambda(n+p-1-\delta)]}, \tag{2.4}$$

$$\sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}\, ka_k \leq \frac{(p+1-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)](n+p)}{(n+p-\alpha-\delta)[1+\lambda(n+p-1-\delta)]}. \tag{2.5}$$

Proof. By using Lemma 2.1, we find from (2.1) that

$$\begin{aligned}(n+p-\alpha-\delta)[1+\lambda(n+p-1-\delta)]&\sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}a_k\\ &\leq \sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}(k-\alpha-\delta)[1+\lambda(k-1-\delta)]a_k\\ &\leq (p+1-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)],\end{aligned} \tag{2.6}$$

which immediately yields the first assertion (2.4) of Lemma 2.2. □

For the proof of second assertion, by appealing to (2.1), we also have

$$\begin{aligned}&[1+\lambda(n+p-\delta-1)]\left\{\sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}ka_k-(\alpha+\delta)\sum_{k=n+p}^{\infty}(k+1-\delta)_{\delta}a_k\right\}\\ &\quad\leq (p+1-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)],\end{aligned} \tag{2.7}$$

by using (2.4) in (2.7), we can easily get the assertion (2.5) of Lemma 2.2.

The distortion inequalities for functions in the class $\mathcal{K}_{n,p}^{\delta}(\lambda,\alpha,\mu)$ are given by Theorem 2.3 below.

Theorem 2.3. *Let a function $f(z) \in \mathcal{T}_n(p)$ be in the class $\mathcal{K}^{\delta}_{n,p}(\lambda, \alpha, \mu)$. Then*

$$|f(z)| \leq |z|^p + \frac{(p+1-\delta)_\delta (p-\alpha-\delta)(p-\delta+\mu)(p-\delta+\mu+1)}{(n+p+1-\delta)_\delta (n+p-\alpha-\delta)(n+p-\delta+\mu)} \frac{1+\lambda(p-\delta-1)}{1+\lambda(n+p-\delta-1)} |z|^{n+p}, \tag{2.8}$$

$$|f(z)| \geq |z|^p - \frac{(p+1-\delta)_\delta (p-\alpha-\delta)(p-\delta+\mu)(p-\delta+\mu+1)}{(n+p+1-\delta)_\delta (n+p-\alpha-\delta)(n+p-\delta+\mu)} \frac{1+\lambda(p-\delta-1)}{1+\lambda(n+p-\delta-1)} |z|^{n+p}. \tag{2.9}$$

Proof. Suppose that a function $f(z) \in \mathcal{T}_n(p)$ is given by (1.1) and also let the function $g(z) \in \mathcal{S}^{\delta}_{n,p}(\lambda, \alpha)$ occurring in the nonhomogenous differential equation (1.13) be given as in the Definitions (1.2) or (1.3) with of course

$$b_k \geq 0 \quad (k = n+p, n+p+1, \ldots). \tag{2.10}$$

Then we easily see from (1.13) that

$$a_k = \frac{(p-\delta+\mu)(p-\delta+\mu+1)}{(k-\delta+\mu)(k-\delta+\mu+1)} b_k \quad (k = n+p, n+p+1, \ldots). \tag{2.11}$$

So that

$$f(z) = z^p - \sum_{k=n+p}^{\infty} a_k z^k = z^p - \sum_{k=n+p}^{\infty} \frac{(p-\delta+\mu)(p-\delta+\mu+1)}{(k-\delta+\mu)(k-\delta+\mu+1)} b_k z^k, \tag{2.12}$$

$$|f(z)| \leq |z|^p + |z|^{n+p} \sum_{k=n+p}^{\infty} \frac{(p-\delta+\mu)(p-\delta+\mu+1)}{(k-\delta+\mu)(k-\delta+\mu+1)} b_k. \tag{2.13}$$

Since $g(z) \in \mathcal{S}^{\delta}_{n,p}(\lambda, \alpha)$, the first assertion (2.4) of Lemma 2.2 yields the following inequality:

$$|b_k| \leq \frac{(p+1-\delta)_\delta (p-\alpha-\delta)[1+\lambda(p-\delta-1)]}{(n+p+1-\delta)_\delta (n+p-\alpha-\delta)[1+\lambda(n+p-\delta-1)]}. \tag{2.14}$$

From (2.13) and (2.14) we have

$$\begin{aligned} |f(z)| \leq |z|^p + |z|^{n+p} & \frac{(p+1-\delta)_\delta (p-\alpha-\delta)[1+\lambda(p-\delta-1)]}{(n+p+1-\delta)_\delta (n+p-\alpha-\delta)[1+\lambda(n+p-\delta-1)]} \\ & \cdot \sum_{k=n+p}^{\infty} \frac{1}{(k-\delta+\mu)(k-\delta+\mu+1)}, \end{aligned} \tag{2.15}$$

and also note that

$$\sum_{k=n+p}^{\infty} \frac{1}{(k-\delta+\mu)(k-\delta+\mu+1)} = \sum_{k=n+p}^{\infty} \left(\frac{1}{k-\delta+\mu} - \frac{1}{k-\delta+\mu+1} \right) = \frac{1}{n+p-\delta+\mu}, \tag{2.16}$$

where $\mu \in \mathbb{R} \setminus \{-n-p, \ -n-p-1, \ldots\}$. The assertion (2.8) of Theorem 2.3 follows at once from (2.15). The assertion (2.9) of Theorem 2.3 can be proven by similarly applying (2.12), (2.14), and (2.15), and also (2.16). □

By setting $\delta := 0$ in Theorem 2.3, we obtain the following Corollary 2.4.

Corollary 2.4 (See Altıntaş et al. [8, Theorem 1]). *If the functions f and g satisfy the nonhomogeneous Cauchy-Euler differential equation* (1.13), *then*

$$|f(z)| \leq |z|^p + \frac{(p-\alpha)(p+\mu)(p+\mu+1)[1+\lambda(p-1)]}{(n+p-\alpha)(n+p+\mu)[1+\lambda(n+p-1)]}|z|^{n+p},$$
$$|f(z)| \geq |z|^p - \frac{(p-\alpha)(p+\mu)(p+\mu+1)[1+\lambda(p-1)]}{(n+p-\alpha)(n+p+\mu)[1+\lambda(n+p-1)]}|z|^{n+p}. \tag{2.17}$$

By letting $\delta := 0$, $\lambda := 0$ and $\delta := 0$, $\lambda := 1$ in Theorem 2.3. We arrive at Corollaries 2.5 and 2.6 (see, [8]).

Corollary 2.5. *If the functions f and g satisfy the nonhomogeneous Cauchy-Euler differential equation* (1.13) *with $g \in \mathcal{S}_n^*(p,\alpha)$, then*

$$|f(z)| \leq |z|^p + \frac{(p-\alpha)(p+\mu)(p+\mu+1)}{(n+p-\alpha)(n+p+\mu)}|z|^{n+p},$$
$$|f(z)| \geq |z|^p - \frac{(p-\alpha)(p+\mu)(p+\mu+1)}{(n+p-\alpha)(n+p+\mu)}|z|^{n+p}. \tag{2.18}$$

Corollary 2.6. *If the functions f and g satisfy the nonhomogeneous Cauchy-Euler differential equation* (1.13) *with $g \in \mathcal{C}_n(p,\alpha)$, then*

$$|f(z)| \leq |z|^p + \frac{p(p-\alpha)(p+\mu)(p+\mu+1)}{(n+p-\alpha)(n+p+\mu)(n+p)}|z|^{n+p},$$
$$|f(z)| \geq |z|^p - \frac{p(p-\alpha)(p+\mu)(p+\mu+1)}{(n+p-\alpha)(n+p+\mu)(n+p)}|z|^{n+p}. \tag{2.19}$$

3. Neighborhoods for the Classes $\mathcal{S}_{n,p}^{\delta}(\lambda,\alpha)$ and $\mathcal{K}_{n,p}^{\delta}(\lambda,\alpha,\mu)$

In this section, we determine inclusion relations for the classes $\mathcal{S}_{n,p}^{\delta}(\lambda,\alpha)$ and $\mathcal{K}_{n,p}^{\delta}(\lambda,\alpha,\mu)$ concerning the (n,p,ε)-neighborhoods is defined by (1.7) and (1.8).

Theorem 3.1. *Let a function $f(z) \in \mathfrak{T}_n(p)$ be in the class $\mathcal{S}_{n,p}^{\delta}(\lambda, \alpha)$. Then*

$$\mathcal{S}_{n,p}^{\delta}(\lambda, \alpha) \subset \mathcal{N}_{n,p}^{\varepsilon}\left(\mathfrak{D}_z^{\delta} h, \mathfrak{D}_z^{\delta} f\right), \tag{3.1}$$

where $h(z)$ is given by (1.9) *and the parameter ε is the given by*

$$\varepsilon := \frac{(n+p)(p-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)]}{(n+p-\alpha-\delta)[1+\lambda(n+p-1-\delta)]}. \tag{3.2}$$

Proof. Assertion (3.1) would follow easily from the definition of $\mathcal{N}_{n,p}^{\varepsilon}(\mathfrak{D}_z^{\delta} h, \mathfrak{D}_z^{\delta} f)$, which is given by (1.8) with $g(z)$ replaced by $f(z)$ and the second assertion (2.5) of Lemma 2.2. □

Theorem 3.2. *Let a function $f(z) \in \mathfrak{T}_n(p)$ be in the class $\mathcal{K}_{n,p}^{\delta}(\lambda, \alpha, \mu)$. Then*

$$\mathcal{K}_{n,p}^{\delta}(\lambda, \alpha, \mu) \subset \mathcal{N}_{n,p}^{\varepsilon}\left(\mathfrak{D}_z^{\delta} g, \mathfrak{D}_z^{\delta} f\right), \tag{3.3}$$

where $g(z)$ is given by (1.13) *and the parameter ε is the given by*

$$\varepsilon := \frac{(n+p)(p-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)][n+(p-\delta+\mu)(p-\delta+\mu+2)]}{(n+p-\alpha-\delta)(n+p-\delta+\mu)[1+\lambda(n+p-1-\delta)]}. \tag{3.4}$$

Proof. Suppose that $f(z) \in \mathcal{K}_{n,p}^{\delta}(\lambda, \alpha, \mu)$. Then, upon substituting from (2.11) into the following coefficient inequality:

$$\sum_{k=n+p}^{\infty} (k-\delta)_{\delta} k \left| b_k - a_k \right| \leq \sum_{k=n+p}^{\infty} (k-\delta)_{\delta} k b_k + \sum_{k=n+p}^{\infty} (k-\delta)_{\delta} k a_k, \tag{3.5}$$

where $a_k \geq 0$ and $b_k \geq 0$, we obtain that

$$\begin{aligned} \sum_{k=n+p}^{\infty} (k-\delta)_{\delta} k \left| b_k - a_k \right| \leq & \sum_{k=n+p}^{\infty} (k-\delta)_{\delta} k b_k \\ & + \sum_{k=n+p}^{\infty} \frac{(p-\delta+\mu)(p-\delta+\mu+1)}{(k-\delta+\mu)(k-\delta+\mu+1)} (k-\delta)_{\delta} k b_k. \end{aligned} \tag{3.6}$$

Since $g(z) \in \mathcal{S}_{n,p}^{\delta}(\lambda, \alpha)$, the second assertion (2.5) of Lemma 2.2 yields that

$$(k-\delta)_{\delta} k b_k \leq \frac{(n+p)(p-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)]}{(n+p-\alpha-\delta)[1+\lambda(n+p-1-\delta)]} \quad (k = n+p, n+p+1, \ldots). \tag{3.7}$$

Finally, by making use of (2.5) as well as (3.7) on the right-hand side of (3.6), we find that

$$\sum_{k=n+p}^{\infty}(k-\delta)_{\delta}k\left|b_k-a_k\right| \leq \frac{(n+p)(p-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)]}{(n+p-\alpha-\delta)[1+\lambda(n+p-1-\delta)]}\left(1+\frac{(p-\delta+\mu)(p-\delta+\mu+1)}{(k-\delta+\mu)(k-\delta+\mu+1)}\right), \tag{3.8}$$

which, by virtue of the identity (2.16), immediately yields that

$$\sum_{k=n+p}^{\infty}(k-\delta)_{\delta}k\left|b_k-a_k\right| \leq \frac{(n+p)(p-\delta)_{\delta}(p-\alpha-\delta)[1+\lambda(p-1-\delta)]}{(n+p-\alpha-\delta)[1+\lambda(n+p-1-\delta)]}\cdot\frac{[n+(p-\delta+\mu)(p-\delta+\mu+2)]}{(n+p-\delta+\mu)} =: \varepsilon. \tag{3.9}$$

Thus, by definition (1.7) with $g(z)$ interchanged by $f(z)$, $f(z) \in \mathcal{N}_{n,p}^{\varepsilon}(\mathfrak{D}_z^{\delta}g, \mathfrak{D}_z^{\delta}f)$. This evidently completes the proof of Theorem 3.2. □

By setting $\delta = 0$ in Theorem 3.2, we receive the following result.

Corollary 3.3. *If the function $f(z) \in \mathcal{T}_n(p)$ is in the class $\mathcal{K}_{n,p}^{0}(\lambda,\alpha,\mu)$. Then*

$$\mathcal{K}_{n,p}^{0}(\lambda,\alpha,\mu) \subset \mathcal{N}_{n,p}^{\varepsilon}(g,f), \tag{3.10}$$

where $g(z)$ is given by (1.13) *and the parameter ε is the given by*

$$\varepsilon := \frac{(n+p)(p-\alpha)[1+\lambda(p-1)][n+(p+\mu)(p+\mu+2)]}{(n+p-\alpha)(n+p+\mu)[1+\lambda(n+p-1)]}. \tag{3.11}$$

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