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*Research Article*

# Differential Subordination Result with the Srivastava-Attiya Integral Operator

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The purpose of this paper is to derive an interested subordination relation which contains the Srivastava-Attiya integral operator $J_{s,b}(f)$ in the open unit disc $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$. Some applications of the main result are also considered.

## 1. Introduction and Definitions

Let $A$ denote the class of functions $f(z)$ normalized by

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k \tag{1.1}$$

which are analytic in the open unit disc $\mathbb{U} = \{z \in \mathbb{C} : |z| < 1\}$.

A function $f(z)$ in the class $A$ is said to be in the class $S^*(\alpha)$ of starlike functions of order $\alpha$ if it satisfies

$$\operatorname{Re}\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \quad (z \in \mathbb{U}), \tag{1.2}$$

for some $\alpha$ $(0 \leq \alpha < 1)$. Also, we write $S(0) = S^*$, the class of starlike functions in $\mathbb{U}$.

For $f(z) \in A$ and $z \in \mathbb{U}$, let the integral operators $A(f), L(f)$, and $L_\gamma(f)$ be defined as

$$A(f)(z) = \int_0^z \frac{f(t)}{t} dt,$$
$$L(f)(z) = \frac{2}{z} \int_0^z f(t)dt, \tag{1.3}$$
$$L_\gamma(f)(z) = \frac{1+\gamma}{z^\gamma} \int_0^z f(t)t^{\gamma-1}dt \quad (\gamma > -1).$$

The operators $A(f)$ and $L(f)$ are Alexander operator and Libera operator which were introduced earlier by Alexander [1] and Libera [2]. $L_\gamma(f)$ is called generalized Bernardi operator; the operator $L_\gamma(f)$ when $\gamma \in \mathbb{N} = \{1, 2, \ldots\}$ was introduced by Bernardi [3].

Jung et al. [4] introduced the following integral operator:

$$I^\sigma(f)(z) = \frac{2^\sigma}{z\Gamma(\sigma)} \int_0^z \left(\log\left(\frac{z}{t}\right)\right)^{\sigma-1} f(t)dt \quad (\sigma > 0, f(z) \in A). \tag{1.4}$$

The operator $I^\sigma(f)$ is closely related to multiplier transformations studied earlier by Flett [5], see also [6–8].

A general Hurwitz-Lerch Zeta function $\varphi(z, s, b)$ defined by (cf., e.g., [9, page 121 et seq.])

$$\varphi(z, s, b) = \sum_{k=0}^{\infty} \frac{z^k}{(k+b)^s}, \tag{1.5}$$

($b \in \mathbb{C} \setminus \mathbb{Z}_0^-, \mathbb{Z}_0^- = \mathbb{Z}^- \cup \{0\} = \{0, -1, -2, \ldots\}$, $s \in \mathbb{C}$ when $z \in \mathbb{U}$, $\mathrm{Re}(s) > 1$ when $|z| = 1$). Recently, several properties of $\varphi(z, s, b)$ have been studied by Choi and Srivastava [10], Ferreira and López [11], Lin and Srivastava [12], Luo and Srivastava [13], and others.

For $f(z) \in A$, $s \in \mathbb{C}$, and $b \in \mathbb{C} \setminus \mathbb{Z}_0^-$, let

$$G_{s,b}(z) = (1+b)^s \left[\varphi(z, s, b) - b^{-s}\right] \quad (z \in \mathbb{U}). \tag{1.6}$$

Srivastava and Attiya [14] defined the operator $J_{s,b}(f)$ as

$$J_{s,b}(f)(z) = G_{s,b}(z) * f(z) \quad (z \in \mathbb{U}; f(z) \in A), \tag{1.7}$$

where the symbol $(*)$ denotes the *Hadamard product* (*or convolution*).

They showed that if $f(z) \in A$ and $z \in \mathbb{U}$, then,

$$
\begin{aligned}
J_{0,b}\,(f)(z) &= f(z),\\
J_{1,0}\,(f)(z) &= \int_0^z \frac{f(t)}{t}dt = A(f)(z),\\
J_{1,1}\,(f)(z) &= \frac{2}{z}\int_0^z f(t)dt = L(f)(z),\\
J_{1,\gamma}\,(f)(z) &= \frac{1+\gamma}{z^{\gamma}}\int_0^z f(t)t^{\gamma-1}dt = L_{\gamma}(f)(z) \quad (\gamma \text{ real}; \gamma > -1),\\
J_{\sigma,1}\,(f)(z) &= z + \sum_{k=2}^{\infty}\left(\frac{2}{k+1}\right)^{\sigma} a_k z^k = I^{\sigma}(f)(z) \quad (\sigma \text{ real}; \sigma > 0\ ).
\end{aligned}
\tag{1.8}
$$

Also, for $f(z) \in A, t_1; t_2; \ldots; t_n;\ z \in \mathbb{U},\ n \in \mathbb{N}$, and $b \in \mathbb{C} \setminus \mathbb{Z}^-$, we have

$$
\begin{aligned}
J_{2,0}\,(f)(z) &= \int_0^z \frac{1}{t_1}\int_0^{t_1} \frac{f(t_2)}{t_2}dt_2\,dt_1,\\
J_{n,0}\,(f)(z) &= \int_0^z \frac{1}{t_1}\int_0^{t_1} \frac{1}{t_2}\int_0^{t_2}\cdots\frac{1}{t_{n-1}}\int_0^{t_{n-1}} \frac{f(t_n)}{t_n}dt_n\,dt_{n-1}\cdots dt_1,\\
J_{2,b}\,(f)(z) &= \frac{(1+b)^2}{z^b}\int_0^z \frac{1}{t_1}\int_0^{t_1} t_2^{b-1} f(t_2)dt_2\,dt_1,\\
J_{n,b}\,(f)(z) &= \frac{(1+b)^n}{z^b}\int_0^z \frac{1}{t_1}\int_0^{t_1} \frac{1}{t_2}\int_0^{t_2}\cdots\ \frac{1}{t_{n-1}}\int_0^{t_{n-1}} t_n^{b-1} f(t_n)dt_n\,dt_{n-1}\cdots dt_1.
\end{aligned}
\tag{1.9}
$$

Now we introduce the following definition.

*Definition 1.1.* For $f(z) \in A, s \in \mathbb{C}$ and $b \in \mathbb{C} \setminus \mathbb{Z}_0^-$. Then the function $f(z)$ is said to be a member of the class $H_{s,b,\alpha}(A,B)$ if it satisfies

$$
\frac{1}{1-\alpha}\left\{\frac{z\left(J_{s,b}(f)(z)\right)'}{J_{s,b}(f)(z)} - \alpha\right\} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}),
\tag{1.10}
$$

for some $\alpha, A, B (0 \le \alpha < 1;\ -1 \le B < A \le 1)$. We note that $H_{0,b,\alpha}(1,-1)$ is the class of starlike functions of order $\alpha$.

We will also need the following definitions.

*Definition 1.2.* Let $f(z)$ and $F(z)$ be analytic functions. The function $f(z)$ is said to be *subordinate* to $F(z)$, written $f(z) \prec F(z)$, if there exists a function $w(z)$ analytic in $\mathbb{U}$, with $w(0) = 0$ and $|w(z)| \le 1$, and such that $f(z) = F(w(z))$. If $F(z)$ is univalent, then $f(z) \prec F(z)$ *if and only if* $f(0) = F(0)$ and $f(\mathbb{U}) \subset F(\mathbb{U})$.

*Definition 1.3.* Let $\Psi : \mathbb{C}^2 \times \mathbb{U} \rightarrow \mathbb{C}$ be analytic in domain $\mathbb{D}$, and let $h(z)$ be univalent in $\mathbb{U}$. If $p(z)$ is analytic in $\mathbb{U}$ with $(p(z), zp'(z); z) \in \mathbb{D}$ when $z \in \mathbb{U}$, then we say that $p(z)$ satisfies a first order differential subordination if:

$$\Psi(p(z), zp'(z); z) \prec h(z) \quad (z \in \mathbb{U}). \tag{1.11}$$

The univalent function $q(z)$ is called *dominant* of the differential subordination (1.11), if $p(z) \prec q(z)$ for all $p(z)$ satisfies (1.11), if $\tilde{q}(z) \prec q(z)$ for all dominant of (1.11), then we say that $\tilde{q}(z)$ is *the best dominant* of (1.11).

## 2. Some Preliminary Lemmas

To prove our main results, we need the following lemmas.

**Lemma 2.1** (Srivastava and Attiya [14]). *If the function $f(z)$ belongs to $A$, then*

$$zJ'_{s+1,b}(f)(z) = (1+b)J_{s,b}(f)(z) - bJ_{s+1,b}(f)(z), \tag{2.1}$$

*for $s \in \mathbb{C}$, $b \in \mathbb{C} \setminus \mathbb{Z}_0^-$ and $z \in \mathbb{U}$.*

**Lemma 2.2** (Wilken and Feng [15], see also [16]). *Let $\mu$ be a positive measure on $[0,1]$ and let $g$ be a complex-valued function defined on $\mathbb{U} \times [0,1]$ such that $g(\cdot, t)$ is analytic in $\mathbb{U}$ for each $t \in [0,1]$, and $g(z, \cdot)$ is $\mu$-integrable on $[0,1]$ for all $z \in U$. In addition, suppose that $\mathrm{Re}\{g(z,t)\} > 0, g(-r,t)$ is real and*

$$\mathrm{Re}\left\{\frac{1}{g(z,t)}\right\} \geq \frac{1}{g(-r,t)}, \tag{2.2}$$

*for $|z| \leq r < 1$ and $t \in [0,1]$. If*

$$g(z) = \int_0^1 g(z,t)\,d\mu(t), \tag{2.3}$$

*then*

$$\mathrm{Re}\left\{\frac{1}{g(z)}\right\} \geq \frac{1}{g(-r)}. \tag{2.4}$$

**Lemma 2.3.** *For real or complex parameters $a, b$, and $c$ $(c \notin \mathbb{Z}_0^-)$,*

$$\int_0^1 t^{b-1}(1-t)^{c-b-1}(1-zt)^{-a}dt = \frac{\Gamma(b)\Gamma(c-b)}{\Gamma(c)}\,{}_2F_1\left(a,b;c;\frac{z}{z-1}\right) \quad (\mathrm{Re}(c) > \mathrm{Re}(b) > 0), \tag{2.5}$$

$${}_2F_1(a,b;c;z) = (1-z)^{-a}\,{}_2F_1\left(a, c-b; c; \frac{z}{z-1}\right), \tag{2.6}$$

*where ${}_2F_1(a,b;c;z)$ is the Gauss hypergeometric function.*

Each of the identities (2.5) and (2.6) asserted by Lemma 2.3 is well known in the literature (cf., e.g., [17, Chapter 9]).

**Lemma 2.4** (Miller and Mocanu [18]). *If* $-1 \leq B < A \leq 1$, $\beta > 0$, *and the complex number* $\gamma$ *is constrained by* $\operatorname{Re}\gamma \geq (-\beta(1-A))/(1-B)$, *then the differential equation*

$$q(z) + \frac{zq'(z)}{\beta q(z) + \gamma} = \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}) \tag{2.7}$$

*has a univalent solution in* $\mathbb{U}$ *given by*

$$q(z) = \begin{cases} \dfrac{z^{\beta+\gamma(1+Bz)^{\beta(A-B)/B}}}{\beta \int_0^z t^{\beta+\gamma-1}(1+Bt)^{\beta(A-B)/B}dt} - \dfrac{\gamma}{\beta}, & B \neq 0, \\[2ex] \dfrac{z^{\beta+\gamma}\exp(\beta Az)}{\beta \int_0^z t^{\beta+\gamma-1}\exp(\beta At)dt} - \dfrac{\gamma}{\beta}, & B = 0. \end{cases} \tag{2.8}$$

*If the function* $\phi(z)$ *given by*

$$\phi(z) = 1 + c_1 z + c_2 z^2 + \cdots \tag{2.9}$$

*is analytic in* $\mathbb{U}$ *and satisfies*

$$\phi(z) + \frac{z\phi'(z)}{\beta\phi(z) + \gamma} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{2.10}$$

*then*

$$\phi(z) \prec q(z) \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}) \tag{2.11}$$

*and* $q(z)$ *is the best dominant of* (2.10).

## 3. Subordination Result and Starlikeness of $J_{s,b}(f)$

**Theorem 3.1.** *For* $s \in \mathbb{C}$, $b \in \mathbb{C} \setminus \mathbb{Z}_0^-$, $0 \leq \alpha < 1$, *and* $-1 \leq B < A \leq 1$. *If the function* $f(z)$ *belongs to the class* $H_{s,b,\alpha}(A,B)$ *which satisfies* $J_{s+1,b}(f)(z)/z \neq 0$. *Also, let*

$$\operatorname{Re} b \geq -\frac{[(1-A) + \alpha(A-B)]}{(1-B)}, \tag{3.1}$$

*then*

$$\frac{1}{1-\alpha}\left\{\frac{z(J_{s+1,b}(f)(z))'}{J_{s+1,b}(f)(z)} - \alpha\right\} \prec q(z) = \frac{1}{1-\alpha}\left\{\frac{1}{M(z)} - \alpha - b\right\} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{3.2}$$

*where*

$$M(z) = \begin{cases} \displaystyle\int_0^1 t^b \left(\frac{1+Btz}{1+Bz}\right)^{(1-\alpha)(A-B)/B} dt, & B \neq 0 \\ \displaystyle\int_0^1 t^b \exp((1-\alpha)(t-1)Az)dt, & B = 0, \end{cases} \tag{3.3}$$

*and $q(z)$ is the best dominant of* (3.2).

*Moreover, if $b$ is real number with $-1 \leq B < 0$, then*

$$J_{s+1,b}(f)(z) \in S^*(\mu), \tag{3.4}$$

*where*

$$\mu = \frac{b+1}{{}_2F_1(1,(1-\alpha)(B-A)/B; b+2, B/(B-1))} - b. \tag{3.5}$$

*The constant factor $\mu$ cannot be replaced by a larger one.*

*Proof.* Let $f(z) \in H_{s,b,\alpha}(A,B)$, also let

$$\phi(z) = \frac{1}{1-\alpha}\left\{\frac{z(J_{s+1,b}(f)(z))'}{J_{s+1,b}(f)(z)} - \alpha\right\} \quad (z \in \mathbb{U}). \tag{3.6}$$

Then $\phi(z)$ is analytic in $\mathbb{U}$ with $\phi(0) = 1$. Using the identity in Lemma 2.1 in (3.6), we have

$$(1+b)\frac{J_{s,b}(f)(z)}{J_{s+1,b}(f)(z)} = (1-\alpha)\phi(z) + \alpha + b. \tag{3.7}$$

Carrying out logarithmic differentiation in (3.7), we deduce that

$$\frac{1}{1-\alpha}\left\{\frac{z(J_{s,b}(f)(z))'}{J_{s,b}(f)(z)} - \alpha\right\} = \phi(z) + \frac{z\phi'(z)}{(1-\alpha)\phi(z)+\alpha+b} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}). \tag{3.8}$$

Hence, by using (3.1) and Lemma 2.4, we find that

$$\phi(z) \prec q(z) \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{3.9}$$

where $q(z)$ given in (3.2) is the best dominant of (3.8). This proves the assertion (3.2) of the theorem. □

Next, in order to prove (3.4), it suffices to show that

$$\inf_{z\in\mathbb{U}}\{\operatorname{Re} q(z)\} = q(-1). \tag{3.10}$$

Putting

$$a = \frac{(1-\alpha)(B-A)}{B}, \tag{3.11}$$

since $B \geq -1$, then from (3.3), by using (2.5) and (2.6), we see that, for $B \neq 0$

$$\begin{aligned} M(z) &= \int_0^1 t^b \left(\frac{1+Btz}{1+Bz}\right)^{(1-\alpha)(A-B)/B} dt \\ &= (1+Bz)^a \int_0^1 t^b (1+Btz)^{-a} dt \\ &= \frac{\Gamma(b+1)}{\Gamma(b+2)}\, {}_2F_1\left(1, a; b+2; \frac{Bz}{Bz+1}\right). \end{aligned} \tag{3.12}$$

To prove (3.10), we need to show that

$$\operatorname{Re}\left\{\frac{1}{M(z)}\right\} \geq \frac{1}{M(-1)} \quad (z \in \mathbb{U}). \tag{3.13}$$

By using (2.5) and (3.12), we have

$$M(z) = \int_0^1 h(z,t)\, d\nu(t), \tag{3.14}$$

where

$$\begin{aligned} h(z,t) &= \frac{1+Bz}{1+(1-t)Bz} \quad (0 \leq t \leq 1), \\ d\nu(t) &= \frac{\Gamma(b+1)}{\Gamma(a)\Gamma(b+2-a)} t^{a-1}(1-t)^{b-a+1}, \end{aligned} \tag{3.15}$$

which is a positive measure on $[0,1]$.

We note that

$$\operatorname{Re} h(z,t) > 0, \quad h(-r,t) \text{ is real} \quad (r \in [0,1)), \tag{3.16}$$

also, for $-1 \leq B < 0$, it implies that

$$\operatorname{Re}\left\{\frac{1}{h(z,t)}\right\} = \operatorname{Re}\left\{\frac{1+(1-t)Bz}{1+Bz}\right\} \geq \frac{1+(1-t)Br}{1+Br} = \frac{1}{h(-r,t)}. \tag{3.17}$$

Therefore by using Lemma 2.4, we have

$$\operatorname{Re}\left\{\frac{1}{M(z)}\right\} \geq \frac{1}{M(-1)} \quad (|z| \leq r < 1), \tag{3.18}$$

which, upon letting $r \to 1^-$, yields

$$\operatorname{Re}\left\{\frac{1}{M(z)}\right\} \geq \frac{1}{M(-1)} \quad (z \in \mathbb{U}). \tag{3.19}$$

Since $q(z)$ is the best dominant of (3.2), therefore the constant factor $\mu$ cannot be replaced by a larger one.

**Corollary 3.2.** *Let $s$ be a complex number, $0 \leq \alpha < 1, -1 \leq B < A \leq 1$ with $-1 \leq B < 0$ and the real number $b$ is constrained by*

$$b \geq \frac{-[(1-A)+\alpha(A-B)]}{(1-B)}. \tag{3.20}$$

*Then*

$$H_{s,b,\alpha}(A,B) \subset H_{s+1,b,\alpha}(1-2\delta,-1), \tag{3.21}$$

*where*

$$\delta = \frac{1}{1-\alpha}\left\{\frac{b+1}{{}_2F_1(1,(1-\alpha)(B-A)/B;b+2,B/(B-1))} - \alpha - b\right\}. \tag{3.22}$$

*The constant factor $\delta$ is the best possible.*

## 4. Applications

Putting $s = 0$, in Theorem 3.1, we have the following result for the operator $L_b(f)$.

**Corollary 4.1.** *For $0 \leq \alpha < 1$, $-1 \leq B < A \leq 1$ and $b$ constrained by* (3.20). *If the function $f(z)$ belongs to the class $H_{0,b,\alpha}(A,B)$ which satisfies $L_b(f)(z)/z \neq 0$, then*

$$\frac{1}{1-\alpha}\left\{\frac{z(L_b(f)(z))'}{L_b(f)(z)} - \alpha\right\} \prec q(z) = \frac{1}{1-\alpha}\left\{\frac{1}{M(z)} - \alpha - b\right\} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{4.1}$$

*where $M(z)$ defined by* (3.3) *and $q(z)$ is the best dominant of* (4.1).

*Moreover, if $-1 \leq B < 0$, then*

$$L_b(f)(z) \in S^*(\mu), \tag{4.2}$$

*where $\mu$ defined by* (3.5). *The constant factor $\mu$ cannot be replaced by a larger one.*

Setting $b = 1$, in Theorem 3.1 and $s \geq 0$; real, we obtain the following property for the operator $I^s(f)$.

**Corollary 4.2.** *Let $s \geq 0$; real, $0 \leq \alpha < 1$ and $-1 \leq B < A \leq 1$. If the function $f(z)$ belongs to the class $H_{s,1,\alpha}(A, B)$ which satisfies $I^{s+1}(f)(z)/z \neq 0$. Then*

$$\frac{1}{1-\alpha}\left\{\frac{z\left(I^{s+1}(f)(z)\right)'}{I^{s+1}(f)(z)} - \alpha\right\} \prec q(z) = \frac{1}{1-\alpha}\left\{\frac{1}{M(z)} - \alpha - 1\right\} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{4.3}$$

*where*

$$M(z) = \begin{cases} \displaystyle\int_0^1 t\left(\frac{1+Btz}{1+Bz}\right)^{(1-\alpha)(A-B)/B} dt, & B \neq 0 \\[2ex] \dfrac{(1-\alpha)Az + \exp(-(1-\alpha)Az) - 1}{(1-\alpha)^2 A^2 z^2} & B = 0, \end{cases} \tag{4.4}$$

*and $q(z)$ is the best dominant of* (4.3).

*Moreover, if $-1 \leq B < 0$, then*

$$I^{s+1}(f)(z) \in S^*(\mu), \tag{4.5}$$

*where*

$$\mu = \frac{2}{{}_2F_1(1, (1-\alpha)(B-A)/B; 3, B/(B-1))} - 1. \tag{4.6}$$

*The constant factor $\mu$ cannot be replaced by a larger one.*

By taking $f(z) = f_0(z) = z/(1-z)$, in Theorem 3.1, we readily obtain the following Hurwitz-Lerch Zeta function property.

**Corollary 4.3.** *Let $s$ be a complex number, $0 \leq \alpha < 1$, $-1 \leq B < A \leq 1$, and $b$ constrained by* (3.20), *also, let $G_{s+1,b}(z)/z \neq 0$. If*

$$\frac{1}{1-\alpha}\left\{\frac{z(G_{s,b}(z))'}{G_{s,b}(z)} - \alpha\right\} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{4.7}$$

*then*

$$\frac{1}{1-\alpha}\left\{\frac{z(G_{s+1,b}(z))'}{G_{s+1,b}(z)} - \alpha\right\} \prec q(z) = \frac{1}{1-\alpha}\left\{\frac{1}{M(z)} - \alpha - b\right\} \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{4.8}$$

*where $M(z)$ defined by* (3.3) *and $q(z)$ is the best dominant of* (4.7).

*Moreover, if* $-1 \leq B < 0$, *then*

$$G_{s+1,b}(z) \in S^*(\mu), \tag{4.9}$$

*where* $\mu$ *is given by* (3.5). *The constant factor* $\mu$ *cannot be replaced by a larger one.*

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