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Research Article

On the Symmetric Properties of the Multivariate p-Adic Invariant Integral on $\mathbb{Z}_p$ Associated with the Twisted Generalized Euler Polynomials of Higher Order

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We study the symmetric properties for the multivariate p-adic invariant integral on $\mathbb{Z}_p$ related to the twisted generalized Euler polynomials of higher order.

1. Introduction

Let p be a fixed prime number. Throughout this paper, the symbols $\mathbb{Z}$, $\mathbb{Z}_p$, $\mathbb{Q}_p$, and $\mathbb{C}_p$ denote the ring of rational integers, the ring of p-adic integers, the field of p-adic rational numbers, and the completion of algebraic closure of $\mathbb{Q}_p$, respectively. Let $\mathbb{N}$ be the set of natural numbers. The normalized valuation in $\mathbb{C}_p$ is denoted by $|\cdot|_p$ with $|p|_p = 1/p$. Let $UD(\mathbb{Z}_p)$ be the space of uniformly differentiable function on $\mathbb{Z}_p$. For $f \in UD(\mathbb{Z}_p)$, the fermionic p-adic invariant integral on $\mathbb{Z}_p$ is defined as

$$I(f) = \int_{\mathbb{Z}_p} f(x)d\mu(x) = \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)\mu\left(x + p^N\mathbb{Z}_p\right) = \lim_{N\to\infty} \sum_{x=0}^{p^N-1} f(x)(-1)^x \tag{1.1}$$

(see [1–25]). For $n \in \mathbb{N}$, we note that

$$\int_{\mathbb{Z}_p} f(x+n)d\mu(x) = (-1)^n \int_{\mathbb{Z}_p} f(x)d\mu(x) + \sum_{l=0}^{n-1} (-1)^{n-1-l} f(l) \quad \text{(see [5])}. \tag{1.2}$$

Let d be a fixed odd positive integer. For $N \in \mathbb{N}$, we set

$$\begin{gathered} X = X_d = \lim_{\overleftarrow{N}} \mathbb{Z}/dp^N\mathbb{Z}, \quad X_1 = \mathbb{Z}_p, \\ X^* = \bigcup_{\substack{0<a<dp \\ (a,p)=1}} (a + dp\mathbb{Z}_p), \\ a + dp^N\mathbb{Z}_p = \left\{ x \in X \mid x \equiv a \left(\operatorname{mod}\, dp^N \right) \right\}, \end{gathered} \tag{1.3}$$

where $a \in \mathbb{Z}$ lies in $0 \leq a < dp^N$ (see [1–13]). It is well known that for $f \in UD(\mathbb{Z}_p)$,

$$\int_X f(x) d\mu(x) = \int_{\mathbb{Z}_p} f(x) d\mu(x). \tag{1.4}$$

For $n \in \mathbb{N}$, let C_{p^n} be the cyclic group of order p^n. That is, $C_{p^n} = \{\xi \mid \xi^{p^n} = 1\}$. The p-adic locally constant space, T_p, is defined by $T_p = \lim_{n\to\infty} C_{p^n} = \bigcup_{n\geq 1} C_{p^n}$.

Let χ be Dirichlet's character with conductor $d \in \mathbb{N}$ and let $\xi \in T_p$. Then the generalized twisted Bernoulli polynomials $B_{n,\chi,\xi}(x)$ attached to χ are defined as

$$\frac{t \sum_{a=0}^{d-1} \chi(a) \xi^a e^{at}}{\xi^d e^{dt} - 1} e^{xt} = \sum_{n=0}^{\infty} B_{n,\chi,\xi}(x) \frac{t^n}{n!} \quad (\text{see [10]}). \tag{1.5}$$

In [4, 7, 10–12], the generalized twisted Bernoulli polynomials of order k attached to χ are also defined as follows:

$$\underbrace{\left(\frac{t \sum_{a=0}^{d-1} \chi(a) \xi^a e^{at}}{\xi^d e^{dt} - 1} \right) \times \cdots \times \left(\frac{t \sum_{a=0}^{d-1} \chi(a) \xi^a e^{at}}{\xi^d e^{dt} - 1} \right)}_{k\text{-times}} e^{xt} = \sum_{n=0}^{\infty} B_{n,\chi,\xi}^{(k)}(x) \frac{t^n}{n!}. \tag{1.6}$$

Recently, the symmetry identities for the generalized twisted Bernoulli polynomials and the generalized twisted Bernoulli polynomials of order k are studied in [4, 12].

In this paper, we study the symmetric properties of the multivariate p-adic invariant integral on $\mathbb{Z}_p$. From these symmetric properties, we derive the symmetry identities for the twisted generalized Euler polynomials of higher order. In [14], Kim gave the relation between the power sum polynomials and the generalized higher-order Euler polynomials. The main purpose of this paper is to give the symmetry identities for the twisted generalized Euler polynomials of higher order using the symmetric properties of the multivariate p-adic invariant integral on $\mathbb{Z}_p$.

2. Symmetry Identities for the Twisted Generalized Euler Polynomials of Higher Order

Let χ be Dirichlet's character with an odd conductor $d \in \mathbb{N}$. That is, $d \in \mathbb{N}$ with $d \equiv 1 \pmod 2$. For $\xi \in T_p$, the twisted generalized Euler polynomials attached to χ, $E_{n,\chi,\xi}(x)$, are defined as

$$\int_X \chi(y)\xi^y e^{(x+y)t} d\mu(y) = \frac{2\sum_{a=0}^{d-1}(-1)^a \chi(a)\xi^a e^{at}}{\xi^d e^{dt}+1} e^{xt} = \sum_{n=0}^{\infty} E_{n,\chi,\xi}(x)\frac{t^n}{n!} \quad (\text{see [12]}). \tag{2.1}$$

In the special case $x = 0$, $E_{n,\chi,\xi} = E_{n,\chi,\xi}(0)$ are called the nth twisted generalized Euler numbers attached to χ.

From (2.1), we note that

$$\int_X \chi(y)\xi^y (x+y)^m d\mu(y) = E_{m,\chi,\xi}(x), \quad m \in \mathbb{N} \cup \{0\}. \tag{2.2}$$

For $n \in \mathbb{N}$ with $n \equiv 1 \pmod 2$, we have

$$\int_X \chi(x)\xi^x e^{(x+nd)t} d\mu(x) + \int_X \chi(x)\xi^x e^{xt} d\mu(x) = 2\sum_{l=0}^{nd-1}(-1)^l \chi(l)\ \xi^l e^{lt}. \tag{2.3}$$

Let $T_{k,\chi,\xi}(n) = \sum_{l=0}^{n}(-1)^l \chi(l)\xi^l l^k$. Then we see that

$$\begin{aligned} &\xi^{nd}\int_X \chi(x)\xi^x e^{(x+nd)t} d\mu(x) + \int_X \chi(x)\xi^x e^{xt} d\mu(x) \\ &\quad = \frac{2\int_X e^{xt}\chi(x)\xi^x d\mu(x)}{\int_X e^{ndxt}\xi^{ndx} d\mu(x)} = 2\sum_{k=0}^{\infty} T_{k,\chi,\xi}(nd-1)\frac{t^k}{k!}. \end{aligned} \tag{2.4}$$

Now we define the twisted generalized Euler polynomials $E_{n,\chi,\xi}^{(k)}(x)$ of order k attached to χ as follows:

$$e^{xt}\left(\frac{2\sum_{a=0}^{d-1}(-1)^a \chi(a)\xi^a e^{at}}{\xi^d e^{dt}+1}\right)^k = \sum_{n=0}^{\infty} E_{n,\chi,\xi}^{(k)}(x)\frac{t^n}{n!}. \tag{2.5}$$

In the special case $x = 0$, $E_{n,\chi,\xi}^{(k)} = E_{n,\chi,\xi}^{(k)}(0)$ are called the nth twisted generalized Euler numbers of order k.

Let $w_1, w_2, d \in \mathbb{N}$ with $w_1 \equiv 1, w_2 \equiv 1$, and $d \equiv 1 \pmod 2$. Then we set

$$\begin{aligned} J_{\chi,\xi}^{(m)}(w_1, w_2 \mid x) &= \left(\frac{\int_{X^m} \left(\prod_{i=1}^m \chi(x_i)\right) \xi^{(\sum_{i=1}^m x_i) w_1} e^{(\sum_{i=1}^m x_i + w_2 x) w_1 t} d\mu(x_1) \cdots d\mu(x_m)}{\int_X \xi^{d w_1 w_2 x} e^{d w_1 w_2 x t} d\mu(x)} \right) \\ &\quad \times \left(\int_{X^m} \left(\prod_{i=1}^m \chi(x_i) \right) \xi^{(\sum_{i=1}^m x_i) w_2} e^{(\sum_{i=1}^m x_i + w_1 y) w_2 t} d\mu(x_1) \cdots d\mu(x_m) \right), \end{aligned} \tag{2.6}$$

where

$$\int_{X^m} f(x_1, \ldots, x_m) d\mu(x_1) \cdots d\mu(x_m) = \underbrace{\int_X \cdots \int_X}_{m\text{-times}} f(x_1, \ldots, x_m) d\mu(x_1) \cdots d\mu(x_m). \tag{2.7}$$

From (2.6), we note that

$$\begin{aligned} J_{\chi,\xi}^{(m)}(w_1, w_2 \mid x) &= \left(\int_{X^m} \left(\prod_{i=1}^m \chi(x_i) \right) \xi^{(\sum_{i=1}^m x_i) w_1} e^{(\sum_{i=1}^m x_i) w_1 t} d\mu(x_1) \cdots d\mu(x_m) \right) e^{w_1 w_2 x t} \\ &\quad \times \left(\frac{\int_X \chi(x_m) \xi^{w_2 x_m} e^{w_2 x_m t} d\mu(x_m)}{\int_X \xi^{d w_1 w_2 x} e^{d w_1 w_2 x t} d\mu(x)} \right) e^{w_1 w_2 y t} \\ &\quad \times \left(\int_{X^{m-1}} \left(\prod_{i=1}^{m-1} \chi(x_i) \right) \xi^{(\sum_{i=1}^{m-1} x_i) w_2} e^{(\sum_{i=1}^{m-1} x_i) w_2 t} d\mu(x_1) \cdots d\mu(x_{m-1}) \right). \end{aligned} \tag{2.8}$$

From (2.4), we can easily derive the following equation:

$$\frac{\int_X \chi(x) \xi^x e^{xt} d\mu(x)}{\int_X \xi^{d w_1 x} e^{d w_1 x t} d\mu(x)} = \sum_{l=0}^{d w_1 - 1} (-1)^l \chi(l) \xi^l e^{lt} = \sum_{k=0}^{\infty} T_{k,\chi,\xi}(d w_1 - 1) \frac{t^k}{k!}. \tag{2.9}$$

It is not difficult to show that

$$\begin{aligned} & e^{w_1 w_2 x t} \left(\int_{X^m} \left(\prod_{i=1}^m \chi(x_i) \right) \xi^{(\sum_{i=1}^m x_i) w_1} e^{(\sum_{i=1}^m x_i) w_1 t} d\mu(x_1) \cdots d\mu(x_m) \right) \\ &= \left(\frac{2 \sum_{a=0}^{d-1} (-1)^a \chi(a) \xi^{a w_1} e^{a w_1 t}}{\xi^{d w_1} e^{d w_1 t} + 1} \right)^m e^{w_1 w_2 x t} = \sum_{k=0}^{\infty} E_{k,\chi,\xi^{w_1}}^{(m)}(w_2 x) \frac{w_1^k t^k}{k!}. \end{aligned} \tag{2.10}$$

By (2.8), (2.9), and (2.10), we see that

$$
\begin{aligned}
J_{\chi,\xi}^{(m)}(w_1,w_2 \mid x) &= \left(\sum_{l=0}^{\infty} E_{l,\chi,\xi^{w_1}}^{(m)}(w_2x)\frac{w_1^l t^l}{l!}\right)\left(\sum_{k=0}^{\infty} T_{k,\chi,\xi^{w_2}}(w_1d-1)\frac{w_2^k t^k}{k!}\right)\\
&\quad\times\left(\sum_{i=0}^{\infty} E_{i,\chi,\xi^{w_2}}^{(m-1)}(w_1y)\frac{w_2^i t^i}{i!}\right)\\
&=\sum_{n=0}^{\infty}\left(\sum_{j=0}^{n}\binom{n}{j}w_2^j w_1^{n-j}E_{n-j,\chi,\xi^{w_1}}^{(m)}(w_2x)\right.\\
&\quad\left.\times\sum_{k=0}^{j}\binom{j}{k}T_{k,\chi,\xi^{w_2}}(w_1d-1)E_{j-k,\chi,\xi^{w_2}}^{(m-1)}(w_1y)\right)\frac{t^n}{n!}.
\end{aligned}
\tag{2.11}
$$

In the viewpoint of the symmetry of $J_{\chi,\xi}^{(m)}(w_1,w_2 \mid x)$ for w_1 and w_2, we have

$$
\begin{aligned}
J_{\chi,\xi}^{(m)}(w_1,w_2 \mid x) &=\sum_{n=0}^{\infty}\left(\sum_{j=0}^{n}\binom{n}{j}w_1^j w_2^{n-j}E_{n-j,\chi,\xi^{w_2}}^{(m)}(w_1x)\right.\\
&\quad\left.\times\sum_{k=0}^{j}\binom{j}{k}T_{k,\chi,\xi^{w_1}}(w_2d-1)E_{j-k,\chi,\xi^{w_1}}^{(m-1)}(w_2y)\right)\frac{t^n}{n!}.
\end{aligned}
\tag{2.12}
$$

Comparing the coefficients on both sides of (2.11) and (2.12), we obtain the following theorem.

Theorem 2.1. *Let $w_1,w_2,d \in \mathbb{N}$ with $w_1 \equiv 1, w_2 \equiv 1$, and $d \equiv 1 \pmod 2$. For $n \in \mathbb{N}\cup\{0\}$ and $m \in \mathbb{N}$, one has*

$$
\begin{aligned}
&\sum_{j=0}^{n}\binom{n}{j}w_2^j w_1^{n-j}E_{n-j,\chi,\xi^{w_1}}^{(m)}(w_2x)\sum_{k=0}^{j}\binom{j}{k}T_{k,\chi,\xi^{w_2}}(w_1d-1)E_{j-k,\chi,\xi^{w_2}}^{(m-1)}(w_1y)\\
&\quad=\sum_{j=0}^{n}\binom{n}{j}w_1^j w_2^{n-j}E_{n-j,\chi,\xi^{w_2}}^{(m)}(w_1x)\sum_{k=0}^{j}\binom{j}{k}T_{k,\chi,\xi^{w_1}}(w_2d-1)E_{j-k,\chi,\xi^{w_1}}^{(m-1)}(w_2y).
\end{aligned}
\tag{2.13}
$$

Let $m=1$ and $y=0$ in Theorem 2.1. Then we also have the following corollary.

Corollary 2.2. *For $w_1,w_2,d \in \mathbb{N}$ with $w_1 \equiv 1, w_2 \equiv 1$, and $d \equiv 1 \pmod 2$, one has*

$$
\begin{aligned}
&\sum_{m=0}^{n}\binom{n}{m}E_{m,\chi,\xi^{w_1}}(w_2x)w_1^m w_2^{n-m}T_{n-m,\chi,\xi^{w_2}}(w_1d-1)\\
&\quad=\sum_{m=0}^{n}\binom{n}{m}E_{m,\chi,\xi^{w_2}}(w_1x)w_1^{n-m}w_2^m T_{n-m,\chi,\xi^{w_1}}(w_2d-1)\qquad (\textit{see } [2]).
\end{aligned}
\tag{2.14}
$$

Let χ be the trivial character and $d=1$. Then we also have the following corollary.

Corollary 2.3. *Let $w_1, w_2 \in \mathbb{N}$ with $w_1 \equiv 1, w_2 \equiv 1 \pmod 2$. Then one has*

$$\begin{aligned}
&\sum_{j=0}^{n}\binom{n}{j}w_1^{n-j}w_2^{j}E_{n-j,\xi^{w_1}}(w_2x)T_{k,\xi^{w_2}}(w_1-1)\\
&\quad=\sum_{j=0}^{n}\binom{n}{j}w_1^{j}w_2^{n-j}E_{n-j,\xi^{w_2}}(w_1x)T_{k,\xi^{w_1}}(w_2-1),
\end{aligned}\tag{2.15}$$

where $E_{n,\xi}(x)$ are the nth twisted Euler polynomials.

If we take $w_2=1$ in Corollary 2.3, then we obtain the following corollary.

Corollary 2.4 (Distribution for the twisted Euler polynomials). *For $w_1 \in \mathbb{N}$ with $w_1 \equiv 1 \pmod 2$, one has*

$$E_{n,\xi}(x)=\sum_{i=0}^{n}\binom{n}{i}w_1^{i}E_{i,\xi^{w_1}}(x)T_{n-i,\xi}(w_1-1).\tag{2.16}$$

From (2.6), we can derive that

$$\begin{aligned}
&J_{\chi,\xi}^{(m)}(w_1,w_2\mid x)\\
&\quad=\Bigg(\sum_{l=0}^{w_1d-1}\chi(l)(-1)^{l}\,\xi^{w_2l}\int_{X^m}\left(\prod_{i=1}^{m}\chi(x_i)\right)\\
&\qquad\qquad\times\xi^{(\sum_{i=1}^{m}x_i)w_1}e^{w_1(\sum_{i=1}^{m}x_i+(w_2/w_1)l+w_2x)t}d\mu(x_1)\cdots d\mu(x_m)\Bigg)\\
&\qquad\times\left(\int_{X^{m-1}}\left(\prod_{i=1}^{m-1}\chi(x_i)\right)\xi^{(\sum_{i=1}^{m-1}x_i)w_2}e^{(\sum_{i=1}^{m-1}x_i)w_2t}d\mu(x_1)\cdots d\mu(x_{m-1})\right)\\
&\quad=\sum_{n=0}^{\infty}\Bigg(\sum_{k=0}^{n}\binom{n}{k}w_1^{k}w_2^{n-k}E_{n-k,\chi,\xi^{w_2}}^{(m-1)}(w_1y)\\
&\qquad\qquad\times\sum_{l=0}^{w_1d-1}\chi(l)(-1)^{l}\xi^{w_2l}E_{k,\chi,\xi^{w_1}}^{(m)}\left(w_2x+\frac{w_2}{w_1}l\right)\Bigg)\frac{t^n}{n!}.
\end{aligned}\tag{2.17}$$

By the symmetry property of $J_{\chi,\xi}^{(m)}(w_1,w_2\mid x)$ in w_1 and w_2, we also see that

$$\begin{aligned}
&J_{\chi,\xi}^{(m)}(w_1,w_2\mid x)\\
&\quad=\sum_{n=0}^{\infty}\left(\sum_{k=0}^{n}\binom{n}{k}w_2^{k}w_1^{n-k}E_{n-k,\chi,\xi^{w_1}}^{(m-1)}(w_2y)\sum_{l=0}^{w_2d-1}\chi(l)(-1)^{l}\xi^{w_1l}E_{k,\chi,\xi^{w_2}}^{(m)}\left(w_1x+\frac{w_1}{w_2}l\right)\right)\frac{t^n}{n!}.
\end{aligned}\tag{2.18}$$

Comparing the coefficients on both sides of (2.17) and (2.18), we obtain the following theorem which shows the relationship between the power sums and the twisted generalized Euler polynomials of higher order.

Theorem 2.5. *Let* $w_1, w_2, d \in \mathbb{N}$ *with* $w_1 \equiv 1, w_2 \equiv 1$*, and* $d \equiv 1 \pmod 2$*. For* $n \in \mathbb{N} \cup \{0\}$ *and* $m \in \mathbb{N}$*, one has*

$$\begin{aligned}&\sum_{k=0}^{n}\binom{n}{k}w_1^k w_2^{n-k}E_{n-k,\chi,\xi^{w_2}}^{(m-1)}(w_1 y)\sum_{l=0}^{w_1 d-1}\chi(l)(-1)^l\xi^{w_2 l}E_{k,\chi,\xi^{w_1}}^{(m)}\left(w_2 x+\frac{w_2}{w_1}l\right)\\&\quad=\sum_{k=0}^{n}\binom{n}{k}w_2^k w_1^{n-k}E_{n-k,\chi,\xi^{w_1}}^{(m-1)}(w_2 y)\sum_{l=0}^{w_2 d-1}\chi(l)(-1)^l\xi^{w_1 l}E_{k,\chi,\xi^{w_2}}^{(m)}\left(w_1 x+\frac{w_1}{w_2}l\right).\end{aligned}\tag{2.19}$$

If we take $x=0$, $y=0$, and $m=1$ in Theorem 2.5, then we have the following identity:

$$\begin{aligned}&\sum_{k=0}^{n}\binom{n}{k}E_{k,\chi,\xi^{w_1}}w_1^k w_2^{n-k}T_{n-k,\chi,\xi^{w_2}}(w_1 d-1)\\&\quad=\sum_{k=0}^{n}\binom{n}{k}E_{k,\chi,\xi^{w_2}}w_2^k w_1^{n-k}T_{n-k,\chi,\xi^{w_1}}(w_2 d-1).\end{aligned}\tag{2.20}$$

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