Hindawi Publishing Corporation
Journal of Inequalities and Applications
Volume 2011, Article ID 531540, 10 pages
doi:10.1155/2011/531540

Research Article

Some Properties of Certain Class of Integral Operators

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Received 17 October 2010; Accepted 10 January 2011

Academic Editor: Andrea Laforgia

The main object of this paper is to derive some inequality properties and convolution properties of certain class of integral operators defined on the space of meromorphic functions.

1. Introduction and Preliminaries

Let Σ denote the class of functions of the form

$$f(z) = \frac{1}{z} + \sum_{k=1}^{\infty} a_k z^k, \tag{1.1}$$

which are *analytic* in the *punctured* open unit disk

$$\mathbb{U}^* := \{z : z \in \mathbb{C},\ 0 < |z| < 1\} =: \mathbb{U} \setminus \{0\}. \tag{1.2}$$

Let $f,\ g \in \Sigma$, where f is given by (1.1) and g is defined by

$$g(z) = \frac{1}{z} + \sum_{k=1}^{\infty} b_k z^k. \tag{1.3}$$

Then the Hadamard product (or convolution) $f * g$ of the functions f and g is defined by

$$(f * g)(z) := \frac{1}{z} + \sum_{k=1}^{\infty} a_k b_k z^k =: (g * f)(z). \tag{1.4}$$

For two functions f and g, analytic in $\mathbb{U}$, we say that the function f is subordinate to g in $\mathbb{U}$ and write

$$f(z) \prec g(z), \tag{1.5}$$

if there exists a Schwarz function ω, which is analytic in $\mathbb{U}$ with

$$\omega(0) = 0, \quad |\omega(z)| < 1 \quad (z \in \mathbb{U}) \tag{1.6}$$

such that

$$f(z) = g(\omega(z)) \quad (z \in \mathbb{U}). \tag{1.7}$$

Indeed, it is known that

$$f(z) \prec g(z) \Longrightarrow f(0) = g(0), \qquad f(\mathbb{U}) \subset g(\mathbb{U}). \tag{1.8}$$

Furthermore, if the function g is univalent in $\mathbb{U}$, then we have the following equivalence:

$$f(z) \prec g(z) \Longleftrightarrow f(0) = g(0), \qquad f(\mathbb{U}) \subset g(\mathbb{U}). \tag{1.9}$$

Analogous to the integral operator defined by Jung et al. [1], Lashin [2] recently introduced and investigated the integral operator

$$Q_{\alpha,\beta} : \Sigma \longrightarrow \Sigma \tag{1.10}$$

defined, in terms of the familiar Gamma function, by

$$\begin{aligned} Q_{\alpha,\beta} f(z) &= \frac{\Gamma(\beta+\alpha)}{\Gamma(\beta)\Gamma(\alpha)} \frac{1}{z^{\beta+1}} \int_0^z t^{\beta} \left(1 - \frac{t}{z}\right)^{\alpha-1} f(t)dt \\ &= \frac{1}{z} + \frac{\Gamma(\beta+\alpha)}{\Gamma(\beta)} \sum_{k=1}^{\infty} \frac{\Gamma(k+\beta+1)}{\Gamma(k+\beta+\alpha+1)} a_k z^k \quad (\alpha > 0;\ \beta > 0;\ z \in \mathbb{U}^*). \end{aligned} \tag{1.11}$$

By setting

$$f_{\alpha,\beta}(z) := \frac{1}{z} + \frac{\Gamma(\beta)}{\Gamma(\beta+\alpha)} \sum_{k=1}^{\infty} \frac{\Gamma(k+\beta+\alpha+1)}{\Gamma(k+\beta+1)} z^k \quad (\alpha > 0;\ \beta > 0;\ z \in \mathbb{U}^*), \tag{1.12}$$

we define a new function $f^{\lambda}_{\alpha,\beta}(z)$ in terms of the Hadamard product (or convolution)

$$f_{\alpha,\beta}(z) * f^{\lambda}_{\alpha,\beta}(z) = \frac{1}{z(1-z)^{\lambda}} \quad (\alpha > 0;\ \beta > 0;\ \lambda > 0;\ z \in \mathbb{U}^*). \tag{1.13}$$

Then, motivated essentially by the operator $Q_{\alpha,\beta}$, Wang et al. [3] introduced the operator

$$Q^{\lambda}_{\alpha,\beta} : \Sigma \longrightarrow \Sigma, \tag{1.14}$$

which is defined as

$$\begin{aligned} Q^{\lambda}_{\alpha,\beta} f(z) :&= f^{\lambda}_{\alpha,\beta}(z) * f(z) \\ &= \frac{1}{z} + \frac{\Gamma(\beta+\alpha)}{\Gamma(\beta)} \sum_{k=1}^{\infty} \frac{(\lambda)_{k+1}}{(k+1)!} \frac{\Gamma(k+\beta+1)}{\Gamma(k+\beta+\alpha+1)} a_k z^k \quad (z \in \mathbb{U}^*;\ f \in \Sigma), \end{aligned} \tag{1.15}$$

where (and throughout this paper unless otherwise mentioned) the parameters α, β, and λ are constrained as follows:

$$\alpha > 0, \qquad \beta > 0, \qquad \lambda > 0 \tag{1.16}$$

and $(\lambda)_k$ is the Pochhammer symbol defined by

$$(\lambda)_k := \begin{cases} 1 & (k = 0), \\ \lambda(\lambda+1)\cdots(\lambda+k-1) & (k \in \mathbb{N} := \{1, 2, \cdots\}). \end{cases} \tag{1.17}$$

Clearly, we know that $Q^{1}_{\alpha,\beta} = Q_{\alpha,\beta}$.

It is readily verified from (1.15) that

$$z\left(Q^{\lambda}_{\alpha,\beta} f\right)'(z) = \lambda Q^{\lambda+1}_{\alpha,\beta} f(z) - (\lambda+1) Q^{\lambda}_{\alpha,\beta} f(z), \tag{1.18}$$

$$z\left(Q^{\lambda}_{\alpha+1,\beta} f\right)'(z) = (\beta+\alpha) Q^{\lambda}_{\alpha,\beta} f(z) - (\beta+\alpha+1) Q^{\lambda}_{\alpha+1,\beta} f(z). \tag{1.19}$$

Recently, Wang et al. [3] obtained several inclusion relationships and integral-preserving properties associated with some subclasses involving the operator $Q^{\lambda}_{\alpha,\beta}$, some subordination and superordination results involving the operator are also derived. Furthermore, Sun et al. [4] investigated several other subordination and superordination results for the operator $Q^{\lambda}_{\alpha,\beta}$.

In order to derive our mainresults, we need the following lemmas.

Lemma 1.1 (see [5]). *Let ϕ be analytic and convex univalent in $\mathbb{U}$ with $\phi(0) = 1$. Suppose also that p is analytic in $\mathbb{U}$ with $p(0) = 1$. If*

$$p(z) + \frac{zp'(z)}{c} \prec \phi(z) \quad (\Re(c) \geqq 0;\ c \neq 0), \tag{1.20}$$

then

$$p(z) \prec cz^{-c} \int_0^z t^{c-1}\phi(t)dt \prec \phi(z), \tag{1.21}$$

and $cz^{-c} \int_0^z t^{c-1}\phi(t)dt$ is the best dominant of (1.20).

Let $\mathbb{P}(\gamma)$ $(0 \leqq \gamma < 1)$ denote the class of functions of the form

$$\mathfrak{p}(z) = 1 + \mathfrak{p}_1 z + \mathfrak{p}_2 z^2 + \cdots, \tag{1.22}$$

which are analytic in $\mathbb{U}$ and satisfy the condition

$$\Re(\mathfrak{p}(z)) > \gamma \quad (z \in \mathbb{U}). \tag{1.23}$$

Lemma 1.2 (see [6]). *Let*

$$\psi_j(z) \in \mathbb{P}(\gamma_j) \quad (0 \leqq \gamma_j < 1;\ j = 1,2). \tag{1.24}$$

Then

$$(\psi_1 * \psi_2)(z) \in \mathbb{P}(\gamma_3) \quad (\gamma_3 = 1 - 2(1-\gamma_1)(1-\gamma_2)). \tag{1.25}$$

The result is the best possible.

Lemma 1.3 (see [7]). *Let*

$$\mathfrak{p}(z) = 1 + \mathfrak{p}_1 z + \mathfrak{p}_2 z^2 + \cdots \in \mathbb{P}(\gamma) \quad (0 \leqq \gamma < 1). \tag{1.26}$$

Then

$$\Re(\mathfrak{p}(z)) > 2\gamma - 1 + \frac{2(1-\gamma)}{1+|z|}. \tag{1.27}$$

In the present paper, we aim at proving some inequality properties and convolution properties of the integral operator $Q^{\lambda}_{\alpha,\beta}$.

2. Main Results

Our first main result is given by Theorem 2.1 below.

Theorem 2.1. *Let* $\mu < 1$ *and* $-1 \leqq B < A \leqq 1$. *If* $f \in \Sigma$ *satisfies the condition*

$$z\left[(1-\mu)Q_{\alpha,\beta}^{\lambda+1}f(z) + \mu Q_{\alpha,\beta}^{\lambda}f(z)\right] \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{2.1}$$

then

$$\Re\left(\left(zQ_{\alpha,\beta}^{\lambda}f(z)\right)^{1/n}\right) > \left(\frac{\lambda}{1-\mu}\int_0^1 u^{\lambda/(1-\mu)-1}\left(\frac{1-Au}{1-Bu}\right)du\right)^{1/n} \quad (n \geqq 1). \tag{2.2}$$

The result is sharp.

Proof. Suppose that

$$p(z) := zQ_{\alpha,\beta}^{\lambda}f(z) \quad \left(z \in \mathbb{U};\ f \in \Sigma\right). \tag{2.3}$$

Then p is analytic in $\mathbb{U}$ with $p(0) = 1$. Combining (1.18) and (2.3), we find that

$$zQ_{\alpha,\beta}^{\lambda+1}f(z) = p(z) + \frac{zp'(z)}{\lambda}. \tag{2.4}$$

From (2.1), (2.3), and (2.4), we get

$$p(z) + \frac{1-\mu}{\lambda}zp'(z) \prec \frac{1+Az}{1+Bz}. \tag{2.5}$$

By Lemma 1.1, we obtain

$$p(z) \prec \frac{\lambda}{1-\mu}z^{-\lambda/(1-\mu)}\int_0^z t^{\lambda/(1-\mu)-1}\left(\frac{1+At}{1+Bt}\right)dt, \tag{2.6}$$

or equivalently,

$$zQ_{\alpha,\beta}^{\lambda}f(z) = \frac{\lambda}{1-\mu}\int_0^1 u^{\lambda/(1-\mu)-1}\left(\frac{1+Au\omega(z)}{1+Bu\omega(z)}\right)du, \tag{2.7}$$

where ω is analytic in $\mathbb{U}$ with

$$\omega(0) = 0, \quad |\omega(z)| < 1 \quad (z \in \mathbb{U}). \tag{2.8}$$

Since $\mu < 1$ and $-1 \leqq B < A \leqq 1$, we deduce from (2.7) that

$$\Re\left(zQ^{\lambda}_{\alpha,\beta}f(z)\right) > \frac{\lambda}{1-\mu}\int_0^1 u^{\lambda/(1-\mu)-1}\left(\frac{1-Au}{1-Bu}\right)du. \tag{2.9}$$

By noting that

$$\Re\left(\varrho^{1/n}\right) \geqq (\Re(\varrho))^{1/n} \quad (\varrho \in \mathbb{C},\ \Re(\varrho) \geqq 0;\ n \geqq 1), \tag{2.10}$$

the assertion (2.2) of Theorem 2.1 follows immediately from (2.9) and (2.10).

To show the sharpness of (2.2), we consider the function $f \in \Sigma$ defined by

$$zQ^{\lambda}_{\alpha,\beta}f(z) = \frac{\lambda}{1-\mu}\int_0^1 u^{\lambda/(1-\mu)-1}\left(\frac{1+Auz}{1+Buz}\right)du. \tag{2.11}$$

For the function f defined by (2.11), we easily find that

$$z\left[(1-\mu)Q^{\lambda+1}_{\alpha,\beta}f(z) + \mu Q^{\lambda}_{\alpha,\beta}f(z)\right] = \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{2.12}$$

it follows from (2.12) that

$$zQ^{\lambda}_{\alpha,\beta}f(z) \longrightarrow \frac{\lambda}{1-\mu}\int_0^1 u^{\lambda/(1-\mu)-1}\left(\frac{1-Au}{1-Bu}\right)du \quad (z \longrightarrow -1). \tag{2.13}$$

This evidently completes the proof of Theorem 2.1. □

In view of (1.19), by similarly applying the method of proof of Theorem 2.1, we get the following result.

Corollary 2.2. *Let $\mu < 1$ and $-1 \leqq B < A \leqq 1$. If $f \in \Sigma$ satisfies the condition*

$$z\left[(1-\mu)Q^{\lambda}_{\alpha,\beta}f(z) + \mu Q^{\lambda}_{\alpha+1,\beta}f(z)\right] \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{2.14}$$

then

$$\Re\left(\left(zQ^{\lambda}_{\alpha+1,\beta}f(z)\right)^{1/n}\right) > \left(\frac{\beta+\alpha}{1-\mu}\int_0^1 u^{(\beta+\alpha)/(1-\mu)-1}\left(\frac{1-Au}{1-Bu}\right)du\right)^{1/n} \quad (n \geqq 1). \tag{2.15}$$

The result is sharp.

For the function $f \in \Sigma$ given by (1.1), we here recall the integral operator

$$\mathcal{Q}_{\upsilon} : \Sigma \longrightarrow \Sigma, \tag{2.16}$$

defined by

$$\mathcal{J}_{\upsilon} f(z) := \frac{\upsilon - 1}{z^{\upsilon}} \int_0^z t^{\upsilon-1} f(t)dt \quad (\upsilon > 1). \tag{2.17}$$

Theorem 2.3. *Let $\mu < 1$, $\upsilon > 1$ and $-1 \leqq B < A \leqq 1$. Suppose also that $\mathcal{J}_{\upsilon}$ is given by* (2.17)*. If $f \in \Sigma$ satisfies the condition*

$$z\left[(1-\mu)Q^{\lambda}_{\alpha,\beta} f(z) + \mu Q^{\lambda}_{\alpha,\beta}\mathcal{J}_{\upsilon} f(z)\right] \prec \frac{1+Az}{1+Bz} \quad (z \in \mathbb{U}), \tag{2.18}$$

then

$$\Re\left(\left(zQ^{\lambda}_{\alpha,\beta}\mathcal{J}_{\upsilon} f(z)\right)^{1/n}\right) > \left(\frac{\upsilon-1}{1-\mu}\int_0^1 u^{(\upsilon-1)/(1-\mu)-1}\left(\frac{1-Au}{1-Bu}\right)du\right)^{1/n} \quad (n \geqq 1). \tag{2.19}$$

The result is sharp.

Proof. We easily find from (2.17) that

$$(\upsilon - 1)Q^{\lambda}_{\alpha,\beta} f(z) = \upsilon Q^{\lambda}_{\alpha,\beta}\mathcal{J}_{\upsilon} f(z) + z\left(Q^{\lambda}_{\alpha,\beta}\mathcal{J}_{\upsilon} f\right)'(z). \tag{2.20}$$

Suppose that

$$q(z) := zQ^{\lambda}_{\alpha,\beta}\mathcal{J}_{\upsilon} f(z) \quad (z \in \mathbb{U};\ f \in \Sigma). \tag{2.21}$$

It follows from (2.18), (2.20) and (2.21) that

$$z\left[(1-\mu)Q^{\lambda}_{\alpha,\beta} f(z) + \mu Q^{\lambda}_{\alpha,\beta}\mathcal{J}_{\upsilon} f(z)\right] = q(z) + \frac{1-\mu}{\upsilon-1}zq'(z) \prec \frac{1+Az}{1+Bz}. \tag{2.22}$$

The remainder of the proof of Theorem 2.3 is much akin to that of Theorem 2.1, we therefore choose to omit the analogous details involved. □

Theorem 2.4. *Let $\mu < 1$ and $-1 \leqq B_j < A_j \leqq 1$ $(j = 1, 2)$. If $f \in \Sigma$ is defined by*

$$Q^{\lambda}_{\alpha,\beta} f(z) = Q^{\lambda}_{\alpha,\beta}(f_1 * f_2)(z), \tag{2.23}$$

and each of the functions $f_j \in \Sigma$ $(j = 1, 2)$ satisfies the condition

$$z\left[(1-\mu)Q^{\lambda+1}_{\alpha,\beta} f_j(z) + \mu Q^{\lambda}_{\alpha,\beta} f_j(z)\right] \prec \frac{1+A_j z}{1+B_j z} \quad (z \in \mathbb{U}), \tag{2.24}$$

then

$$\Re\left(z\left[(1-\mu)Q_{\alpha,\beta}^{\lambda+1}f(z)+\mu Q_{\alpha,\beta}^{\lambda}f(z)\right]\right)>1-\frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1-\frac{\lambda}{1-\mu}\int_0^1\frac{u^{\lambda/(1-\mu)-1}}{1+u}du\right). \tag{2.25}$$

The result is sharp when $B_1 = B_2 = -1$.

Proof. Suppose that $f_j \in \Sigma$ $(j = 1, 2)$ satisfy conditions (2.24). By setting

$$\psi_j(z) := z\left[(1-\mu)Q_{\alpha,\beta}^{\lambda+1}f_j(z)+\mu Q_{\alpha,\beta}^{\lambda}f_j(z)\right] \quad (z\in\mathbb{U};\ j=1,2), \tag{2.26}$$

it follows from (2.24) and (2.26) that

$$\psi_j \in \mathbb{P}(\gamma_j) \quad \left(\gamma_j=\frac{1-A_j}{1-B_j};\ j=1,2\right). \tag{2.27}$$

Combining (1.18) and (2.26), we get

$$Q_{\alpha,\beta}^{\lambda}f_j(z)=\frac{\lambda}{1-\mu}z^{-\lambda/(1-\mu)}\int_0^z t^{\lambda/(1-\mu)-1}\psi_j(t)dt \quad (j=1,2). \tag{2.28}$$

For the function $f \in \Sigma$ given by (2.23), we find from (2.28) that

$$\begin{aligned} Q_{\alpha,\beta}^{\lambda}f(z) &= Q_{\alpha,\beta}^{\lambda}(f_1 * f_2)(z)\\ &= \left(\frac{\lambda}{1-\mu}z^{-\lambda/(1-\mu)}\int_0^z t^{\lambda/(1-\mu)-1}\psi_1(t)dt\right)*\left(\frac{\lambda}{1-\mu}z^{-\lambda/(1-\mu)}\int_0^z t^{\lambda/(1-\mu)-1}\psi_2(t)dt\right)\\ &= \frac{\lambda}{1-\mu}z^{-\lambda/(1-\mu)}\int_0^z t^{\lambda/(1-\mu)-1}\psi(t)dt, \end{aligned} \tag{2.29}$$

where

$$\psi(z)=\frac{\lambda}{1-\mu}z^{-\lambda/(1-\mu)}\int_0^z t^{\lambda/(1-\mu)-1}(\psi_1*\psi_2)(t)dt. \tag{2.30}$$

By noting that $\psi_1 \in \mathbb{P}(\gamma_1)$ and $\psi_2 \in \mathbb{P}(\gamma_2)$, it follows from Lemma 1.2 that

$$(\psi_1*\psi_2)(z)\in\mathbb{P}(\gamma_3) \quad (\gamma_3=1-2(1-\gamma_1)(1-\gamma_2)). \tag{2.31}$$

Furthermore, by Lemma 1.3, we know that

$$\Re\big((\varphi_1 * \varphi_2)(z)\big) > 2\gamma_3 - 1 + \frac{2(1-\gamma_3)}{1+|z|}. \tag{2.32}$$

In view of (2.24), (2.30), and (2.32), we deduce that

$$\begin{aligned}
&\Re\Big(z\Big[(1-\mu)Q_{\alpha,\beta}^{\lambda+1}f(z) + \mu Q_{\alpha,\beta}^{\lambda}f(z)\Big]\Big) \\
&\quad = \Re(\varphi(z)) = \frac{\lambda}{1-\mu}\int_0^1 u^{\lambda/(1-\mu)-1}\Re\big((\varphi_1 * \varphi_2)(uz)\big)du \\
&\quad \geqq \frac{\lambda}{1-\mu}\int_0^1 u^{\lambda/(1-\mu)-1}\left(2\gamma_3 - 1 + \frac{2(1-\gamma_3)}{1+u|z|}\right)du \\
&\quad = 1 - \frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1 - \frac{\lambda}{1-\mu}\int_0^1 \frac{u^{\lambda/(1-\mu)-1}}{1+u}du\right).
\end{aligned} \tag{2.33}$$

When $B_1 = B_2 = -1$, we consider the functions $f_j \in \Sigma$ $(j = 1,2)$ which satisfy conditions (2.24) and are given by

$$Q_{\alpha,\beta}^{\lambda}f_j(z) = \frac{\lambda}{1-\mu}z^{-\lambda/(1-\mu)}\int_0^z t^{\lambda/(1-\mu)-1}\left(\frac{1+A_jt}{1-t}\right)dt \quad (j = 1,2). \tag{2.34}$$

It follows from (2.26), (2.28), (2.30), and (2.34) that

$$\varphi(z) = \frac{\lambda}{1-\mu}\int_0^1 u^{\lambda/(1-\mu)-1}\left[1 - (1+A_1)(1+A_2) + \frac{(1+A_1)(1+A_2)}{1-uz}\right]du. \tag{2.35}$$

Thus, we have

$$\varphi(z) \longrightarrow 1 - (1+A_1)(1+A_2)\left(1 - \frac{\lambda}{1-\mu}\int_0^1 \frac{u^{\lambda/(1-\mu)-1}}{1+u}du\right) \quad (z \longrightarrow -1). \tag{2.36}$$

The proof of Theorem 2.4 is evidently completed. □

With the aid of (1.19), by applying the similar method of the proof of Theorem 2.4, we obtain the following result.

Corollary 2.5. *Let $\mu < 1$ and $-1 \leqq B_j < A_j \leqq 1$ $(j = 1,2)$. If $f \in \Sigma$ is defined by (2.23) and each of the functions $f_j \in \Sigma$ $(j = 1,2)$ satisfies the condition*

$$z\Big[(1-\mu)Q_{\alpha,\beta}^{\lambda}f_j(z) + \mu Q_{\alpha+1,\beta}^{\lambda}f_j(z)\Big] \prec \frac{1+A_jz}{1+B_jz} \quad (z \in \mathbb{U}), \tag{2.37}$$

then

$$\Re\left(z\left[(1-\mu)Q^{\lambda}_{\alpha,\beta}f(z)+\mu Q^{\lambda}_{\alpha+1,\beta}f(z)\right]\right)>1-\frac{4(A_1-B_1)(A_2-B_2)}{(1-B_1)(1-B_2)}\left(1-\frac{\beta+\alpha}{1-\mu}\int_0^1\frac{u^{(\beta+\alpha)/(1-\mu)-1}}{1+u}du\right). \tag{2.38}$$

The result is sharp when $B_1 = B_2 = -1$.

Acknowledgments

This work was supported by the *National Natural Science Foundation under Grant* 11026205, the *Science Research Fund of Guangdong Provincial Education Department* under Grant LYM08101, the *Natural Science Foundation of Guangdong Province* under Grant 10452800001004255, and the *Excellent Youth Foundation of Educational Committee of Hunan Province* under Grant 10B002 of the People's Republic of China.

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