Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2007, Article ID 14504, 12 pages
doi:10.1155/2007/14504

Research Article

Delay Analysis of an $M/G/1/K$ Priority Queueing System with Push-out Scheme

Yutae Lee, Bong Dae Choi, Bara Kim, and Dan Keun Sung

Received 16 March 2007; Accepted 18 October 2007

Recommended by Giuseppe Rega

This paper considers an $M/G/1/K$ queueing system with push-out scheme which is one of the loss priority controls at a multiplexer in communication networks. The loss probability for the model with push-out scheme has been analyzed, but the waiting times are not available for the model. Using a set of recursive equations, this paper derives the Laplace-Stieltjes transforms (LSTs) of the waiting time and the push-out time of low-priority messages. These results are then utilized to derive the loss probability of each traffic type and the mean waiting time of high-priority messages. Finally, some numerical examples are provided.

1. Introduction

Recently, many protocols and architectures to support a wide variety of communication services with different quality of service(QoS) requirements have been proposed and implemented so far. In this communication environment, various types of traffic sources are statistically multiplexed to utilize the network resources efficiently. Due to a consequence of statistical multiplexing, the network sometimes encounters random and unpredictable overflows. The congestion control is necessary to enhance the efficiency of system resource and to satisfy different QoS requirements of various types of traffic. The QoS is mainly measured by two parameters: delay and loss probability [1, 2]. Therefore, the priority disciplines in telecommunication networks can be categorized into two major types: delay priority discipline [3] and loss priority discipline. As for loss priority disciplines, the partial buffer-sharing scheme (or buffer-reservation scheme) [4–6] and the push-out scheme [7, 6, 8–15] have attracted considerable attention in the literature. In the partial buffer-sharing scheme, a threshold is fixed and low-priority messages are accepted in a buffer only when the current buffer occupancy is not more than the threshold value. Consequently, the buffer is partitioned in two parts: the first one for all incoming messages and the second one for only high-priority messages. In the push-out scheme, low-priority messages in the buffer are replaced by newly arrived high priority messages when the buffer is full [9]. Therefore, no high-priority messages are discarded until the buffer is filled only with high-priority messages. Heyman [13] investigated an $M/M/c/c$ queue with a push-out scheme and derived the expected number of messages of each class in service and the loss probabilities for both classes. Hebuterne and Gravey [12] considered an $M/D/1/K$ queue with a push-out scheme and derived the loss probabilities for both classes. Kröner et al. [6] derived the loss probabilities for a partial buffer-sharing scheme and for a push-out scheme in the case of $M/G/1/K$ queueing system. Saito [14] derived the loss probabilities for both classes in the case of $MMPP/G/1/K$ queueing system with a push-out scheme. Chang and Tan [9] derived the loss probabilities for both classes when bursty traffic is generated by a multiple number of three-state discrete-time Markov sources. Gui and Fan [11] derived the loss probabilities for a push-out scheme by a discrete time-queueing model.

In the above literatures, only the loss probabilities for a push-out scheme with two classes of messages are relatively well studied. Kasahara et al. [7] considered an $M/G/1/K$ system with only one class of messages, where any new message finding the system full pushes out a message in the buffer. The buffering policy in [7] is different from the push-out priority discipline considered in this paper, where we consider two classes of messages based on the loss-sensitivity and only new loss-sensitive messages, finding the system full can push out a loss-insensitive message in the buffer. To the best of our knowledge on the push-out scheme with two classes of messages, the distributions of the waiting time and the push-out time, which is defined as the time period for a low-priority message to stay from its arrival until being pushed out, have not been studied. When the push-out priority scheme is implemented, the waiting time of each class can be important for designing the buffer.

This paper considers an $M/G/1/K$ priority queue with two classes of messages, where the system is controlled by a push-out scheme. We find the LST's of the waiting time and the push-out time of a low-priority message by using simple recursive equations. These results are then utilized to derive the loss probabilities for both classes. Then, from the above results, the mean waiting time of high-priority messages can also be obtained.

The remaining part of the paper is organized in the following manner. In Section 2, a detailed description of the queueing model with a push-out scheme is given. Section 3 presents the computational methods for the waiting time, the push-out time, and the loss probabilities. The results on the waiting time and the push-out time are new. Even mean waiting times have not been studied yet. Some numerical examples are presented in Section 4.

2. Queueing model

In this section, a queueing model with a push-out scheme is described. We consider an $M/G/1/K$ queueing system with a push-out scheme where two classes of messages are served. It is assumed that each arriving message is given a priority value based on the loss sensitivity. The priority classes are labeled 1 and 2 with class 1 favored over class 2. Due to a large number of sources, it may be assumed that the arrival process for class i, $i = 1,2$, is a Poisson process with rate λ_i. Messages from both classes are multiplexed in the same buffer and identically processed. We denote by $B(x)$ the probability distribution function of the service time with mean b. The arrival processes are independent of each other and of the service times.

We apply the following push-out buffering policy. All messages are stored in the buffer as long as there is a space available at the buffer. When the buffer is full, an arriving class-2 message is discarded, while an arriving class-1 message is allowed to join the queue by pushing out a waiting class-2 message if it finds at least one class-2 message in the buffer. The class-2 message, being pushed out is, lost. The class-1 message will be lost only when the buffer is full and there are no class-2 messages in the buffer upon its arrival. It is assumed that, when a class-2 message has to be discarded, the last one which joined the buffer is pushed out.

3. Performance analysis

3.1. Waiting time of a served class-2 message. First, we will find the waiting-time distribution of a served class-2 message. The position of a tagged class-2 message and the number of class-1 messages behind the tagged message will be observed immediately before every completion of service. The performance measures of the tagged class-2 message are not influenced by any class-2 message arriving after the arrival of the tagged message. Formulas on the waiting time will appear in a recursive form in terms of the position of the tagged class-2 message and the number of class-1 messages behind the tagged message.

Let us number the positions in the buffer by $1,2,\ldots,K$ in order of near position to the server. Let us observe the movement of a tagged class-2 message. If the arriving tagged message finds the system empty, then the message goes to the server and its waiting time in the buffer is zero. If it finds the system nonempty and occupies the position i, this tagged message in the position i moves to the position $i-1$ after a service completion. An arriving class-1 message joins the queue when the buffer is not full. When the buffer is full and there are no class-2 messages behind the tagged message, an arriving class-1 message joins the buffer in the position K and the tagged message is pushed out. Thus, the movement of the tagged message depends on both its own position and the number of class-1 messages behind the tagged message.

The waiting time of a served class-2 message consists of the remaining service time of the message in service and the service times of messages in the buffer upon its arrival. First, we will compute the second component of the waiting time. We observe the position of the tagged message immediately before every completion of a service as the server proceeds serving the messages. Let us denote by (j,k), $1 \le j \le K$, $0 \le k \le K-j$ the state that the position of the tagged message is j and the number of class-1 messages behind the tagged message is k immediately before the completion of a service.

Let

$$W_L(t \mid j,k) \equiv P\left\{\begin{array}{l}\text{a tagged class-2 message in state } (j,k) \\ \text{will finally reach the server and} \\ \text{the time until it reaches the server} \\ \text{will not exceed } t\end{array}\right\}, \tag{3.1}$$

$$W_L^*(s \mid j,k) \equiv \int_{0^-}^{\infty} e^{-st} dW_L(t \mid j,k), \tag{3.2}$$

where $1 \le j \le K$ and $0 \le k \le K - j$. Given that the tagged message has reached state (j,k), $1 \le j \le K$, $0 \le k \le K - j$, immediately before the completion of a service, the number of class-1 arrivals during the next service time must be less than or equal to $K - j - k + 1$ in order for the tagged message not to be pushed out during the next service time. In the case that there are l class-1 arrivals, $(l \le K - j - k + 1)$ during the next service time, the tagged message will move to state $(j - 1, k + l)$ immediately before the completion of the service and the time to reach the server from the position j will be the sum of the service time and the amount of time until it reaches the server from state $(j - 1, k + l)$ without being pushed out. Thus, we have the following recursions for $W_L^*(s \mid j,k)$, $1 \le j \le K$, $0 \le k \le K - j$:

$$W_L^*(s \mid 1,k) = 1, \quad 0 \le k < K, \tag{3.3}$$

$$W_L^*(s \mid j,k) = \sum_{l=0}^{K-j-k+1} a_l^*(s) W_L^*(s \mid j-1, k+l), \quad 1 < j \le K, 0 \le k \le K - j, \tag{3.4}$$

where

$$a_l^*(s) \equiv \int_0^{\infty} e^{-(s+\lambda_1)t} \frac{(\lambda_1 t)^l}{l!} dB(t), \quad l = 0,1,2,\ldots, \tag{3.5}$$

represents the joint distribution of a service time and the number of class-1 arrivals during the service time. Note that we can compute $W_L^*(s \mid j,k)$ for all j and k from the recursive formula (3.4) with the initial condition (3.3).

In order to find the LST $W_L^*(s)$ of the waiting time of a served class-2 message, we need the joint probability distribution of the total number N of messages in the system and the remaining service time X_+ of a message in service at an arbitrary time. Let us denote the joint probability distribution of N and X_+ by

$$\Pi_j(t) \equiv P\{N = j, X_+ \le t\}, \quad 1 \le j \le K + 1, \tag{3.6}$$

and $\Pi_0 = P\{N = 0\}$. The LST of $\Pi_j(t)$, $1 \le j \le K + 1$, is denoted by $\Pi_j^*(s)$. From the conservation law for aggregated steady-state probabilities, the joint probability distribution of N and X_+ at an arbitrary time is identical to that for the ordinary $M/G/1/K$ queueing system with arrival rate $\lambda = \lambda_1 + \lambda_2$ and no priority scheme. Thus, $\Pi_j^*(s)$, $1 \le j \le K + 1$, is obtained from the $M/G/1/K$ queueing system with no priority scheme [16]. Hence, the

LST $W_L^*(s)$ of the waiting time of a served class-2 message is given by

$$W_L^*(s) = \frac{1}{P\{\text{served}\}}\left[\Pi_0 + \sum_{j=1}^{K}\sum_{k=0}^{K-j}\Pi_{j:k}^*(s)W_L^*(s \mid j,k)\right], \tag{3.7}$$

where

$$\begin{aligned}\Pi_{j:k}^*(s) &= \int_0^\infty e^{-(s+\lambda_1)t}\frac{(\lambda_1 t)^k}{k!}d\Pi_j(t), \quad 1 \le j \le K,\, 0 \le k \le K-j,\\ P\{\text{served}\} &= \Pi_0 + \sum_{j=1}^{K}\sum_{k=0}^{K-j}\Pi_{j:k}^*(0)W_L^*(0 \mid j,k).\end{aligned} \tag{3.8}$$

3.2. Push-out time of a class-2 message. The push-out time defined as the time period for a class-2 message to stay from its arrival until being pushed out is used to find the mean waiting time of a served class-1 message. By the similar way, as for the waiting time of a served class-2 message, we now obtain the LST $W_P^*(s)$ of the push-out time of a class-2 message which is pushed out eventually.

We first define

$$\begin{aligned}W_P(t \mid j,k) &\equiv P\left\{\begin{array}{l}\text{a tagged class-2 message in state } (j,k)\\ \text{will be pushed out eventually and}\\ \text{the time until it will be pushed out}\\ \text{will not exceed } t\end{array}\right\},\\ W_P^*(s \mid j,k) &\equiv \int_{0^-}^\infty e^{-st}dW_P(t \mid j,k),\end{aligned} \tag{3.9}$$

where $1 \le j \le K$ and $0 \le k \le K-j$. Given that the tagged message has reached state (j,k), $1 \le j \le K$ and $0 \le k \le K-j$, immediately before the completion of a service, case (3.1), it will be pushed out during the next service time if there are l class-1 arrivals during the service time, $l > K-j-k+1$; case (3.2), it will move forward to state $(j-1, k+l)$ immediately before the completion of the next service if $l \le K-j-k+1$ and it will be pushed out later eventually. In the case (3.1), the time to be pushed out will be the time from the beginning of the service to the arrival epoch of the $(K-j-k+2)$th class-1 message during the service time. In the case (3.2), the time to be pushed out will be the sum of the service time and the amount of time until the tagged message is pushed out from state $(j-1, k+l)$.

Thus, we obtain the following recursions for $W_P^*(s \mid j,k)$:

$$\begin{aligned}W_P^*(s \mid 1,k) &= 0, \quad 0 \le k < K,\\ W_P^*(s \mid j,k) &= O^*(s \mid j,k) + \sum_{l=0}^{K-j-k+1} a_l^*(s)W_P^*(s \mid j-1,k+l),\\ &\qquad 1 < j \le K,\, 0 \le k \le K-j,\end{aligned} \tag{3.10}$$

where

$$O^*(s \mid j,k) = \int_0^\infty \sum_{l=K-j-k+2}^{\infty} Y^*_{K-j-k+2}(s \mid l,t) \frac{(\lambda_1 t)^l e^{-\lambda_1 t}}{l!} dB(t), \tag{3.11}$$

$$\begin{aligned} Y^*_k(s \mid l,t) &= \int_0^t e^{-sy} \frac{l!}{(l-k)!(k-1)!} \frac{1}{t}\left(\frac{y}{t}\right)^{k-1}\left(1-\frac{y}{t}\right)^{l-k} dy \\ &= \frac{l!}{\left(k-1\right)!t^k} \sum_{m=0}^{l-k} \frac{(-1)^m}{(l-k-m)!m!t^m} \int_0^t e^{-sy} y^{k+m-1} dy \end{aligned} \tag{3.12}$$

represents the LST of the kth-order statistic among l-order statistics from a uniform distribution over $[0,t]$.

If we define

$$O^*(s \mid j) = \int_0^\infty \sum_{l=K-j+1}^{\infty} Y^*_{K-j+1}(s \mid l,t) \frac{(\lambda_1 t)^l e^{-\lambda_1 t}}{l!} d\Pi_j(t), \tag{3.13}$$

the LST $W^*_P(s)$ of the push-out time of a class-2 message which is pushed out is given by

$$W^*_P(s) = \frac{1}{P\{\text{pushed-out}\}} \sum_{j=1}^{K} \left[O^*(s \mid j) + \sum_{k=0}^{K-j+1} \Pi^*_{j:k}(s) W^*_P(s \mid j,k) \right], \tag{3.14}$$

where the probability of a class-2 message being pushed out is given by

$$P\{\text{pushed-out}\} = 1 - P\{\text{served}\} - \Pi^*_{K+1}(0). \tag{3.15}$$

3.3. Loss probabilities. The loss probability P_L of a class-2 message is

$$P_L = 1 - P\{\text{served}\}. \tag{3.16}$$

Let P_T denote the total loss probability. Since, for every arriving message which finds the buffer completely occupied, either the arriving message itself or the replaced message is lost, P_T is the same as the loss probability of the ordinary $M/G/1/K$ system with arrival rate $\lambda_1 + \lambda_2$. Hence,

$$P_T = \Pi^*_{K+1}(0). \tag{3.17}$$

Since our system is work conservative, it leads to

$$\lambda_1 P_H + \lambda_2 P_L = (\lambda_1 + \lambda_2) P_T, \tag{3.18}$$

where P_H is the loss probability for a class-1 message. Hence,

$$P_H = \frac{\lambda_1 + \lambda_2}{\lambda_1} P_T - \frac{\lambda_2}{\lambda_1} P_L. \tag{3.19}$$

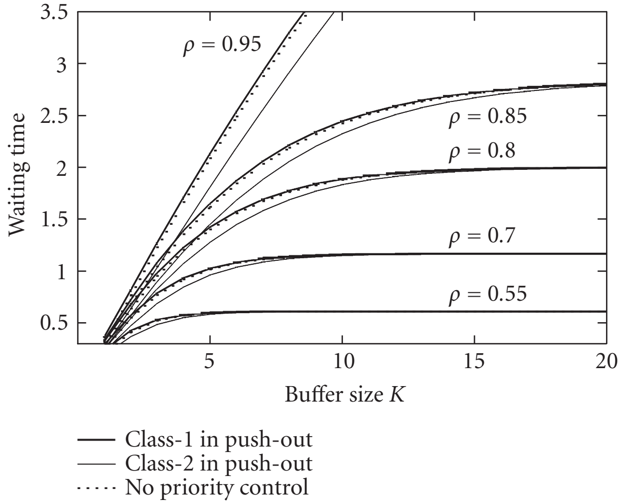

FIGURE 4.1. Mean waiting time versus buffer size K: $\rho_2 = \rho/5$.

3.4. Mean waiting time of a served class-1 message. The mean waiting time $E[W_H]$ for a served class-1 message can be calculated by applying Little's theorem [17, 18] to these messages present in the buffer. The mean number N_B of messages in the buffer is given by

$$N_B = \sum_{j=1}^{K} j\Pi_{j+1}^{*}(0). \tag{3.20}$$

On the other hand, by Little's theorem,

$$N_B = \lambda_1(1-P_H)E[W_H] - \lambda_2\Big[P\{\text{served}\}W_L^{*'}(0) + P\{\text{pushed-out}\}W_P^{*'}(0)\Big]. \tag{3.21}$$

Hence,

$$E[W_H] = \frac{1}{\lambda_1(1-P_H)}\Big[N_B + \lambda_2\Big(P\{\text{served}\}W_L^{*'}(0) + P\{\text{pushed-out}\}W_P^{*'}(0)\Big)\Big]. \tag{3.22}$$

4. Numerical examples

In this section, we give the numerical examples for the mean and the coefficient of variation of the waiting time and the loss probabilities for both classes. In all numerical examples discussed in this section, we assume that the service time of each message is the fixed value 1.

Figure 4.1 displays the mean waiting times as functions of the buffer size when the total traffic load is fixed and the traffic load ρ_2 of class-2 messages is given by $\rho_2 = \rho/5$. For comparison purposes, we show the results for the system with five different values ρ, $\rho = 0.55, 0.70, 0.80, 0.85, 0.95$, and also show the results for the system with no priority control. As the total traffic load increases, the mean waiting times also increase. The mean

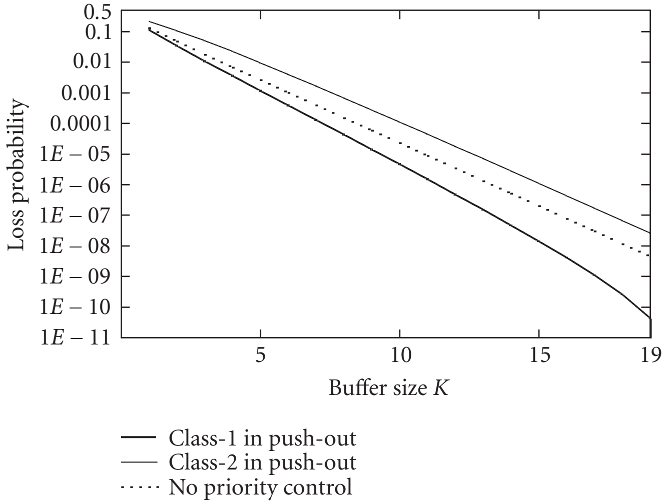

FIGURE 4.2. Loss probabilities versus buffer size K: $\rho_1 = 0.5$, $\rho_2 = 0.1$.

waiting time for class 2 in the push-out scheme is smaller than that in no priority control, and the mean waiting time for class 1 in the push-out scheme is slightly larger than that in no priority control. Under the push-out scheme, since the number of messages being pushed out decreases as the buffer size increases, the larger the buffer size, the smaller the difference between the mean waiting times for class-1 and class-2 messages.

In Figure 4.2, we see a typical influence of the push-out scheme on loss probability. We assume the traffic load of class 1 and class 2 are $\rho_1 = 0.5$ and $\rho_2 = 0.1$, respectively. Loss probabilities for both classes are displayed versus buffer size. The buffer size of the multiplexer has much influence on the loss probabilities. When the buffer size is constant, P_H is much smaller than P_T and P_T is smaller than P_L because of the use of the push-out scheme. With the increasing of the buffer size, P_H, P_L and P_T decrease sharply and it shows that the buffer size is a key factor of the push-out scheme.

Figure 4.3 displays the mean waiting times as functions of the total traffic load when the traffic load ρ_2 of class 2 is given by $\rho_2 = \rho/5$ and the buffer size K is 7. The mean waiting times increase apparently as the total arrival rate increases, as we expected.

Figure 4.4 (resp., Figure 4.5) clearly illustrates the tradeoffs between the mean waiting time of class-1 (resp., class-2) messages and the loss probability of class-2 (resp., class-1) messages under the push-out scheme. Both the total traffic load and the buffer size are varied while the ratio between the traffic loads of both classes is fixed. Results are shown for the cases that the traffic loads ρ_1 of class 1 traffic are equal to 0.2, 0.4, and 0.6, and the traffic load ρ_2 of class 2 is given by $\rho_2 = \rho_1/4$. Moving from left top to right bottom along a curve of constant arrival rate, point values are shown for buffer sizes of $K = 1, 2, \ldots, 14$. An important result shown in Figures 4.4 and 4.5 is that a relatively large decrease in the loss probability is obtained at the expense of a relatively small increase in the mean waiting time as the buffer size increases.

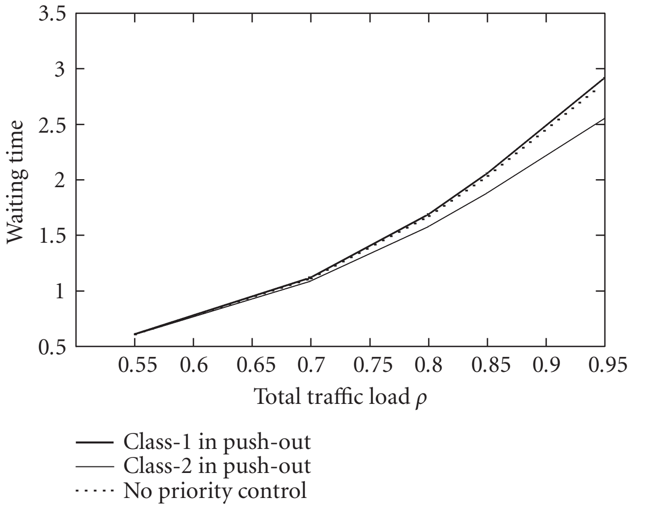

FIGURE 4.3. Mean waiting time versus total traffic load ρ: $\rho_2 = \rho/5$, $K = 7$.

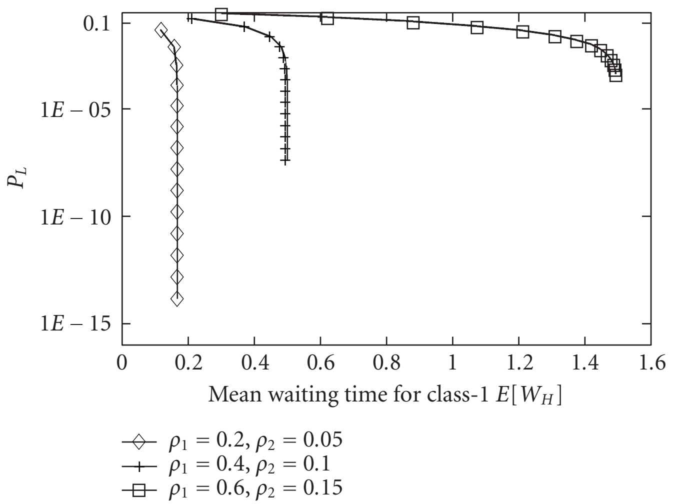

FIGURE 4.4. Push out: P_L versus $E[W_H]$ for various traffic loads.

Figure 4.6 shows a similar trend for the case with no priority control in the same setting as in Figures 4.4 and 4.5. We compare the performance of push-out scheme with that of the system with no priority control. In Figures 4.4, 4.5, and 4.6, we see a typical influence of the push-out scheme on the loss probability. The push-out scheme dramatically improves the loss of class 1 at the expense of a considerable increase in the loss of class 2. Furthermore, the push-out scheme decreases the mean waiting time of class-2 messages

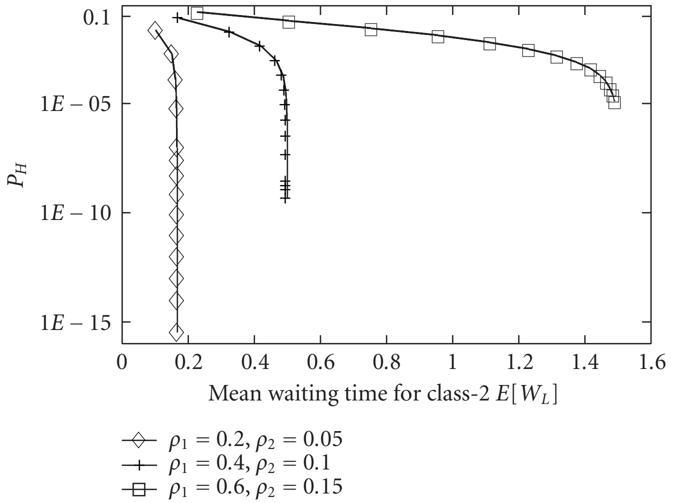

FIGURE 4.5. Push out: P_H versus $E[W_L]$ for various traffic loads.

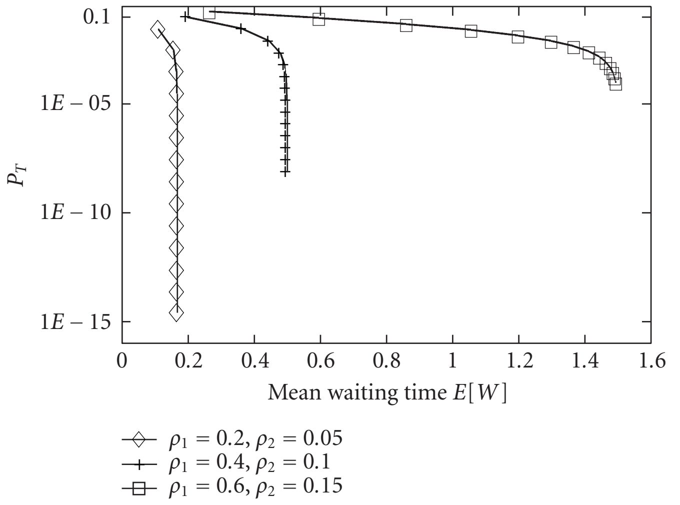

FIGURE 4.6. No priority control: P_T versus $E[W]$ for various traffic loads.

at the expense of a little increase in the mean waiting time of class-1 messages. The difference between the mean waiting time (resp., loss probability) of class-2 (resp., class-1) messages in the push-out scheme and that in the system with no priority control gets larger as the total arrival rate becomes larger. Hence, for a given ratio between the arrival rates of both classes, the larger the total arrival rate, the more advantageous the push-out scheme, as we expected.

Figure 4.7 illustrates the coefficient of variation of the waiting time of the served class-2 message with and without push-out scheme. We note that the variation under push-out scheme is not larger than that under the system with no priority control for most K's.

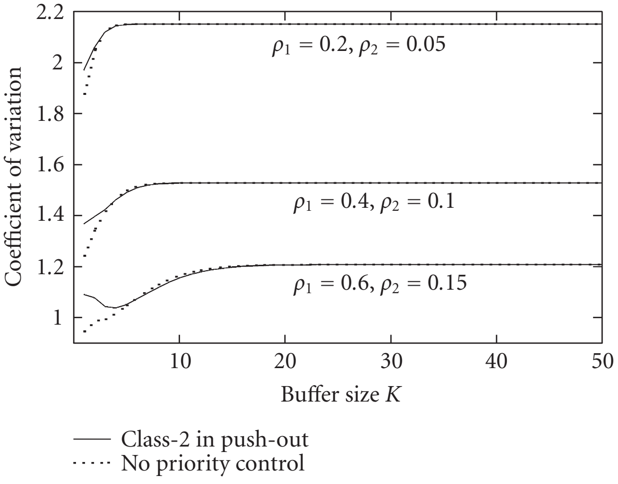

FIGURE 4.7. Coefficient of variation for waiting time versus K for various traffic loads.

5. Conclusions

This paper has analyzed an $M/G/1/K$ queueing system with push-out scheme. We have found the LST of the waiting time of a served class-2 message and derived the loss probabilities for both classes. Then, the mean waiting time of class-1 messages has also been obtained. On the basis of the analytical model, some numerical examples have been discussed and the effectiveness of the push-out scheme has been proved.

Acknowledgments

This work was partially supported by the LG Yonam Foundation, and the MIC (Ministry of Information and Communication), Korea, under the ITRC (Information Technology Research Center) support program supervised by the IITA (Institute of Information Technology Assessment).

References

[1] H. Armbruster and G. Arndt, "broadband communication and its realization with broadband ISDN," *IEEE Communications Magazine*, vol. 25, no. 11, pp. 8–19, 1987.

[2] H. Takagi, "Explicit delay distribution in first-come first-served M/M/m/K and M/M/m/K/n queue and a mixed loss-delay system," in *Proceedings of Asia-Pacific Symposium on Queueing Theory and Its Applications to Telecommunication Networks*, Seoul, Korea, 2006.

[3] Y. Kim and J. Kim, "Performance analysis of an ATM multiplexer with multiple QoS VBR traffic," *ETRI Journal*, vol. 19, no. 1, pp. 12–23, 1997.

[4] A. Gravey and G. Hebuterne, "Mixing time and loss priorities in a single server queue," in *Proceedings of the 13th International Teletraffic Congress (ITC '91)*, vol. 13, pp. 47–52, Stockholm, Sweden, June 1991.

[5] A. Gravey, P. Boyer, and G. Hebuterne, "Tagging versus strict rate enforcement in ATM networks," in *IEEE Global Telecommunications Conference (GLOBECOM '91)*, vol. 1, pp. 271–275, Phoenix, Ariz, USA, December 1991.

[6] H. Kröner, G. Hebuterne, P. Boyer, and A. Gravey, "Priority management in ATM switching nodes," *IEEE Journal on Selected Areas in Communications*, vol. 9, no. 3, pp. 418–427, 1991.

[7] S. Kasahara, H. Takagi, Y. Takahashi, and T. Hasegawa, "$M/G/1/K$ system with push-out scheme under vacation policy," *Journal of Applied Mathematics and Stochastic Analysis*, vol. 9, no. 2, pp. 143–157, 1996.

[8] K. V. Cardoso, J. F. de Rezende, and N. L. S. Fonseca, "On the effectiveness of push-out mechanisms for the discard of TCP packets," in *Proceedings of IEEE International Conference on Communications (ICC '02)*, vol. 4, pp. 2636–2640, New York, NY, USA, April-May 2002.

[9] C. G. Chang and H. H. Tan, "Queueing analysis of explicit policy assignment push-out buffer sharing schemes for ATM networks," in *Proceedings of the 13th IEEE Networking for Global Communications*, vol. 2, pp. 500–509, Toronto, Canada, June 1994.

[10] X. Cheng and I. F. Akyildiz, "A finite buffer two class queue with different scheduling and push-out schemes," in *Proceedings of the the 11th Annual Conference of the IEEE Computer and Communications Societies (INFOCOM '92)*, vol. 1, pp. 231–241, Florence, Italy, May 1992.

[11] L. Gui and C. Fan, "Analysis of a priority cell discarding method for ATM networks," *Telecommunication Systems*, vol. 4, no. 1, pp. 51–60, 1995.

[12] G. Hebuterne and A. Gravey, "Space priority queuing mechanism for multiplexing ATM channels," *Computer Networks and ISDN Systems*, vol. 20, no. 1–5, pp. 37–43, 1990.

[13] D. P. Heyman, "The push-out priority queue discipline," *Operations Research*, vol. 33, no. 2, pp. 397–403, 1985.

[14] H. Saito, "Queueing analysis of cell loss probability control in ATM networks," in *Proceedings of the 13th International Teletraffic Congress (ITC '91)*, vol. 13, pp. 19–24, Stockholm, Sweden, June 1991.

[15] G.-L. Wu and J. W. Mark, "A buffer allocation scheme for ATM networks: complete sharing based on virtual partition," *IEEE/ACM Transactions on Networking*, vol. 3, no. 6, pp. 660–670, 1995.

[16] H. Takagi, *Queueing Analysis: A Foundation of Performance Evaluation. Vol. 2. Finite Systems*, North-Holland, Amsterdam, The Netherlands, 1991.

[17] L. Kleinrock, *Queueing Systems, Volume 1: Theory*, John Wiley & Sons, New York, NY, USA, 1975.

[18] L. Kleinrock, *Queueing Systems, Volume 2: Computer Applications*, John Wiley & Sons, New York, NY, USA, 1976.

Yutae Lee: Department of Information and Communication Engineering, Dongeui University, Busanjin-gu, Busan 614-714, South Korea
Email address: ytalee@ucdavis.edu

Bong Dae Choi: Department of Mathematics, College of Science, Korea University, Sungbuk-gu, Seoul 136-701, South Korea
Email address: queue@korea.ac.kr

Bara Kim: Department of Mathematics, College of Science, Korea University, Sungbuk-gu, Seoul 136-701, South Korea
Email address: bara@korea.ac.kr

Dan Keun Sung: School of Electrical Engineering and Computer Science, College of Engineering, Korea Advanced Institute of Science and Technology (KAIST), Yuseong-gu, Daejeon 305-701, South Korea
Email address: dksung@ee.kaist.ac.kr