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Research Article

A Note on Variable Viscosity and Chemical Reaction Effects on Mixed Convection Heat and Mass Transfer Along a Semi-Infinite Vertical Plate

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In the present study, an analysis is carried out to study the variable viscosity and chemical reaction effects on the flow, heat, and mass transfer characteristics in a viscous fluid over a semi-infinite vertical porous plate. The governing boundary layer equations are written into a dimensionless form by similarity transformations. The transformed coupled nonlinear ordinary differential equations are solved numerically by using the shooting method. The effects of different parameters on the dimensionless velocity, temperature, and concentration profiles are shown graphically. In addition, tabulated results for the local skin-friction coefficient, the local Nusselt number, and the local Sherwood number are presented and discussed.

1. Introduction

In many transport processes existing in nature and in industrial applications in which heat and mass transfer is a consequence of buoyancy effects caused by diffusion of heat and chemical species. The study of such processes is useful for improving a number of chemical technologies, such as polymer production and food processing. In nature, the presence of pure air or water is impossible. Some foreign mass may be present either naturally or mixed with the air or water. The effect of the presence of foreign mass on the free convection flow past a semi-infinite vertical plate was studied by Gebhart and Pera [1]. The presence of a foreign mass in air or water causes some kind of chemical reaction. During a chemical reaction between two species, heat is also generated [2]. In most cases of chemical reaction, the reaction rate depends on the concentration of the species itself. A reaction is said to be first order if the rate of reaction is directly proportional to concentration itself [3]. Chemical reaction effects on heat and mass transfer laminar boundary layer flow have been studied by many authors [4–11] in different situations.

The previous studies are based on the constant physical properties of the fluid. For most realistic fluids, the viscosity shows a rather pronounced variation with temperature. It is known that the fluid viscosity changes with temperature [12]. Then it is necessary to take into account the variation of viscosity with temperature in order to accurately predict the heat transfer rates. The effect of temperature-dependent viscosity on the mixed convection flow from vertical plate is investigated by several authors [13–17].

The aim of this work is to study the effects of chemical reaction and variable viscosity on flow, heat, and mass transfer on a semi-infinite vertical plate with suction. The order of chemical reaction in this work is taken as first-order reaction.

2. Mathematical formulation

Consider a steady, viscous incompressible Newtonian fluid past a semi-infinite vertical porous plate which is aligned parallel to a uniform free stream with velocity u_∞. It is assumed that the viscous dissipation and the heat generated during chemical reaction can be neglected. Also, it is assumed that the fluid has a constant properties except the density in the buoyancy term of the momentum equation and in the fluid viscosity which is assumed to be an inverse linear function of temperature [13].

Under the above assumptions and Boussinesq's approximation, the boundary layer equations governing the flow can be expressed as [18]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{1}{\rho_\infty}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)+g\beta(T-T_\infty)+g\beta^*(C-C_\infty), \tag{2.2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=k\frac{\partial^2 T}{\partial y^2}, \tag{2.3}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_1(C-C_\infty). \tag{2.4}$$

The appropriate boundary conditions are

$$\begin{aligned} &\text{at } y=0: \quad u=0, \quad v=-v_w, \quad C=C_w, \quad T=T_w,\\ &\text{as } y\longrightarrow\infty: \quad u\longrightarrow u_\infty, \quad C\longrightarrow C_\infty, \quad T\longrightarrow T_\infty, \end{aligned} \tag{2.5}$$

where the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normally to the plate. u and v are the velocity components along x and y, respectively. T is the fluid temperature, C is the species concentration, T_∞ is the ambient temperature, C_∞ is the ambient concentration, ρ_∞ is the ambient density, g is the gravitational acceleration, β is the coefficient of thermal expansion, β^* is the coefficient of concentration expansion, μ_∞ is the ambient viscosity, μ is the viscosity of the fluid, k is the thermal diffusivity, D is the molecular diffusion coefficient, K_1 is the chemical reaction parameter, and $v_w(>0)$ is the suction velocity.

Introducing the similarity variables,

$$\eta = \sqrt{\frac{u_\infty}{\nu_\infty x}}y, \qquad \psi(x,y) = \sqrt{\nu_\infty u_\infty x} f(\eta). \tag{2.6}$$

The velocity components are given by

$$u = \frac{\partial \psi}{\partial y}, \qquad v = -\frac{\partial \psi}{\partial x}. \tag{2.7}$$

It can be easily verified that the continuity equation (2.1) is identically satisfied. Equations (2.2)–(2.4) reduce to

$$f''' + \frac{1}{2}\left(\frac{\theta_r - \theta}{\theta_r}\right) f f'' + \frac{\theta'}{(\theta_r - \theta)} f'' + \frac{G_r}{R_e^2}\left(\frac{\theta_r - \theta}{\theta_r}\right)\theta + \frac{G_c}{R_e^2}\left(\frac{\theta_r - \theta}{\theta_r}\right)\phi = 0, \tag{2.8}$$

$$\theta'' + \frac{1}{2} P_r f \theta' = 0, \tag{2.9}$$

$$\phi'' + \frac{1}{2} S_c f \phi' - \gamma S_c R_e \phi = 0, \tag{2.10}$$

where $\theta = (T - T_\infty)/(T_w - T_\infty)$, $\phi = (C - C_\infty)/(C_w - C_\infty)$, $\theta_r = -1/\alpha(T_w - T_\infty)$ is the viscosity parameter, $R_e = u_\infty x/\nu_\infty$ is the local Reynolds number, $G_r = g\beta(T_w - T_\infty)x^3/\nu_\infty^2$ is the local Grashof number, $G_c = g\beta^*(C_w - C_\infty)x^3/\nu_\infty^2$ is the local modified Grashof number, $\gamma = K_1\nu_\infty/u_\infty^2$ is the chemical reaction parameter, $P_r = \mu_\infty c_p/k$ is the Prandtl number, $S_c = \nu_\infty/D$ is the Schmidt number, α is a constant (> 0 for liquids) and (< 0 for gases), $\nu_\infty = \mu_\infty/\rho_\infty$, c_p is the specific heat at constant pressure, and the primes denote differentiation with respect to η.

The transformed boundary conditions are given by

$$\begin{gathered} \eta = 0 : f = f_\omega, \quad f' = 0, \quad \theta = 1, \quad \phi = 1, \\ \eta \longrightarrow \infty : f' \longrightarrow 1, \quad \theta \longrightarrow 0, \quad \phi \longrightarrow 0, \end{gathered} \tag{2.11}$$

where $f_w = \sqrt{x/\nu_\infty u_\infty}\, \nu_w$ (suction parameter > 0).

The physical quantities of interest in this problem are the local skin-friction coefficient C_{fx}, the local Nussselt number N_{ux}, and the local Sherwood number S_{hx} which are defined by

$$C_{\text{fx}} = \frac{2\theta_r}{(\theta_r - 1)} R_e^{-1/2} f''(0,\theta_r), \qquad N_{\text{ux}} = -R_e^{1/2}\theta'(0,\theta_r), \qquad S_{\text{hx}} = -R_e^{1/2}\varphi'(0,\theta_r). \tag{2.12}$$

3. Numerical solution and discussion

Equations (2.8)–(2.10) with the boundary conditions (2.11) were solved numerically using the shooting method. To verify the accuracy of the numerical method, we require

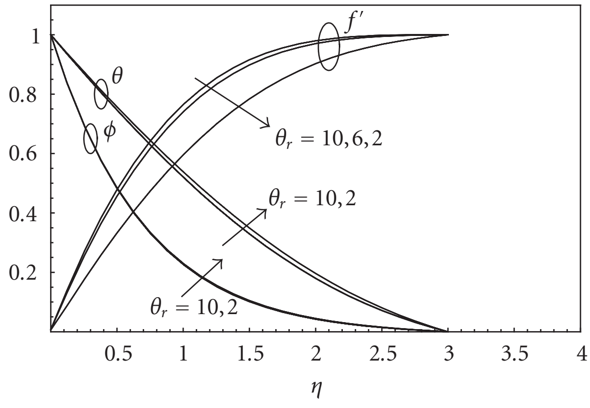

Figure 3.1. Velocity, temperature, and concentration profiles for various values of θ_r.

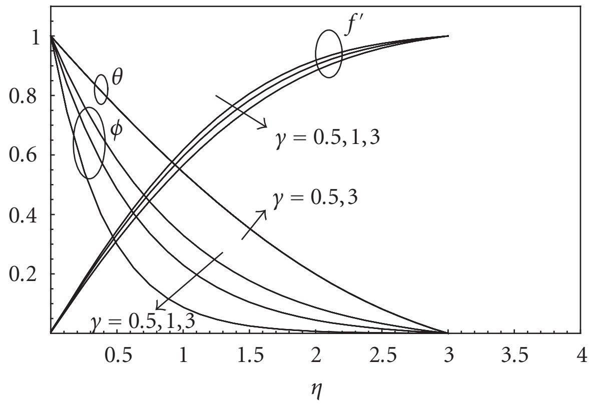

Figure 3.2. Velocity, temperature, and concentration profiles for various values of γ.

$f''(0)$ and $-\theta'(0)$ to have exactly the same values as those reported by Hady et al. [15]. Equations (2.8) and (2.9) in our work are the same as those of Hady et al. [15] with $A_r = 0$ if we take $G_r = G_c = 0$. In the present, study our results for $f''(0)$ and $-\theta'(0)$ for $G_r = G_c = 0$, $\theta_r = 0.2$, and $p_r = 0.7$ are -0.19064, 0.26415, respectively which are exactly as reported in [15].

The velocity, temperature, and concentration profiles obtained in the dimensionless form are presented in Figures 3.1–3.3 for $p_r = 0.71$ which represents air at temperature 20°C and for $S_c = 0.6$ which corresponds to water vapor that represents a diffusion chemical species of most common interest in air. Grashof number for heat transfer is chosen to be $G_r = 5$, modified Grashof number for mass transfer is chosen to be $G_c = 4$, since these values correspond to a cooling problem, and Reynolds number $R_e = 3.0$. The values of γ

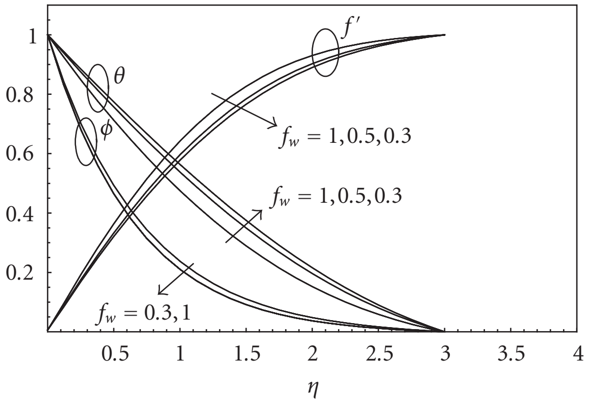

Figure 3.3. Velocity, temperature, and concentration profiles for various values of f_w.

is chosen to be 0.5, 1, and 3. It is important to note that θ_r is negative for liquids and positive for gases when $T_w - T_\infty$ is positive. The values of θ_r (for air $\theta_r > 0$) are chosen to be 2, 6, and 10. The values of f_w are chosen to be 0.3, 0.5, and 1. The effects of the viscosity parameter θ_r on the velocity, temperature, and concentration profiles are shown through Figure 3.1. It is seen from this figure that the velocity increases with the increase of the viscosity parameter while the thermal boundary layer thickness decreases as the viscosity parameter increases. So, the increase of viscosity parameter accelerates the fluid motion and reduces the temperature of the fluid along the wall. Also, one sees that the concentration of the fluid is almost not affected with increase of the viscosity parameter. Figure 3.2 displays the influence of the chemical reaction parameter γ on the velocity, temperature, and concentration profiles. It is clear from this figure that increasing the values of γ produces a decrease in the velocity. This means that in the case of suction, the chemical reaction decelerates the fluid motion while the temperature of the fluid is almost not affected with increase of γ. Also, the chemical reaction decelerates the concentration of the fluid in the case of suction. Figure 3.3 shows the influence of the suction parameter f_w on the velocity, temperature and concentration profiles in the boundary layer. From this figure one sees that the thickness of the velocity boundary layer increases as the suction parameter increases. On the contrary, the thickness of the temperature boundary layer and the thickness of the concentration boundary layer decreases with an increase in the suction parameter f_w. Table 3.1 presents the effects of θ_r, γ, and f_w on the values of the skin-friction coefficient $f^{//}(0)$, the Nusselt number $-\theta^{/}(0)$, and the Sherwood number $-\phi^{/}(0)$. The results show that the skin-friction coefficient, the Nusselt number and the Sherwood number increase as θ_r or f_w increases. Also, as the chemical parameter γ increases, the skin-friction coefficient and the Nusselt number decrease while the Sherwood number decreases.

4. Conclusion

The effects of variable viscosity, suction, and chemical reaction on flow, mass, and heat transfer of a steady incompressible Newtonian fluid past a vertical plate have been studied

Table 3.1. Effects of various parameters on $f''(0)$, $-\theta'(0)$, and $-\phi'(0)$ with $R_e = 3.0$.

θ_r	γ	f_w	$f''(0)$	$-\theta'(0)$	$-\phi'(0)$
2	1	0.5	0.7265	0.50857	1.4331
6	1	0.5	1.1245	0.52738	1.4377
10	1	0.5	1.19899	0.53059	1.4386
2	0.5	0.5	0.754003	0.51109	1.0576
2	1	0.5	0.726512	0.50857	1.43315
2	3	0.5	0.68402	0.505075	2.4045
2	1	0.3	0.70173	0.46411	1.4011
2	1	0.5	0.72652	0.50857	1.4331
2	1	1	0.79371	0.62896	1.51613

numerically using the shooting method. From the previous results and discussion, we conclude the following.

(1) The velocity increases with the increase of the viscosity or the suction parameter.
(2) The chemical reaction parameter decelerates the fluid motion.
(3) The temperature decreases as the viscosity or the suction parameter increases.
(4) The concentration decreases with the increase of the chemical reaction or the suction parameter.
(5) The skin-friction coefficient, the Nusselt, and the Sherwood number increase as the viscosity or the suction parameter increases.
(6) The skin-friction coefficient and the Nusselt number decrease with the increase of the chemical reaction parameter but the Sherwood number increases as the chemical reaction parameter increases.

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References

[1] B. Gebhart and L. Pera, "The nature of vertical natural convection flows resulting from the combined buoyancy effects of thermal and mass diffusion," *International Journal of Heat and Mass Transfer*, vol. 14, no. 12, pp. 2025–2050, 1971.
[2] R. Byon Bird, W. E. Stewart, and E. N. Lightfoot, *Transport Phenomena*, John Wiley & Sons, New York, NY, USA, 1992.
[3] E. L. Cussler, *Diffusion Mass Transfer in Fluid Systems*, Cambridge University Press, London, UK, 1988.
[4] A. Apelblat, "Mass transfer with a chemical reaction of the first order. Effect of axial diffusion," *The Chemical Engineering Journal*, vol. 23, no. 2, pp. 193–203, 1982.
[5] U. N. Das, R. Deka, and V. M. Soundalgekar, "Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction," *Forschung im Ingenieurwesen*, vol. 60, no. 10, pp. 284–287, 1994.

[6] H. I. Andersson, O. R. Hansen, and B. Holmedal, "Diffusion of a chemically reactive species from a stretching sheet," *International Journal of Heat and Mass Transfer*, vol. 37, no. 4, pp. 659–664, 1994.
[7] J. R. Fan, J. M. Shi, and X. Z. Xu, "Similarity solution of mixed convection over a horizontal moving plate," *Heat and Mass Transfer*, vol. 32, no. 3, pp. 199–206, 1997.
[8] H. S. Takhar, A. J. Chamkha, and G. Nath, "Flow and mass transfer on a stretching sheet with a magnetic field and chemically reactive species," *International Journal of Engineering Science*, vol. 38, no. 12, pp. 1303–1314, 2000.
[9] R. Muthucumaraswamy and P. Ganesan, "Effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate," *Journal of Applied Mechanics and Technical physics*, vol. 42, no. 4, pp. 665–671, 2001.
[10] A. J. Chamkha, "MHD flow of a uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction," *International Communications in Heat and Mass Transfer*, vol. 30, no. 3, pp. 413–422, 2003.
[11] R. Kandasamy, K. Periasamy, and K. K. Sivagnana Prabhu, "Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection," *International Journal of Heat and Mass Transfer*, vol. 48, no. 7, pp. 1388–1394, 2005.
[12] H. Herwing and G. Wickern, "The effect of variable properties on laminar boundary layer flow," *Wärme-und Stoffübctragung*, vol. 20, pp. 47–54, 1986.
[13] F. C. Lai and F. A. Kulacki, "The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium," *International Journal of Heat and Mass Transfer*, vol. 33, no. 5, pp. 1028–1031, 1990.
[14] N. G. Kafoussius and E. W. Williams, "The effect of temperature-dependent viscosity on free-forced convective laminar boundary layer flow past a vertical isothermal flat plate," *Acta Mechanica*, vol. 110, no. 1–4, pp. 123–137, 1995.
[15] F. M. Hady, A. Y. Bakier, and R. S. R. Gorla, "Mixed convection boundary layer flow on a continuous flat plate with variable viscosity," *Heat and Mass Transfer*, vol. 31, no. 3, pp. 169–172, 1996.
[16] N. G. Kafoussias, D. A. S. Rees, and J. E. Daskalakis, "Numerical study of the combined free-forced convective laminar boundary layer flow past a vertical isothermal flat plate with temperature-dependent viscosity," *Acta Mechanica*, vol. 127, no. 1–4, pp. 39–50, 1998.
[17] A. Hossain and S. Munir, "Mixed convection flow from a vertical flat plate with temperature dependent viscosity," *International Journal of Thermal Sciences*, vol. 39, no. 2, pp. 173–183, 2000.
[18] I. Pop and D. B. Ingham, *Convective Heat Transfer, Mathematical and Computational Modelling of Viscous Fluid and Porous Media*, Pergamon, London, UK, 2001.

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