Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2007, Article ID 57128, 24 pages
doi:10.1155/2007/57128

Research Article

Quality Evaluation in Flexible Manufacturing Systems: A Markovian Approach

Jingshan Li and Ningjian Huang

Received 16 March 2007; Accepted 9 May 2007

Recommended by P. T. Kabamba

The flexible manufacturing system (FMS) has attracted substantial amount of research effort during the last twenty years. Most of the studies address the issues of flexibility, productivity, cost, and so forth. The impact of flexible lines on product quality is less studied. This paper intends to address this issue by applying a Markov model to evaluate quality performance of a flexible manufacturing system. Closed expressions to calculate good part probability are derived and discussions to maintain high product quality are carried out. An example of flexible fixture in machining system is provided to illustrate the applicability of the method. The results of this study suggest a possible approach to investigate the impact of flexibility on product quality and, finally, with extensions and enrichment of the model, may lead to provide production engineers and managers a better understanding of the quality implications and to summarize some general guidelines of operation management in flexible manufacturing systems.

1. Introduction

Manufacturing system design and product quality have been studied extensively during the last 50 years. However, most of the studies address the problems independently. In other words, the majority of the publications on quality research seek to maintain and improve product quality while ignoring the production system concerns. Similarly, the majority of the production system research seeks to maintain the desired productivity while neglecting the question of quality. Little research attention has been paid to investigate the coupling or interaction between production system design and product quality. However, it has been shown in [1] that production system design and product quality are tightly coupled, that is, production system design has a significant impact on product quality as well as other factors. The analysis in this area, which is important but largely unexplored, will open a new direction of research in production systems engineering. To stimulate research in this area, [1] presents several research opportunities from the automotive industry perspective, and flexibility is one of them.

To satisfy the rapidly changing markets and varying customer demands, manufacturing systems are becoming more and more flexible. For example, in automotive industry, flexible manufacturing is "becoming even more critical" [2]. Substantial amount of research effort and practices have been devoted to flexible manufacturing systems (FMSs), and it has taken an explicit role in production system design. Much of the work related to flexibility addresses the issues of investment cost, flexibility measurement, and the trade-offs between productivity and flexibility. However, interactions not only exist between flexibility and productivity, but also between flexibility and quality (as suggested by [1]). The latter one is much less studied.

For example, in many flexible machining systems, a flexible fixture restricts and is the core enabler to flexibility of the whole system, and the cost of designing and fabricating fixtures can amount to 10%–20% of the total manufacturing system cost [3, 4]. A flexible fixture often is a programmable fixture designed to support multiple distinguished parts being manufactured (assembled or machined) on the same line. With the flexible fixture, system flexibility can be achieved with little or no loss of production. In automotive industry, a flexible fixture might be clamps/locators held by robots or other "smart" mobile apparatuses. The challenge, however, with the flexible fixture is the accuracy of the locator measured by the variance. Whenever there is a product change, the fixture needs to adapt itself to the desired corresponding location. As we know, the quality of the manufacturing operation heavily depends on the fixture. The discrepancy of the fixture location from its "ideal" one, in many cases, dominates the quality of the products. For instance, consider a production line producing two products, A and B. Assuming that the fixture is located in a "good" position, that is, within the nominal tolerance, for product A, then if the subsequent parts belong to product A, it is more likely that good quality parts can be produced. Analogously, if the fixture is in a "bad" location, then more defective parts can be produced. However, when the subsequent part is switched to product B, then the fixture needs to readjust its location and either good quality or defective parts may be produced (more detailed description is introduced in Section 4). Therefore, the quality characteristic of the current part is dependent on the part type and quality of the previous one. A study to evaluate that the quality performance in flexible machining environment is valuable, however, has been missing in current literature.

An automotive paint shop is typically capable of painting different models with desired colors. However, the number of available paint colors can significantly impact product quality [2]. Whenever a color change happens, previous paints and solvent need to be purged and spray guns need to be cleaned to remove any residue. The paint quality may temporarily decline after the switch [5]. Thus, the previous vehicle's color may affect next vehicle's quality, as well as other factors (e.g., paint mixing, vehicle cleaning, dirty air, and equipment, etc.). Therefore, vehicles with the same colors are usually grouped into a batch before entering the painting booths without sacrificing much on vehicle delivery. In addition, it is typical to sequence the light color vehicles before the darker ones [6]. Through this, the change-over time (or paint purging time) and the cost of paint purging are reduced. More importantly, the paint quality can be improved by reducing the possibility of incomplete cleaning during purging [7]. However, no analytical study has been found to investigate how flexibility (in terms of number of colors) impacts paint quality, and what would the appropriate batch size and batch sequence be to obtain good paint quality and to satisfy throughput and order delivery requirements as well.

Additional examples can be found in welding, assembly operations, and so forth, as well. These examples suggest that flexibility and quality are tightly coupled and much more work is needed to fully understand this coupling. Such an issue is very important but almost neglected. We believe that quality should be integrated into the considerations when designing production systems as well as objectives of productivity and flexibility. The goal of this study is to investigate the coupling between flexibility and product quality, and to provide production engineers and managers a better understanding of the quality implications in flexible manufacturing systems and to offer some general guidelines for management of flexible operations. To start such a study, a simple Markovian model to analyze the quality performance of a flexible manufacturing system is developed. Specifically, a closed-form expression is derived to evaluate the system quality in terms of good part probability and some discussions are carried out based on the analysis. Although inventory, flow control, scheduling, and so forth are also important parts of FMS studies, we limit our work in this paper to quality performance only. Enrichment of the model by integrating quality with other performance measures (e.g., throughput, inventory, cost, etc.) will be a topic for future work.

The rest of the paper is structured as follows. Section 2 reviews the related literature. Models and analysis are developed and carried out in Section 3. Using the method developed, an example of quality performance evaluation in a flexible machining system is introduced in Section 4. Finally, Section 5 concludes the paper. All proofs are presented in the appendix.

2. Literature review

Although significant research effort has been devoted separately to manufacturing system design and product quality, the coupling or interaction between them has not been studied intensively. Paper [1] reviews the related literature and suggests that this is an open area with promising research opportunities. Limited work addressing this coupling can be found in [8–14]. Specifically, [8] studies the perturbation in the average steady state production rate by quality inspection machines for an asymptotically reliable two-machine one-buffer line. The tradeoffs between productivity and product quality as well as their impact on optimal buffer designs are investigated in [9]. Paper [10] delineates the tradeoff between throughput and quality for a robot whose repeatability deteriorates with speed. Paper [11] uses stochastic search techniques (generic algorithms and simulated annealing) to investigate the impact of inspection allocation in manufacturing systems (serial and nonserial) from the cost perspective. The competing effects of large or small batch sizes are studied in [12] and a model for the interaction between batch size and quality is developed. In addition, [13] uses quantitative measures to deduce that U-shaped lines produce better quality products. A new line balancing approach is proposed in [14] to improve quality by reducing work overload. The recent advances in this area are contained in [15–19]. In [15], a multistage variation propagation model is presented. Paper [16] studies a transfer production line with Andon. It is shown that to produce more good quality parts, Andon is preferable only when average repair time is short and the line should be stopped to repair all the defects. The impact of repair capacity and first time quality on the quality buy rate of an automotive paint is analyzed in [17, 18]. Paper [19] introduces an integrated model of a two-machine one-buffer line with inspection and information feedback to study both quality and quantity performances in terms of good production rate.

Flexibility has attracted a significant amount of research in the last two decades. Most of the work related to flexibility focus on the definition, meaning, and measurement of manufacturing flexibility, and performance modeling of flexible manufacturing systems, and so forth (see, e.g., monographs [20–23], and review papers [24–31]). However, as pointed out in [3], most of the flexibility studies assume that quality-related issues, such as rejects, rework, have minimal impact and that only products of acceptable quality are produced. The production of high quality parts in an FMS requires significant effort and investments. Only a few publications are found discussing the impact of manufacturing flexibility on product quality [32–35]. Specifically, a measure of productivity, quality, and flexibility for production systems is presented in [32]. Paper [33] studies the issues of flexibility, productivity, and quality from an extensive search and analysis of empirical studies. In [34], a method is developed to model the fuzzy flexibility elements such as quality level, efficiency, versatility, and availability. In addition, paper [35] surveys the existing literature related to mass customization. In particular, it points out that quality control issues should be taken into account and current literature lacks in-depth study on how to assure quality in mass-customized products.

In spite of the above effort, the current literature does not provide a quantitative model which enables us to investigate the correlation between quality and number of products and to predict the quality performance of a flexible manufacturing system. We still need to fully understand the coupling or interactions between flexible manufacturing system design and product quality. An in-depth analytical study of the impact of flexibility on quality is necessary and important. This paper is intended to contribute to this end.

3. Models and analysis

3.1. One product type. Consider a flexible manufacturing system producing one product type and let g and d denote the states that the system is producing a good quality part or a defective part in steady states, respectively. Note that here we only study the working or production period of the system. In other words, machine breakdowns are not considered. When the system is in state g, it has a transition probability λ to produce a defective part in the next cycle, and probability $1-\lambda$ to continue producing a good part. Similarly, when the system is in state d, it can produce a good part with probability μ and a defective part with probability $1-\mu$ in the next cycle (see Figure 3.1). λ and μ can be viewed as *quality failure and repair probabilities*, respectively. Similar to throughput analysis, constant transition probabilities are assumed to simplify the analysis for steady state operations.

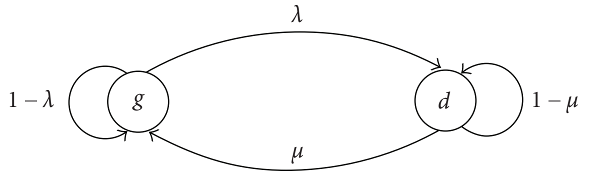

Figure 3.1. State transition diagram in one-product-type case.

Let $P(g,t)$ and $P(d,t)$ denote the probabilities that the system is in states g or d at cycle t, respectively. Clearly, states g and d are similar to the up- and down-states in throughput analysis. Therefore, by extending the method used in throughput analysis to study quality performance, we obtain

$$\begin{aligned} P(g,t+1) &= P\big(\text{produce a good part at } t+1 \mid \text{produce a good part at } t\big)P(g,t) \\ &\quad + P\big(\text{produce a good part at } t+1 \mid \text{produce a defective part at } t\big)P(d,t) \\ &= P(g,t+1 \mid g,t)P(g,t) + P(g,t+1 \mid d,t)P(d,t) \\ &= (1-\lambda)P(g,t) + \mu P(d,t). \end{aligned} \tag{3.1}$$

In terms of the steady states, $P(g)$ and $P(d)$ are used to denote the probabilities to produce a good or a defective part during a cycle, respectively, that is,

$$\lim_{t\to\infty} P(g,t) := P(g), \qquad \lim_{t\to\infty} P(d,t) := P(d). \tag{3.2}$$

It follows that

$$P(g) = (1-\lambda)P(g) + \mu P(d), \tag{3.3}$$

which implies that

$$P(d) = \frac{\lambda}{\mu}P(g). \tag{3.4}$$

From the fact that total probability equals 1,

$$P(g) + P(d) = 1, \tag{3.5}$$

it follows that the system good product ratio is

$$P(g) = \frac{\mu}{\lambda+\mu}. \tag{3.6}$$

Clearly, as expected, (3.6) has a similar form as machine efficiency in throughput analysis. Below, we will extend this study to multiple-product-types case.

3.2. Two product types. Now we consider a flexible system producing two types of products, types 1 and 2. Introduce $P(g_i)$ and $P(d_i)$ as the probabilities to produce a good part type i, $i = 1,2$, or defective part type i, $i = 1,2$, during a cycle, respectively. Again $P(g)$ and $P(d)$ are used to represent the good or defective part probability (of both products). Then we obtain

$$P(g_1) + P(g_2) = P(g), \qquad P(d_1) + P(d_2) = P(d). \tag{3.7}$$

In addition, introduce the following assumptions.

(i) A flexible system has four states: producing good part type 1, type 2, and producing defective part type 1 and type 2, denoted as g_1, g_2, d_1, and d_2, respectively.

(ii) The transition probabilities from good states g_i, $i = 1,2$, to defective states d_j, $j = 1,2$, are determined by λ_{ij}. The system has probabilities ν_{ij} to stay in good states g_j, $j = 1,2$. Similarly, when the system is in defective states d_i, $i = 1,2$, it has probabilities μ_{ij} to transit to good states g_j, $j = 1,2$, and probabilities η_{ij} to stay in defective states d_j, $j = 1,2$.

Remark 3.1. Similar to one-product-type case, λ_{ii} and μ_{ii}, $i = 1,2$, can be viewed as *nonswitching quality failure and repair probabilities*, respectively (i.e., product types are not switched). Analogously, λ_{ij} and μ_{ij}, $i,j = 1,2$, $i \neq j$, can be viewed as *switching quality failure and repair rates*, respectively.

(iii) When incoming parts are in random order without correlations (nonsequenced), the part flow is identically and uniformly distributed with probabilities $P(1)$ and $P(2)$ for part types 1 and 2, respectively. In other words, every cycle the system has probability $P(1)$ or $P(2)$ to work on part types 1 and 2, respectively.

Remark 3.2. Assumptions (ii) and (iii) imply that probabilities $P(1)$ and $P(2)$ are embedded in the transition probabilities λ_{ij}, μ_{ij}, ν_{ij}, and η_{ij}, $i,j = 1,2$. For example, λ_{ij} defines the transition probability that the incoming part is type j and the system produces a defective part at cycle $t+1$ given that it produces a good type i part at cycle t.

Based on the above assumptions, we can describe the system using a discrete Markov chain illustrated in Figure 3.2. In addition, since total probabilities equal 1, we have

$$\begin{gathered} P(1) + P(2) = 1, \qquad P(g_1) + P(d_1) = P(1), \qquad P(g_2) + P(d_2) = P(2), \\ \lambda_{11} + \lambda_{12} + \nu_{11} + \nu_{12} = 1, \qquad \lambda_{22} + \lambda_{21} + \nu_{22} + \nu_{21} = 1, \\ \mu_{11} + \mu_{12} + \eta_{11} + \eta_{12} = 1, \qquad \mu_{22} + \mu_{21} + \eta_{22} + \eta_{21} = 1. \end{gathered} \tag{3.8}$$

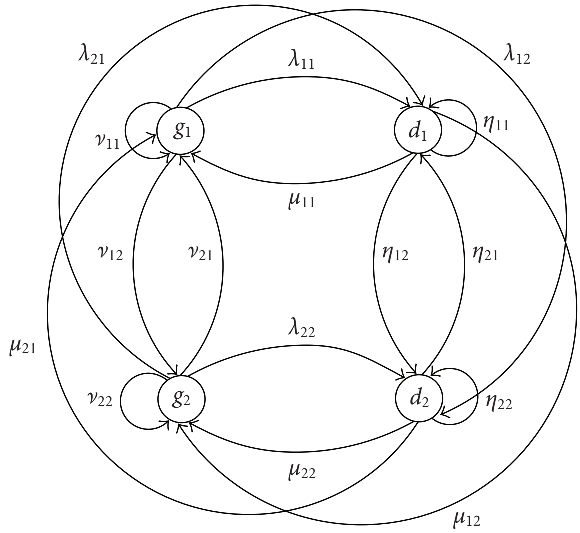

Figure 3.2. State transition diagram in two-product-type case.

Analogously to Section 3.1, the transitions to state g_1 can be described as

$$\begin{aligned} P(g_1,t+1) &= P(g_1,t+1 \mid g_1,t)P(g_1,t) + P(g_1,t+1 \mid d_1,t)P(d_1,t) \\ &\quad + P(g_1,t+1 \mid g_2,t)P(g_2,t) + P(g_1,t+1 \mid d_2,t)P(d_2,t) \\ &= \nu_{11}P(g_1,t) + \nu_{21}P(g_2,t) + \mu_{11}P(d_1,t) + \mu_{21}P(d_2,t). \end{aligned} \tag{3.9}$$

Considering the steady state probability $P(g_1)$, we have

$$P(g_1) = \nu_{11}P(g_1) + \nu_{21}P(g_2) + \mu_{11}P(d_1) + \mu_{21}P(d_2). \tag{3.10}$$

Similarly,

$$P(g_2) = \nu_{12}P(g_1) + \nu_{22}P(g_2) + \mu_{12}P(d_1) + \mu_{22}P(d_2), \tag{3.11}$$

$$P(d_1) = \lambda_{11}P(g_1) + \lambda_{21}P(g_2) + \eta_{11}P(d_1) + \eta_{21}P(d_2), \tag{3.12}$$

$$P(d_2) = \lambda_{12}P(g_1) + \lambda_{22}P(g_2) + \eta_{12}P(d_1) + \eta_{22}P(d_2). \tag{3.13}$$

Solving the above equations, we obtain a closed formula to calculate the probability of good quality part $P(g)$.

Theorem 3.3. *Under assumptions (i)–(iii), the good part probability $P(g)$ can be calculated as*

$$P(g) = \frac{\mathcal{F}}{\mathcal{F}+\mathcal{G}}, \tag{3.14}$$

where

$$\begin{aligned}
\mathcal{F} = {} & (\lambda_{11}-\lambda_{21})(\mu_{12}\mu_{21}-\mu_{11}\mu_{22}) + (1-\nu_{22}+\nu_{12})[(1-\eta_{11})\mu_{21}+\eta_{21}\mu_{11}] \\
& + (1-\nu_{11}+\nu_{21})[(1-\eta_{11})\mu_{22}+\eta_{21}\mu_{12}], \\
\mathcal{G} = {} & (\mu_{21}-\mu_{11})[(1-\nu_{22})\lambda_{11}+\nu_{12}\lambda_{21}] - (\mu_{12}-\mu_{22})[(1-\nu_{11})\lambda_{21}+\lambda_{11}\nu_{21}] \\
& + [(1-\nu_{11})(1-\nu_{22})-\nu_{12}\nu_{21}](1-\eta_{11}+\eta_{21}).
\end{aligned} \tag{3.15}$$

For the proof, see the appendix.

3.3. Multiple ($n > 2$) **product types.** Now consider a flexible manufacturing system producing more than two types of product. The same assumptions and notations in Section 3.2 will be used with the exception that now $i = 1,\dots,n$, denoting n product types. Therefore, we have

$$\begin{gathered}
\sum_{i=1}^{n} P(i) = 1, \qquad \sum_{i=1}^{n} P(g_i) = P(g), \qquad \sum_{i=1}^{n} P(d_i) = P(d), \\
P(g_i) + P(d_i) = P(i), \quad i = 1,\dots,n, \qquad \sum_{i=1}^{n} P(g_i) + \sum_{i=1}^{n} P(d_i) = 1, \\
\sum_{j=1}^{n} (\lambda_{ij}+\nu_{ij}) = 1, \quad i = 1,\dots,n, \qquad \sum_{j=1}^{n} (\mu_{ij}+\eta_{ij}) = 1, \quad i = 1,\dots,n.
\end{gathered} \tag{3.16}$$

Analogously to Section 3.2, we obtain the following transition equations:

$$\begin{aligned}
P(g_j) &= \sum_{i=1}^{n} \nu_{ij} P(g_i) + \sum_{i=1}^{n} \mu_{ij} P(d_i), \quad j = 1,\dots,n, \\
P(d_j) &= \sum_{i=1}^{n} \lambda_{ij} P(g_i) + \sum_{i=1}^{n} \eta_{ij} P(d_i), \quad j = 1,\dots,n-1, \\
1 &= \sum_{i=1}^{n} P(g_i) + \sum_{i=1}^{n} P(d_i).
\end{aligned} \tag{3.17}$$

Rearranging them and writing into a matrix form, we have

$$AX = B, \tag{3.18}$$

where

$$A=\begin{pmatrix} \nu_{11}-1 & \nu_{21} & \dots & \nu_{n1} & \mu_{11} & \mu_{21} & \dots & \mu_{n-1,1} & \mu_{n1} \\ \nu_{12} & \nu_{22}-1 & \dots & \nu_{n2} & \mu_{12} & \mu_{22} & \dots & \mu_{n-1,2} & \mu_{n2} \\ & \vdots & & \vdots & & \vdots & & \vdots & \\ \nu_{1n} & \nu_{2n} & \dots & \nu_{nn}-1 & \mu_{1n} & \mu_{2n} & \dots & \mu_{n-1,n} & \mu_{nn} \\ \lambda_{11} & \lambda_{21} & \dots & \lambda_{n1} & \eta_{11}-1 & \eta_{21} & \dots & \eta_{n-1,1} & \eta_{n1} \\ \lambda_{12} & \lambda_{22} & \dots & \lambda_{n2} & \eta_{12} & \eta_{22}-1 & \dots & \eta_{n-1,2} & \eta_{n2} \\ & \vdots & & \vdots & & \vdots & & \vdots & \\ \lambda_{1,n-1} & \lambda_{2,n-1} & \dots & \lambda_{n,n-1} & \eta_{1,n-1} & \eta_{2,n-1} & \dots & \eta_{n-1,n-1}-1 & \eta_{n,n-1} \\ 1 & 1 & \dots & 1 & 1 & 1 & \dots & & 1 \end{pmatrix}, \tag{3.19}$$

$$X=\left(P(g_1),P(g_2),\dots,P(g_n),P(d_1),P(d_2),\dots,P(d_n)\right)^T, \tag{3.20}$$

$$B=(0,0,\dots,1)^T. \tag{3.21}$$

Therefore, we obtain the following.

THEOREM 3.4. *Under assumptions (i)–(iii), the good part probability $P(g)$ can be calculated from*

$$P(g)=\sum_{i=1}^{n}P(g_i)=\sum_{i=1}^{n}x_i, \tag{3.22}$$

where $x_i=P(g_i)$, $i=1,\dots,n$, are the elements in X and can be solved from

$$X=A^{-1}B, \tag{3.23}$$

and A, B are defined in (3.19) and (3.21), respectively.

Note that the inverse of matrix A exists due to the fact that an irreducible Markov chain with finite number of states has a unique stationary distribution [36].

In the case of "equal product types," that is, n product types are equally composed ($1/n$ each) and have identical transition probabilities, we have

$$\begin{aligned} &\mu_{11}=\mu_{ii}, \quad \nu_{11}=\nu_{ii}, \quad \lambda_{11}=\lambda_{ii}, \quad \eta_{11}=\eta_{ii}, \quad i=1,\dots,n, \\ &\mu_{12}=\mu_{ij}, \quad \nu_{12}=\nu_{ij}, \quad \lambda_{12}=\lambda_{ij}, \quad \eta_{12}=\eta_{ij}, \quad i,j=1,\dots,n,\ i\neq j, \end{aligned} \tag{3.24}$$

which implies that the transitions from one product type to another are reversible (or equivalent) in terms of quality. Then we obtain the following.

COROLLARY 3.5. *Under assumptions (i)–(iii), the good part probability $P(g)$ for n equal product types is described by*

$$P(g)=\frac{\mu_{11}+(n-1)\mu_{12}}{\lambda_{11}+\mu_{11}+(n-1)(\lambda_{12}+\mu_{12})}. \tag{3.25}$$

In addition, $P(g)$ is monotonically increasing and decreasing with respect to μ_{1i} and λ_{1i}, $i=1,2$, respectively.

For the proof, see the appendix.

In order to avoid messy notations, the following discussions are limited to equal product types only.

3.4. Discussions

3.4.1. Single versus multiple product types. Similar to throughput analysis (e.g., [20, 21]), let

$$e_{1i} = \frac{\mu_{1i}}{\lambda_{1i}+\mu_{1i}}, \quad i = 1,2, \tag{3.26}$$

where e_{12} and e_{11} denote the "switching and nonswitching quality efficiencies," respectively. In other words, e_{1i} represents the efficiency to produce a good quality part if product type is kept constant ($i = 1$) or changed ($i = 2$). By comparing the results with the results of one product case, the following is derived.

Corollary 3.6. *Under assumptions (i)–(iii), the following statements hold for the equal product-type case:*

(a)

$$P(g) = \frac{\mu_{11}}{\lambda_{11}+\mu_{11}} \quad \text{if } e_{11} = e_{12}, \tag{3.27}$$

(b)

$$P(g) < \frac{\mu_{11}}{\lambda_{11}+\mu_{11}} \quad \text{if } e_{11} > e_{12}, \tag{3.28}$$

(c)

$$P(g) > \frac{\mu_{11}}{\lambda_{11}+\mu_{11}} \quad \text{if } e_{11} < e_{12}. \tag{3.29}$$

From (3.26), we have $\lambda_{1i}+\mu_{1i} = \mu_{1i}/e_{1i}$, $i = 1,2$. Then expression (3.25) can be rewritten into

$$P(g) = \frac{\mu_{11}+(n-1)\mu_{12}}{\mu_{11}/e_{11}+(n-1)(\mu_{12}/e_{12})} = e_{11}\left[\frac{\mu_{11}+(n-1)\mu_{12}}{\mu_{11}+(n-1)\mu_{12}\cdot(e_{11}/e_{12})}\right]. \tag{3.30}$$

The statements follow immediately by replacing e_{12} with e_{11} in the denominator.

Corollary 3.6 implies that when $e_{11} = e_{12}$, that is, quality efficiency does not change whether the product types are changed or not, we can obtain $P(g)$ with the same method as in one product case. In other words, if introducing a new product does not change the quality failure or repair probabilities and the product mix does not affect the quality efficiency, then the same quality performance can be achieved, which agrees with our intuition. However, if $e_{11} > e_{12}$, that is, switching quality efficiency is decreased compared to nonswitching, then introducing an additional product will lead to a decrease in system quality performance. Finally, a flexible system can perform better on different products in terms of quality only when the switching quality efficiency is improved with the additional products, that is, $e_{12} > e_{11}$.

Since in many cases much more effort may be needed to keep e_{12} the same as or larger than e_{11}, this result indicates that frequently changing product types may lead to quality degradation in a multiple-product environment. Therefore, using batch operation to reduce product transitions may be an alternative solution to keep both product flexibility and high quality performance

3.4.2. Less versus more product types. Now we consider how the number of product types may affect quality. This is based on the investigation of the monotonic property of $P(g)$ as a function of number of product types n.

Corollary 3.7. *Under assumptions (i)–(iii), the good part probability $P(g)$ is monotonically decreasing or increasing with respect to the number of product types n if $e_{11} > e_{12}$ or $e_{11} < e_{12}$, respectively.*

For the proof, see the appendix.

Corollary 3.7 suggests that when the switching quality efficiency is not as good as non-switching efficiency, introducing more products may be harmful for overall quality performance of the system. Therefore, to ensure maintaining desired quality performance, every effort has to be made to achieve $e_{12} \geq e_{11}$.

3.4.3. Random versus sequenced part flows. To further investigate this phenomenon, consider the following two systems, A and B, both producing n equal part types. System A adopts a sequencing policy with part types 1 to n being mixed randomly with uniform distribution (as described in assumption (iii)), while system B keeps strict alternative sequences $1,2,\dots,n,1,2,\dots,n,1,2,\dots$, that is, product type changes at the end of every cycle. Clearly, from (3.25),

$$P(g)^A = \frac{\mu_{11} + (n-1)\mu_{12}}{\lambda_{11} + \mu_{11} + (n-1)(\lambda_{12} + \mu_{12})}, \tag{3.31}$$

where $P(g)^A$ defines the good job probability of system A. For system B, product type is changed at every cycle, therefore,

$$P(g)^B = \frac{\mu_{12}}{\lambda_{12} + \mu_{12}}. \tag{3.32}$$

Comparing $P(g)^A$ and $P(g)^B$, we have

$$\begin{aligned}
P(g)^A - P(g)^B &= \frac{\mu_{11} + (n-1)\mu_{12}}{\lambda_{11} + \mu_{11} + (n-1)(\lambda_{12} + \mu_{12})} - \frac{\mu_{12}}{\lambda_{12} + \mu_{12}} \\
&= \frac{\lambda_{12}\mu_{11} - \lambda_{11}\mu_{12}}{[\lambda_{11} + \mu_{11} + (n-1)(\lambda_{12} + \mu_{12})](\lambda_{12} + \mu_{12})} \\
&= \frac{\mu_{11}\mu_{12}(e_{11} - e_{12})}{e_{11}e_{12}[\lambda_{11} + \mu_{11} + (n-1)(\lambda_{12} + \mu_{12})](\lambda_{12} + \mu_{12})}.
\end{aligned} \tag{3.33}$$

Therefore, if $e_{11} > e_{12}$, we obtain $P(g)^A > P(g)^B$. It implies that when quality efficiency is decreased for changing products, using randomly mixed sequence has better quality performance than using strictly alternating sequence policy, since the former one has less

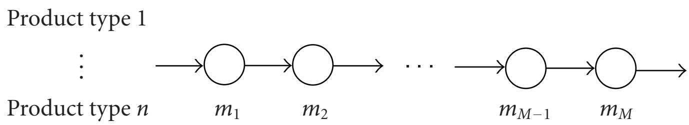

Figure 3.3. M-machine line.

transitions among products. Again, it indicates that using batch processing may lead to a better quality performance than the sequencing policy. A thorough investigation of batch production is important and is a topic in future work.

3.5. Extensions to multistage flexible systems. Now we consider a flexible system consisting of multistages as shown in Figure 3.3, where the circles represent each stage. Introduce the following additional assumption.

(iv) Each stage of flexible system, m_i, only performs its own function, and therefore each stage is independent. In other words, downstream stages could not correct the defects introduced by upstream stages.

Let $P(g(i))$, $i = 1,\dots,M$, be the probability of producing a good part at stage i, then the overall probability to produce a good part for an M-stage flexible line would be

$$P(G) = \prod_{i=1}^{M} P(g(i)). \tag{3.34}$$

Introduce $\lambda_{i,kj}$ and $\mu_{i,kj}$, $i = 1,\dots,M$, $k, j = 1,\dots,n$, to be the transition probabilities from state g_k to state d_j, or from d_k to g_j for machine i. Then for the case of n equal part types, we obtain

$$P(G) = \prod_{i=1}^{M} \frac{\mu_{i,11} + (n-1)\mu_{i,12}}{\lambda_{i,11} + \mu_{i,11} + (n-1)(\lambda_{i,12} + \mu_{i,12})}. \tag{3.35}$$

In the case where all stages are identical, the first subscripts in $\lambda_{i,kj}$ and $\mu_{i,kj}$ can be omitted, we have

$$P(G) = [P(g(i))]^M = \left[\frac{\mu_{11} + (n-1)\mu_{12}}{\lambda_{11} + \mu_{11} + (n-1)(\lambda_{12} + \mu_{12})}\right]^M. \tag{3.36}$$

Similar insights can be obtained when we compare the results with the single-stage multiple-product-type case (where quality performance is $[\mu_{11}/(\lambda_{11} + \mu_{11})]^M$). In other words, when switching quality efficiency is kept the same as nonswitching in mixed products environment, that is, $e_{11} = e_{12}$, the same quality performance as single product case can be achieved. However, if quality efficiency is decreased for changing products, $e_{11} > e_{12}$, then additional product type can decrease the system quality performance. Only when $e_{12} > e_{11}$, multiple-product system has better quality performance. Therefore, to ensure a flexible manufacturing system having high quality performance, the quality efficiency for changing products must be equivalent to or better than that for single product.

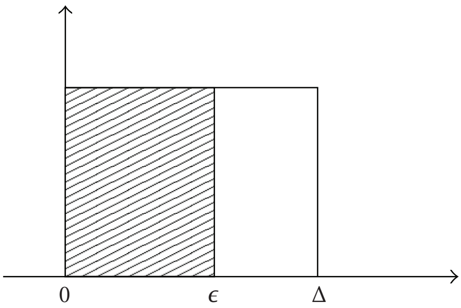

Figure 4.1. Locator discrepancy and tolerance range.

Remark 3.8. Note that in the above multistage model of flexible systems, only quality performance is addressed and the issues of buffers and inventory are not investigated. However, even if buffers are considered, since the current formulation does not include machine breakdown, all parts, no matter good or defective, will flow into and out of the buffer without interruptions. Moreover, even when productivity (e.g., machine breakdowns) is taken into consideration, a *separation principle* can be applied, that is, as long as there are no actions (e.g., scrap, rework, etc.) taking at each stage, we can simply *separate* the analysis of quality and productivity (similar to the separation principle in control theory) by evaluating the good part probability and production volume independently. Only when we reach the stage where some actions are taken, integrated analysis is needed. Such integrated study will be a topic of future work.

4. An example in flexible machining system

Consider a drilling operation in a flexible machining system that drills a hole on part type A and part type B. The system has a flexible fixture. When a job comes in, the fixture can adapt itself to predesigned locations (referred to as L_a and L_b for part types A and B, resp.) in order to hold the part, then the drill will take place. Now assuming incoming parts are in a random order mixed with types A and B (assumption (iii)), then the fixture may move to location L_a when part type A is coming, then to L_b when B is coming, and may return to L_a after some time to process A again. Since the fixture is not perfect, the L_as (correspondingly, L_bs) may not be the same as the designed L_a (correspondingly, L_b). One way of evaluating it is to measure the distance between the real L_a (correspondingly, L_b) and the ideal location. Figure 4.1 shows discrepancy of a locator from its nominal position, assuming the locator can be anywhere between the "ideal" location 0 and distance Δ_a or Δ_b with uniform distribution for parts A and B, respectively. It is clear that when the locator (e.g., L_a) is too far from the designed (ideal) location, the hole will be drilled on a wrong place, which will cause a quality defect. On the other hand, when the locator is within the designed tolerance (shown in Figure 4.1 as ϵ), it will not hurt the hole drilling.

Now we assume that the flexible fixture is the only factor that causes quality defects. (It is common that the locating error is much larger than the tooling error.) Then the probability of a part with good quality is ϵ/Δ_a for part type A (correspondingly, ϵ/Δ_b

for part type B), denoted as δ_a (correspondingly, δ_b), indicating the probability that the locator moves to a satisfactory location.

Assuming δ_a and δ_b are independent of the locator's starting location, then the transition matrix of the states of this problem (making part A and part B) becomes

$$P_{\text{transition}} = \begin{pmatrix} \nu_{11} & \lambda_{11} & \nu_{12} & \lambda_{12} \\ \mu_{11} & \eta_{11} & \mu_{12} & \eta_{12} \\ \nu_{21} & \lambda_{21} & \nu_{22} & \lambda_{22} \\ \mu_{21} & \eta_{21} & \mu_{22} & \eta_{22} \end{pmatrix} \tag{4.1}$$

$$= \begin{pmatrix} P(\overline{\text{move}}, \text{good})P_a & P(\overline{\text{move}}, \text{bad})P_a & P(\text{ move}, \text{good})P_b & P(\text{ move}, \text{bad})P_b \\ P(\overline{\text{move}}, \text{good})P_a & P(\overline{\text{move}}, \text{bad})P_a & P(\text{move}, \text{good})P_b & P(\text{move}, \text{bad})P_b \\ P(\text{move}, \text{good})P_a & P(\text{move}, \text{bad})P_a & P(\overline{\text{move}}, \text{good})P_b & P(\overline{\text{move}}, \text{bad})P_b \\ P(\text{move}, \text{good})P_a & P(\text{move}, \text{bad})P_a & P(\overline{\text{move}}, \text{good})P_b & P(\overline{\text{move}}, \text{bad})P_b \end{pmatrix}, \tag{4.2}$$

where P_a and P_b are the probabilities that the next job is part A or B, respectively, and $P_a + P_b = 1$. $P(\text{move}, \text{good})$ and $P(\text{move}, \text{bad})$ are the probabilities that the locator has moved and is in a "good" or "bad" location, respectively. Similarly, $P(\overline{\text{move}}, \text{good})$ and $P(\overline{\text{move}}, \text{bad})$ are the probabilities that the locator has not moved and is in a "good" or "bad" location, respectively.

This matrix can be simplified. For example, when the locator is in "good" location producing part A, then it does not move if the next job is still part A, and the transition probability of making a good part A (correspondingly, bad part A) will be only determined by P_a (correspondingly, 0). (Note that here we assume that location error is the only source for defects.) This is because when the locator is in a good position and the next job belongs to the same type, the probability of making another good job is 1. Similarly, if it is in the "good" location producing part A, but the next job is part B, the locator will move. The probability of moving to a "good" position (making a good part B) is δ_b. Therefore the transition probability from good A location to good B location is $\delta_b P_b$. Repeat this process, and finally we can obtain a simplified transition matrix:

$$P_{\text{transition}} = \begin{pmatrix} \nu_{11} & \lambda_{11} & \nu_{12} & \lambda_{12} \\ \mu_{11} & \eta_{11} & \mu_{12} & \eta_{12} \\ \nu_{21} & \lambda_{21} & \nu_{22} & \lambda_{22} \\ \mu_{21} & \eta_{21} & \mu_{22} & \eta_{22} \end{pmatrix} = \begin{pmatrix} P_a & 0 & \delta_b P_b & (1-\delta_b)P_b \\ 0 & P_a & \delta_b P_b & (1-\delta_b)P_b \\ \delta_a P_a & (1-\delta_a)P_a & P_b & 0 \\ \delta_a P_a & (1-\delta_a)P_a & 0 & P_b \end{pmatrix}. \tag{4.3}$$

With the above relationship, we obtain values for variables λ_{ij}, μ_{ij}, ν_{ij} and η_{ij}, $i, j = 1,2$. Then, using Theorem 3.3, the good part probability is obtained as

$$P(g) = \frac{\mathcal{F}}{\mathcal{F} + \mathcal{G}}, \tag{4.4}$$

where

$$
\begin{aligned}
\mathcal{F} &= -P_a(1-\delta_a)\delta_b P_b \delta_a P_a + (1-P_b+\delta_b P_b)(1-P_a)\delta_a P_a + (1-P_a+\delta_a P_a)(1-\delta_a)P_a\delta_b P_b \\
&= P_a[-(1-\delta_a)\delta_a\delta_b P_a P_b + (1-P_b+\delta_b P_b)(1-P_a)\delta_a + (1-P_a+\delta_a P_a)(1-\delta_a)\delta_b P_b] \\
&= P_a(1-P_a)[(1-\delta_a)\delta_b P_b + \delta_a(1-P_b+\delta_b P_b)] \\
&= P_a P_b(\delta_a P_a + \delta_b P_b), \\
\mathcal{G} &= \delta_a P_a \delta_b P_b (1-\delta) P_a - \delta_b P_b (1-P_a)(1-\delta_a) P_a \\
&\quad + [(1-P_a)(1-P_b) - \delta_b P_b \delta_a P_a][1-P_a+(1-\delta_a)P_a] \\
&= \delta_a\delta_b P_a^2 P_b (1-\delta_a) - \delta_b P_a P_b^2 (1-\delta_a) + P_a P_b (1-\delta_a\delta_b)[P_b + (1-\delta_a)P_a] \\
&= P_a P_b [P_a\delta_a\delta_b(1-\delta_a) - P_b\delta_b(1-\delta_a) + (1-\delta_a\delta_b)(P_b+(1-\delta_a)P_a)] \\
&= P_a P_b [(1-\delta_a)(P_a\delta_a - P_b)\delta_b + (1-\delta_a\delta_b)(1-P_a\delta_a)].
\end{aligned}
\tag{4.5}
$$

It follows that

$$
\begin{aligned}
\mathcal{F}+\mathcal{G} &= P_a P_b [(\delta_a P_a + \delta_b P_b) + (1-\delta_a)(P_a\delta_a - P_b)\delta_b + (1-\delta_a\delta_b)(1-P_a\delta_a)] \\
&= P_a P_b (1 - \delta_a\delta_b + P_a\delta_a\delta_b + P_b\delta_a\delta_b) = P_a P_b.
\end{aligned}
\tag{4.6}
$$

Therefore, we obtain

$$
P(g) = \frac{P_a(1-P_a)[\delta_a(1-P_b)+\delta_b P_b]}{P_a P_b} = \delta_a P_a + \delta_b P_b. \tag{4.7}
$$

Furthermore, it is reasonable to assume that Δ_a and Δ_b would be the same in many cases. Therefore $\delta_a = \delta_b = \delta$, and we obtain $P(g) = \delta$, that is, the probability of making a good part depends only on the flexible locators, which is consistent with our intuition.

Applying the same concept to three-product case, we assume that three products A, B, and C are manufactured with the flexible locator. For simplicity, here we only consider the case of $\delta_a = \delta_b = \delta_c = \delta$. We compose the matrix A in (3.19) and simplify it as follows:

$$
\begin{aligned}
A &= \begin{pmatrix}
\nu_{11}-1 & \nu_{21} & \nu_{31} & \mu_{11} & \mu_{21} & \mu_{31} \\
\nu_{12} & \nu_{22}-1 & \nu_{32} & \mu_{12} & \mu_{22} & \mu_{32} \\
\nu_{13} & \nu_{23} & \nu_{33}-1 & \mu_{13} & \mu_{23} & \mu_{33} \\
\lambda_{11} & \lambda_{21} & \lambda_{31} & \eta_{11}-1 & \eta_{21} & \eta_{31} \\
\lambda_{12} & \lambda_{22} & \lambda_{32} & \eta_{12} & \eta_{22}-1 & \eta_{32} \\
1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix} \\
&= \begin{pmatrix}
P_a-1 & \delta P_a & \delta P_a & 0 & \delta P_a & \delta P_a \\
\delta P_b & P_b-1 & \delta P_b & \delta P_b & 0 & \delta P_b \\
\delta P_c & \delta P_c & P_c-1 & \delta P_c & \delta P_c & 0 \\
0 & (1-\delta)P_a & (1-\delta)P_a & P_a-1 & (1-\delta)P_a & (1-\delta)P_a \\
(1-\delta)P_b & 0 & (1-\delta)P_b & (1-\delta)P_b & P_b-1 & (1-\delta)P_b \\
1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}.
\end{aligned}
\tag{4.8}
$$

After some simplification and rearrangement (see the appendix for details), we can finally reach

$$P(g_a) = \delta P_a, \qquad P(g_b) = \delta P_c, \qquad P(g_c) = \delta P_c, \tag{4.9}$$

where g_a, g_b, and g_c denote that the system is in good states producing parts A, B, and C, respectively. Therefore, the probability of making a good part is

$$P(g) = P(g_a) + P(g_b) + P(g_c) = \delta(P_a + P_b + P_c) = \delta. \tag{4.10}$$

This result again is consistent with the one of two-product case and matches our expectation. It also verifies the analysis presented in Section 3.

For more than three-product case, assume there are n products, and all $\delta_i = \delta$, $i = 1,\dots,n$. By induction, we can show that $P(g) = \delta$ holds again. The idea of the proof is as follows. We first show that the base case ($n = 2$) is true (4.7). Next, we assume that the case $n = k - 1$ is true. Then for case $n = k$, we can group the first $k - 1$ products into an aggregated product since they result in good part probability equal to δ. Now we only have two products, the aggregated product and product k. Using the results for $n = 2$, we prove that the case $n = k$ is also true, which will lead to the good part probability equal to δ for n products as well.

It is not surprising that the probability of making a good part is not dependent on the number of products nor the penetration of each product, since we assume that the quality is only determined by the locators with the same δ. This implies that once we can control the flexible fixture (locator), introducing more products will not hurt product quality. However, when δ's are not identical for different products, then the system quality performance will be dependent on the number of products, their respective δ, and different ratios of product mix.

5. Conclusions

Manufacturing system design has a significant impact on product quality as well as other factors. The quality performance of a flexible manufacturing system is less studied and often assumed unchanged compared to dedicated production lines. In this paper, we develop a quantitative model to evaluate the quality performance of a flexible manufacturing system using a discrete Markov chain. We derive closed formulas to calculate good part probability and show that the quality of a flexible system depends on the quality efficiency during transitions of different products. An example in a flexible machining system is presented to illustrate the applicability of the method and verify the results obtained in the paper.

The work presented in this paper provides a possible approach for further investigation of the coupling between flexibility and product quality. The future work can be directed to, first, extend the model to multiple-stage flexible lines with correlated quality propagations (e.g., variation stack-up), where the quality performance of a flexible system is also dependent on the condition of incoming parts; second, extend the model to investigate flexible lines with batch or sequenced production to evaluate the impacts of different scheduling and control policies on quality; third, integrate with online and offline inspections, quality repair, and maintenance scheduling, and so forth; fourth, integrate with multiple-product throughput analysis models with quality control devices and study the tradeoffs among productivity, quality, and order delivery; and finally, apply the method to model and analyze different flexible manufacturing systems. The results of such study will provide production engineers and managers a better understanding of the quality implications and to summarize some general guidelines for operation management in flexible manufacturing systems.

Appendices

A. Proofs

Due to page limitation, we provide here only the sketches of the proofs. The complete proof can be found in [37].

Proof of Theorem 3.3. From transition equation (3.10), we have

$$P(d_2) = \frac{1}{\mu_{21}}[(1-\nu_{11})P(g_1) - \nu_{21}P(g_2) - \mu_{11}P(d_1)]. \tag{A.1}$$

Substituting (A.1) into (3.11), we have

$$\begin{aligned}(1-\nu_{22})\mu_{21}P(g_2) = \nu_{12}\mu_{21}P(g_1) + \mu_{12}\mu_{21}P(d_1) + \mu_{22}(1-\nu_{11})P(g_1) \\ - \mu_{22}\nu_{21}P(g_2) - \mu_{22}\mu_{11}P(d_1),\end{aligned} \tag{A.2}$$

which leads to

$$P(d_1) = \frac{[(1-\nu_{22})\mu_{21} + \mu_{22}\nu_{21}]P(g_2) - [(1-\nu_{11})\mu_{22} + \mu_{21}\nu_{12}]P(g_1)}{\mu_{12}\mu_{21} - \mu_{11}\mu_{22}}. \tag{A.3}$$

Substituting into (A.1), we obtain

$$P(d_2) = \frac{[\mu_{11}\nu_{12} + \mu_{12}(1-\nu_{11})]P(g_1) - [\mu_{11}(1-\nu_{22}) + \mu_{12}\nu_{21}]P(g_2)}{\mu_{12}\mu_{21} - \mu_{11}\mu_{22}}. \tag{A.4}$$

Rewriting (3.12), we have

$$(1-\eta_{11})P(d_1) = \lambda_{11}P(g_1) + \lambda_{21}P(g_2) + \eta_{21}P(d_2). \tag{A.5}$$

Again substituting (A.3) and (A.4), we obtain

$$\begin{aligned}(1-\eta_{11})&\frac{[(1-\nu_{22})\mu_{21} + \mu_{22}\nu_{21}]P(g_2) - [(1-\nu_{11})\mu_{22} + \mu_{21}\nu_{12})P(g_1)}{\mu_{12}\mu_{21} - \mu_{11}\mu_{22}} \\ &= \lambda_{11}P(g_1) + \lambda_{21}P(g_2) \\ &\quad + \eta_{21}\frac{[\mu_{11}\nu_{12} + \mu_{12}(1-\nu_{11})]P(g_1) - [\mu_{11}(1-\nu_{22}) + \mu_{12}\nu_{21}]P(g_2)]}{\mu_{12}\mu_{21} - \mu_{11}\mu_{22}}.\end{aligned} \tag{A.6}$$

It follows that

$$\begin{aligned}
&[(1-\eta_{11})(1-\nu_{22})\mu_{21}+(1-\eta_{11})\mu_{22}\nu_{21}-\lambda_{21}(\mu_{12}\mu_{21}-\mu_{11}\mu_{22})\\
&\quad+\eta_{21}\mu_{11}(1-\nu_{22})+\eta_{21}\mu_{12}\nu_{21}]P(g_2)\\
&\qquad=[\lambda_{11}(\mu_{12}\mu_{21}-\mu_{11}\mu_{22})+\eta_{21}\mu_{11}\nu_{12}+\eta_{21}\mu_{12}(1-\nu_{11})\\
&\qquad\quad+(1-\eta_{11})(1-\nu_{11})\mu_{22}+(1-\eta_{11})\mu_{21}\nu_{12}]P(g_1).
\end{aligned}\tag{A.7}$$

Therefore,

$$\begin{aligned}
&P(g_2)\\
&\quad=\frac{\lambda_{11}(\mu_{12}\mu_{21}-\mu_{11}\mu_{22})+\eta_{21}[\mu_{11}\nu_{12}+\mu_{12}(1-\nu_{11})]+(1-\eta_{11})[(1-\nu_{11})\mu_{22}+\mu_{21}\nu_{12}]}{(1-\eta_{11})[(1-\nu_{22})\mu_{21}+\mu_{22}\nu_{21}]-\lambda_{21}(\mu_{12}\mu_{21}-\mu_{11}\mu_{22})+\eta_{21}[\mu_{11}(1-\nu_{22})+\mu_{12}\nu_{21}]}P(g_1).
\end{aligned}\tag{A.8}$$

From total probabilities equal to 1, that is,

$$P(g_1)+P(g_2)+P(d_1)+P(d_2)=1,\tag{A.9}$$

we obtain

$$\begin{aligned}
&P(g_1)+P(g_2)+\frac{[(1-\nu_{22})\mu_{21}+\mu_{22}\nu_{21}]P(g_2)-[(1-\nu_{11})\mu_{22}+\mu_{21}\nu_{12}]P(g_1)}{\mu_{12}\mu_{21}-\mu_{11}\mu_{22}}\\
&\quad+\frac{[\mu_{11}\nu_{12}+\mu_{12}(1-\nu_{11})]P(g_1)-[\mu_{11}(1-\nu_{22})+\mu_{12}\nu_{21}]P(g_2)}{\mu_{12}\mu_{21}-\mu_{11}\mu_{22}}=1,
\end{aligned}\tag{A.10}$$

which implies that

$$\begin{aligned}
\mu_{12}\mu_{21}-\mu_{11}\mu_{22}&=P(g_1)[\mu_{12}(1-\nu_{11}+\mu_{21})-\mu_{22}(1-\nu_{11}+\mu_{11})+(\mu_{11}-\mu_{21})\nu_{12}]\\
&\quad+P(g_2)[\mu_{21}(1-\nu_{22}+\mu_{12})-\mu_{11}(1-\nu_{22}+\mu_{22})+\nu_{21}(\mu_{22}-\mu_{12})].
\end{aligned}\tag{A.11}$$

For simplification purpose, introduce the following notations:

$$\begin{aligned}
\mathcal{A}&=\mu_{12}\mu_{21}-\mu_{11}\mu_{22},\\
\mathcal{B}&=(1-\nu_{11})(\mu_{12}-\mu_{22})+(\mu_{11}-\mu_{21})\nu_{12}+\mathcal{A},\\
\mathcal{C}&=(1-\eta_{11})[(1-\nu_{22})\mu_{21}+\mu_{22}\nu_{21}]-\lambda_{21}\mathcal{A}+\eta_{21}[\mu_{11}(1-\nu_{22})+\mu_{12}\nu_{21}],\\
\mathcal{D}&=\lambda_{11}\mathcal{A}+\eta_{21}[\mu_{11}\nu_{12}+\mu_{12}(1-\nu_{11})]+(1-\eta_{11})[(1-\nu_{11})\mu_{22}+\mu_{21}\nu_{12}],\\
\mathcal{E}&=(1-\nu_{22})(\mu_{21}-\mu_{11})+(\mu_{22}-\mu_{12})\nu_{21}+\mathcal{A}.
\end{aligned}\tag{A.12}$$

Replacing into (A.11), we obtain

$$\mathcal{A}=\mathcal{B}P(g_1)+\mathcal{E}P(g_2).\tag{A.13}$$

From (A.8), we have

$$P(g_2)=\frac{\mathcal{D}}{\mathcal{C}}P(g_1),\tag{A.14}$$

then

$$\mathcal{A} = \mathcal{B}P(g_1) + \frac{\mathcal{D}}{\mathcal{C}}\mathcal{E}P(g_1). \tag{A.15}$$

It follows that

$$\begin{aligned} P(g_1) &= \frac{\mathcal{A}}{\mathcal{B} + (\mathcal{D}/\mathcal{C})\mathcal{E}} = \frac{\mathcal{A}\mathcal{C}}{\mathcal{B}\mathcal{C} + \mathcal{D}\mathcal{E}}, \\ P(g_2) &= \frac{\mathcal{D}}{\mathcal{C}}P(g_1) = \frac{\mathcal{A}\mathcal{D}}{\mathcal{B}\mathcal{C} + \mathcal{D}\mathcal{E}}. \end{aligned} \tag{A.16}$$

Therefore,

$$P(g) = P(g_1) + P(g_2) = \frac{\mathcal{A}(\mathcal{C} + \mathcal{D})}{\mathcal{B}\mathcal{C} + \mathcal{D}\mathcal{E}}. \tag{A.17}$$

To continue simplifying the equations, we obtain

$$\begin{aligned} \mathcal{C} + \mathcal{D} = {} & (1 - \nu_{22} + \nu_{12})[(1 - \eta_{11})\mu_{21} + \eta_{21}\mu_{11}] + (\lambda_{11} - \lambda_{21})\mathcal{A} \\ & + (1 - \nu_{11} + \nu_{21})[(1 - \eta_{11})\mu_{22} + \eta_{21}\mu_{12}], \\ \mathcal{B}\mathcal{C} + \mathcal{D}\mathcal{E} = {} & \mathcal{A}(\mathcal{C} + \mathcal{D}) \\ & + \mathcal{A}[(\mu_{21} - \mu_{11})[(1 - \nu_{22})\lambda_{11} + \nu_{12}\lambda_{21}] - (\mu_{12} - \mu_{22}) \\ & \times [(1 - \nu_{11})\lambda_{21} + \lambda_{11}\nu_{21}] + [(1 - \nu_{11})(1 - \nu_{22}) - \nu_{12}\nu_{21}](1 - \eta_{11} + \eta_{21})]. \end{aligned} \tag{A.18}$$

Introduce notations $\mathcal{F}$ and $\mathcal{G}$:

$$\begin{aligned} \mathcal{F} &= \mathcal{C} + \mathcal{D}, \\ \mathcal{G} &= (\mu_{21} - \mu_{11})[(1 - \nu_{22})\lambda_{11} + \nu_{12}\lambda_{21}] - (\mu_{12} - \mu_{22})[(1 - \nu_{11})\lambda_{21} + \lambda_{11}\nu_{21}] \\ &\quad + [(1 - \nu_{11})(1 - \nu_{22}) - \nu_{12}\nu_{21}](1 - \eta_{11} + \eta_{21}). \end{aligned} \tag{A.19}$$

Then

$$\mathcal{B}\mathcal{C} + \mathcal{D}\mathcal{E} = \mathcal{A}(\mathcal{C} + \mathcal{D}) + \mathcal{A}\mathcal{G}. \tag{A.20}$$

Finally, we obtain

$$P(g) = \frac{\mathcal{A}\mathcal{F}}{\mathcal{F}\mathcal{A} + \mathcal{G}\mathcal{A}} = \frac{\mathcal{F}}{\mathcal{F} + \mathcal{G}}. \tag{A.21}$$

□

Proof of Corollary 3.5. First we aggregate all the good states g_i, $i = 1,\dots,n$, and all the defective states d_i, $i = 1,\dots,n$, into aggregated good state g_{agg} and defective state d_{agg}, respectively. Following the logic in Section 3.1, we have

$$P(g_{\text{agg}}) = P(g_{\text{agg}})(1 - \lambda_{\text{agg}}) + P(d_{\text{agg}})\mu_{\text{agg}}, \tag{A.22}$$

where λ_{agg} and μ_{agg} are the aggregated quality failure and repair probabilities. (The state transition diagram is equivalent to that of Figure 3.1 with subscripts "agg" in all notations.)

Therefore,

$$P(g_{\text{agg}}) = \frac{\mu_{\text{agg}}}{\lambda_{\text{agg}} + \mu_{\text{agg}}}. \tag{A.23}$$

In addition,

$$\begin{aligned}
\lambda_{\text{agg}} &= \sum_{i=1}^{n} \lambda_{ii} P(g_i) P(d_i) + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \lambda_{ij} P(g_i) P(d_j) \\
&= n\lambda_{11} P(g_1) P(d_1) + n(n-1)\lambda_{12} P(g_1) P(d_2), \\
\mu_{\text{agg}} &= \sum_{i=1}^{n} \mu_{ii} P(d_i) P(g_i) + \sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} \mu_{ij} P(d_i) P(g_j) \\
&= n\mu_{11} P(d_1) P(g_1) + n(n-1)\mu_{12} P(d_1) P(g_2).
\end{aligned} \tag{A.24}$$

Since all products are equally distributed, we have

$$P(d_2) = P(d_1), \qquad P(g_2) = P(g_1). \tag{A.25}$$

Therefore,

$$\begin{aligned}
\lambda_{\text{agg}} &= n\lambda_{11} P(g_1) P(d_1) + n(n-1)\lambda_{12} P(g_1) P(d_1), \\
\mu_{\text{agg}} &= n\mu_{11} P(d_1) P(g_1) + n(n-1)\mu_{12} P(d_1) P(g_1).
\end{aligned} \tag{A.26}$$

Substituting into (A.23), we obtain

$$\begin{aligned}
P(g_{\text{agg}}) &= \frac{nP(d_1)P(g_1)[\mu_{11} + n(n-1)\mu_{12}]}{nP(g_1)P(d_1)[\lambda_{11} + (n-1)\lambda_{12}] + nP(d_1)P(g_1)[\mu_{11} + (n-1)\mu_{12}]} \\
&= \frac{\mu_{11} + (n-1)\mu_{12}}{\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}}.
\end{aligned} \tag{A.27}$$

Moreover, from

$$\begin{aligned}
\frac{\partial P(g)}{\partial \mu_{11}} &= \frac{1}{\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}} - \frac{\mu_{11} + (n-1)\mu_{12}}{[\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}]^2} \\
&= \frac{\lambda_{11} + (n-1)\lambda_{12}}{[\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}]^2} > 0, \\
\frac{\partial P(g)}{\partial \mu_{12}} &= \frac{n-1}{\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}} - \frac{[\mu_{11} + (n-1)\mu_{12}](n-1)}{[\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}]^2} \\
&= \frac{[\lambda_{11} + (n-1)\lambda_{12}](n-1)}{[\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}]^2} > 0, \\
\frac{\partial P(g)}{\partial \lambda_{11}} &= -\frac{\mu_{11} + (n-1)\mu_{12}}{[\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}]^2} < 0, \\
\frac{\partial P(g)}{\partial \lambda_{12}} &= -\frac{[\mu_{11} + (n-1)\mu_{12}](n-1)}{[\lambda_{11} + (n-1)\lambda_{12} + \mu_{11} + (n-1)\mu_{12}]^2} < 0,
\end{aligned} \tag{A.28}$$

we obtain the monotonicities of $P(g)$ with respect to μ_{1i} and λ_{1i}, $i = 1,2$. □

Proof of Corollary 3.7. From

$$\begin{aligned}\frac{\partial P(g)}{\partial n} &= \frac{\mu_{12}}{\lambda_{11}+(n-1)\lambda_{12}+\mu_{11}+(n-1)\mu_{12}} - \frac{[\mu_{11}+(n-1)\mu_{12}](\lambda_{12}+\mu_{12})}{[\lambda_{11}+(n-1)\lambda_{12}+\mu_{11}+(n-1)\mu_{12}]^2}\\ &= -\frac{\mu_{11}\mu_{12}(e_{11}-e_{12})}{[\lambda_{11}+(n-1)\lambda_{12}+\mu_{11}+(n-1)\mu_{12}]^2},\end{aligned} \tag{A.29}$$

we obtain

$$\frac{\partial P(g)}{\partial n} < 0 \quad \text{if } e_{11} > e_{12}, \qquad \frac{\partial P(g)}{\partial n} > 0 \quad \text{if } e_{11} < e_{12}. \tag{A.30}$$

Therefore, $P(g)$ is monotonically decreasing or increasing with respect to n if $e_{11} > e_{12}$ or $e_{11} < e_{12}$, respectively. □

B. Solution procedure for three-product case in Section 4

Using matrix A in (4.8), we further simplify (3.18) as follows:

$$\begin{pmatrix} \frac{P_a-1-\delta P_a}{\delta P_a} & 0 & 0 & -1 & 0 & 0\\ 0 & \frac{P_b-1-\delta P_b}{\delta P_b} & 0 & 0 & -1 & 0\\ 0 & 0 & \frac{P_c-1-\delta P_c}{\delta P_c} & 0 & 0 & -1\\ -1 & 0 & 0 & \frac{\delta P_a-1}{(1-\delta)P_a} & 0 & 0\\ 0 & -1 & 0 & 0 & \frac{\delta P_b-1}{(1-\delta)P_b} & 0\\ 1 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} P(g_a)\\ P(g_b)\\ P(g_c)\\ P(d_a)\\ P(d_b)\\ P(d_c) \end{pmatrix} = \begin{pmatrix} -1\\ -1\\ -1\\ -1\\ -1\\ 1 \end{pmatrix}. \tag{B.1}$$

Therefore, we obtain

$$\begin{aligned} \frac{P_a-1-\delta P_a}{\delta P_a}P(g_a) - P(d_a) &= -1,\\ \frac{P_b-1-\delta P_b}{\delta P_b}P(g_b) - P(d_b) &= -1,\\ \frac{P_c-1-\delta P_c}{\delta P_c}P(g_c) - P(d_c) &= -1,\\ -P(g_a) + \frac{\delta P_a-1}{(1-\delta)P_a}P(d_a) &= -1,\\ -P(g_b) + \frac{\delta P_b-1}{(1-\delta)P_b}P(d_b) &= -1. \end{aligned} \tag{B.2}$$

Rearranging the first equation, we have

$$(P_a-1-\delta P_a)P(g_a) - \delta P_a P(d_a) = -\delta P_a. \tag{B.3}$$

Using $P(g_a)+P(d_a)=P_a$, it follows that

$$(P_a-1)P(g_a)-\delta P_a^2=-\delta P_a, \tag{B.4}$$

which leads to

$$P(g_a)=\frac{\delta P_a^2-\delta P_a}{P_a-1}=\delta P_a. \tag{B.5}$$

Similarly, we obtain

$$P(g_b)=\delta P_b, \qquad P(g_c)=\delta P_c. \tag{B.6}$$

Then the probability of making a good part is

$$P(g)=P(g_a)+P(g_b)+P(g_c)=\delta(P_a+P_b+P_c)=\delta. \tag{B.7}$$

Acknowledgment

The authors thank Dr. Samuel P. Marin of General Motors Research and Development Center for his valuable comments and suggestions.

References

[1] R. R. Inman, D. E. Blumenfeld, N. Huang, and J. Li, "Designing production systems for quality: research opportunities from an automotive industry perspective," *International Journal of Production Research*, vol. 41, no. 9, pp. 1953–1971, 2003.
[2] D. E. Zoia, "Harbour Outlines Who's Winning and Why," September 2005, http://WardsAuto.com/.
[3] J. Payne and V. Cariapa, "A fixture repeatability and reproducibility measure to predict the quality of machined parts," *International Journal of Production Research*, vol. 38, no. 18, pp. 4763–4781, 2000.
[4] Z. M. Bi and W. J. Zhang, "Flexible fixture design and automation: review, issues and future directions," *International Journal of Production Research*, vol. 39, no. 13, pp. 2867–2894, 2001.
[5] A. Bolat and C. Yano, "Procedures to analyze the tradeoffs between costs of setup and utility work for automobile assembly lines," Tech. Rep. 89-3, Department of Industrial and Operations Engineering, The University of Michigan, Ann Arbor, Mich, USA, 1989.
[6] "Sector Notebook—Profile of the Fabricated Metal Products Industry—Part 2," December 2005, http://www.epa.gov/compliance/resources/publications/assistance/sectors/notebooks/fabmetsnpt2.pdf.
[7] "Automobile Production at OPEL, Bochum (Gemany)," http://www.profibus.com/pall/applications/casestudies/article/3043/.
[8] D. A. Jacobs and S. M. Meerkov, "Asymptotically reliable serial production lines with a quality control system," *Computers & Mathematics with Applications*, vol. 21, no. 11-12, pp. 85–90, 1991.
[9] A. A. Bulgak, "Impact of quality improvement on optimal buffer designs and productivity in automatic assembly systems," *Journal of Manufacturing Systems*, vol. 11, pp. 124–136, 1992.
[10] M. Khouja, G. Rabinowitz, and A. Mehrez, "Optimal robot operation and selection using quality and output trade-off," *International Journal of Advanced Manufacturing Technology*, vol. 10, no. 5, pp. 342–355, 1995.

[11] N. Viswanadham, S. M. Sharma, and M. Taneja, "Inspection allocation in manufacturing systems using stochastic search techniques," *IEEE Transactions on Systems, Man, and Cybernetics Part A: Systems and Humans.*, vol. 26, no. 2, pp. 222–230, 1996.

[12] T. L. Urban, "Analysis of production systems when run length influences product quality," *International Journal of Production Research*, vol. 36, no. 11, pp. 3085–3094, 1998.

[13] C. H. Cheng, J. Miltenburg, and J. Motwani, "The effect of straight- and U-shaped lines on quality," *IEEE Transactions on Engineering Management*, vol. 47, no. 3, pp. 321–334, 2000.

[14] S. A. I. Matanachai and C. A. Yano, "Balancing mixed-model assembly lines to reduce work overload," *IIE Transactions*, vol. 33, no. 1, pp. 29–42, 2001.

[15] Y. Ding, J. Jin, D. Ceglarek, and J. Shi, "Process-oriented tolerancing for multi-station assembly systems," *IIE Transactions*, vol. 37, no. 6, pp. 493–508, 2005.

[16] J. Li and D. E. Blumenfeld, "Quantitative analysis of a transfer production line with Andon," *IIE Transactions*, vol. 38, no. 10, pp. 837–846, 2006.

[17] J. Li, D. E. Blumenfeld, and S. P. Marin, "Manufacturing system design to improve quality buy rate: an automotive paint shop application study," *IEEE Transactions on Automation Science and Engineering*, vol. 4, no. 1, pp. 75–79, 2007.

[18] J. Li, D. E. Blumenfeld, and S. P. Marin, "Production system design for quality robustness: theory and application in automotive paint shops," *IIE Transactions*, vol. 39, 2007.

[19] J. Kim and S. B. Gershwin, "Integrated quality and quantity modeling of a production line," *OR Spectrum*, vol. 27, no. 2-3, pp. 287–314, 2005.

[20] N. Viswanadham and Y. Narahari, *Performance Modeling of Automated Manufacturing System*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1992.

[21] J. A. Buzacott and J. G. Shantikumar, *Stochastic Models of Manufacturing Systems*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1993.

[22] H. Tempelmeier and H. Kuhn, *Flexible Manufacturing Systems: Decision Support for Design and Operation*, John Wiley & Sons, New York, NY, USA, 1993.

[23] M. Zhou and K. Venkatesh, *Modeling, Simulation and Control of Flexible Manufacturing Systems: A Petri Net Approach*, World Scientific, Singapore, 1999.

[24] J. A. Buzacott, "The fundamental principles of flexibility in manufacturing systems," in *Proceedings of the 1st International Conference on Flexible Manufacturing Systems*, pp. 13–22, Brighton, UK, 1982.

[25] J. A. Buzacott and D. D. Yao, "Flexible manufacturing systems: a review of analytical models," *Management Science*, vol. 32, no. 7, pp. 890–905, 1986.

[26] A. K. Sethi and S. P. Sethi, "Flexibility in manufacturing: a survey," *International Journal of Flexible Manufacturing Systems*, vol. 2, no. 4, pp. 289–328, 1990.

[27] N. Viswanadham, Y. Narahari, and T. L. Johnson, "Stochastic modelling of flexible manufacturing systems," *Mathematical and Computer Modelling*, vol. 16, no. 3, pp. 15–34, 1992.

[28] M. Barad and S. Y. Nof, "CIM flexibility measures: a review and a framework for analysis and applicability assessment," *International Journal of Computer Integrated Manufacturing*, vol. 10, no. 1–4, pp. 296–308, 1997.

[29] A. De Toni and S. Tonchia, "Manufacturing flexibility: a literature review," *International Journal of Production Research*, vol. 36, no. 6, pp. 1587–1617, 1998.

[30] R. Beach, A. P. Muhlemann, D. H. R. Price, A. Paterson, and J. A. Sharp, "A review of manufacturing flexibility," *European Journal of Operational Research*, vol. 122, no. 1, pp. 41–57, 2000.

[31] D. Shi and R. L. Daniels, "A survey of manufacturing flexibility: implications for e-business flexibility," *IBM Systems Journal*, vol. 42, no. 3, pp. 414–427, 2003.

[32] Y. K. Son and C. S. Park, "Economic measure of productivity, quality and flexibility in advanced manufacturing systems," *Journal of Manufacturing Systems*, vol. 6, no. 3, pp. 193–207, 1987.

[33] F. F. Chen and E. E. Adam Jr., "The impact of flexible manufacturing systems on productivity and quality," *IEEE Transactions on Engineering Management*, vol. 38, no. 1, pp. 33–45, 1991.

[34] N. Van Hop and K. Ruengsak, "Fuzzy estimation for manufacturing flexibility," *International Journal of Production Research*, vol. 43, no. 17, pp. 3605–3617, 2005.

[35] G. Da Silveira, D. Borenstein, and F. S. Fogliatto, "Mass customization: literature review and research directions," *International Journal of Production Economics*, vol. 72, no. 1, pp. 1–13, 2001.

[36] P. G. Hoel, S. C. Port, and C. J. Stone, *Introduction to Stochastic Processes*, Houghton Mifflin, Boston, Mass, USA, 1972.

[37] J. Li and N. Huang, "A Markovian model to evaluate quality performance in flexible manufacturing systems," Tech. Rep. R&D-10274, General Motors Research & Development Center, Warren, Mich, USA, 2005.

Jingshan Li: Department of Electrical and Computer Engineering and Center for Manufacturing, University of Kentucky, Lexington, KY 40506, USA
Email address: jingshan@engr.uky.edu

Ningjian Huang: Manufacturing Systems Research Lab, General Motors Research and Development Center, General Motors Corporation, Warren, MI 48090-9055, USA
Email address: ninja.huang@gm.com