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Letter to the Editor

Comments on "Flow of a Newtonian Fluid in a Symmetrically Heated Channel: Effect of Viscosity and Viscous Dissipation"

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Recommended by Mehrdad Massoudi

In the above paper [1] an analysis has been carried out to obtain results in the Poiseuille flow of a Newtonian fluid with viscous dissipation and temperature-dependent viscosity. The fluid viscosity is an exponential function of temperature. The equations are solved numerically with the finite difference method. However, there are some fundamental errors in this paper which are presented below.

1. The title is wrong. The channel is not symmetrically heated because the plate temperatures are not equal. The upper plate has temperature T_b and the lower plate temperature T_0 (see [1, Figure 1.1, equations (2.5) and (2.6)]). Except that all the presented results are given for different values of $\alpha = T_b - T_0$. In addition the temperature profile shown in [1, Figure 1.1] is wrong.

2. In the abstract it is mentioned that "the coupled nonlinear differential equations arising in the planar Poiseuille flow are not amendable to analytical solutions." In [1, Figure 1.1] it is shown that the two plates are motionless. In page 2 it is mentioned that "The coupled nonlinear momentum and energy equations arising in planar Poiseuille flow are solved numerically." In page 4 it is mentioned that "In this section, numerical solutions of plane Poiseuille flow are presented." In page 7 it is mentioned that "The numerical solutions of the coupled nonlinear equations arising from planar Poiseuille flow are also presented." From the above it is clear that the present work concerns a Poiseuille flow, which in fluid mechanics, is defined as the flow between two motionless plates, or the flow in a pipe driven by a pressure gradient. However, in page 3 it is mentioned that "U_0 is the constant velocity at moving surface." This is a contradiction because in

Poiseuille flow there is no moving plate and therefore U_0 is zero. If we suppose that the authors treated a Couette-Poiseuille flow and they forgot to mention it, the velocity profiles [1, Figures 4.1 and 4.3] show clearly that the fluid velocity at the two plates is zero. If the upper plate was moving, the dimensionless velocity at the upper plate in [1, Figures 4.1 and 4.3] should be 1. If we suppose that the authors treated a Poiseuille pipe flow, this problem has been already solved for a more complicated fluid (a third grade fluid) by Massoudi and Christie [2]. So we consider the present flow as Poiseuille flow between parallel plates. Taking into account this fact, the dimensionless velocity and dimensionless pressure included in [1, equation (2.5)] cannot be defined because U_0 is zero. Otherwise the authors should explain how this velocity U_0 is defined. In addition, if the present problem is a planar Poiseuille flow (which is the most probable scenario), then this problem has been already solved for a more complicated fluid (a third grade fluid) by Szeri and Rajagopal [3]. Except that the problem treated by the authors is actually stated and/or solved (in similar situations) in textbooks. For example, see Constantinescu [4, pages 208–214] and Slattery [5, pages 330–336].

3. In page 3, it is mentioned that the flow is symmetric. This is not valid because the temperatures at the two plates are unequal. The boundary conditions

$$\begin{aligned} \frac{du(0)}{dr} &= 0 \quad \text{(middle of plates)}, \\ \frac{d\theta(0)}{dr} &= 0 \quad \text{(middle of plates)}, \\ \theta(1) &= 0 \quad \text{(upper plate)} \end{aligned} \tag{1}$$

are all wrong. The first two conditions are not valid because the flow is not symmetric and the correct form, the third condition, should be

$$\theta(1) = 1 \quad \text{(upper plate)}. \tag{2}$$

4. We see that both velocity and temperature [1, Figures 4.1, 4.2, 4.3] are symmetric. However, due to unequal plate temperatures, the velocity and temperature profiles should be nonsymmetric. The correct form of the temperature profiles should be as shown in Figure 1.

5. In two recent papers, Costa and Mecedonio [6, 7] mentioned that in a steady state fully developed Poiseuille flow of an incompressible fluid with temperature-dependent viscosity and viscous dissipation, there is a dimensionless parameter G which is an important criterion for this flow. If $G > G_{\text{crit}}$ the system does not admit a solution, whereas when $G < G_{\text{crit}}$, the system has two solutions, one of which (the solution with greater temperature) may be unstable. The existence of a critical quantity which characterizes the flow in ducts for fluids with temperature-dependent viscosity including viscous heating was suggested by Grundfest [8]. A very good review on this matter is given by Sukanek

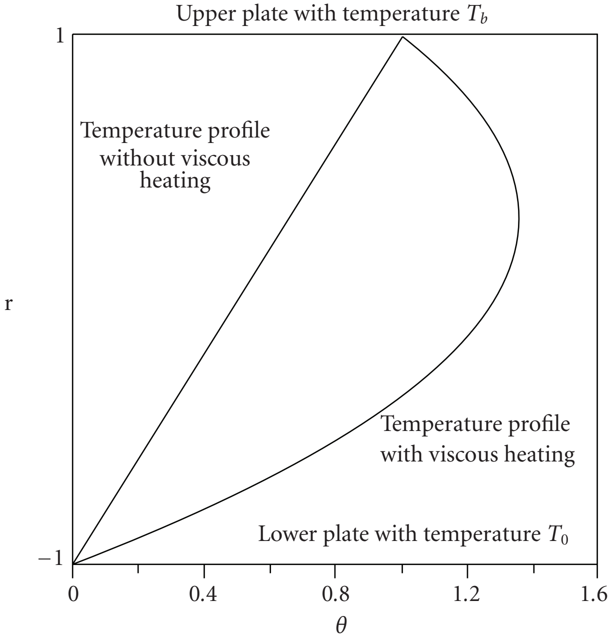

FIGURE 1. Temperature distribution in a Poiseuille flow with unequal plate temperatures and viscous heating.

and Laurence [9]. The authors ignore these criteria and mention nothing in their paper about it.

6. In conclusion this is a completely wrong paper.

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