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Research Article

Model Validation Using Coordinate Distance with Performance Sensitivity

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This paper presents an innovative approach to model validation for a structure with significant parameter variations. Model uncertainty of the structural dynamics is quantified with the use of a singular value decomposition technique to extract the principal components of parameter change, and an interval model is generated to represent the system with parameter uncertainty. The coordinate vector, corresponding to the identified principal directions, of the validation system is computed. The coordinate distance between the validation system and the identified interval model is used as a metric for model validation. A beam structure with an attached subsystem, which has significant parameter uncertainty, is used to demonstrate the proposed approach.

1. Introduction

Model validation of structural dynamics is of great interest to both government and industry [1]. Recently, a model validation workshop [2, 3] was organized by Sandia National Laboratories to address the problem of certification of structures under various forms of uncertainty. Following their formulation, an integrated system consisting of a beam structure and an attached subsystem, shown in Figure 1, is the test structure used for study. In this model the physical elements of the attached three degrees of freedom subsystem are the only ones exhibiting significant parameter variations, all other parameters are known. The substructure, along with its nonlinear connection, is considered for calibration, and data are provided as a basis for the calibration of the substructure model [2].

In the process of certifying structures for use in harsh dynamic environments, it is often required that not only the main structure be capable of withstanding the loads but also all the attached substructures. To ensure survivability of all the substructures, Sandia in [2] has chosen a performance metric in terms of the maximum acceleration magnitude of mass 3, top of the substructure, under a shock force at position x_8. For this study, the uncertain

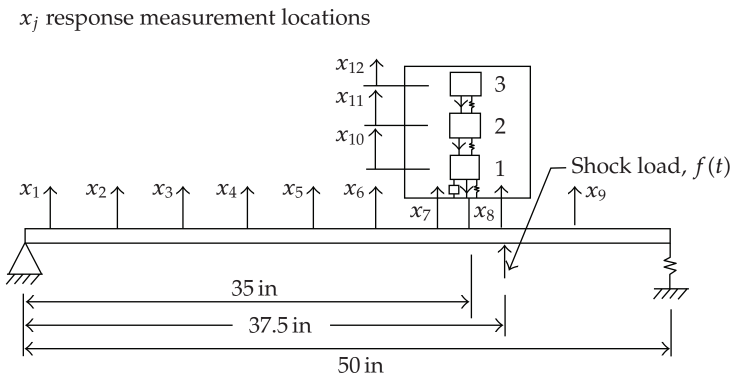

Figure 1: A beam structure with an attached subsystem.

parameters are the identified modal parameters (frequency, damping, and mode shape) of subsystem, 15 parameters total.

This paper presents a model validation methodology based on an interval modeling technique for the structural dynamics problem proposed by Sandia [2]. A singular value decomposition technique [4] is applied to extract the principal components of parameter change, where the sensitivity of performance is included in the SVD process. From this process, an interval model is generated and each interval corresponds to one identified bounded uncertainty parameter with its associated principal direction. This interval modeling technique can precisely quantify the uncertainty of a system with significant parameter uncertainty [4]. The coordinate vector, corresponding to the identified principal directions, of the validation system can be computed. The coordinate distance between the validation system and the identified interval model is used as the metric for model validation [5].

2. Model validation

In the model validation process, first an interval modeling technique, given in the appendix, is applied for uncertainty quantification. The data used for model uncertainty quantification are based on the identified modal parameters from 60 virtual experiments [2], generated from 20 identical systems selected from a virtual pool and three levels of random excitation applied at mass 2. The modal parameter vector of the subsystem is defined as

$$p = \left[\omega_1 \;\; \omega_2 \;\; \omega_3 \;\; \xi_1 \;\; \xi_2 \;\; \xi_3 \;\; \phi_{11} \;\; \phi_{21} \;\; \phi_{31} \;\; \phi_{12} \;\; \cdots \;\; \phi_{33}\right]^T, \tag{2.1}$$

where ω_i is the ith natural frequency, ξ_i is the ith damping ratio, and ϕ_{ji} is the jth component of the ith mode shape. The interval modeling technique in the appendix is applied to generate an interval model as

$$P = \left\{ p \mid p = p_0 + \sum_{j=1}^{15} \alpha_j q_j,\ \alpha_j \in \left[\alpha_j^-, \alpha_j^+\right] \right\}, \tag{2.2}$$

where p_0 is the nominal parameter vector, and α_j is the jth identified bounded uncertainty parameter corresponding to the basis vector q_j. The coordinate vector of any validation system with parameter vector p^v can be computed as

$$\beta^v = U^{-1} \Delta p^v, \tag{2.3}$$

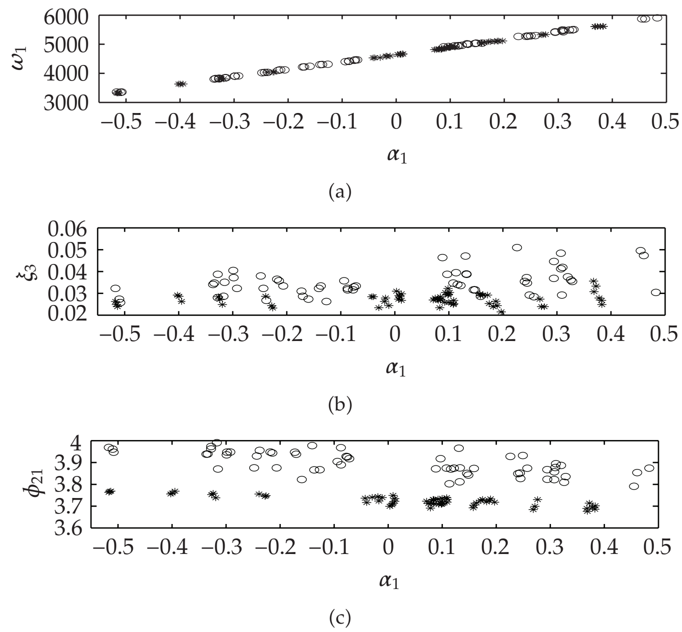

Figure 2: Modal parameters of subsystems: (a) natural frequency (rad/sec) of 1st mode, (b) damping ratio of 3rd mode, (c) 2nd mode shape coefficient of 1st mode. (circels) 60 calibration systems; (asterisks) 60 validation systems.

with

$$\Delta p^v = p^v - p_0, \qquad U = \left[q_1 \ \cdots \ q_{15} \right], \tag{2.4}$$

where U is the basis matrix. The coordinate distance between a validation system and the interval model is defined as

$$d^v = \min\left\{ \sqrt{\left[\beta^v - \beta(p)\right]^T \left[\beta^v - \beta(p)\right]},\ p \in P \right\}, \tag{2.5}$$

where $\beta(p)$ is the coordinate vector of the subsystem with parameter vector p. This distance represents a metric of performance deviation between a validation system and the identified interval model since the weighting of performance sensitivity is included in SVD process [4, 5].

3. Discussion of results

There are 60 sets of identified modal parameters used for model validation [2], generated from 20 identical systems selected from a virtual pool with three levels of shock input at mass 1. Figure 2 shows three modal parameters of 60 calibration systems and 60 validation systems as functions of the first uncertainty parameter α_1. Variations in the natural frequencies are significant, around 100%, and increase linearly as the first uncertainty parameter α_1 increases. Natural frequencies of calibration systems and validation systems share same variation characteristics. Damping and mode shape coefficients of validation systems show bias from those of calibration systems. For example, the mean value of ξ_3 of the validation systems

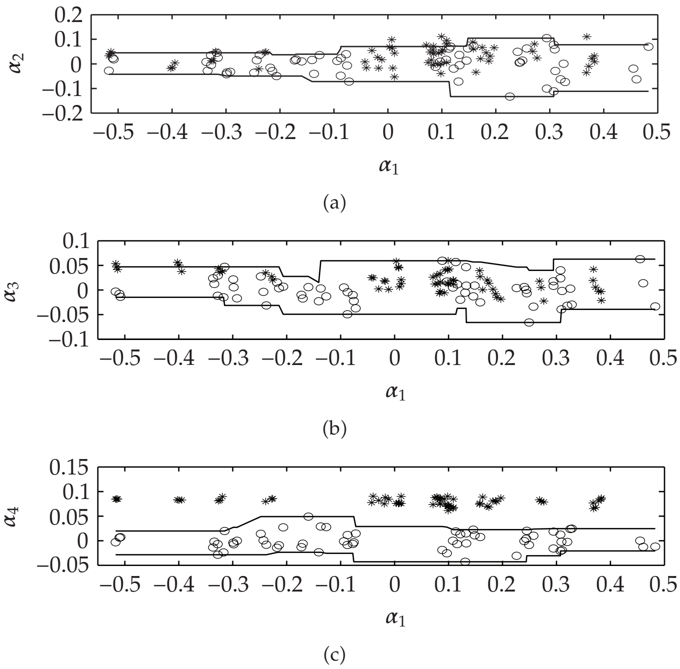

Figure 3: Coordinates and parameter bounds of uncertainty parameters: (a) 2nd uncertainty parameter, (b) 3rd uncertainty parameter, (c) 4th uncertainty parameter. (circles) 60 calibration systems; (asterisks) 60 validation systems; — parameter bounds of interval model.

is around 30% lower than that of the calibration systems when α_1 is 0.1. For the second mode shape coefficient of the first mode in Figure 2, the mean value for the validation systems is always around 5% lower than that of the calibration systems. Figure 3 shows the uncertainty parameters α_2–α_4 and the identified interval bounds as functions of the first uncertainty parameter α_1. The third interval length, normalized to the first interval length (i.e., $\alpha_1 = 1$), drops to less than 10% (see Figure 3) of the first interval length [4]. The model uncertainty is dominated by the first uncertainty parameter α_1. Natural frequency variations are the dominant uncertainty corresponding to variations in α_1. In contrast to frequency variations, damping and mode shape variations behave more like random variables, and they correspond to secondary uncertainties [4]. All α_2 and α_3 of validation systems are inside the bounds or close to the boundary of the identified interval model. All α_4 of validation systems are outside the bounds of the interval model, and this bias is mainly contributed from the bias of mode shape and damping. Figure 4 shows the coordinate distance of 60 validation systems from interval model. The distance is mainly due to the bias of α_4.

Figure 5 shows the performance sensitivity to the identified uncertainty parameters α_i. The sensitivity of performance to the jth uncertainty parameters α_j of the ith chosen subsystem p^i is defined as

$$s_{ij}^{\alpha} = \frac{1}{a(p^i)}\left|\frac{\partial a(p^i)}{\partial \alpha_j}\right|, \quad i = 1, \ldots, n_s, \tag{3.1}$$

where $a(p^i)$ is the maximum acceleration magnitude of the integrated system with subsystem parameter vector p^i, and n_s is the number of parameter vectors. This sensitivity represents a

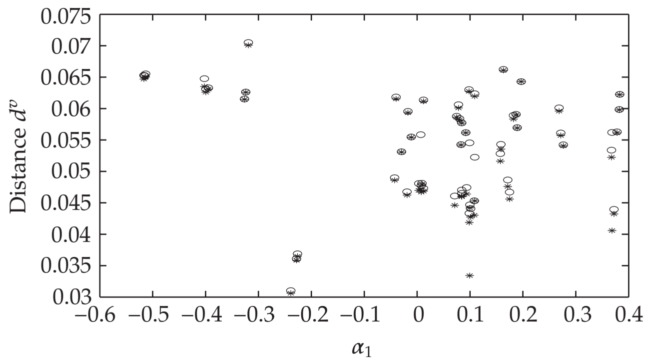

Figure 4: Coordinate distance of 60 validation systems from interval model: (circls) distance from interval model; (asterisks) distance contributed from α_4 bias.

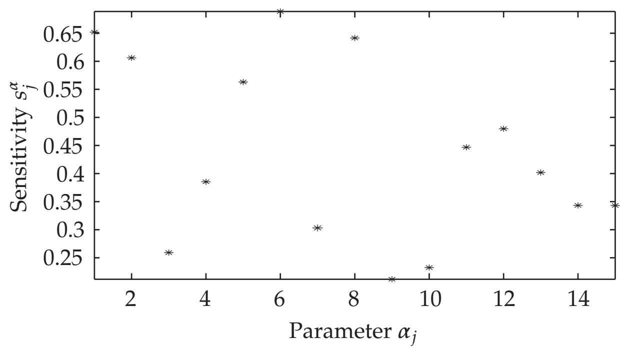

Figure 5: Sensitivity of performance to identified uncertainty parameters α_j.

percentage change. The average sensitivity corresponding to the jth uncertainty parameters α_j is defined as

$$s_j^\alpha = \frac{1}{n_s}\sum_{i=1}^{n_s}\left|s_{ij}^\alpha\right|. \tag{3.2}$$

Figure 5 shows that this sensitivity is between 21% and 69%, corresponding to the original maximum acceleration magnitude, and the sensitivity to α_4 is 39% of the maximum acceleration. Coordinate distance of all the validation systems is between 0.03 and 0.07. This means that the maximum acceleration deviation between the validation system and a system in interval model is insignificant (around 1% to 3%), based on the sensitivity in Figure 5. All the validation systems are acceptable, based on the coordinate distance corresponding to performance index of maximum acceleration.

Figure 6 shows the maximum acceleration of the integrated systems with the identified interval model, 60 calibration systems, and 60 validation systems when an impulse force is applied at x_8 position. The results show that the identified interval model well represents and covers 60 calibration systems. The maximum acceleration of all the validation systems is inside the envelope or close to the boundary of the interval model. As expected, the validation systems are acceptable, based on the coordinate distance results shown in Figure 4. This

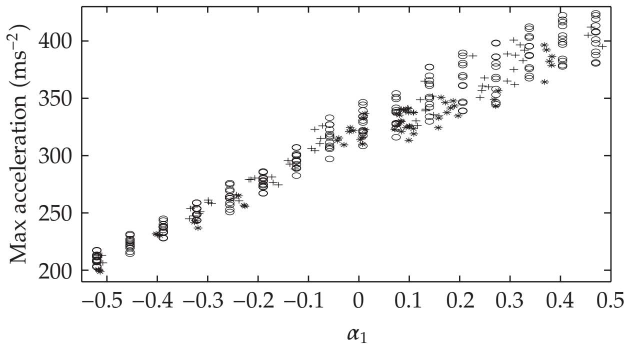

Figure 6: Maximum acceleration with impulse input: (circls) interval system; + 60 calibration systems; (asterisks) 60 validation systems.

coordinate distance represents a metric of the maximum acceleration deviation (percentage difference) between a validation system and the identified interval model.

4. Concluding remarks

This paper presents a novel approach for model validation of a system with an attached subsystem that is exhibiting significant parameter uncertainty. An interval modeling technique is applied for uncertainty quantification with the performance sensitivity weighting in SVD process. The coordinate distance, between the validation system and the identified interval model, is defined as a metric for model validation. This distance represents a metric of the possible performance deviation of the validation system from a system in interval model. The results show that all the validation systems provided by Sandia are acceptable, based on this distance metric. This demonstrates an efficient tool for model validation, based on the interval model analysis. The proposed technique in this paper can be extended to probability framework.

Appendix

Model uncertainty quantification

The sensitivity of performance index a, such as maximum acceleration magnitude, to the jth component of the ith chosen subsystem p^i is defined as

$$s_{ij} = \frac{1}{a(p^i)}\left|\frac{\partial a(p^i)}{\partial p^{ij}}\right|\sigma_j, \quad i = 1,\ldots,n_s, \tag{A.1}$$

where p^{ij} is the jth component of parameter vector p^i, and σ_j is the standard deviation of the jth vector component. This sensitivity represents a percentage change including the factor σ_j to account for the size of the parameter variation. The average sensitivity corresponding to the jth vector component is defined as

$$s_j = \frac{1}{n_s}\sum_{i=1}^{n_s}\left|s_{ij}\right|. \tag{A.2}$$

To quantify the parameter uncertainty, an uncertainty matrix is defined as

$$\Delta P = \left[\Delta p_1 \ \ \Delta p_2 \ \ \cdots \ \ \Delta p_n\right], \tag{A.3}$$

with

$$\Delta p_j = p_j - p_0, \quad j = 1, \ldots, n, \ p_0 = \frac{1}{n}\sum_{j=1}^{n} p_j, \tag{A.4}$$

where p_j is the jth identified parameter vector, and p_0 is the nominal parameter vector, which is computed as the average from $n = 60$ experiments.

A singular value decomposition (SVD) technique [4] is used to generate an optimal linear interval model. This SVD process involves the following computational steps.

(1) Compute an initial weighting matrix as

$$\Delta P^1 = W_1^{-1}\Delta P, \tag{A.5}$$

where W_1 is a diagonal matrix with its jth diagonal element as the standard deviation σ_j.

(2) Compute the weighting matrix including sensitivity as

$$\Delta P^W = W_2 \Delta P^1, \tag{A.6}$$

where W_2 is a diagonal matrix with its jth diagonal element s_j.

(3) Use SVD to compute the basis matrix U^W for ΔP^W,

$$\Delta P^W = U^W S V^T, \quad S = \operatorname{diag}\left[d_1 \ \ \cdots \ \ d_{15}\right]. \tag{A.7}$$

(4) Compute the basis matrix U for ΔP,

$$U = W_1 W_2^{-1} U^W, \quad U = \left[q_1 \ \ \cdots \ \ q_{15}\right]. \tag{A.8}$$

The singular values d_j are in descending order, this leads to a descending order of perturbation distribution in q_j.

(5) Compute the coordinate vector of Δp_i corresponding to the basis vectors q_j,

$$\beta_i = U^{-1}\Delta p_i. \tag{A.9}$$

(6) Represent each parameter vector as

$$p_i = p_0 + \sum_{l=1}^{15} \beta_i(l) q_l, \tag{A.10}$$

where $\beta_i(l)$ is the lth element of the coordinate vector β_i.

(7) Compute the parameter bounds as

$$\begin{aligned} \alpha_j^+ &= \max\left\{\beta_1(j), \beta_2(j), \ldots, \beta_n(j)\right\}, \\ \alpha_j^- &= \min\left\{\beta_1(j), \beta_2(j), \ldots, \beta_n(j)\right\}. \end{aligned} \tag{A.11}$$

All the basis vectors, coordinates, and parameter bounds are normalized to the first interval length [6].

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