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Research Article

Analytic Approximate Solutions for Unsteady Two-Dimensional and Axisymmetric Squeezing Flows between Parallel Plates

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The flow of a viscous incompressible fluid between two parallel plates due to the normal motion of the plates is investigated. The unsteady Navier-Stokes equations are reduced to a nonlinear fourth-order differential equation by using similarity solutions. Homotopy analysis method (HAM) is used to solve this nonlinear equation analytically. The convergence of the obtained series solution is carefully analyzed. The validity of our solutions is verified by the numerical results obtained by fourth-order Runge-Kutta.

1. Introduction

The problem of unsteady squeezing of a viscous incompressible fluid between two parallel plates in motion normal to their own surfaces independent of each other and arbitrary with respect to time is a fundamental type of unsteady flow which is met frequently in many hydrodynamical machines and apparatuses. Some practical examples of squeezing flow include polymer processing, compression, and injection molding. In addition, the lubrication system can also be modeled by squeezing flows. Stefan [1] published a classical paper on squeezing flow by using lubrication approximation. In 1886, Reynolds [2] obtained a solution for elliptic plates, and Archibald [3] studied this problem for rectangular plates. The theoretical and experimental studies of squeezing flows have been conducted by many researchers [4, 4–14]. Earlier studies of squeezing flow are based on Reynolds equation. The inadequacy of Reynolds equation in the analysis of porous thrust bearings and squeeze films involving high velocity has been demonstrated by Jackson [13], Ishizawa [14]. The general study of the problem with full Navier-Stokes equations involves extensive numerical study requiring more computer time and larger memory. However, many of the important features of this problem can be grasped by prescribing the relative velocity of the plates suitably.

If the relative normal velocity is proportional to $(1 - \alpha t)^{1/2}$, where t is the time and α a constant of dimension $[T^{-1}]$ which characterizes unsteadiness, then the unsteady Navier–Stokes equations admit similarity solution.

In 1992, Liao [15] employed the basic ideas of the homotopy in topology to propose a general analytic method for nonlinear problems, namely, homotopy analysis method (HAM) [16–21]. Based on homotopy of topology, the validity of the HAM is independent of whether or not there exist small parameters in the considered equation. Therefore, the HAM can overcome the foregoing restrictions and limitations of perturbation methods [22]. Furthermore, the HAM always provides us with a family of solution expressions in the auxiliary parameter $\hbar$, the convergence region, and the rate of each solution might be determined conveniently by the auxiliary parameter $\hbar$. The HAM also avoids discretization and provides an efficient numerical solution with high accuracy, minimal calculation, and avoidance of physically unrealistic assumptions. Besides, the HAM is rather general and contains the homotopy perturbation method (HPM) [21], the Adomian decomposition method (ADM) [23], and δ-expansion method. In fact, HPM and ADM are always special cases of HAM when $\hbar = -1$. The convergence of HAM solution series is dependent upon three factors, that is, the initial guess, the auxiliary linear operator, and the auxiliary parameter $\hbar$. However, as a special case of homotopy analysis method when $\hbar = -1$, the convergence of HPM solution series is only dependent upon two factors: the auxiliary linear operator and the initial guess. So, given the initial guess and the auxiliary linear operator, HPM cannot provide other ways to ensure that the solution is convergent. HAM provides us with a family of solution expression in the auxiliary parameter $\hbar$ and the solution given by ADM is only one of them.

In recent years, the HAM has been successfully employed to solve many types of nonlinear problems such as the nonlinear equations arising in heat transfer [24], the nonlinear model of diffusion and reaction in porous catalysts [25], the chaotic dynamical systems [26], the nonhomogeneous Blasius problem [27], the generalized three-dimensional MHD flow over a porous stretching sheet [28], the wire coating analysis using MHD Oldroyd 8-constant fluid [29], the axisymmetric flow and heat transfer of a second-grade fluid past a stretching sheet [30], the MHD flow of a second-grade fluid in a porous channel [31], the generalized Couette flow [32], the Glauert-jet problem [33], the Burger and regularized long wave equations [34], the laminar viscous flow in a semiporous channel in the presence of a uniform magnetic field [35], and other problems. All of these successful applications verified the validity, effectiveness, and flexibility of the HAM.

In this paper, we use homotopy analysis method to investigate the problem of unsteady squeezing of a viscous incompressible fluid between two parallel plates. The paper is organized as follows. In Section 2, the mathematical formulation is presented. In Section 3, we extend the application of the HAM to construct the approximate solutions for the governing equation. The convergence of the obtained series solution is carefully analyzed in Section 4. Section 5 contains the results and discussion. The conclusions are summarized in Section 6.

2. Mathematical formulation

Let the position of the two plates be at $z = \pm\ell(1 - \alpha t)^{1/2}$, where ℓ is the position at time $t = 0$ as shown in Figure 1. We assume that the length 1 (in the two-dimensional case) or the diameter D (in the axisymmetric case) is much larger than the gap width $2z$ at any time that the end effects can be neglected. When α is positive the two plates are squeezed until

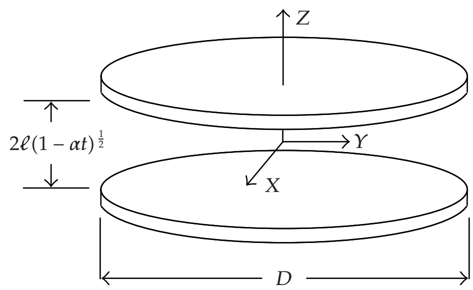

Figure 1: Schematic diagram of the problem.

they touch at $t = 1/\alpha$. When α is negative the two plates are separated. Let u, v, and w be the velocity components in the x, y, and z directions, respectively. For two-dimensional flow, Wang introduced the following transforms [36]:

$$u = \frac{\alpha x}{[2(1-\alpha t)]} f'(\eta),$$
$$w = \frac{-\alpha \ell}{[2(1-\alpha t)^{1/2}]} f(\eta), \tag{2.1}$$

where

$$\eta = \frac{z}{[\ell(1-\alpha t)^{1/2}]}. \tag{2.2}$$

Substituting (2.1) into the unsteady two-dimensional Navier-Stokes equations yields a nonlinear ordinary differential equation in form

$$f'''' + \mathrm{S}\{-\eta f''' - 3f'' - f'f'' + ff'''\} = 0, \tag{2.3}$$

where $\mathrm{S} = \alpha\ell^2/2\nu$ (squeeze number) is the nondimensional parameter. The flow is characterized by this parameter. It should be mentioned that ν is the kinematic viscosity. The boundary conditions are such that on the plates the lateral velocities are zero and the normal velocity is equal to the velocity of the plate, that is,

$$f(0) = 0, \qquad f''(0) = 0,$$
$$f(1) = 1, \qquad f'(1) = 0. \tag{2.4}$$

Similarly, Wang's transforms [36] for axisymmetric flow are

$$u = \frac{\alpha x}{[4(1-\alpha t)]} f'(\eta),$$

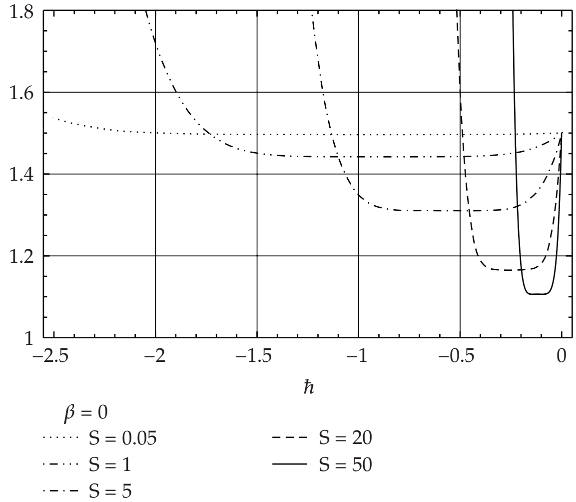

Figure 2: The $\hbar$-curves of $f'(0)$ given by the 15th-order approximation for the axisymmetric case for the different values of the squeeze number S.

$$v = \frac{\alpha y}{\left[4(1-\alpha t)\right]} f'(\eta),$$
$$w = \frac{-\alpha \ell}{\left[2(1-\alpha t)^{1/2}\right]} f(\eta). \tag{2.5}$$

Using transforms (2.5), unsteady axisymmetric Navier–Stokes equations reduce to

$$f'''' + S\{-\eta f''' - 3f'' + f f'''\} = 0, \tag{2.6}$$

subject to the boundary conditions (2.4).

Consequently, we should solve the nonlinear ordinary differential equation

$$f'''' + S\{-\eta f''' - 3f'' - \beta f' f'' + f f'''\} = 0, \tag{2.7}$$

where

$$\beta = \begin{cases} 0, & \text{axisymmetric}, \\ 1, & \text{two-dimensional}, \end{cases} \tag{2.8}$$

and subject to boundary conditions (2.4).

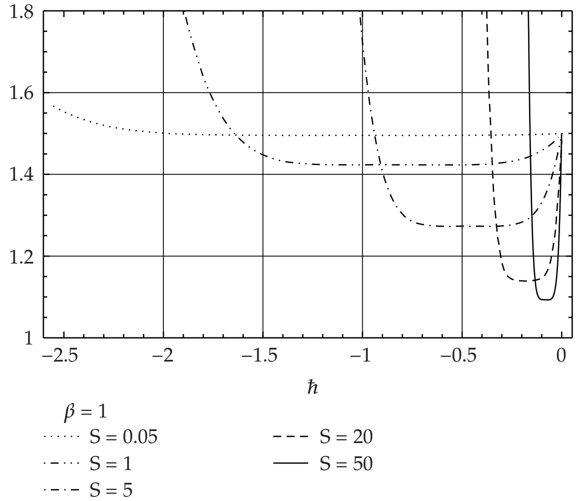

Figure 3: The $\hbar$-curves of $f'(0)$ given by the 15th-order approximation for the two-dimensional case for the different values of the squeeze number S.

3. HAM solution

To investigate the explicit and totally analytic solutions of (2.7) by using HAM, we choose

$$f_0(\eta) = \frac{1}{2}\left(3\eta - \eta^3\right), \tag{3.1}$$

as initial approximation of $f(\eta)$, which satisfies the boundary conditions (2.4). Besides, we select the auxiliary linear operator $\mathcal{L}(f)$ as

$$\mathcal{L}(f) = f''''. \tag{3.2}$$

It is easy to check that this operator satisfies the following equation:

$$\mathcal{L}\left(c_1\eta^3 + c_2\eta^2 + c_3\eta + c_4\right) = 0, \tag{3.3}$$

where $c_i,\ 1 \le i \le 4$, are arbitrary constants. Based on (2.7), we are led to define the following nonlinear operator:

$$\begin{aligned} &\mathcal{N}[\varphi(\eta;p)] \\ &\quad = \frac{\partial^4\varphi(\eta;p)}{\partial\eta^4} + \mathrm{S}\left(-\eta\frac{\partial^3\varphi(\eta;p)}{\partial\eta^3} - 3\frac{\partial^2\varphi(\eta;p)}{\partial\eta^2} - \beta\frac{\partial\varphi(\eta;p)}{\partial\eta}\frac{\partial^2\varphi(\eta;p)}{\partial\eta^2} + \varphi(\eta;p)\frac{\partial^3\varphi(\eta;p)}{\partial\eta^3}\right). \end{aligned} \tag{3.4}$$

Using these operators, we can construct the so-called zeroth-order deformation equation as

$$(1-p)\mathcal{L}[\varphi(\eta;p)-f_0(\eta)] = p\hbar\mathcal{N}[\varphi(\eta;p)], \tag{3.5}$$

where $p \in [0,1]$ is an embedding parameter and $\hbar$ is an auxiliary nonzero parameter. It should be emphasized that one has great freedom to choose the initial guess, the auxiliary linear operator and the auxiliary parameter $\hbar$. However, (3.5), the original equation of HAM, is the origin of the mathematical term "homotopy" (parameter $\hbar$ is the head letter of "homotopy"). In addition, if $\hbar = -1$, (3.5) will always change to original equation of HPM. The boundary conditions for (3.5) are

$$\begin{aligned} &\varphi(0;p) = 0, \quad \frac{\partial^2\varphi(0;p)}{\partial\eta^2} = 0, \\ &\varphi(1;p) = 1, \quad \frac{\partial\varphi(1;p)}{\partial\eta} = 0. \end{aligned} \tag{3.6}$$

Obviously, when $p = 0$ and $p = 1$, the above zeroth-order deformation equation has the following solutions:

$$\varphi(\eta;0) = f_0(\eta), \qquad \varphi(\eta;1) = f(\eta). \tag{3.7}$$

As p increases from 0 to 1, $\varphi(\eta;p)$ varies from $f_0(\eta)$ to $f(\eta)$. Now expanding $\varphi(\eta;p)$ by its Taylor series in terms of p, one would obtain

$$\varphi(\eta;p) = f_0(\eta) + \sum_{m=1}^{+\infty} f_m(\eta)p^m, \tag{3.8}$$

where

$$f_m(\eta) = \frac{1}{m!}\frac{\partial^m\varphi(\eta;p)}{\partial p^m}\bigg|_{p=0}. \tag{3.9}$$

As pointed out by Liao [19], the convergence of series (3.8) strongly depends upon the auxiliary parameter $\hbar$. Assume that $\hbar$ is selected such that series (3.8) is convergent at $p = 1$, then due to (3.7), the final series solution becomes

$$f(\eta) = f_0(\eta) + \sum_{m=1}^{+\infty} f_m(\eta). \tag{3.10}$$

For the mth-order deformation equation, we differentiate (3.5) m times with respect to p, divide by $m!$, and then set $p = 0$. The resulting deformation equation at the mth-order is

$$\mathcal{L}[f_m(\eta) - \chi_m f_{m-1}(\eta)] = \hbar R_m(\eta), \tag{3.11}$$

with the following boundary conditions

$$f_m(0) = 0, \qquad f''_m(0) = 0,$$
$$f_m(1) = 0, \qquad f'_m(1) = 0, \tag{3.12}$$

where

$$R_m(\eta) = \frac{\partial^4 f_{m-1}(\eta)}{\partial\eta^4} + S\Bigg(-\eta\frac{\partial^3 f_{m-1}(\eta)}{\partial\eta^3} - 3\frac{\partial^2 f_{m-1}(\eta)}{\partial\eta^2} + \sum_{n=0}^{m-1}\bigg(-\beta\frac{\partial f_n(\eta)}{\partial\eta}\frac{\partial^2 f_{m-1-n}(\eta)}{\partial\eta^2} + f_n(\eta)\frac{\partial^3 f_{m-1-n}(\eta)}{\partial\eta^3}\bigg)\Bigg), \tag{3.13}$$

$$\chi_m = \begin{cases} 0, & m \le 1, \\ 1, & m > 1. \end{cases}$$

We use the symbolic software MATHEMATICA to solve the system of linear equations (3.11) with the boundary conditions (3.12), and successively obtain

$$\begin{aligned}
f_1(\eta) &= \frac{\hbar S}{560}\{(37+15\beta)\eta - (73+33\beta)\eta^3 + (35+21\beta)\eta^5 - (-1+3\beta)\eta^7\},\\
f_2(\eta) &= \frac{\hbar S}{15523200}\{(1025640 + 415800\beta + 1025640\hbar + 415800\hbar\beta + 153060\hbar S + 126789\hbar S\beta\\
&\quad + 25875\hbar S\beta^2)\eta\\
&\quad - (2023560 + 914760\beta + 2023560\hbar + 914760\hbar\beta + 349010\hbar S + 325392\hbar S\beta\\
&\qquad + 73998\hbar S\beta^2)\eta^3\\
&\quad + (970200 + 582120\beta + 970200\hbar + 582120\hbar\beta + 227304\hbar S + 281358\hbar S\beta\\
&\qquad + 79002\hbar S\beta^2)\eta^5\\
&\quad - (-27720 + 83160\beta - 27720\hbar + 83160\hbar\beta + 20196\hbar S + 92268\hbar S\beta\\
&\qquad + 40392\hbar S\beta^2)\eta^7\\
&\quad + (10780\hbar S + 8085\hbar S\beta + 10395\hbar S\beta^2)\eta^9 - (378\hbar S - 1428\hbar S\beta + 882\hbar S\beta^2)\eta^{11}\},\\
&\ \vdots
\end{aligned} \tag{3.14}$$

Table 1: The analytic results of $f(\eta)$ at different orders of approximation compared with the numerical results obtained by the fourth-order Runge-Kutta for the axisymmetric case.

S, $\hbar$	η	2nd-order	4th-order	6th-order	7th-order	8th-order	Numerical
−1.5, − 1.3	0.2	0.319474	0.319526	0.319526	0.319526	0.319526	0.319526
	0.4	0.603652	0.603825	0.603830	0.603830	0.603830	0.603830
	0.6	0.822574	0.822863	0.822875	0.822876	0.822876	0.822876
	0.8	0.956580	0.956789	0.956800	0.956801	0.956801	0.956801
−0.5, − 1	0.2	0.302545	0.302582	0.302582	0.302582	0.302582	0.302582
	0.4	0.578028	0.578082	0.578082	0.578082	0.578082	0.578082
	0.6	0.800737	0.800780	0.800780	0.800780	0.800780	0.800780
	0.8	0.947686	0.947702	0.947702	0.947702	0.947702	0.947702
0.5, − 1	0.2	0.290353	0.290322	0.290322	0.290322	0.290322	0.290322
	0.4	0.559299	0.559253	0.559252	0.559252	0.559252	0.559252
	0.6	0.784341	0.784304	0.784303	0.784303	0.784303	0.784303
	0.8	0.940717	0.940704	0.940703	0.940703	0.940703	0.940703
1.5, − 0.8	0.2	0.281032	0.281010	0.281010	0.281010	0.281010	0.281010
	0.4	0.544851	0.544780	0.544779	0.544779	0.544779	0.544779
	0.6	0.771493	0.771374	0.771371	0.771371	0.771371	0.771371
	0.8	0.935127	0.935038	0.935036	0.935036	0.935036	0.935036
2.5, − 0.7	0.2	0.273767	0.273683	0.273682	0.273682	0.273682	0.273682
	0.4	0.533516	0.533255	0.533247	0.533246	0.533246	0.533246
	0.6	0.761299	0.760868	0.760848	0.760847	0.760847	0.760847
	0.8	0.930617	0.930299	0.930281	0.930280	0.930280	0.930280

Therefore, like (3.10), the analytical solution of the problem can be expressed as an infinite series of the form (see [37])

$$\begin{aligned} f(\eta) &= f_0(\eta) + \lim_{M\to\infty}\left[\sum_{m=1}^{M}\left(\sum_{n=1}^{2m+2} a_{m,n}\eta^{2n-1}\right)\right], \\ f(\eta) &= \lim_{M\to\infty}\left[\sum_{m=0}^{M}\left(\sum_{n=1}^{2m+2} a_{m,n}\eta^{2n-1}\right)\right]. \end{aligned} \tag{3.15}$$

4. Convergence of HAM solution

The series solution contains the auxiliary parameter $\hbar$. The validity of the method is based on such an assumption that series (3.8) converges at $p = 1$. It is the auxiliary parameter $\hbar$ which ensures that this assumption can be satisfied. In general, by means of the so-called $\hbar$-curve, it is straightforward to choose a proper value of $\hbar$ which ensures that the series solution is convergent. For the different values of the squeeze number S, the $\hbar$-curves obtained by the 15th-order approximation for the axisymmetric ($\beta = 0$) and two-dimensional ($\beta = 1$) cases are shown in Figures 2 and 3, respectively. From these figures, the valid regions of $\hbar$ correspond to the line segments nearly parallel to the horizontal axis. Figures 2 and 3 elucidate that the size of the valid region strongly depends on S. In fact, the interval for admissible values of $\hbar$

Table 2: The analytic results of $f(\eta)$ at different orders of approximation compared with the numerical results obtained by the fourth-order Runge-Kutta for the two-dimensional case.

S, $\hbar$	η	2nd-order	4th-order	6th-order	7th-order	8th-order	Numerical
−1.5, −1.3	0.2	0.332883	0.333591	0.333617	0.333618	0.333618	0.333618
	0.4	0.623190	0.624315	0.624358	0.624358	0.624358	0.624358
	0.6	0.838219	0.839284	0.839324	0.839325	0.839325	0.839325
	0.8	0.962441	0.962961	0.962983	0.962984	0.962984	0.962984
−0.5, −1	0.2	0.305436	0.305543	0.305545	0.305545	0.305545	0.305545
	0.4	0.582314	0.582468	0.582470	0.582470	0.582470	0.582470
	0.6	0.804271	0.804390	0.804392	0.804392	0.804392	0.804392
	0.8	0.949065	0.949107	0.949108	0.949108	0.949108	0.949108
0.5, −1	0.2	0.288347	0.288261	0.288260	0.288260	0.288260	0.288260
	0.4	0.556268	0.556145	0.556143	0.556143	0.556143	0.556143
	0.6	0.781768	0.781670	0.781671	0.781671	0.781671	0.781671
	0.8	0.939674	0.939641	0.939640	0.939640	0.939640	0.939640
1.5, −0.8	0.2	0.276526	0.276433	0.276432	0.276432	0.276432	0.276432
	0.4	0.537929	0.537754	0.537752	0.537752	0.537752	0.537752
	0.6	0.765463	0.765252	0.765249	0.765249	0.765249	0.765249
	0.8	0.932607	0.932474	0.932471	0.932471	0.932471	0.932471
2.5, −0.7	0.2	0.268041	0.267797	0.267791	0.267791	0.267791	0.267791
	0.4	0.524558	0.524057	0.524045	0.524045	0.524045	0.524045
	0.6	0.753285	0.752627	0.752605	0.752605	0.752605	0.752605
	0.8	0.927166	0.926724	0.926704	0.926703	0.926703	0.926703

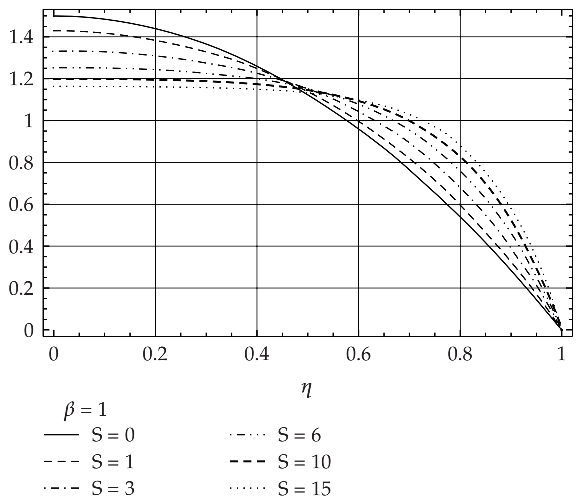

Figure 4: The influence of positive S on $f'(\eta)$ for the two-dimensional case, when $\hbar = -0.3$.

shrinks toward zero by increasing the squeeze number. As mentioned above, the homotopy analysis method is rather general and always contains the homotopy perturbation method (HPM) [21] and the Adomian decomposition method (ADM) [23] when $\hbar = -1$. From Figures 2 and 3, $\hbar = -1$ is not valid for the large values of S.

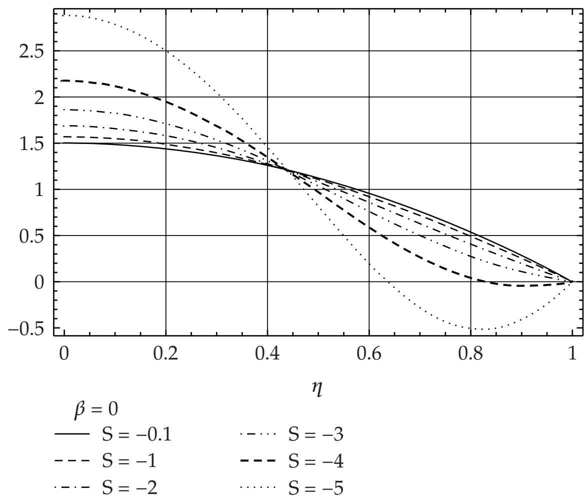

Figure 5: The influence of negative S on $f'(\eta)$ for the axisymmetric case, when $\hbar = -2$.

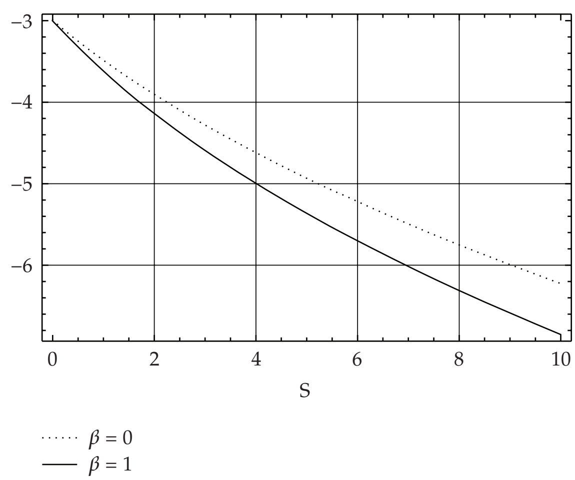

Figure 6: The skin friction $(f''(1))$ for the axisymmetric and two-dimensional cases, when $\hbar = -0.4$. From $\hbar$-curves, the series solution for $\hbar = -0.4$ converges in the whole region $0 \leq S \leq 10$.

5. Results and discussion

Our main concern is the various values of $f(\eta)$ and $f'(\eta)$. These quantities describe the flow behavior. For several values of S, the function $f(\eta)$ obtained by the different orders of approximation for the axisymmetric and two-dimensional cases are compared with the numerical results in Tables 1 and 2, respectively. It is worth mentioning that the numerical results have been obtained using the fourth-order Runge-Kutta in C++ program. We can see a very good agreement between the purely analytic results of the HAM and numerical results. The variation of $f'(\eta)$ with the change in the positive values of S for the two-dimensional case is plotted in Figure 4. Figure 5 shows the influence of negative S on $f'(\eta)$ for the axisymmetric case. Note that for the large negative values of S, the results of similarity analysis are not

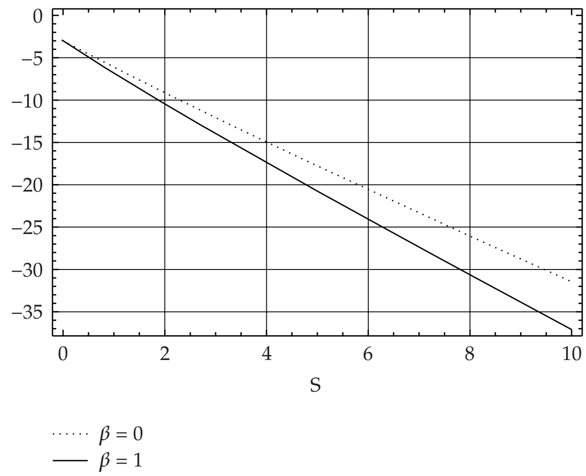

Figure 7: The pressure gradient ($f'''(1)$) for the axisymmetric and two-dimensional cases, when $\hbar = -0.4$. From $\hbar$-curves, the series solution for $\hbar = -0.4$ converges in the whole region $0 \leq S \leq 10$.

reliable. $f''(1)$ gives skin friction, and $f'''(1)$ represents pressure gradient. $f''(1)$ and $f'''(1)$ as functions of S are illustrated in Figures 6 and 7, respectively.

6. Conclusions

In this paper, the unsteady axisymmetric and two-dimensional squeezing flows between two parallel plates are studied analytically using the HAM. The convergence of the results is explicitly shown. Graphical results and tables are presented to investigate the influence of the squeeze number S on the velocity, skin friction, and pressure gradient. The solution obtained by means of the HAM is an infinite power series for appropriate initial approximation, which can be, in turn, expressed in a closed form. Unlike perturbation methods, the HAM does not depend on any small physical parameters. Thus, it is valid for both weakly and strongly nonlinear problems. Besides, different from all other analytic methods, the HAM provides us with a simple way to adjust and control the convergence region of the series solution by means of auxiliary parameter $\hbar$. Thus the auxiliary parameter $\hbar$ plays an important role within the frame of the HAM which can be determined by the so-called $\hbar$-curves. Consequently, the present success of the homotopy analysis method for the highly nonlinear problem of squeezing flows verifies that the method is a useful tool for nonlinear problems in science and engineering.

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