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Research Article

Modelling and Quasilinear Control of Compressor Surge and Rotating Stall Vibrations

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An unsteady nonlinear and extended version of the Moore-Greitzer model is developed to facilitate the synthesis of a quasilinear stall vibration controller. The controller is synthesised in two steps. The first step defines the equilibrium point and ensures that the desired equilibrium point is stable. In the second step, the margin of stability at the equilibrium point is tuned or increased by an appropriate feedback of change in the mass flow rate about the steady mass flow rate at the compressor exit. The relatively simple and systematic non-linear modelling and linear controller synthesis approach adopted in this paper clearly highlights the main features on the controller that is capable of inhibiting compressor surge and rotating stall vibrations. Moreover, the method can be adopted for any axial compressor provided its steady-state compressor and throttle maps are known.

1. Introduction

Compressor surge and rotating stall vibrations place fundamental limitations on aircraft engine performance and remain persistent problems in the development of axial compressor and fan stages. Compressor surge and rotating stall are purely fluid mechanic instabilities, while blade flutter, stall flutter, and surge flutter and their variants are aeroelastic instabilities involving both blade vibrations and fluid motion. Although both rotating stall flutter and rotating stall tend to occur when the blades of a compressor or fan are operating at high-incidence angles and/or speed, and unsteady viscous flow separation plays a key role in both of these phenomena, the various fluttering phenomena are precursors to compressor surge.

Surge is characterized by large amplitude fluctuations of the pressure in unsteady, circumferentially uniform, annulus-averaged mass flow. It is a one-dimensional instability that spreads through the compression system as a whole and culminates in a limit cycle oscillation in the compressor map. In most situations surge is initiated in a compressor when the compressor mass flow is obstructed and throttled. The frequency of surge oscillations is relatively in a low-frequency band (<25–30 Hz) which could couple with the aeroelastic modes of vibration. The performance of the compressor in surge is characterised by a loss in efficiency leading to high-aeroelastic vibrations in the blade as well as influence the stress levels in the casing. In jet engines, surge can lead to the so-called flame-out of the combustor which could involve reverse flow and chaotic vibrations.

Based on the amplitude of mass flow and pressure fluctuations, surge was classified into four distinct categories: mild surge, classical surge, modified surge, and deep surge by de Jager [1]. This classification is now widely accepted and is used to differentiate between different forms of surge and rotating stall vibrations. During mild surge, the frequency of oscillations is around the Helmholtz frequency associated with the resonance within a cavity, that is, the resonance frequency of the compressor duct and the plenum volume connected to the compressor. This frequency is typically over an order of magnitude smaller than the maximal rotating stall frequency which is normally of the same order as the rotor frequency. Classical surge is a nonlinear phenomenon such as bifurcation and chaos with larger oscillations and at a lower frequency than mild surge, but the mass flow fluctuations remain positive. Modified surge is a mix of both classical surge and rotating stall. Deep surge, which is associated with reverse flow over part of the cycle, is associated with a frequency of oscillation well below the Helmholtz frequency and is induced by transient nonlinear processes within the plenum. Mild surge may be considered as the first stage of a complex nonlinear phenomenon which bifurcates into other types of surge by throttling the flow to compressor to lower mean mass flows. Mild surge is generally a relatively low-frequency phenomenon (≈ 5–10 Hz) while rotating stall is a relatively higher-frequency phenomenon (≈ 25–30 Hz).

There are two modes of stable control of a compressor, the first is based on surge avoidance which involves operating the compressor in a instability free domain (Epstein et al. [2], and Gu et al. [3]). Most control systems currently used in industry are based on this control strategy. In this simple strategy, a control point is defined in parameter space with a redefined stability margin from the conditions for instability defined in terms of stall point. This stability margin is defined by (i) typical uncertainties in the location of the stall point, (ii) typical disturbances including load variations, inlet distortions, and combustion noise, and (iii) a consideration of the available sensors and actuators and their limitations. Generally, a bleed valve or another form of bleeding or recycling of the flow is used to negate the effect of throttling the flow. The control is either the valve position or if one employs an on/off approach as in pulse width modulation, the relative full opening times of the bleed valve in a cycle. Such an approach achieves stability at the expense of performance and the approach is not particularly suitable when the flow is compressible. In short, the surge avoidance approach is not performance optimal. There are also problems associated with the detection of instability. The second mode of control involves continuous feedback control of the mass flow by introducing a control valve or an independently controlled fan. This method involves stability augmentation as the changes in the mass flow will effectively change the conditions for instability and thus increase the stability margin. Rather than operating away from the domain of instability, the domain is pushed further away from the operating point. Based on the experiments performed by a number of earlier researchers (see, e.g., Greitzer [4]), a 20% increase in mass flow is deemed achievable by this means of stability augmentation.

Several attempts have been made to incorporate the influence of blade dynamics into model for stall prediction. Compressor surge by itself places a fundamental limitation on performance. Hence active control methods that tend to suppress the various forms of stall will allow the system to be effectively employed over the parameter space prior to the occurrence of surge. Moreover, it is important to consider the various forms of stall in a holistic and integrated fashion as it would be quite impossible to design individual control systems to eliminate each of the individual instabilities. To this end it is also important to develop a holistic and integrated dynamic model. The model developed by Moore and Greitzer [5] based on the assumptions that the system is incompressible except in a plenum which is assumed to enclose the compressor and turbine stages, and that radial variations are unimportant, represents the compressor surge as a Helmholtz-type hydrodynamic instability. In the original Moore and Greitzer model, an empirical, semiactuator disk representation of the compressor was used, incorporating Hawthorne and Horlock's [6] original actuator disc model of an axial compressor and it served as the basic model incorporating rotating stall. By introducing a semiempirical actuator disk theory into the model, Moore and Greitzer were able to predict rotating stall and surge. The advantage of the Moore and Greitzer model is the analyst ability to incorporate a host of hysteresis models into the compressor characteristics that permit the prediction of a variety of limit cycle response characteristics. Gravdahl and Egeland [7] extended the Moore and Greitzer model by including the spool dynamics and the input torque into the same framework as the original model, thus permitting the inclusion of the control inputs into the dynamics. The models may be derived by the application of finite volume type analysis and may also be extended to the case of rotating stall instability and rotating stall-induced flutter. In the Moore and Greitzer model, the downstream flow field is assumed to be a linearized flow with vorticity, so a solution of a form similar to the upstream solution can be found. The plenum chamber is assumed to be an isentropic compressible chamber in which the flow is negligibly small and perturbations are completely mixed and distributed. Thus the plenum acts merely as a "fluid spring". The throttle is modelled as a simple quasisteady device across which the drop in pressure is only a function of the mass flow rate. Flow variations across the compressor are subject to fluid-inertia lags in both the rotor and the stator, and these lags determine the rotation rate of rotating stall. Stability of rotating stall is determined by the slope of the compressor total-to-static pressure rise map. Greitzer [4] discussed the possibility of the active control of both stall and rotating stall by controlling the relevant Helmholtz cavity resonance frequencies which could be achieved by structural feedback.

Apart from the numerous methods of synthesizing control laws that have been proposed by the application of linear control law synthesis methods, which are only suitable for the guaranteed stabilisation of mild surge, a few nonlinear control law synthesis methods have also been proposed. In order to design an active feedback controller that can control deep surge, an inherently nonlinear surge-control model is essential. A number of nonlinear models have been proposed (Chen et al. [8], Krstic et al. [9], Nayfeh and Abed [10], Paduano et al. [11], and Young et al. [12]), and but almost all of these are oriented towards rotating stall control synthesis and include the dynamics of the amplitude of the leading circumferential mode. Many of these models (Gu et al. [13] and Hõs et al. [14]) have been employed to perform a bifurcation analysis to explore the behaviour of the postinstability dynamics.

In this paper, an unsteady nonlinear and extended version of the Moore-Greitzer model is developed to facilitate the synthesis of a surge and stall controller. The motivation is the need for a comprehensive and yet low-order model to describe the various forms of stall as well as the need to independently represent the transient disturbance and control inputs in the compressor pressure rise dynamics. Furthermore, the extended version of the Moore-Greitzer model is developed by reducing the number of independent model parameters to a minimum. Our preliminary studies indicate that model can effectively capture the dynamics of the phenomenon of compressor surge and that its poststall instability behaviour is a well representative of the observed behaviours in real axial flow compressors. The controller is synthesised in two steps. In the first step, the desired equilibrium throttle position and the desired equilibrium value of the ratio of the nondimensional pressure rise at minimum flow to a quarter of the peak to peak variation of the pressure fluctuation at the compressor exit are established. This defines the equilibrium point and ensures that the desired equilibrium point is stable. In the second step, the margin of stability at the equilibrium point is tuned or increased by an appropriate feedback of change in the mass flow rate about the steady mass flow rate at the compressor exit. The first step may be considered to be an equilibrium point controller while the second corresponds to stability augmentation. Such a two-step process then ensures that both the desired equilibrium solution is reachable and that any perturbations about the equilibrium point are sufficiently stable.

2. Fundamental Model Equations

The unsteady and steady fluid mechanics of the flow upstream and downstream of the compressor is considered while the viscous effects are limited to within the actuator disc of the compressor which allows one to define nondimensional total to static pressure rise map. Compressibility is assumed to be confined to the plenum chamber downstream of the compressor where the compression is assumed to be uniform and isentropic. The throttle map sets the mass flow through the system and is a function of the plenum pressure and the throttle opening. It is essential in defining the flow characteristics of the compressor. The rate of change of the plenum pressure is determined from the one-dimensional continuity conditions and is a function of difference in the compressor flow averaged over the face of the compressor and the throttle flow. The second equation is defined by the one-dimensional rate of change of momentum which relates to the dynamic pressure. Two other equations complete the definition of the complete dynamics of the Moore-Greitzer model; the first relates to the rate of change of the throttle flow and the second defines the compressor dynamics and is based on an unsteady adaptation of the actuator disc model. These equations were first proposed by Greitzer [15] in 1976.

The dimensionless compressor mass flow is assumed to be ϕ_c and ψ is the dimensionless plenum pressure rise. Furthermore, $\Psi_{c,ss}$ is the dimensionless steady-state compressor pressure rise given in the compressor map, whereas Ψ_c is the dimensionless dynamic compressor pressure rise. The dimensionless throttle mass flow is ϕ_t and dimensionless pressure drop across the throttle is Ψ_t

$$
\begin{aligned}
\frac{1}{B}\frac{d}{d\tau}\phi_c &= \Psi_c - \psi, \\
\frac{G}{B}\frac{d}{d\tau}\phi_t &= \psi - \Psi_t, \\
B\frac{d}{d\tau}\psi &= \phi_c - \phi_t, \\
\tau_c\frac{d}{d\tau}\Psi_c &= \Psi_{c,ss} - \Psi_c,
\end{aligned}
\tag{2.1}
$$

where $\phi_c = \dot{m}_c/(\rho_a A_c U_t)$, $\phi_t = \dot{m}_t/(\rho_a A_c U_t)$, $\Psi_c = 2\Delta p_c/(\rho_a U_t^2)$, $\Psi_t = 2\Delta p_t/(\rho_a U_t^2)$, B is the Greitzer parameter given by $B = U_t/2\omega_H L_c$, $\omega_H = a\sqrt{A_c/(V_p L_c)}$ is the Helmholtz cavity resonance frequency for the plenum, τ is the non-dimensional time defined in terms of the Helmholtz frequency and the time t, in seconds as, $\tau = \omega_H t$, G is the geometry ratio parameter of the throttle duct and control volume given by $G = (L_t/A_t)/(L_c/A_c)$, and τ_c is the time constant of the compression system that would be different for stall and for rotating stall. In the preceding definitions of the model parameters, $\dot{m}_c$ is the mass flow rate through the compressor, $\dot{m}_t$ is the mass flow rate through the throttle, Δp_c is the pressure rise across the compressor, Δp_t is the pressure drop across the throttle, ρ_a is the ambient air density, a is the speed of sound corresponding to ambient conditions, A_c is the cross-sectional area of the control volume, L_c is the length of the control volume, A_t is the cross-sectional area of the throttle duct, L_t is the length of the throttle duct, V_p is the volume of the plenum chamber, and U_t is the rotor tip speed.

The compressor map in steady flow is a plot of the non-dimensional pressure with the non-dimensional mass flow rate through the compressor for each rotation speed. However, the plots are self-similar and can be reduced to single plot by scaling the non-dimensional mass flow rate and the non-dimensional dynamic pressure rise. The compressor surge line is obtained simply by linking the maximum point on each compressor characteristic for a particular rotational speed. Representing the compressor characteristics in a non-dimensional manner for each rotation speed and appropriately scaling the axes simply reduces the "surge line" to a single point which is the maximum point on the characteristic. Following, Hõs et al. [14], the scaled compressor map in steady flow when $\phi_c = \phi_{cs}$ is assumed to be

$$\Psi_{c,ss}(\phi_{cs}) = \Psi_{c0} + \frac{H}{2}\left(2 + 3\left(\frac{\phi_{cs}}{F} - 1\right) - \left(\frac{\phi_{cs}}{F} - 1\right)^3\right). \tag{2.2}$$

In (2.2), H defines half the peak-to-peak variation of the pressure fluctuation at the compressor exit or the amplitude of the pressure fluctuation while F is half the change in the steady mass flow rate, ϕ_{cs} is required for the pressure to change from the minimum to the maximum. The definitions of the parameters H and F are illustrated in Figure 1.

The throttle map in steady flow when $\phi_t = \phi_{ts}$ is taken to be

$$\Psi_{t,ss} = \left(\frac{\phi_{ts}}{C_t\gamma}\right)^2, \tag{2.3}$$

where the dimensionless throttle parameter C_t is a coefficient defining the capacity of the fully opened throttle and γ is the dimensionless throttle position.

Following Gravdahl and Egeland [7], the input torque to the compressor may be included and the dynamics of the spool as another state equation is given by

$$I\left(\frac{d\omega}{dt}\right) = T_{\text{ext}} - T_c, \tag{2.4}$$

where I is the mass moment of inertia of the compressor rotor, ω the angular velocity which may be expressed in terms of the Greitzer parameter and tip radius as $\omega = U_t/R_t = 2\omega_H L_c B/R_t$, T_{ext} is the external torque input, and T_c is the torque necessary to drive the

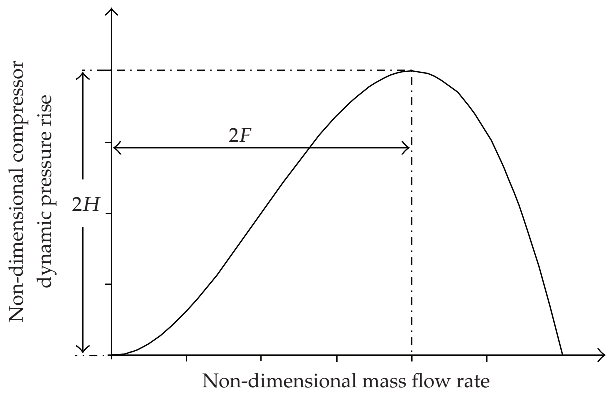


Figure 1: Definitions of the compressor characteristic parameters H and F.

compressor which may be expressed in terms of the slip ratio σ, as $T_c = \rho_a A_c U_t^2 R_t \phi_c \sigma$. The slip ratio σ can be defined as the ratio of the tangential velocity of the fluid at the compressor exit guide vanes and the tip speed. The external torque may be expressed in a non-dimensional form as, $\Gamma_{\text{ext}} = T_{\text{ext}}/\rho_a A_c U_t^2 R_t$. Hence (2.4) may be expressed in a non-dimensional form as

$$\frac{dB}{d\tau} = \frac{B^2(\Gamma_{\text{ext}} - \phi_c \sigma)}{\mu}, \tag{2.5}$$

where $\mu = I/2\rho_a R_t^2 A_c L_c$ is the non-dimensional inertia parameter, and Γ_{ext} is the non-dimensional torque input.

In this analysis all controls are initially assumed to be fixed as the uncontrolled dynamics is considered first. For this reason, any bleed valve that may have been included is closed and all control pressure perturbations are assumed to be equal to zero.

3. Steady Flow Analysis

Assuming the conditions of steady flow, the equations are

$$\frac{1}{B}\frac{d}{d\tau}\phi_c = \Psi_c - \psi = 0, \tag{3.1a}$$

$$\frac{G}{B}\frac{d}{d\tau}\phi_t = \psi - \Psi_t = 0, \tag{3.1b}$$

$$B\frac{d}{d\tau}\psi = \phi_c - \phi_t = 0, \tag{3.1c}$$

$$\tau_c\frac{d}{d\tau}\Psi_c = \Psi_{c,ss} - \Psi_c = 0, \tag{3.1d}$$

$$\frac{dB}{d\tau} = \frac{B^2}{\mu}(\Gamma_{\text{ext}} - \phi_c \sigma) = 0. \tag{3.1e}$$

From the third of the above equations, (3.1c), in steady flow, let

$$\phi_{cs} = \phi_{ts} = \phi_{s0}. \tag{3.2}$$

The steady flow conditions are obtained from the first two of the above equations, (3.1a) and (3.1b), and are given by $\Psi_{c,ss} = \Psi_{t,ss}$; that is,

$$\Psi_{c,ss}(\phi_{s0}) = \Psi_{c0} + \frac{H}{2}\left(2 + 3\left(\frac{\phi_{s0}}{F} - 1\right) - \left(\frac{\phi_{s0}}{F} - 1\right)^3\right) = \left(\frac{\phi_{s0}}{C_t\gamma}\right)^2. \tag{3.3}$$

A parameter p is defined as

$$p = \left(\frac{2}{H}\right)\left(\frac{F}{C_t\gamma_n}\right)^2, \tag{3.4}$$

where p is the throttle non-dimensional pressure rise at minimum flow and a parameter p_0

$$p_0 = \frac{2}{H}\Psi_{c0} + 2 \tag{3.5}$$

which is the ratio of the non-dimensional pressure rise at minimum flow to a quarter of the peak-to-peak variation of the pressure fluctuation at the compressor exit, then (3.3) reduces to

$$\Psi_{c,ss}(\phi_{s0}) = \frac{H}{2}\left(p_0 + 3x - x^3\right) = p\frac{H}{2}(1 + x)^2, \tag{3.6a}$$

where the variable x is

$$x = \left(\frac{\phi_{s0}}{F}\right) - 1. \tag{3.6b}$$

If one assumes that with the minimum flow through the compressor and the throttle, the flow is always steady, then with $\phi_{s0}/F = 1$, one obtains from (3.4),

$$p_0 = p. \tag{3.7}$$

Assuming that the position of the throttle γ is set to a nominal value $\gamma = \gamma_n$ when (3.6a)-(3.6b), and (3.7) are satisfied, (3.6a)-(3.6b) may be rearranged and written as

$$\Psi_{c0} = \frac{H}{2}(p - 2). \tag{3.8}$$

Eliminating Ψ_{c0}, the steady flow characteristic may be defined entirely in terms of the compressor and throttle map parameters, H, F and the product $\gamma_n C_t$ and is

$$3x - x^3 = px(x+2), \tag{3.9}$$

and (3.8) may be expressed as

$$x\left(x^2 + px + 2p - 3\right) = 0. \tag{3.10}$$

From the first factor of (3.10) the assumed solution, $\phi_{s0}/F = 1$, is recovered. Assuming $x \neq 0$ and solving for p

$$p = \frac{\left(3 - (\phi_{s0}/F - 1)^2\right)}{(\phi_{s0}/F + 1)}. \tag{3.11}$$

If one assumes that with the flow through the compressor and the throttle either minimum or below minimum, it is always steady, then $x = x_0$. Then it follows that,

$$\frac{H}{2}\left(p_0 + 3x_0 - x_0^3\right) = p\frac{H}{2}(1 + x_0)^2. \tag{3.12}$$

Eliminating p_0, one obtains

$$\left(3(x - x_0) - \left(x^3 - x_0^3\right)\right) = 2p(x - x_0) + p\left(x^2 - x_0^2\right). \tag{3.13}$$

Solving for p, one obtains

$$p = \frac{(3 - (x^2 + xx_0 + x_0^2))}{(x + x_0 + 2)}. \tag{3.14}$$

When $x_0 = 0$, (3.14) reduces to (3.11).

4. Unsteady NonLinear Extended Moore-Greitzer Model

Rather than combining the quasisteady and transient components of compressor pressure rise, the independent contributions from these two components of the pressure rise are separately identified. If one defines $\Delta\Psi_c = \Psi_c - \Psi_{c,qs}$ as the transient disturbance and control pressure component of the compressor pressure rise, the first three unsteady equations may be expressed as

$$\frac{d\phi_c}{Bd\tau} = \Psi_{c,qs} - \psi + \Delta\Psi_c, \tag{4.1a}$$

$$\frac{Gd\phi_t}{Bd\tau} = \psi - \Psi_{t,qs}, \tag{4.1b}$$

$$\frac{Bd\psi}{d\tau} = \phi_c - \phi_t. \tag{4.1c}$$

The compressor transient disturbance and control dynamics, in the absence of a control pressure input, is defined entirely in terms of $\Delta\Psi_c$ as

$$\frac{\tau_c d\Delta\Psi_c}{d\tau} = -\Delta\Psi_c + \Psi_{c,ss} - \Psi_{c,qs}, \tag{4.2}$$

where the unsteady compressor characteristics, $\Psi_{c,qs}$, and the unsteady throttle map, $\Psi_{t,qs}$, are assumed to satisfy the quasisteady model equations given by

$$\Psi_{c,qs}(\phi_c) = \Psi_{c0} + \frac{H}{2}\left(2 + 3\left(\frac{\phi_c}{F} - 1\right) - \left(\frac{\phi_c}{F} - 1\right)^3\right), \tag{4.3a}$$

$$\Psi_{c0} = \frac{H}{2}(p-2), \qquad \Psi_{t,qs} = \left(\frac{\phi_t}{C_t\gamma}\right)^2. \tag{4.3b}$$

Furthermore

$$\Psi_{c,ss}(\phi_{s0}) = \frac{H}{2}\left(p + 3\left(\frac{\phi_{s0}}{F} - 1\right) - \left(\frac{\phi_{s0}}{F} - 1\right)^3\right). \tag{4.4}$$

Further from the definition of the parameter, p, one may write

$$C_t^2\gamma_n^2 = \frac{2F^2}{pH}. \tag{4.5}$$

In (4.5) one considers the throttle's non-dimensional nominal position, $\gamma = \gamma_n$, to be fixed and any perturbations to it must be considered as a deviation. If $\Delta\gamma$ is the deviation of the throttle position from the nominal position, $\gamma = \gamma_n$, then in the general case (4.5) may be written as

$$C_t^2\gamma^2 = \left(\sqrt{\frac{2F^2}{pH}} + C_t\Delta\gamma\right)^2. \tag{4.6}$$

Considering the last equation for the dynamics of the compressor spool, one assumes that that the non-dimensional torque input, Γ_{ext}, is provided by a non-dimensional power input and can be defined by $\Gamma_{\text{ext}} = \Pi_{\text{ext}}/B$. The equation for the spool dynamics is

$$\frac{dB}{d\tau} = \frac{B}{\mu}\left(\Pi_{\text{ext}} - B\phi_c\sigma\right), \tag{4.7}$$

where the non-dimensional power input is related to the real power, P_{ext}, by the equation

$$\Pi_{\text{ext}} = \frac{P_{\text{ext}}}{2\rho_a U_t^2} A_c \omega_H L_c. \tag{4.8}$$

In most practical situations involving jet engines, it is power that is delivered to a turbine driving the compressor by a combustor and this can be modelled independently.

Using (3.14) to (4.7), the complete unsteady nonlinear equations not including the control inputs may be expressed in terms of the five states ϕ_c, ϕ_t, ψ, $\Delta\Psi_c$, and B, as

$$\begin{aligned}
\frac{d\phi_c}{d\tau} &= B\Psi_{c,qs} + B\left(\Delta\Psi_c - \psi\right), \\
\frac{d}{d\tau}\phi_t &= -\frac{B}{G}\frac{\phi_t^2}{\left(\sqrt{2F^2/pH} + C_t\Delta\gamma\right)^2} + \frac{B\psi}{G}, \\
\frac{d}{d\tau}\psi &= \frac{\left(\phi_c - \phi_t\right)}{B}, \\
\frac{d\Delta\Psi_c}{d\tau} + \frac{\Delta\Psi_c}{\tau_c} &= \frac{\left(\Psi_{c,ss} - \Psi_{c,qs}\right)}{\tau_c}, \\
\frac{dB}{d\tau} &= \frac{B}{\mu}\left(\Pi_{\text{ext}} - B\phi_c\sigma\right)
\end{aligned} \tag{4.9}$$

with

$$p = \frac{\left(3 - \left(\phi_{s0}/F - 1\right)^2\right)}{\left(\phi_{s0}/F + 1\right)}. \tag{4.10}$$

The eight model parameters are $\phi_{s0}/F, H, G, \tau_c, F, C_t\Delta\gamma, \mu$, and σ. The input to the model is defined by Π_{ext}, the non-dimensional power input to the compressor.

5. Application to Rotating Stall Vibrations

Equations (4.9) describe surge in our one-dimensional model but do not include rotating stall. The extension needed is derived and explained in detail by Moore and Greitzer [5] by Galerkin projection, and only the essence of the method is presented here. The Galerkin projection procedure represents the reduction of the differential equation by a set of basic or coordinate functions to capture the behaviour in the circumferential direction with a finite set of modes. One-mode truncation via Galerkin projection results in an additional equation in terms of a new variable J that must be included with (4.9). The square of the new variable J represents the amplitude of the first Galerkin mode. Following Hõs et al. [14], the dynamics of J is described by

$$\tau_J \frac{dJ}{d\tau} = \frac{H}{F} J \left(1 - \left(\frac{\phi_c}{F} - 1\right)^2 - \frac{1}{4} J\right), \tag{5.1}$$

where the time constant τ_J is related to the time constant of an N-stage compressor τ_c and the slope of the compressor duct flow parameter m, by the relations

$$\tau_J = \frac{\omega_H R(1 + ma)}{3aU_t}, \qquad \text{with } a = \frac{R}{\tau_c U_t}. \tag{5.2}$$

The presence of rotating stall influences the compressor characteristic (2.3), and following Hõs et al. [14], it is modified as

$$\Psi_{c,ss}(\phi_{cs}) = \Psi_{c0} + \frac{H}{2}\left(2 + 3\left(\frac{\phi_{cs}}{F} - 1\right)\left(1 - \frac{J}{2}\right) - \left(\frac{\phi_{cs}}{F} - 1\right)^3\right). \tag{5.3}$$

Conditions for steady flow now require additionally that either $J = J_s = 0$, corresponding to an equilibrium with no rotating stall disturbance, or $J = J_s = 4(1 - x^2)$, corresponding to an equilibrium with a rotating stall disturbance. Since J represents the amplitude of rotating stall amplitude, to avoid rotating stall J must tend to zero. If it tends to any other finite value the rotating stall amplitude is nonzero, implying that rotating stall exists. In the case when the rotating stall amplitude is nonzero, (3.4) and (3.5) are unchanged but (3.10) and (3.11) are, respectively, modified, in case J is given by the latter non-zero equilibrium point as

$$\begin{gathered} 5x^2 - xp - 3 - 2p = 0, \\ p = \frac{\left(5(\phi_{s0}/F - 1)^2 - 3\right)}{(\phi_{s0}/F + 1)}, \end{gathered} \tag{5.4}$$

where the definition of the parameter p is unchanged. In the model, it should be noted that the Greitzer parameter B is no longer a parameter but a slowly varying state. In this respect, our analysis is different from that of Moore and Greitzer [5] who treated it as a parameter and stated the conditions for surge in terms of this parameter. For control applications, particularly when the external control input is due to a control torque, it is most appropriate to allow the Greitzer parameter B to vary. However, when the Greitzer parameter B is assumed to be variable, it is essential that both the compressor steady characteristic parameters, H and F, are not constant but functions of B. Based on a set of typical characteristics, the parameters, H and F, are assumed to be linear functions of the Greitzer parameter B and given by

$$H = H_0 + H_B B, \qquad F = F_B B, \tag{5.5a}$$

where H_0, H_B, and F_B are assumed to be constants. Thus in steady state, when $B = B_0$, H and F are given by

$$H_s = H_0 + H_B B_0, \qquad F_s = F_B B_0. \tag{5.5b}$$

If one defines the change in J by $\Delta J = J - J_s$ in the unsteady case, (4.9) are now modified as

$$\frac{d\phi_c}{d\tau} = B\Psi_{c,qs} + B\left(\Delta\Psi_c - \psi\right), \tag{5.6a}$$

$$\frac{d}{d\tau}\phi_t = -\frac{B}{G}\frac{\phi_t^2}{\left(\sqrt{2F^2/pH} + C_t\Delta\gamma\right)^2} + \frac{B\psi}{G}, \tag{5.6b}$$

$$\frac{d}{d\tau}\psi = \frac{\left(\phi_c - \phi_t\right)}{B}, \tag{5.6c}$$

$$\frac{d\Delta\Psi_c}{d\tau} + \frac{\Delta\Psi_c}{\tau_c} = \frac{\left(\Psi_{c,ss} - \Psi_{c,qs}\right)}{\tau_c}, \tag{5.6d}$$

$$\frac{dB}{d\tau} = \frac{B}{\mu}\left(\Pi_{\text{ext}} - B\phi_c\sigma\right), \tag{5.6e}$$

$$\tau_J\frac{d\Delta J}{d\tau} = \frac{H}{F}(J_s + \Delta J)\left(1 - \left(\frac{\phi_c}{F} - 1\right)^2 - \frac{1}{4}(J_s + \Delta J)\right) \tag{5.6f}$$

with

$$J_s = 0 \tag{5.7}$$

or

$$J_s = 4\left(\left(1 - \left(\frac{\phi_{s0}}{F}\right) - 1\right)^2\right), \tag{5.8}$$

where the parameter F is evaluated under steady conditions. Only the former is used and it also required the equilibrium point to be stable. Moreover, there is now an additional parameter τ_J, which may be related to τ_c as

$$\tau_J = \frac{\tau_c\omega_H\left(1 + mR\tau_c/U_t\right)}{3}, \tag{5.9}$$

but will be treated as an independent parameter. Equations (5.6a)–(5.6f) represent a six-state dynamic model of the dynamics of the compressor system.

Table 1: Typical parameter and initial state values for simulation.

Parameter	Primary value	State/input	Initial value
ϕ_{s0}	0.375	ϕ_c	0.4
F_B	0.625	ϕ_t	0.3
H_0	0.06	ψ	0.0
H_B	0.3	$\Delta\Psi_c$	1.0
G	2	B	0.4
σ	0.9	ΔJ	3.1 or 0.1
$C_t\Delta\gamma$	0.0	J_s	0
μ	40	Π_{ext}	0.17
τ_c	0.05	τ_J	0.5

6. Model Response and Instability

Although our primary interest is in establishing a nonlinear model for synthesizing an active surge controller, one needs to understand the dynamic response of the uncontrolled model not only in the vicinity of the domain of instability but also in the postinstability domains in the parameter space. For this reason, the dynamic response of the model proposed in the preceding section is considered, without including any controls which could include a bleed valve or a feedback controller that influences the transient dynamics of the compressor. The rotating stall dynamics is ignored in the first instance.

Table 1 lists the nominal typical values of the parameters, initial values of the states, and the inputs used in the simulation of the dynamic response, for which the system was stable. The parameter p is not shown in Table 1 as it is computed from the parameters in the table. It is however an important parameter as a high value represents greater levels of throttling and a reduced mass flow rate through the throttle. The system was not unstable unless either H was negative or $\gamma < \gamma_n$. A typical example of a stable response is shown in Figure 2.

The first case considered was with $C_t\Delta\gamma = 0$. In this case, no chaotic behaviour was observed although both stable and unstable behaviours were observed. When the compressor was stable, the behaviour was always lightly damped and oscillatory. Choosing the parameter $\gamma = \gamma_n$ represents a case of tuning or matching the throttle to the compressor. In most cases the instability could be eliminated by proper tuning of the parameters and no active stabilisation was deemed necessary.

When H is locally negative, it corresponds to the case of negative slope in the characteristic that was considered by Hõs et al. [14]. When H is negative and the parameter, $\gamma > \gamma_n$, the throttle mass flow is not matched to the compressor mass flow. Although the system was unstable, no chaos was observed. When H is negative and $\gamma < \gamma_n$, there was a clear incidence of chaos in the flow through the compressor, which was identified by a one-dimensional Poincaré map. The chaotic response with a negative H is significant as it represents the case of flame-out in jet-engines. However, this case is not of much practical importance for controller synthesis as the compressor becomes unstable before it becomes chaotic.

The responses of B and J, when H is negative and $\gamma > \gamma_n$ in the rotating stall case, are illustrated in Figure 3. Apparently the Greitzer "parameter" is itself stable in this case but the sustained response in J away from the trivial equilibrium solution ($J = 0$) represents the presence of rotating stall disturbances.

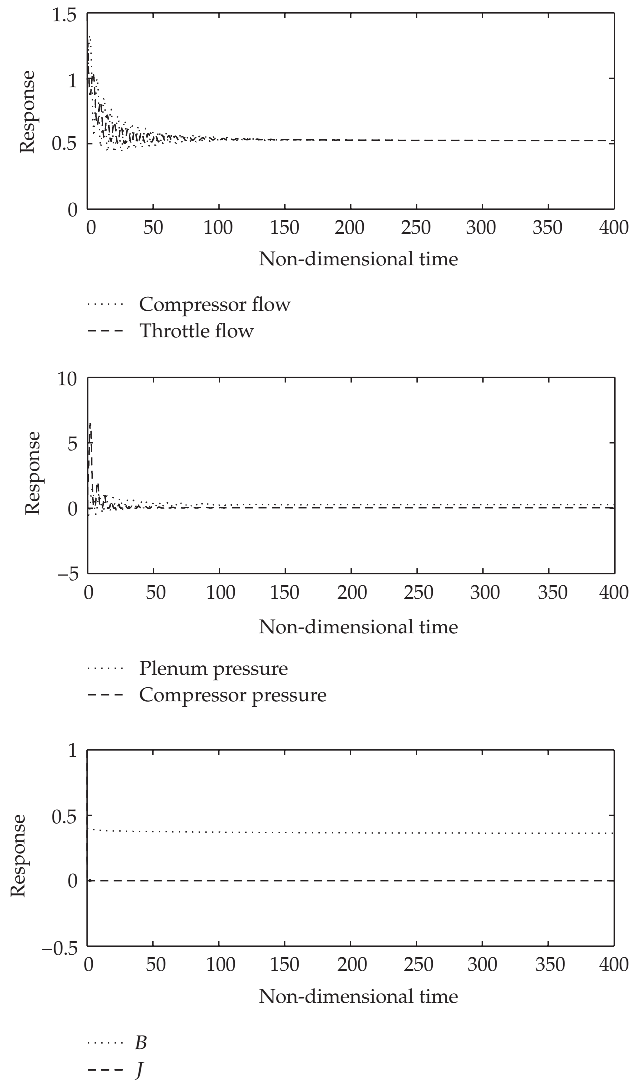


Figure 2: Stable responses of states to disturbance, for the nominal typical values of the parameters.

Considering the case of rotating stall with H positive and $C_t \Delta\gamma = 0$, the system always exhibited stability in the sense that the response converged to a steady state. With $\gamma \neq \gamma_n$ or $\gamma = \gamma_n$, H positive, and $\phi_{s0}/F < 2$, the equilibrium solution jumps from one with $J_s = 0$ to one with $J_s = 1$ and this is followed by the pressure in the plenum chamber falling to zero. The state responses in this case are illustrated in Figure 4(a). The corresponding unsteady compressor map and the operating point on the map are shown in Figure 4(b).

Although, when the compressor flow and throttle flow were matched, that is, with $\gamma = \gamma_n$, the system is stable; it is also important to maintain J at zero, as it represents the amplitude of the rotating stall disturbance amplitude. It can be concluded that open-loop stability is not enough to drive the operating point to $\gamma = \gamma_n$ and also suppress rotating stall disturbances, by using a controller such as an automatically controlled bleed valve. The bleed valve by itself is not always adequate to maintain J at zero and additional feedback is essential

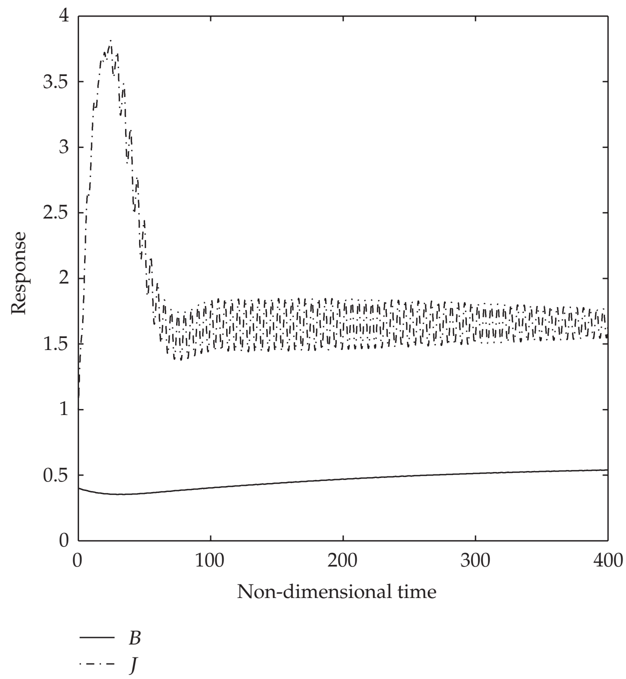


Figure 3: Typical responses of B and J when H is negative and $\gamma < \gamma_n$ in the rotating stall case.

to suppress the rotating stall disturbance by changing the operating equilibrium point. Some authors (Gu et al. [13]) have referred to this requirement as "bifurcation control".

7. Control Law for Throttle Setting

To design the throttle controller, one rewrites (5.6b) as

$$\frac{d}{d\tau}\phi_t = -\frac{B}{G}\frac{\phi_t^2}{\left(\sqrt{2F^2/p_d H} + C_t \Delta\gamma\right)^2} + \frac{B\psi}{G}, \tag{7.1}$$

where p_d is the desired set value for p. The first step in designing a controller is to choose an appropriate value for p_d. The next step is to gradually wash out $\Delta\gamma$ according to some dynamic law such as

$$\tau_u \frac{d\Delta\gamma}{d\tau} = -\Delta\gamma, \tag{7.2}$$

where τ_u is an appropriate time constant so the washout does not interfere with the plant dynamics.

If one further chooses $x > 1$, the equilibrium with $J = J_s = 0$ is stable. To establish the controller parameter p_d, a suitable choice may be made by first choosing x_0 and the operating point x and using (3.14). A typical choice could be $x_0 = 0$ and $x > 1$ giving a value for

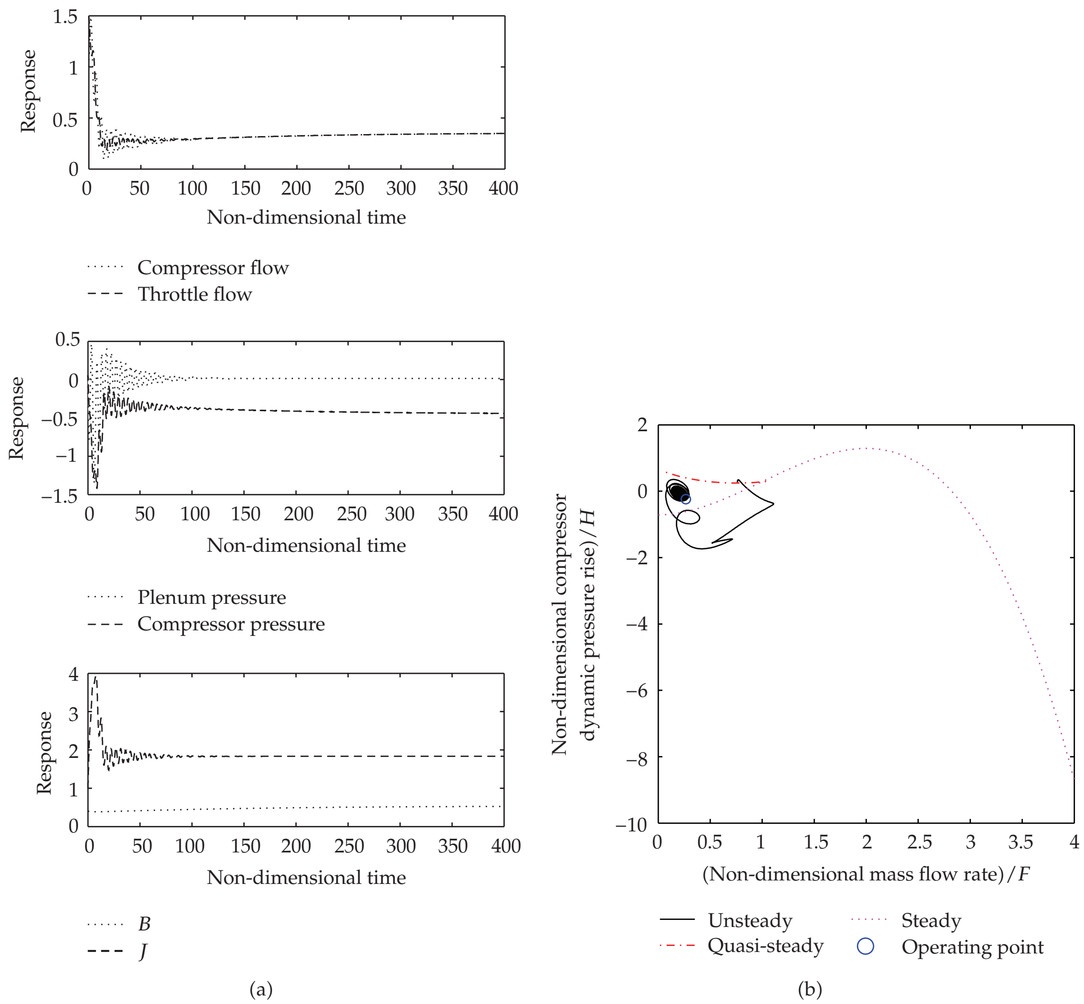


Figure 4: (a) Typical open-loop state responses when H is positive and $\phi_{s0}/F < 2$ in the rotating stall case. (b) Unsteady, quasisteady and steady characteristics of the compressor corresponding to (a).

$p_d < 0.666$. If the initial value of p is p_0 and is greater than this value, then the steady state value of $\Delta\Psi_c$ must be increased by

$$\Delta\Psi_{c,ss} = \frac{H}{2}(p_d - p_0) = \frac{H\Delta p}{2}. \tag{7.3}$$

The corresponding initial condition for $\Delta\gamma$ is then given by

$$\Delta\gamma(t)\big|_{t=0} = \gamma - \gamma_n = \gamma - \frac{\sqrt{2F^2/p_d H}}{C_t}. \tag{7.4}$$

8. Control of the Rotating Stall Vibration Amplitude

To increase the steady state value of $\Delta\Psi_c$, it is important to increase the steady flow delivered by the compressor. This can be done by increasing the input to the compressor. To incorporate such a feature in our model, one assumes a distribution of pressure sources at the inlet to the compressor and write the compressor unsteady pressure dynamics equation with a source control term included as

$$\frac{d\Delta\Psi_c}{d\tau} + \frac{\Delta\Psi_c}{\tau_c} = \frac{(\Psi_{c,ss} - \Psi_{c,qs})}{\tau_c} + \frac{\Delta u_0}{\tau_c}, \tag{8.1a}$$

where the control input is a distribution of pressure sources which are integrated over the inlet area of the compressor and chosen according to the control law

$$\Delta u_0 = \frac{H}{2}(p_d - p_0) + \Delta u = \frac{H\Delta p}{2} + \Delta u, \tag{8.1b}$$

where Δu is the control input perturbation to provide feedback. The complete model equations (5.6a)–(5.6f) including the controller may be expressed as

$$\frac{d\phi_c}{d\tau} = B\Psi_c(p_0, \phi_c) + B(\Delta\Psi_c - \psi), \tag{8.2a}$$

$$\frac{d}{d\tau}\phi_t = -\frac{B}{G}\frac{\phi_t^2}{\left(\sqrt{2F^2/p_d H} + C_t\Delta\gamma\right)^2} + \frac{B\psi}{G}, \tag{8.2b}$$

$$\frac{d}{d\tau}\psi = \frac{(\phi_c - \phi_t)}{B}, \tag{8.2c}$$

$$\frac{d\Delta\Psi_c}{d\tau} + \frac{\Delta\Psi_c}{\tau_c} = \frac{(\Psi_{c,ss}(p_0, \phi_{cs}) - \Psi_c(p_0, \phi_c))}{\tau_c} + \frac{\Delta u_0}{\tau_c}, \tag{8.2d}$$

$$\frac{dB}{d\tau} = \frac{B}{\mu}(\Pi_{\text{ext}} - B\phi_c\sigma), \tag{8.2e}$$

$$\tau_J\frac{d\Delta J}{d\tau} = \frac{H}{F}(J_s + \Delta J)\left(1 - \left(\frac{\phi_c}{F} - 1\right)^2 - \frac{1}{4}(J_s + \Delta J)\right), \tag{8.2f}$$

$$\tau_u\frac{dC_t\Delta\gamma}{d\tau} = -C_t\Delta\gamma, \tag{8.2g}$$

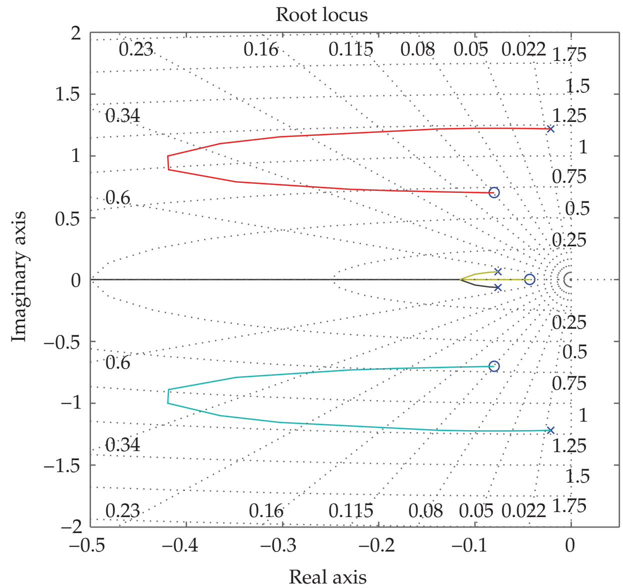


Figure 5: Root locus plot illustrating the effect of the negative feedback of $\Delta\phi_c$.

where

$$\Psi_c(p_0, \phi_c) = \frac{H}{2}\left(p_0 + \left(\frac{\phi_c}{F} - 1\right)\left(1 - \frac{J}{2}\right) - \left(\frac{\phi_c}{F} - 1\right)^3\right),$$
$$\Psi_{c,ss}(p_0, \phi_{cs}) = \Psi_c(p_0, \phi_c)\big|_{t\to\infty}, \tag{8.3}$$
$$\Delta u_0 = \frac{H}{2}(p_d - p_0) + \Delta u = \frac{H\Delta p}{2} + \Delta u.$$

To implement such a controller the parameter p_0 must be known. This parameter must therefore be identified offline *a priori* or adaptively, so the control input can be synthesised.

9. Stability of Controlled Equilibrium

An important step in the validation of the controller is the assessment of the stability of the closed loop equilibrium. To determine the stability of the controlled equilibrium, one first linearises (8.2a)–(8.2f), about the controlled equilibrium solution which is characterised by $p = p_d$ and $\phi_c = \phi_t = \phi_{sd}$. Perturbing the state vector and the control input and linearising (8.2a)–(8.2g) about the equilibrium states result in

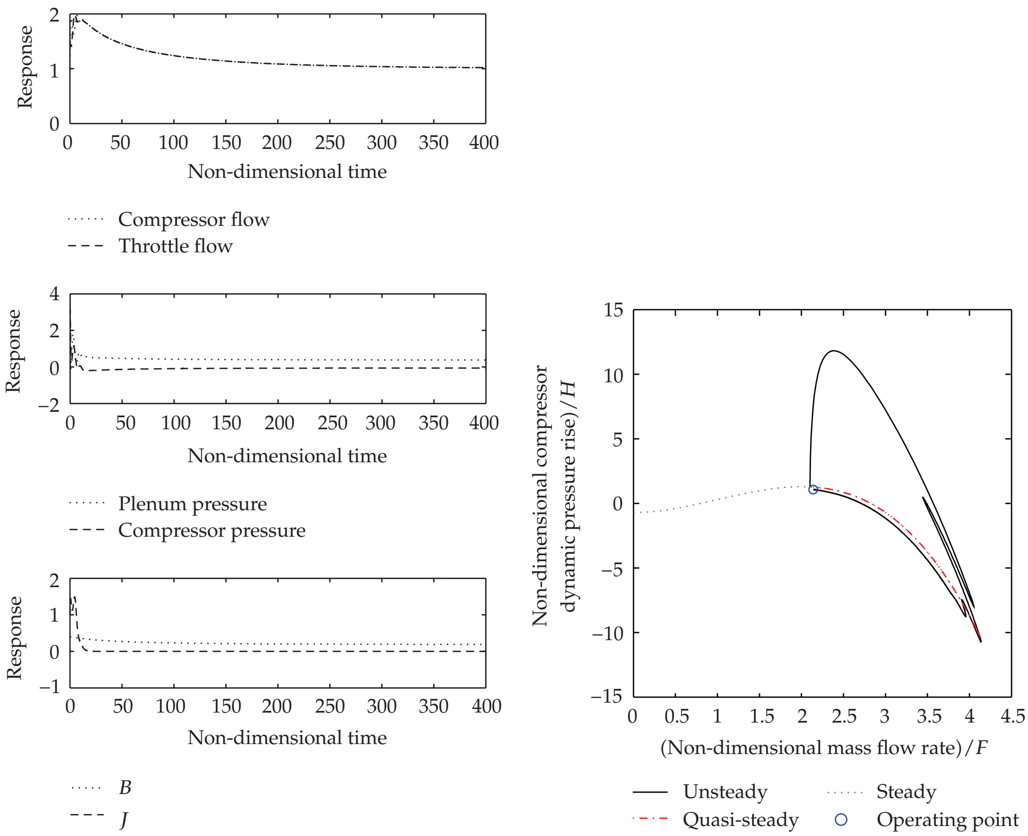
$$\frac{d\Delta\phi_c}{d\tau} = \left(\Psi_c(p_d, \phi_{sd}) + B_0\frac{d\Psi_c(p_d, \phi_{sd})}{dB}\right)\Delta B$$
$$+ B_0\left(\frac{d\Psi_c(p_d, \phi_{sd})}{d\phi_c}\Delta\phi_c + \frac{d\Psi_c(p_d, \phi_{sd})}{dJ}\Delta J + \Delta\Psi_c - \Delta\varphi\right),$$

$$
\begin{aligned}
\frac{d\Delta\phi_t}{d\tau} &= \frac{B_0\Delta\psi}{G} \\
&\quad - \frac{B_0}{G}\frac{p_d H_s \phi_{sd}}{2F_s^2}\left(2\Delta\phi_t - \frac{2\phi_{sd}C_t\Delta\gamma}{\sqrt{2F_s^2/p_d H_s}} - \left(\frac{F_B}{F_s} - \frac{H_B}{2H_s}\right)\frac{\phi_{sd}}{\sqrt{2F_s^2/p_d H_s}}\Delta B\right), \\
\frac{d}{d\tau}\Delta\psi &= \frac{(\Delta\phi_c - \Delta\phi_t)}{B_0}, \\
\frac{d\Delta\Psi_c}{d\tau} + \frac{\Delta\Psi_c}{\tau_c} &= -\frac{1}{\tau_c}\left(\frac{d\Psi_c(p_d,\phi_{sd})}{d\phi_c}\Delta\phi_c + \frac{d\Psi_c(p_d,\phi_{sd})}{dJ}\Delta J + \frac{d\Psi_c(p_d,\phi_{sd})}{dB}\Delta B\right) + \frac{\Delta u}{\tau_c}, \\
\frac{d\Delta B}{d\tau} &+ \frac{\Delta B}{\mu}(2B_0\phi_{sd}\sigma - \Pi_{\mathrm{ext}}) \\
&= -\frac{B_0^2 F_s\sigma}{\mu}\frac{\Delta\phi_c}{F_s}, \\
\tau_J\frac{d\Delta J}{d\tau} &= -\frac{H_s}{F_s}\left(2J_s\left(\frac{\phi_{cs}}{F_s} - 1\right)\frac{\Delta\phi_c}{F_s}\right) \\
&\quad - \frac{H_s}{F_s}\left(\left(\left(\frac{\phi_{cs}}{F_s} - 1\right)^2 - 1 + \frac{J_s}{4}\right)J_s\left(\frac{H_B}{H_s} - \frac{F_B}{F_s}\right)\Delta B\right. \\
&\qquad \left. + \left(\left(\frac{\phi_{cs}}{F_s} - 1\right)^2 - 1 + \frac{J_s}{2}\right)\Delta J,\right) \\
\tau_u\frac{dC_t\Delta\gamma}{d\tau} &= -C_t\Delta\gamma,
\end{aligned}
\tag{9.1}
$$

where Δu is the control input perturbation and $\Delta\phi_c$, $\Delta\phi_t$, $\Delta\psi$, $\Delta\Psi_c$, ΔJ, and ΔB are the perturbations to the corresponding states.

From (9.1) observe that the last three of the linearised perturbation equations are only weakly coupled with the first four. An analysis of the stability indicates that the controlled system is stable. Assume that the compressor perturbation mass flow ($\Delta\phi_c$) is measured; the root locus plot is obtained and shown in Figure 5. The two lightly damped poles correspond to modes associated primarily with $\Delta\phi_c$ and $\Delta\psi$. To increase the stability margins, one could include stability augmentation negative feedback (gain = 3.3) and this is implemented in calculating the closed loop response in the next section. The chosen value of the gain corresponds to the maximum stability margin based on root locus plot.

The controller can now be tested by simulating it and the complete nonlinear plant. The case of a compressor with the parameters as listed in Table 1 is considered. The desired compressor flow ratio is chosen to be $\phi_{sd}/F = 2.1$. The desired value of the parameter $p = p_d$ is then estimated from (3.14). The initial value for $\Delta\gamma$ is chosen to be -0.2. The results of the closed loop simulation including negative feedback are illustrated in Figure 6(a) which corresponds to the same case as the one shown in Figure 4(a) without feedback.



(a) (b)

Figure 6: (a) Typical closed-loop state responses when H is positive and $\phi_{sd}/F > 2$ in the rotating stall case. (b) Unsteady, quasisteady and steady characteristics of the closed-loop compressor.

Figure 6(b) illustrates the unsteady characteristics of the closed-loop compressor which are compared with the steady-state characteristics. Also shown in the figure is the steady-state closed loop operating point. The results clearly indicate that the compressor now operates with the equilibrium $J = J_s = 0$ being stable. Thus the rotating stall disturbance is eliminated.

10. Conclusions

The dynamics of compressor stall has been reparameterised in a form that would facilitate the construction of a nonlinear control law for the active nonlinear control of compressor stall. The regions of stable performance in parameter space ($\gamma = \gamma_n$, $H > 0$, $J = J_s = 0$) and unstable performance ($\gamma \neq \gamma_n$ or $H < 0$, $J \neq 0$) were identified. This has led to the belief that a control law that maintains both $\gamma = \gamma_n$, $H > 0$ and $J = J_s = 0$ would actively stabilize the compressor. One observes that by merely setting the throttle at its optimum equilibrium position does not maintain, $J = J_s = 0$. An additional control input must aim to manipulate the transient and control pressure dynamics defined by (8.2d) which would involve control inputs to the compressors inlet guide vanes or some other means of feedback control. That in turn points to a need for a better compressor pressure rise model incorporating the control input dynamics. Yet the relatively simple and systematic approach adopted in this paper clearly highlights the main features on the controller that is capable of inhibiting compressor surge and rotating stall. Moreover, the method can be adopted for any axial compressor provided its steady-state compressor and throttle maps are known. Furthermore, the linear perturbation controller synthesised in the previous section could be substituted by a nonlinear controller synthesised by applying the backstepping approach as demonstrated by Krstic et al. [9]. Preliminary implementations of such a controller have supported the view that there is a need for an improved, matching, nonlinear compressor pressure rise model including disturbance and uncertainty effects and the results of this latter study involving a robust complimentary nonlinear H_∞ optimal control law will be reported elsewhere. Coupled with the views expressed by Greitzer [4], the active structural control of surge and rotating stall could be effectively achieved by realistic low-order modelling of the compressor dynamics.

References

[1] B. de Jager, "Rotating stall and surge control: a survey," in *Proceedings of the 34th IEEE Conference on Decision and Control*, vol. 2, pp. 1857–1862, New Orleans, La, USA, 1995.

[2] A. H. Epstein, J. E. F. Williams, and E. M. Greitzer, "Active suppression of aerodynamic instabilities in turbomachines," *Journal of Propulsion and Power*, vol. 5, no. 2, pp. 204–211, 1989.

[3] G. Gu, S. Banda, and A. Sparks, "An overview of rotating stall and surge control for axial flow compressors," in *Proceedings of the IEEE Conference on Decision and Control*, pp. 2786–2791, Kobe, Japan, 1996.

[4] E. M. Greitzer, "Some aerodynamic problems of aircraft engines: fifty years after-the 2007 IGTI scholar lecture-," *Journal of Turbomachinery*, vol. 121, Article ID 031101, pp. 1–13, 2009.

[5] F. K. Moore and E. M. Greitzer, "A theory of post-stall transients in axial compressors—Part I. Development of the equations," *Journal of Engineering for Gas Turbines and Power*, vol. 108, no. 1, pp. 68–76, 1986.

[6] W. R. Hawthorne and J. H. Horlock, "Actuator disc theory of the Incompressible flow in axial compressor," *Proceedings of Institution of Mechanical Engineers, London*, vol. 196, no. 30, p. 798, 1962.

[7] J. T. Gravdahl and O. Egeland, "A Moore-Greitzer axial compressor model with spool dynamics," in *Proceedings of the 36th IEEE Conference on Decision and Control*, vol. 5, pp. 4714–4719, 1997.

[8] L. Chen, R. Smith, and G. Dullerud, "A linear perturbation model for a non-linear system linearized at an equilibrium neighbourhood," in *Proceedings of the 37th IEEE Conference on Decision and Control*, pp. 4105–4106, Tampa, Fla, USA, 1998.

[9] M. Krstic, J. M. Protz, J. D. Paduano, and P. V. Kokotovic, "Backstepping designs for jet engine stall and surge control," in *Proceedings of the IEEE Conference on Decision and Control*, vol. 3, pp. 3049–3055, 1995.

[10] M. A. Nayfeh and E. H. Abed, "High-gain feedback control of surge and rotating stall in axial flow compressors," in *Proceedings of the American Control Conference*, vol. 4, pp. 2663–2667, San Diego, Calif, USA, 1999.

[11] J. D. Paduano, A. H. Epstein, L. Valavani, J. P. Longley, E. M. Greitzer, and G. R. Guenette, "Active control of rotating stall in a low-speed axial compressor," *Journal of Turbomachinery*, vol. 115, no. 1, pp. 48–56, 1993.

[12] S. Young, Y. Wang, and R. M. Murray, "Evaluation of bleed valve rate requirements in non-linear control of rotating stall on axial flow compressors," Tech. Rep. CALTECH CDS TR 98-001, Division of Engineering and Applied Science, California Institute of Technology, Calif, USA, 1998.

[13] G. Gu, A. Sparks, and S. Banda, "Bifurcation based nonlinear feedback control for rotating stall in axial flow compressors," *International Journal of Control*, vol. 68, no. 6, pp. 1241–1257, 1997.

[14] C. Hős, A. Champneys, and L. Kullmann, "Bifurcation analysis of surge and rotating stall in the Moore-Greitzer compression system," *IMA Journal of Applied Mathematics*, vol. 68, no. 2, pp. 205–228, 2003.

[15] E. M. Greitzer, "Surge and rotating stall in axial flow compressors—part I, II," *Journal of Engineering for Power*, vol. 98, no. 2, pp. 190–217, 1976.