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Research Article

Generate *n*-Scroll Attractor via Composite Switching Controls

Baiyu Ou[1] and Desheng Liu[2]

[1] *Department of Engineering, Army Aviation Institute, Beijing 101123, China*
[2] *State Key Laboratory for Turbulence and Complex System and College of Engineering, Peking University, Beijing 100871, China*

Correspondence should be addressed to Desheng Liu, dsliupku@gmail.com

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We propose a method for designing chaos generators. We introduce a switched system with three-dimensional space functions for generating a new type of chaotic attractor, and then we introduce saturated function series for generating *n*-scroll chaotic attractor. Moreover, we present some examples with numerical simulations that illustrate the efficiency of our method. The statistic behavior is also discussed, which reveals the regularities in the complex dynamics.

1. Introduction

Chaos is a very interesting nonlinear phenomenon, which has been extensively investigated during the last four decades. Since the 1960s, many chaotic (strange) attractors in dynamical systems have been found numerically and experimentally [1–3], such as Lorenz attractor [4], Rössler attractor [5], and Chua's attractor [6–11]. This proved that pursuing systems that can exhibit chaos in form of chaotic attractors is very helpful in understanding complex behavior of nonlinear dynamical systems, and is especially important for chaos communication technology [12], in which a chaos generator is prerequisite for chaos communication.

The Chua's circuit system that has double-scroll attractor is probably the best known and the simplest chaotic system that roots in a concrete physical system and has been extensively studied up to date [6, 13]. Moreover, multiscroll attractors are also found in some simple systems, to which nonlinear scalar functions, such as saturated function series [14] and switching scalar linear feedback [15], are implemented. Besides *n*-scroll attractors, patterns of strange attractors, such as multiple stripes [16], and spherical pattern [17], have been constructed using simple structures.

In [18], a chaotic attractor in a new funnel-shape is introduced, simply by designing a switched system with hysteresis switching signal. It also could be regarded as a method of chaotic attractor generation with one-dimensional space function. Motivated by this work, we introduce a switched system with three-dimensional space functions for generating a new type of chaotic attractor, and then we have made further effort to generate more chaotic behaviors, by introducing saturated function series, that is, a switched system with saturated function series approach is investigated. To our happiness, n-scroll chaotic attractor is observed. And moreover, the attractor with funnel-shape is also emerged in each scroll. The statistic behavior is also discussed, which reveal the regularities in the complex dynamics.

The rest of this paper is organized as follows. In Section 2, the concept of function series is proposed and the basic system framework is introduced. Section 3 introduces a switched system with three-dimensional space functions for generating a new type of chaotic attractor. Then n-scroll attractor generation analysis and simulation are given in Section 4. Finally, a brief conclusion is given in Section 5.

2. Preliminaries

In this section, we introduce the concept of integral function series and saturated function series and then apply them to our basic system framework.

2.1. Integral Function Series and Saturated Function Series

Consider the following integral function series:

$$h(x,m) = i \quad \text{if } im \leq x < (i+1)m,\ i,m \in Z,\ x \in R, \tag{2.1}$$

where m is positive integer, $h(x,m)$ is switched from i to $i+1$ if mx reaches the threshold $i+1$ from below and is switched from $i+1$ to i if mx reaches $i+1$ from above.

Consider the following saturated function:

$$f(x;k,e) = \begin{cases} k, & \text{if } x > e, \\ kx, & \text{if } |x| \leq e, \\ -k, & \text{if } x < -e, \end{cases} \tag{2.2}$$

where k is positive and e is the infinitesimal; we need to use saturated function series to simulate the integral function series. Based on it, the saturated function series can be described as:

$$f(x;k,m,e) = \begin{cases} 2ki, & \text{if } im + e < x < (1+i)m - e, \\ \dfrac{k(x-im)}{e} - k + 2ki, & \text{if } |x - im| \leq e. \end{cases} \tag{2.3}$$

We should point out that hysteresis function series and saturated function series are not continuous in switching methods.

2.2. System Formulation

We consider the following linear systems:

$$\dot{X} = A_1 X + b_1, \tag{2.4}$$

$$\dot{X} = A_2 X + b_2, \tag{2.5}$$

where X is a 3-dimensional state vector, $X = (x_1, x_2, x_3)^T$, A_1, A_2 are 3×3 constant matrices, and b_1, b_2 are 3-dimensional constant vectors.

We consider the switched systems (2.4) and (2.5) with the following specifications:

$$A_1 = \begin{pmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & c \end{pmatrix}, \qquad b_1 = \begin{pmatrix} 0 \\ 0 \\ \varepsilon \end{pmatrix}, \tag{2.6}$$

$$A_2 = \begin{pmatrix} f & 0 & 0 \\ 0 & g & h \\ 0 & -h & g \end{pmatrix}, \qquad b_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{2.7}$$

where we assume that $a > 0$, $c \neq 0$, $f < 0$, $g < 0$, $\varepsilon > 0$. System (2.4) with A_1 and b_1 given by (2.6) will be referred to as system (2.6). Similarly, system (2.5) with A_2 and b_2 given by (2.7) will be referred to as system (2.7).

By solving $\dot{X} = 0$, the equilibrium of system (2.6) is found to be $X_1^* = (0, 0, -\varepsilon/c)$. The Jacobian matrix J at this point is

$$J = \begin{pmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & c \end{pmatrix}, \tag{2.8}$$

whose eigenvalues are $\lambda_{1,2} = a \pm bi$ and $\lambda_3 = c$. Therefore, the equilibrium $(0, 0, -\varepsilon/c)$ is unstable.

The equilibrium of system (2.7) is $X_2^* = (0, 0, 0)$. The eigenvalues of A_2 are $\lambda_{1,2} = g \pm hi$, and $\lambda_3 = f$. Thus, the equilibrium $(0, 0, 0)$ is stable.

3. Generating Chaotic Attractor by Three-Dimensional Space Functions

This section presents the switched systems (2.6) and (2.7) which can generate chaotic attractor by three-dimensional space functions.

The switching rule is constructed as follows. Firstly, we introduce three-dimensional space function as follows:

$$g(x_1, x_2, x_3) = x_1^2 + x_2^2 + x_3^2. \tag{3.1}$$

Secondly, we assume two approximate integrals $G_1 = 25$, $G_2 = 900$. Finally, when system (2.6) is active, it will switch to system (2.7) at time t_1 if $g(x(t_1), y(t_1), z(t_1)) = G_1$. Similarly, when system (2.7) is active, it will switch to system (2.6) at time t_2 if $g(x(t_2), y(t_2), z(t_2)) = G_2$.

With this switching rule, we can generate chaos or chaos-like behavior by the system parameters are chosen as follows:

$$a = 1, \quad b = 11, \quad c = 0.5, \quad \varepsilon = 0.01, \quad f = -0.5, \quad g = -1, \quad h = 3. \tag{3.2}$$

As shown in Figure 1, the maximum Lyapunov exponent is 0.0026.

From this, we can see that the proposed space functions are quite effective in the generation of attractor with obviously quasi-periodic or chaotic behaviors and other chaotic attractors can be easy generated based on the change of parameters.

4. Generating n-Scroll Chaotic Attractor

This section presents novel systems—the switched systems with saturated function series—which can generate n-scroll funnel attractors. To generate n-scroll chaos, a nonlinear controller, saturated function series is added to system (2.6) and system (2.7). So the whole systems can be written as

$$\dot{X} = A_1 X + b_1 + F_1, \tag{4.1}$$

$$\dot{X} = A_2 X + b_2 + F_2, \tag{4.2}$$

where

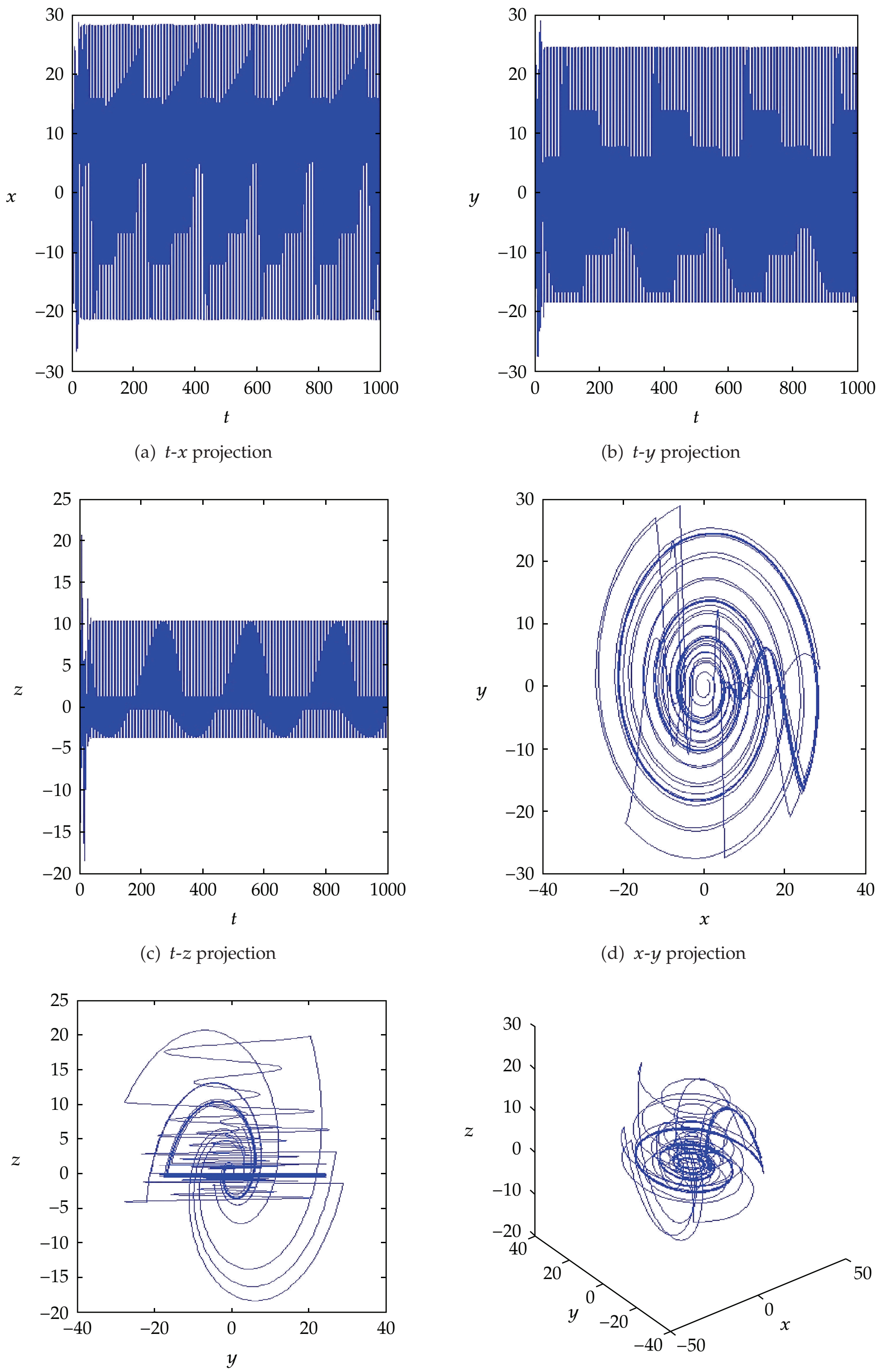
$$F_1 = \begin{pmatrix} f_1^1(X) \\ f_1^2(X) \\ f_1^3(X) \end{pmatrix}, \qquad F_2 = \begin{pmatrix} f_2^1(X) \\ f_2^2(X) \\ f_2^3(X) \end{pmatrix}. \tag{4.3}$$

4.1. Generating Double-Scroll Chaotic Attractors

Similar to the form of (2.3), we introduce the following saturated function:

$$f_0(x; k, m, e) = \begin{cases} 2k, & \text{if } x > m + e, \\ \dfrac{k(x-m)}{e} + k, & \text{if } |x - m| \le e, \\ 0, & \text{if } x < m - e, \end{cases} \tag{4.4}$$

$f_0(x)$ is the abbreviation for $f_0(x; k, m, e)$ in the following work for convenience.



(a) t-x projection (b) t-y projection

(c) t-z projection (d) x-y projection

(e) y-z projection (f) Chaotic attractor

Figure 1: Chaotic attractor generated by three-dimensional space function.

For generating double-scroll attractors located at x-axis, $f_1^1(\mathrm{X})$ is set to $-af_0(x_1)$, $f_1^2(\mathrm{X})$ is set to $bf_0(x_1)$ and $f_1^3(\mathrm{X})$ is set to 0, the system (4.1) can be written as:

$$\begin{aligned} \dot{x_1} &= a\left[x_1 - f_0(x_1)\right] + bx_2, \\ \dot{x_2} &= -b\left[x_1 - f_0(x_1)\right] + ax_2, \\ \dot{x_3} &= cx_3 + \varepsilon. \end{aligned} \tag{4.5}$$

Similarly, $f_2^1(\mathrm{X})$ is set to $-f * f_0(x_1)$, $f_2^2(\mathrm{X})$ and $f_2^3(\mathrm{X})$ are set to 0, and the system (4.2) can be written as

$$\begin{aligned} \dot{x_1} &= f\left[x_1 - f_0(x_1)\right], \\ \dot{x_2} &= gx_2 + hx_3, \\ \dot{x_3} &= -hx_2 + gx_3. \end{aligned} \tag{4.6}$$

Here we select parameter as (3.2) and

$$\begin{gathered} a = 1, \qquad b = 11, \qquad c = 0.5, \qquad \varepsilon = e = 0.01, \qquad f = -0.5, \\ g = -1, \qquad h = 3, \qquad k = m = 5. \end{gathered} \tag{4.7}$$

Figure 2 shows a double-scroll attractor, where 2 scrolls are generated in $(-10, 0)$ and $(0, 0)$. Figure 2(d) illustrates that the variable $x(t)$ spirals around the equilibrium points -10 and 0, which exhibits a random behavior.

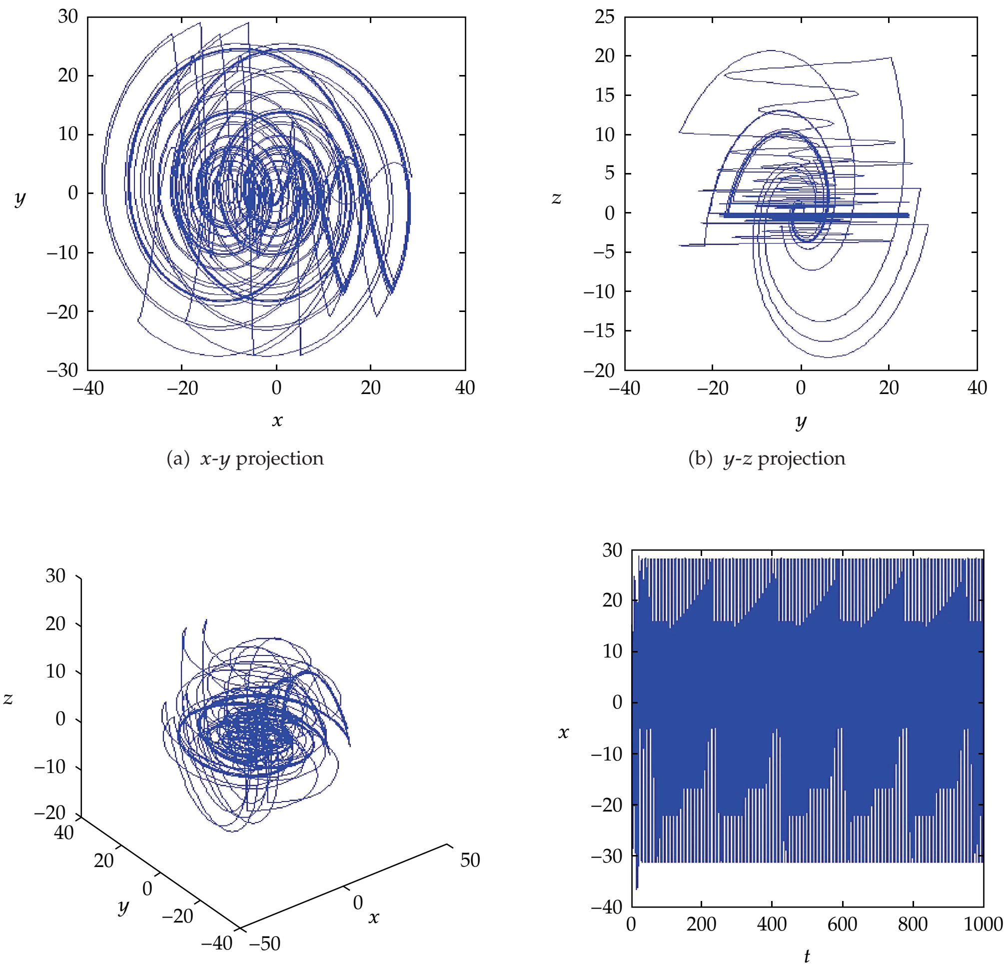
Under the parameter set (4.7), there exist two equilibrium points $(-10, 0)$ and $(0, 0)$, corresponding to saturated plateaus and saturated slope, respectively. It's noticed that the scrolls are generated only around the saturated plateaus.

4.2. Generating n-Scroll Chaotic Attractors

Similar to the form of (2.3), we introduce the following saturated functions:

$$\begin{aligned} f_1(x; k_1, m_1, e) &= \begin{cases} 2k_1, & \text{if } x > m_1 + e, \\ \dfrac{k_1(x - m_1)}{e} + k_1, & \text{if } |x - m_1| \le e, \\ 0, & \text{if } x < m_1 - e, \end{cases} \\ f_2(x; k_2, m_2, e) &= \begin{cases} 2k_2, & \text{if } x > m_2 + e, \\ \dfrac{k_2(x - m_2)}{e} + k_2, & \text{if } |x - m_2| \le e, \\ 0, & \text{if } x < m_2 - e, \end{cases} \end{aligned} \tag{4.8}$$

$f_1(x)$ and $f_2(x)$ are the abbreviations for $f_1(x; k, m, e)$, and $f_2(x; k, m, e)$ in the following work for convenience.



(a) x-y projection

(b) y-z projection

(c) Chaotic attractor

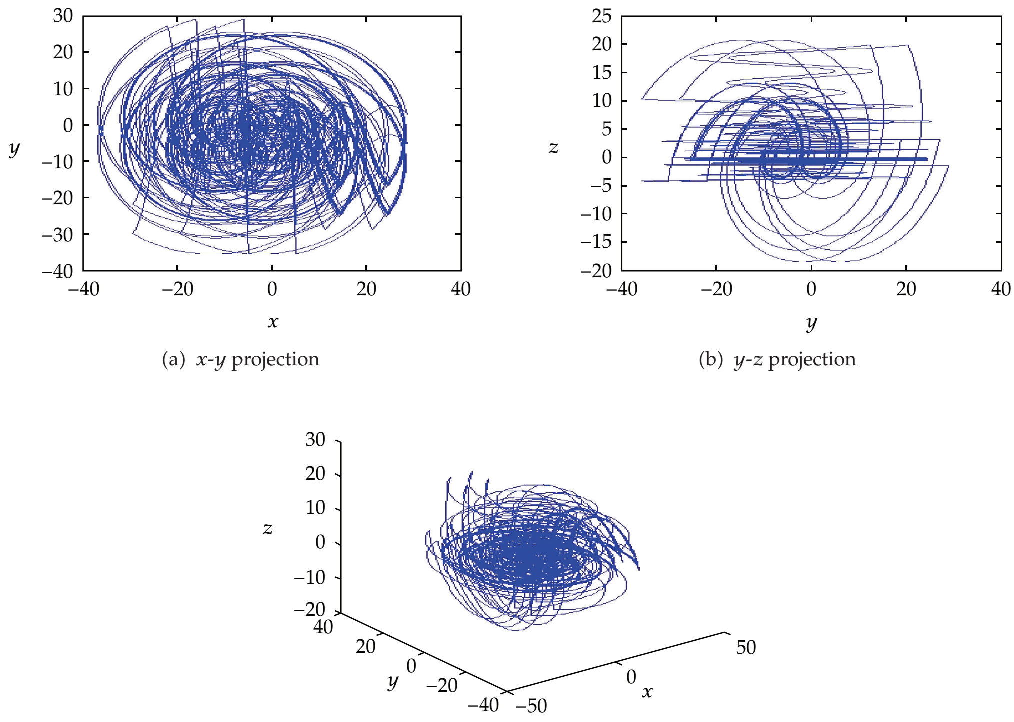
(d) Dynamics of variable $x(t)$

Figure 2: Double-scroll chaotic attractor generated by $f_o(x)$.

In the following, saturated function series controller is added to system (2.6) and system (2.7), aiming to generated n-scroll chaotic attractors. The form of the controller is similar to the one used in last section, the difference is that F_1 and F_2 are chosen to saturated function series defined by (2.3).

As a simple sample, we generate 2×2-scroll attractors located at x-axis and y-axis, $f_1^1(X)$ is set to $-af_1(x_1) - bf_2(x_2)$, $f_1^2(X)$ is set to $bf_1(x_1) - af_2(x_2)$, and $f_1^3(X)$ is set to 0, the system (4.1) can be written as

$$\begin{aligned}
\dot{x_1} &= a\left[x_1 - f_1(x_1)\right] + b\left[x_2 - f_2(x_2)\right], \\
\dot{x_2} &= -b\left[x_1 - f_1(x_1)\right] + a\left[x_2 - f_2(x_2)\right], \\
\dot{x_3} &= cx_3 + \varepsilon.
\end{aligned} \tag{4.9}$$



(a) x-y projection

(b) y-z projection

(c) Chaotic attractor

Figure 3: 2×2-scroll chaotic attractor generated by $f_1(x)$ and $f_2(x)$.

Similarly, $f_2^1(X)$ is set to $-f * f_1(x_1)$, $f_2^2(X)$ is set to $-gf_2(x_2)$, and $f_2^3(X)$ is set to $h * f_2(x_2)$, the system (4.2) can be written as

$$\begin{aligned} \dot{x_1} &= f\left[x_1 - f_1(x_1)\right], \\ \dot{x_2} &= g\left[x_2 - f_2(x_2)\right] + hx_3, \\ \dot{x_3} &= -h\left[x_2 - f_2(x_2)\right] + gx_3. \end{aligned} \tag{4.10}$$

Here we select parameter as (3.2) and

$$\varepsilon = e = 0.01, \qquad k_1 = m_1 = 5, \qquad k_2 = m_2 = 4. \tag{4.11}$$

Figure 3(a) shows a 2×2-grid scroll chaotic attractor, where 2×2 scrolls can be confirmed at the equilibrium points in $(0, 0)$, $(-10, 0)$, $(0, -8)$, $(-10, -8)$, and the maximum Lyapunov exponent is 0.0016.

From these numerical simulations it is shown that we can easily generate n-scroll chaotic attractors by switched systems with three-dimensional space functions and saturated function series. From this, we can also get other n-scroll chaotic attractors based on the change of parameters. Moreover, the statistic behavior is also researched by giving the largest Lyapunov exponents.

5. Conclusion

This paper introduces a switched system with three-dimensional space functions for generating a new type of chaotic attractor, and then, it also introduces n-scroll attractors' generation which by two switched systems with three-dimensional space functions and saturated function series. The generated attractors include 1D n-scroll and 2D $n \times m$-grid scroll. And moreover, new funnel-shape attractor is emerged in each scroll, which adds stochastic properties of the chaos signal. Furthermore, the statistic behavior is also discussed, which reveal the regularities in the complex dynamics. In addition, the method has been developed in this paper can also applied to nonlinear dynamical systems and other fields. It is desirable that one could design more chaos generators by means of the method proposed in this paper.

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