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Research Article

Robust Reliable Stabilization of Switched Nonlinear Systems with Time-Varying Delays and Delayed Switching

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This paper is concerned with the problem of robust reliable stabilization of switched nonlinear systems with time-varying delays and delayed switching is investigated. The parameter uncertainties are allowed to be norm-bounded. The switching instants of the controller experience delays with respect to those of the system. The purpose of this problem is to design a reliable state feedback controller such that, for all admissible parameter uncertainties and actuator failure, the system state of the closed-loop system is exponentially stable. We show that the addressed problem can be solved by means of algebraic matrix inequalities. The explicit expression of the desired robust controllers is derived in terms of linear matrix inequalities (LMIs).

1. Introduction

A switched system is composed of a family of continuous-time or discrete-time subsystems and a rule specifying the switching among them. Switched systems have received increasing attentions in the past few years, since many real-word systems such as mechanical systems, automotive industry, aircraft, and air traffic control systems, chemical processes can be modelled as switched systems (see [1–3]). A large number of results have been reported for such systems (see [4–8]).

On the other hand, time delay systems have continuously been receiveing considerable attention over the past decades. The main reason is that many kinds of engineering systems, for instance, long-distance transportation systems, hydraulic pressure systems, network control systems, and so on, include time delay phenomena in their dynamics. Many valuable results have been obtained for switched systems with time delay (see [9–15]). On the other hand, the actuators may be subjected to failures in real practice, therefore, it is of practical importance to design a control system which can tolerate faults of actuators. Several design approaches to the reliable controller have been proposed for linear and nonlinear systems (see [16–19]), and these results have been extended to switched systems (see [20–22]).

Recently, the asynchronous switching control problem of switched systems has stirred renewed research interests, and a varity of switched systems have been investigated by different approaches [23–26]. However, to the best of the authors' knowledge, the issue of reliable stabilization of switched nonlinear systems with time-varying delay under asynchronous switching has not been fully investigated, which motivated the present study.

In this paper, we are interested in designing the robust reliable controller for uncertain switched nonlinear system with time-varying delays and delayed switching. The remainder of the paper is organized as follows. In Section 2, problem formulation is presented and the failure model of actuator in switched system is introduced briefly. In addition, some necessary lemmas are given. In Section 3, based on the average dwell-time approach, controller design for switched nonlinear system with time-varying delays and delayed switching is developed, and sufficient conditions for the existence of the controller are formulated in terms of a set of matrix inequalities. Concluding remarks are given in Section 4.

Notation. Throughout this paper, the superscript "T" denotes the transpose, $\|\cdot\|$ denotes the Euclidean norm. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ denote the maximum and minimum eigenvalues of matrix P, respectively, I is an identity matrix with appropriate dimension. $\operatorname{diag}\{a_i\}$ denotes diagonal matrix with the diagonal elements a_i, $i = 1, 2, \ldots, n$. The asterisk $*$ in a matrix is used to denote term that is induced by symmetry. The set of positive integers is represented by Z^+.

2. Problem Formulation and Preliminaries

Consider the following uncertain nonlinear switched system with actuator fault

$$\begin{gathered}\dot{x}(t) = \widehat{A}_{\sigma(t)}x(t) + \widehat{A}_{d\sigma(t)}x\big(t - d_{\sigma(t)}(t)\big) + B_{\sigma(t)}u^f(t) + f_{\sigma(t)}(x(t), t), \\ x(t) = \varphi(t), \quad t \in [t_0 - d, t_0],\end{gathered} \tag{2.1}$$

where $x(t) \in R^n$ is the state vector, $u^f(t) \in R^l$ is the control input of actuator fault, $\varphi(t)$ is a continuous vector-valued function. The function $\sigma(t) : [t_0, \infty) \to \underline{N} = \{1, 2, \ldots, N\}$ is the switching signal which is deterministic, piecewise constant and right continuous, that is, $\sigma : \{(t_0, \sigma(t_0)), (t_1, \sigma(t_1)), \ldots\}$, $k \in Z^+$, where $t_0 \geq 0$ is the initial time, and t_k denotes the kth switching instant. $d_{\sigma(t)}(t)$ denotes the time-varying state delay satisfying $0 < d_{\sigma(t)}(t) \leq d$, $\dot{d}_{\sigma(t)}(t) \leq \tau$ for constants d and τ. Moreover, $\sigma(t) = i$ means that the ith subsystem is activated. N denotes the number of subsystems. $f_i(x(t), t)$ $(i \in \underline{N})$ are nonlinear functions satisfying

$$\big\| f_i(x(t), t)\big\| \leq \|U_i x(t)\|, \tag{2.2}$$

where U_i are known real constant matrices.

$\widehat{A}_i, \widehat{A}_{di}$ for $i \in \underline{N}$ are uncertain real-valued matrices with appropriate dimensions, and have the following form:

$$\left[\widehat{A}_i \ \ \widehat{A}_{di}\right] = \left[A_i \ \ A_{di}\right] + H_i F_i(t)\left[E_{1i} \ \ E_{2i}\right], \tag{2.3}$$

where $A_i,\ A_{di}, B_i, H_{1i}, E_{1i}, E_{2i}$ are known real constant matrices with proper dimensions, and H_{1i}, E_{1i}, E_{2i} denote the structure of the uncertainties, $F_i(t)$ are unknown time-varying matrices which satisfy

$$F_i^T(t)F_i(t) \leq I. \tag{2.4}$$

The control input of actuator fault $u^f(t)$ can be described as

$$u^f(t) = M_{\sigma(t)}u(t), \tag{2.5}$$

where $u(t) = K_{\sigma(t)}x(t)$ is the switching controller which will be designed, M_i $(i \in \underline{N})$ are the actuator fault matrices with the following form:

$$M_i = \operatorname{diag}\{m_{i1}, m_{i2}, \ldots, m_{il}\}, \quad 0 \leq \underline{m}_{ik} \leq m_{ik} \leq \overline{m}_{ik},\ \ \overline{m}_{ik} \geq 1,\ \ k = 1, 2, \ldots, l. \tag{2.6}$$

For simplicity, we introduce the following notation

$$M_{i0} = \operatorname{diag}\{\widetilde{m}_{i1}, \widetilde{m}_{i2}, \ldots, \widetilde{m}_{il}\}, \qquad J_i = \operatorname{diag}\{j_{i1}, j_{i2}, \ldots, j_{il}\}, \qquad L_i = \operatorname{diag}\{l_{i1}, l_{i2}, \ldots, l_{il}\}, \tag{2.7}$$

where $\widetilde{m}_{ik} = 1/2(\overline{m}_{ik} + \underline{m}_{ik})$, $j_{ik} = (\overline{m}_{ik} - \underline{m}_{ik})/(\overline{m}_{ik} + \underline{m}_{ik})$, $l_{ik} = (m_{ik} - \widetilde{m}_{ik})/\widetilde{m}_{ik}$.

From (2.6)-(2.7), we have

$$M_i = M_{i0}(I + L_i), \quad |L_i| \leq J_i \leq I, \tag{2.8}$$

where $|L_i| = \operatorname{diag}\{|l_{i1}|, |l_{i2}|, \ldots, |l_{il}|\}$.

Remark 2.1. $m_{ik} = 1$ means normal operation of the kth actuator signal of the ith subsystem. When $m_{ik} = 0$, it covers the case of the complete failure of the kth actuator signal of the ith subsystem. When $\underline{m}_{ik} > 0$ and $m_{ik} \neq 1$, it corresponds to the case of partial failure of the kth actuator signal of the ith subsystem.

The delayed switching of the controller can be shown in Figure 1.

We can see from Figure 1 that the controller K_i operates the ith subsystem in $[t_{k-1} + \Delta_{k-1}, t_k)$, and operates the jth subsystem in $[t_k, t_k + \Delta_k)$.

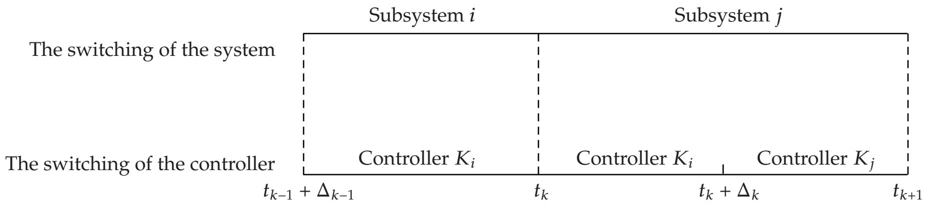

Figure 1: Diagram of the delayed switching.

Let $\sigma'(t)$ denote the switching signal of the controller, the switching instants of the controller can be described as

$$t_1 + \Delta_1, t_2 + \Delta_2, \ldots, t_k + \Delta_k, \ldots, k \in Z^+, \tag{2.9}$$

where $\Delta_k < \inf_{k\in Z^+}(t_{k+1} - t_k)$, Δ_k represents the delayed period, and it is said to be mismatched period.

Remark 2.2. Mismatched period $\Delta_k < \inf_{k\in Z^+}(t_{k+1} - t_k)$ guarantees that there always exists a period that the controller and the system operate synchronously, and this period is said to be matched period in the later section.

Due to the delayed switching, the real input of actuator fault can be written as

$$u^f(t) = M_{\sigma'(t)} K_{\sigma'(t)} x(t). \tag{2.10}$$

Under switching controller (2.10), the resulting closed-loop system is given by

$$\begin{gathered} \dot{x}(t) = \left(A_{\sigma(t)} + B_{\sigma(t)} M_{\sigma'(t)} K_{\sigma'(t)}\right) x(t) + A_{d\sigma(t)} x\left(t - d_{\sigma(t)}(t)\right) + f_{\sigma(t)}(x(t), t), \\ x(t) = \varphi(t), \quad t \in [t_0 - d, t_0]. \end{gathered} \tag{2.11}$$

System (2.1) without uncertainties and actuator fault can be written as

$$\begin{gathered} \dot{x}(t) = A_{\sigma(t)} x(t) + A_{d\sigma(t)} x\left(t - d_{\sigma(t)}(t)\right) + B_{\sigma(t)} u(t) + f_{\sigma(t)}(x(t), t), \\ x(t) = \varphi(t), \quad t \in [t_0 - d, t_0]. \end{gathered} \tag{2.12}$$

Definition 2.3 (see [13]). If there exists switching signal $\sigma(t)$, such that the trajectory of system (2.1) satisfies $\|x(t)\| \le \alpha \|x(t_0)\|_h e^{-\beta(t-t_0)}$, then system (2.1) is said to be exponentially stable with convergent rate β, where $\alpha \ge 1$, $\beta > 0$, $t \ge t_0$, $\|x(t)\|_h = \sup_{-d\le\theta\le 0}\{\|x(t+\theta)\|, \|\dot{x}(t+\theta)\|\}$.

Definition 2.4 (see [27]). For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$ on an interval (T_1, T_2). If

$$N_\sigma(T_1, T_2) \leq N_0 + \frac{T_2 - T_1}{\tau_a} \tag{2.13}$$

hold for given $N_0 \geq 0$, $\tau_a > 0$, then the constant τ_a is called the average dwell time and N_0 is the chatter bound.

The following lemmas play an important role in the later development.

Lemma 2.5 (see [28]). *For given vectors a, b and the positive matrix $X > 0$, there exists the matrix M with appropriate dimension, such that*

$$-2a^T b \leq \inf_{X>0,M} \left\{ \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & XM \\ M^T X & (M^T X + I)X^{-1}(XM + I) \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \right\}. \tag{2.14}$$

Lemma 2.6 (see [29]). *For matrices X, Y of appropriate dimension and $Q > 0$, we have*

$$X^T Y + Y^T X \leq X^T Q X + Y^T Q^{-1} Y. \tag{2.15}$$

Lemma 2.7 (see [30]). *Let U, V, W, and X be real matrices of appropriate dimensions with X satisfying $X = X^T$, then for all $V^T V \leq I$,*

$$X + UVW + W^T V^T U^T < 0 \tag{2.16}$$

if and only if there exists a scalar $\varepsilon > 0$ such that

$$X + \varepsilon U U^T + \varepsilon^{-1} W^T W < 0. \tag{2.17}$$

Lemma 2.8 (see [31]). *For matrices R_1, R_2 with appropriate dimension, there exists a positive scalar $\beta > 0$, such that*

$$R_1 \Sigma(t) R_2 + R_2^T \Sigma^T(t) R_1^T \leq \beta R_1 U R_1^T + \beta^{-1} R_2^T U R_2 \tag{2.18}$$

hold, where $\Sigma(t)$ is time-varying diagonal matrix, U is known real-value matrix satisfying $|\Sigma(t)| \leq U$.

The objective of this paper is to design a reliable controller for system (2.1) with delayed switching such that the resulting closed-loop system is robust exponentially stable.

3. Main Results

To obtain the main results of this paper, we first consider the stability of the following nonlinear delay system

$$\dot{x}(t) = Ax(t) + A_d x(t - d(t)) + f(x(t), t), \tag{3.1}$$

$$x(t) = \varphi(t), \quad t \in [t_0 - d, t_0], \tag{3.2}$$

where $x(t) \in R^n$ is the state vector, $\varphi(t)$ is a continuous vector-valued initial function, A, A_d are real-valued matrices with appropriate dimensions, $f(\cdot,\cdot) : R^n \times R \to R^n$ is unknown nonlinear functions satisfying

$$\| f(x(t), t) \| \leq \left\| \overline{U} x(t) \right\|, \tag{3.3}$$

where $\overline{U}$ is known real constant matrix.

3.1. Stability Analysis

Lemma 3.1. *Consider system* (3.1)-(3.2)*, for given positive constant* α, ρ_1, ρ_2, ε_1, η_1, *if there exist positive definite symmetric matrices* P, S, Q, R, *and any matrices* G, W *with appropriate dimensions, such that*

$$Q \leq \rho_1 I, \qquad P \leq \rho_2 I, \tag{3.4}$$

$$\begin{bmatrix} \Sigma & G^T & W^T A_d & A^T Q & 0 & d(W^T + P) & A^T Q & 0 & \overline{U}^T \\ * & -(1-\tau)e^{-\alpha d} R - G - G^T & -G & A_d^T Q & 0 & 0 & 0 & A_d^T Q & 0 \\ * & * & -d^{-1} e^{-\alpha d} Q & 0 & A_d^T S & 0 & 0 & 0 & 0 \\ * & * & * & -d^{-1} Q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -d^2 S & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -S & 0 & 0 & 0 \\ * & * & * & * & * & * & -d^{-1}\varepsilon_1 Q & 0 & 0 \\ * & * & * & * & * & * & * & -d^{-1}\eta_1 Q & 0 \\ * & * & * & * & * & * & * & * & \Phi \end{bmatrix} < 0, \tag{3.5}$$

holds, then for Lyapunov functional candidate

$$V(x(t)) = x^T(t) P x(t) + \int_{t-d(t)}^{t} \dot{x}^T(s) e^{-\alpha(t-s)} R \dot{x}(s) ds + \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(r) e^{-\alpha(t-r)} Q \dot{x}(r) dr\, d\theta, \tag{3.6}$$

along the trajectory of system (3.1), *there holds the following inequality:*

$$V(x(t)) < e^{-\alpha(t-t_0)}V(x(t_0)), \tag{3.7}$$

where $\Sigma = (A + A_d)^T P + P(A + A_d) + (\alpha + 1)P + R,\ \Phi = -[\tau\rho_1(1 + \varepsilon_1 + \eta_1) + \rho_2]^{-1}I$.

Proof. Let $V_1(x(t)) = x^T(t)Px(t)$, $V_2(x(t)) = \int_{t-d(t)}^{t} \dot{x}^T(s)e^{-\alpha(t-s)}R\dot{x}(s)ds$, $V_3(x(t)) = \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^T(r)e^{-\alpha(t-r)}Q\dot{x}(r)dr\,d\theta$.

Notice that $x(t) - x(t - d(t)) = \int_{t-d(t)}^{t} \dot{x}(r)dr$, (3.1) can be written as

$$\dot{x}(t) = (A + A_d)x(t) + f(x(t), t) - A_d\int_{t-d(t)}^{t} \dot{x}(r)dr. \tag{3.8}$$

Along the trajectory of system (3.1), the time derivative of $V_1(x(t))$ is given by

$$\dot{V}_1(x(t)) = 2x^T(t)P\{(A + A_d)x(t) + f(x(t), t)\} - \int_{t-d(t)}^{t} 2x^T(t)PA_d\dot{x}(r)dr. \tag{3.9}$$

Let $a = A_d\dot{x}(r),\ b = Px(t)$, from Lemma 2.5, we can obtain

$$\begin{aligned}-2x^T(t)PA_d\dot{x}(r) &\le \dot{x}^T(r)A_d^TXA_d\dot{x}(r) + 2\dot{x}^T(r)A_d^TXMPx(t)\\ &\quad + x^T(t)P\left(M^TX + I\right)X^{-1}(XM + I)Px(t).\end{aligned} \tag{3.10}$$

Substituting (3.10) into (3.9) leads to

$$\begin{aligned}\dot{V}_1(x(t)) &\le x^T(t)\left\{(A + A_d)^TP + P(A + A_d) + d(t)P\left(M^TX + I\right)X^{-1}(MX + I)P\right\}x(t)\\ &\quad + x^T(t)Pf(x(t), t) + f^T(x(t), t)Px(t) + 2x^T(t)PM^TXA_d\int_{t-d(t)}^{t} \dot{x}(r)dr\\ &\quad + \int_{t-d(t)}^{t} \dot{x}^T(r)A_d^TXA_d\dot{x}(r)dr\\ &\le x^T(t)\left\{(A + A_d)^TP + P(A + A_d) + \tau P\left(M^TX + I\right)X^{-1}(XM + I)P\right\}x(t)\\ &\quad + 2x^T(t)Pf(x(t), t) + 2x^T(t)PM^TXA_d\int_{t-d(t)}^{t} \dot{x}(r)dr + \int_{t-d(t)}^{t} \dot{x}^T(r)A_d^TXA_d\dot{x}(r)dr.\end{aligned} \tag{3.11}$$

Differentiating $V_2(x(t))$ and $V_3(x(t))$ along the trajectory of system (3.1), we have

$$\begin{aligned}
\dot{V}_2(x(t)) &\leq -\alpha \int_{t-d(t)}^{t} e^{-\alpha(t-s)} \dot{x}^T(s) R \dot{x}(s) ds + x^T(t) R x(t) \\
&\quad - (1-\tau) e^{-\alpha d} x^T(t-d(t)) R x(t-d(t)), \\
\dot{V}_3(x(t)) &= d\dot{x}^T(t) Q \dot{x}(t) - \int_{t-d}^{t} \dot{x}^T(r) e^{-\alpha(t-r)} Q \dot{x}(r) dr - \alpha \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(r) e^{-\alpha(t-r)} Q \dot{x}(r) dr\, d\theta \\
&\leq d\dot{x}^T(t) Q \dot{x}(t) - e^{-\alpha d} \int_{t-d(t)}^{t} \dot{x}^T(r) Q \dot{x}(r) dr - \alpha \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(r) e^{-\alpha(t-r)} Q \dot{x}(r) dr\, d\theta \\
&= dx^T(t) A^T Q A x(t) + 2dx^T(t-d(t)) A_d^T Q A x(t) + dx^T(t-d(t)) A_d^T Q A_d x(t-d(t)) \\
&\quad - e^{-\alpha d} \int_{t-d(t)}^{t} \dot{x}^T(r) Q \dot{x}(r) dr - \alpha \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(r) e^{-\alpha(t-r)} Q \dot{x}(r) dr\, d\theta \\
&\quad + d\Big[2x^T(t) A^T Q f(x(t),t) + 2x^T(t-d(t)) A_d^T Q f(x(t),t) + f^T(x(t),t) Q f(x(t),t)\Big].
\end{aligned} \tag{3.12}$$

By Lemma 2.6, we have

$$\begin{gathered}
2x^T(t) P f(x(t)) \leq x^T(t) P x(t) + f^T(x(t)) P f(x(t)), \\
2x^T(t) A^T Q f(x(t),t) \leq \varepsilon_1^{-1} x^T(t) A^T Q A x(t) + \varepsilon_1 f^T(x(t),t) Q f(x(t),t), \\
2x^T(t-d(t)) A_d^T Q f(x(t),t) \leq \eta_1^{-1} x^T(t-d(t)) A_d^T Q A_d x(t-d(t)) + \eta_1 f^T(x(t),t) Q f(x(t),t).
\end{gathered} \tag{3.13}$$

Therefore

$$\begin{aligned}
&\dot{V}(x(t)) + \alpha V(x(t)) \\
&\quad = \dot{V}_1(x(t)) + \dot{V}_2(x(t)) + \alpha V(x(t)) \\
&\quad \leq x^T(t) \Big\{ (A + A_d)^T P + P(A + A_d) + dP\Big(M^T X + I\Big) X^{-1} (XM + I) P \\
&\qquad\qquad + (\alpha + 1) P + d\Big(1 + \varepsilon_1^{-1}\Big) A^T Q A \Big\} x(t) \\
&\qquad + 2dx^T(t-d(t)) A_d^T Q A x(t) + d\Big(1 + \eta_1^{-1}\Big) x^T(t-d(t)) A_d^T Q A_d x(t-d(t)) \\
&\qquad + 2x^T(t) P M^T X A_d \int_{t-d(t)}^{t} \dot{x}(r) dr \\
&\qquad + \int_{t-d(t)}^{t} \dot{x}^T(r) \Big(A_d^T X A_d - e^{-\alpha d} Q\Big) \dot{x}(r) dr + d(1 + \varepsilon_1 + \eta_1) f^T(x(t),t) \\
&\qquad \times Q f(x(t),t) + f^T(x(t)) P f(x(t))
\end{aligned}$$

$$\begin{aligned}&\leq x^T(t)\Big\{(A+A_d)^TP+P(A+A_d)+dP\Big(M^TX+I\Big)X^{-1}(XM+I)P\\&\qquad+(\alpha+1)P+d\Big(1+\varepsilon_1^{-1}\Big)A^TQA+d(1+\varepsilon_1+\eta_1)\overline{U}^TQ\overline{U}+\overline{U}^TP\overline{U}\Big\}x(t)\\&\quad+2dx^T(t-d(t))A_d^TQAx(t)+d\Big(1+\eta_1^{-1}\Big)x^T(t-d(t))A_d^TQA_dx(t-d(t))\\&\quad+2x^T(t)PM^TXA_d\int_{t-d(t)}^t\dot{x}(r)dr+\int_{t-d(t)}^t\dot{x}^T(r)\Big(A_d^TXA_d-e^{-\alpha d}Q\Big)\dot{x}(r)dr.\end{aligned} \tag{3.14}$$

Notice that $2x^T(t-d(t))G[x(t)-x(t-d(t))-\int_{t-d(t)}^t\dot{x}(r)dr]=0$, where G is any matrix with appropriate dimension, we have

$$\dot{V}(x(t))+\alpha V(x(t))\leq\frac{1}{d(t)}\int_{t-d(t)}^t\xi^T(t,r)Z\xi(t,r)dr, \tag{3.15}$$

where $\xi^T(t,r)=[\,x^T(t)\ \ x^T(t-d)\ \ \dot{x}^T(r)\,]$,

$$Z=\begin{bmatrix}\Psi-\overline{U}^T\Phi_1^{-1}\overline{U} & \tau A^TQA_d+G^T & d(t)W^TA_d\\ \tau A_d^TQA+G & \tau A_d^TQA_d-(1-\tau)e^{-\alpha d}R-G-G^T & -d(t)G\\ d(t)A_d^TW & -d(t)G^T & d(t)\left(\tau^{-1}A_d^TSA_d-e^{-\alpha\tau}Q\right)\end{bmatrix}, \tag{3.16}$$

where $\Psi=(A+A_d)^TP+P(A+A_d)+dP(M^TX+I)X^{-1}(XM+I)P+(\alpha+1)P+R+d(1+\varepsilon_1^{-1})A^TQA$.

Let $W=XMP, S=\tau X$, by Schur complement lemma, (3.5) is equivalent to the following inequality:

$$\begin{bmatrix}\Psi & dA^TQA_d+G^T & W^TA_d & \overline{U}^T\\ dA_d^TQA+G & dA_d^TQA_d-(1-\tau)e^{-\alpha d}-G-G^T & -G & 0\\ A_d^TW & -G^T & d^{-2}A_d^TSA_d-d^{-1}e^{-\alpha\tau}Q & 0\\ \overline{U} & 0 & 0 & \Phi\end{bmatrix}<0. \tag{3.17}$$

Using $\mathrm{diag}\{I,I,d(t)I,I\}$ to pre- and post- multiply the left term of (3.17), respectively, we can obtain $Z<0$. Therefore, $\dot{V}(x(t))+\alpha V(x(t))<0$.

The proof is completed. □

Lemma 3.2. *Consider system* (3.1)-(3.2)*, for given positive constant* β*,* ρ_1*,* ρ_2*,* ε_1*,* η_1*, if there exist positive definite symmetric matrices* P, S, Q*, and any matrices* G, W *with appropriate dimensions, such that*

$$Q \leq \rho_1 I, \qquad P \leq \rho_2 I, \tag{3.18}$$

$$\begin{bmatrix} \Sigma & G^T & W^T A_d & A^T Q & 0 & d(W^T + P) & A^T Q & 0 & \overline{U}^T \\ * & -G - G^T & -G & A_d^T Q & 0 & 0 & 0 & A_d^T Q & 0 \\ * & * & -d^{-1} e^{-\alpha d} Q & 0 & A_d^T S & 0 & 0 & 0 & 0 \\ * & * & * & -d^{-1} Q & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -d^2 S & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -S & 0 & 0 & 0 \\ * & * & * & * & * & * & -d^{-1}\varepsilon_1 Q & 0 & 0 \\ * & * & * & * & * & * & * & -d^{-1}\eta_1 Q & 0 \\ * & * & * & * & * & * & * & * & \Phi \end{bmatrix} < 0 \tag{3.19}$$

holds, then for Lyapunov functional candidate

$$V(x(t)) = x^T(t) P x(t) + \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(r) e^{-\alpha(t-r)} Q \dot{x}(r) dr\, d\theta, \tag{3.20}$$

along the trajectory of system (3.1)*, there holds the following inequality*

$$V(x(t)) < e^{\beta(t-t_0)} V(x(t_0)), \tag{3.21}$$

where $\Sigma = (A + A_d)^T P + P(A + A_d) - (\beta - 1)P$.

Proof. Similarly to the proof line of Lemma 3.1, we can obtain Lemma 3.2. □

3.2. Stabilizing Controller Design

In this subsection, we will design a stabilizing controller for system (2.12) with delayed switching.

In our design approach we only require the subsystems to be stable during matched period, and the subsystems are allowed to be unstable during mismatched period. Under delayed switching controller $u(t) = K_{\sigma'(t)} x(t)$, the corresponding closed-loop system is given by

$$\dot{x}(t) = \left(A_{\sigma(t)} + B_{\sigma(t)} K_{\sigma'(t)}\right) x(t) + A_{d\sigma(t)} x\left(t - d_{\sigma(t)}(t)\right) + f_{\sigma(t)}(x(t), t), \tag{3.22}$$

$$x(t) = \varphi(t), \quad t \in [t_0 - d, t_0]. \tag{3.23}$$

Let $T^{+}(t_0,t)$ denote the total mismatched period during $[t_0,t)$, $T^{-}(t_0,t)$ denote the total matched period during $[t_0,t)$, then we have the following result.

Theorem 3.3. *Consider system* (2.12)*, for given positive constants* α, β, ε_1, η_1, ε_2, η_2, ρ_1, ρ_2, ρ_3, ρ_4, *if there exist positive definite symmetric matrices* $X_i, Z_i, S_i, P_{ij}, Q_{ij}, S_{ij}$, *and any matrices* Y_i, G_{ij}, W_{ij} *with appropriate dimensions, such that, for* $i, j \in \underline{N},\ i \neq j$,

$$Z_i \geq \rho_1^{-1} I, \qquad X_i \geq \rho_2^{-1} I, \qquad Q_{ij} \leq \rho_3 I, \qquad P_{ij} \leq \rho_4 I, \tag{3.24}$$

$$\begin{bmatrix} \Pi_i & X_i & A_{di}Z_i & \Xi_i & 0 & 2dS_i & \Xi_i & 0 & X_iU_i^T \\ * & -2Z_i & -Z_i & Z_iA_{di}^T & 0 & 0 & 0 & Z_iA_{di}^T & 0 \\ * & * & -d^{-1}e^{-\alpha d}Z_i & 0 & Z_iA_{di}^T & 0 & 0 & 0 & 0 \\ * & * & * & -d^{-1}Z_i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -d^2S_i & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -S_i & 0 & 0 & 0 \\ * & * & * & * & * & * & -d^{-1}\varepsilon_1 Z_i & 0 & 0 \\ * & * & * & * & * & * & * & -d^{-1}\eta_1 Z_i & 0 \\ * & * & * & * & * & * & * & * & \Phi_1 \end{bmatrix} < 0, \tag{3.25}$$

$$\begin{bmatrix} \Lambda_{ij} & G_{ij}^T & W_{ij}^TA_{dj} & \Xi_{ij} & 0 & d\left(W_{ij}^T+P_{ij}\right) & \Xi_{ij} & 0 & U_j^T \\ G_{ij} & -G_{ij}-G_{ij}^T & -G_{ij} & A_{dj}^TQ_{ij} & 0 & 0 & 0 & A_{dj}^TQ_{ij} & 0 \\ A_{dj}^TW_{ij} & -G_{ij}^T & -d^{-1}Q_{ij} & 0 & A_{dj}^TS_{ij} & 0 & 0 & 0 & 0 \\ \Xi_{ij}^T & Q_{ij}A_{dj} & 0 & -d^{-1}Q_{ij} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{ij}A_{dj} & 0 & -d^2S_{ij} & 0 & 0 & 0 & 0 \\ \tau(W_{ij}+P_{ij}) & 0 & 0 & 0 & 0 & -S_{ij} & 0 & 0 & 0 \\ \Xi_{ij}^T & 0 & 0 & 0 & 0 & 0 & -d^{-1}\varepsilon_2Q_{ij} & 0 & 0 \\ 0 & Q_{ij}A_{dj} & 0 & 0 & 0 & 0 & 0 & -d^{-1}\eta_2Q_{ij} & 0 \\ U_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_2 \end{bmatrix} < 0 \tag{3.26}$$

holds, then under the switching controller $u(t) = K_{\sigma'(t)}x(t), K_i = Y_iX_i^{-1}$, *and the following average dwell-time scheme:*

$$\inf_{t>t_0} \frac{T^{-}(t_0,t)}{T^{+}(t_0,t)} \geq \frac{\beta+\lambda^*}{\alpha-\lambda^*}, \qquad \tau_a > \tau_a^* = \frac{\ln(\mu_1\mu_2)}{\lambda^*}, \tag{3.27}$$

the corresponding closed-loop system is exponentially stable, where $0 < \lambda^* < \alpha$, $\mu_1, \mu_2 \geq 1$ *satisfying* $X_i^{-1} < \mu_1 P_{ij}$, $P_{ij} < \mu_2 X_i^{-1}$, $Z_i^{-1} < \mu_1 Q_{ij}$, $Q_{ij} < \mu_2 Z_i^{-1}$, $\Phi_1 = -[d\rho_1(1+\varepsilon_1+\eta_1)+\rho_2]^{-1}I$, $\Xi_i = X_iA_i^T + Y_i^TB_i^T$, $\Pi_i = (A_i+A_{di})X_i + X_i(A_i+A_{di})^T + (\alpha+1)X_i + B_iY_i + Y_i^TB_i^T + R_i$, $\Phi_2 = -[d\rho_3(1+\varepsilon_2+\eta_2)+\rho_4]^{-1}I$, $\Xi_{ij} = (A_j + B_jY_iX_i^{-1})^TQ_{ij}$, $\Lambda_{ij} = (A_j+A_{dj})^TP_{ij} + P_{ij}(A_j+A_{dj}) - (\beta-1)P_{ij} + P_{ij}B_jY_iX_i^{-1} + X_i^{-1}Y_i^TB_j^TP_{ij}$.

Proof. Suppose that the ith subsystem is activated at the switching instant t_{k-1}, the jth subsystem is activated at the switching instant t_k, then the corresponding switchings of the controller occur at the switching instant $t_{k-1} + \Delta_{k-1}$ and $t_k + \Delta_k$, respectively.

When $t \in [t_{k-1} + \Delta_{k-1}, t_k)$, system (3.22) can be written as

$$\dot{x}(t) = A_{Ki}x(t) + A_{di}x(t - d_i(t)) + f_i(x(t), t), \tag{3.28}$$

where $A_{Ki} = A_i + B_iK_i$.

Consider Lyapunov functional candidate as follows:

$$V_i(x(t)) = x^T(t)P_ix(t) + \int_{t-d(t)}^{t} \dot{x}^T(s)e^{-\alpha(t-s)}R_i\dot{x}(s)ds + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{x}^T(r)e^{-\alpha(t-r)}Q_i\dot{x}(r)dr\,d\theta. \tag{3.29}$$

For given positive constants α, ρ_1, ρ_2, ε_1, η_1, if there exist positive definite symmetric matrices $P_i, \widetilde{S}_i, Q_i, \widetilde{R}_i$, and any matrices G_i, W_i with appropriate dimensions such that the follows matrix inequalities (3.30)-(3.31) hold, then from Lemma 3.1 we have $\dot{V}_i(x(t)) + \alpha V_i(x(t)) < 0$:

$$Q_i \leq \rho_1 I, \qquad P_i \leq \rho_2 I, \tag{3.30}$$

$$\begin{bmatrix}
\Theta_i & G_i^T & W_i^TA_{di} & A_{Ki}^TQ_i & 0 & d(W_i^T+P_i) & A_{Ki}^TQ_i & 0 & U_i^T \\
* & -(1-\tau)e^{-\alpha d}\widetilde{R}_i-G_i-G_i^T & -G_i & A_{di}^TQ_i & 0 & 0 & 0 & A_{di}^TQ_i & 0 \\
* & * & -d^{-1}e^{-\alpha d}Q_i & 0 & A_{di}^T\widetilde{S}_i & 0 & 0 & 0 & 0 \\
* & * & * & -d^{-1}Q_i & 0 & 0 & 0 & 0 & 0 \\
* & * & * & * & -d^2\widetilde{S}_i & 0 & 0 & 0 & 0 \\
* & * & * & * & * & -\widetilde{S}_i & 0 & 0 & 0 \\
* & * & * & * & * & * & -d^{-1}\varepsilon_1Q_i & 0 & 0 \\
* & * & * & * & * & * & * & -d^{-1}\eta_1Q_i & 0 \\
* & * & * & * & * & * & * & * & \Phi_1
\end{bmatrix} < 0, \tag{3.31}$$

where

$$\begin{gathered}
\Theta_i = (A_{Ki} + A_{di})^TP_i + P_i(A_{Ki} + A_{di}) + (\alpha + 1)P_i + \widetilde{R}_i \\
\Phi_1 = -\left[d\rho_1(1 + \varepsilon_1 + \eta_1) + \rho_2\right]^{-1}I.
\end{gathered} \tag{3.32}$$

Using $\mathrm{diag}\{P_i^{-1}, Q_i^{-1}, Q_i^{-1}, Q_i^{-1}, S_i^{-1}, S_i^{-1}, Q_i^{-1}, Q_i^{-1}, I\}$ to pre- and postmultiply the left term of (3.31), and denoting $P_i^{-1} = X_i$, $K_iP_i^{-1} = Y_i$, $Q_i^{-1} = Z_i$, $\widetilde{S}_i^{-1} = S_i$, $W_i = P_i$, $G_i = Q_i$, $\widetilde{R}_i = 0$, we have that (3.31) is equivalent to (3.25).

Therefore, we have

$$V_i(x(t)) < e^{-\alpha(t-t_0^i)} V_i\left(x\left(t_0^i\right)\right), \tag{3.33}$$

where t_0^i represents the initial value of the ith subsystem.

When $t \in [t_k, t_k + \Delta_k)$, system (3.22) can be written as

$$\dot{x}(t) = A_{Kij} x(t) + A_{dj} x\left(t - d_j(t)\right) + f_j(x(t), t), \tag{3.34}$$

where $A_{Kij} = A_j + B_j K_i$.

Consider Lyapunov functional candidate for system (3.34) as follows:

$$V_{ij}(x(t)) = x^T(t) P_{ij} x(t) + \int_{-\tau}^{0} \int_{t+\theta}^{t} \dot{x}^T(r) e^{\beta(t-r)} Q_{ij} \dot{x}(r) dr\, d\theta. \tag{3.35}$$

For given positive constants β, ρ_3, ρ_4, ε_2, η_2, if there exist positive definite symmetric matrices P_{ij}, S_{ij}, Q_{ij}, and any matrices G_{ij}, W_{ij} with appropriate dimensions such that the follows matrix inequalities (3.36)-(3.37) hold, then from Lemma 3.2 we have $\dot{V}_{ij}(x(t)) - \beta V_{ij}(x(t)) < 0$:

$$Q_{ij} \le \rho_3 I, \qquad P_{ij} \le \rho_4 I, \tag{3.36}$$

$$\begin{bmatrix} \Theta_{ij} & G_{ij}^T & W_{ij}^T A_{dj} & A_{Kij}^T Q_{ij} & 0 & \tau\left(W_{ij}^T + P_{ij}\right) & A_{Kij}^T Q_{ij} & 0 & U_j^T \\ G_{ij} & -G_{ij} - G_{ij}^T & -G_{ij} & A_{dj}^T Q_{ij} & 0 & 0 & 0 & A_{dj}^T Q_{ij} & 0 \\ A_{dj}^T W_{ij} & -G_i^T & -\tau^{-1} Q_{ij} & 0 & A_{dj}^T S_{ij} & 0 & 0 & 0 & 0 \\ Q_{ij} A_{Kij} & Q_{ij} A_{dj} & 0 & -\tau^{-1} Q_{ij} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & S_{ij} A_{dj} & 0 & -\tau^2 S_{ij} & 0 & 0 & 0 & 0 \\ \tau(W_{ij} + P_{ij}) & 0 & 0 & 0 & 0 & -S_{ij} & 0 & 0 & 0 \\ Q_{ij} A_{Kij} & 0 & 0 & 0 & 0 & 0 & -\tau^{-1}\varepsilon_2 Q_{ij} & 0 & 0 \\ 0 & Q_{ij} A_{dj} & 0 & 0 & 0 & 0 & 0 & -\tau^{-1}\eta_2 Q_{ij} & 0 \\ U_j & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \Phi_2 \end{bmatrix} < 0, \tag{3.37}$$

where $\Theta_{ij} = (A_{Kij} + A_{dj})^T P_{ij} + P_{ij}(A_{Kij} + A_{dj}) - (\beta - 1) P_{ij}, \Phi_2 = -[d\rho_3(1 + \varepsilon_2 + \eta_2) + \rho_4]^{-1} I$.

Denoting $K_i = Y_i X_i^{-1}$, we know that (3.37) is equivalent to (3.26). Thus, we have

$$V_{ij}(x(t)) < e^{\beta(t-t_0^{ij})} V_{ij}\left(x\left(t_0^{ij}\right)\right), \tag{3.38}$$

where t_0^{ij} represents the initial value of the jth subsystem.

Let $t_0, t_1, \dots, t_k$ denote the switching instants in $[t_0, t)$, from (3.33) and (3.38), for $t \geq t_k + \Delta_k$, we have

$$\begin{aligned} V(t) &< e^{-\alpha(t-t_k-\Delta_k)} V(t_k + \Delta_k) \\ &< (\mu_1\mu_2)^k e^{-\alpha[(t-t_k-\Delta_k)+(t_k-t_{k-1}-\Delta_{k-1})+\cdots+(t_2-t_1-\Delta_1)+(t_1-t_0-\Delta_0)]+\beta(\Delta_k+\Delta_{k-1}+\cdots+\Delta_1+\Delta_0)} V(t_0) \\ &= (\mu_1\mu_2)^k e^{-\alpha T^-(t_0,t)+\beta T^+(t_0,t)} V(t_0). \end{aligned} \tag{3.39}$$

By Definition 2.4, we have

$$k \leq N_0 + \frac{t - t_0}{\tau_a}. \tag{3.40}$$

From (3.27), it follows that

$$-T^-(t_0, t)\lambda^- + T^+(t_0, t)\lambda^+ \leq -\lambda^*(t - t_0). \tag{3.41}$$

Substituting (3.40) and (3.41) into (3.39), we have

$$\begin{aligned} V(t) &< (\mu_1\mu_2)^{N_0+(t-t_0)/\tau_a} e^{-\lambda^*(t-t_0)} V(t_0) \\ &= (\mu_1\mu_2)^{N_0} e^{[\ln(\mu_1\mu_2)/\tau_a-\lambda^*](t-t_0)} V(t_0). \end{aligned} \tag{3.42}$$

Thus

$$\|x(t)\| < \sqrt{\frac{b}{a}} \cdot (\mu_1\mu_2)^{N_0/2} e^{[\ln(\mu_1\mu_2)/\tau_a-\lambda^*](t-t_0)/2} \|x(t_0)\|_h, \tag{3.43}$$

where $a = \min_{i,j\in\underline{N},i\neq j}\{\lambda_{\min}(X_i^{-1}), \lambda_{\min}(P_{ij})\}$, $b = \max_{i,j\in\underline{N},i\neq j}\{\lambda_{\max}(X_i^{-1}) + (\tau^2/2)\lambda_{\max}(Z_i^{-1}), \lambda_{\max}(P_{ij}) + (\tau^2/2)\lambda_{\max}(Q_{ij})\}$.

The proof is completed. □

Remark 3.4. When $\mu_1 = \mu_2 = 1$, we have $\tau_a^* = 0$, which implies that switching signals can be arbitrary.

3.3. Robust Reliable Controller Design

Now, we are in a position to present sufficient conditions for the existence of robust reliable controller for system (2.1) with delayed switching.

Theorem 3.5. *Consider system* (2.1)*, for given positive constants* α*,* β*,* ε_1*,* η_1*,* ρ_1*,* δ_1*,* ε_2*,* η_2*,* ρ_2*,* δ_2*,* ρ_3*,* δ_3*,* ρ_4*,* δ_4*, if there exist positive definite symmetric matrices* X_i*,* Z_i*,* R_i*,* P_{ij}*,* Q_{ij}*,* S_{ij}*, and any matrices* Y_i*,* G_{ij}*,* W_{ij} *with appropriate dimensions, such that, for* $i, j \in \underline{N}, i \neq j$*,*

$$Z_i \geq \rho_1^{-1} I, \qquad X_i \geq \rho_2^{-1} I, \qquad Q_{ij} \leq \rho_3 I, \qquad P_{ij} \leq \rho_4 I \tag{3.44}$$

$$\begin{bmatrix} \Omega_i & \vartheta_i & \Phi_i \\ * & \Psi_i & 0 \\ * & * & \Gamma_i \end{bmatrix} < 0, \tag{3.45}$$

$$\begin{bmatrix} \Omega_{ij} & \vartheta_{ij} & \Phi_{ij} \\ * & \Psi_{ij} & \Delta_{ij} \\ * & * & \Gamma_{ij} \end{bmatrix} < 0 \tag{3.46}$$

hold, then under the reliable switching controller $u(t) = K_{\sigma'(t)} x(t)$*,* $K_i = Y_i X_i^{-1}$*, and the following average dwell-time scheme:*

$$\inf_{t > t_0} \frac{T^-(t_0, t)}{T^+(t_0, t)} \geq \frac{\beta + \lambda^*}{\alpha - \lambda^*}, \qquad \tau_a > \tau_a^* = \frac{\ln(\mu_1 \mu_2)}{\lambda^*}, \tag{3.47}$$

the corresponding closed-loop system is exponentially stable,where $0 < \lambda^* < \alpha, \mu_1, \mu_2 \geq 1$ *satisfying* $X_i^{-1} < \mu_1 P_{ij}, P_{ij} < \mu_2 X_i^{-1}$*,* $Z_i^{-1} < \mu_1 Q_{ij}$*,*

$$Q_{ij} < \mu_2 Z_i^{-1}, \qquad \Omega_i = \begin{bmatrix} \Omega_{i11} & X_i & A_{di} Z_i \\ * & -2Z_i & -Z_i \\ * & * & -d^{-1} e^{-\alpha d} Z_i \end{bmatrix},$$

$$\Omega_{i11} = (A_i + A_{di}) X_i + X_i (A_i + A_{di})^T + (\alpha + 1) X_i + B_i Y_i + Y_i^T B_i^T + \delta_1 H_i H_i^T + \delta_2 B_i J_i B_i^T,$$

$$\vartheta_i = \begin{bmatrix} X_i A_i^T + Y_i^T B_i^T & 0 & 2dR_i & X_i A_i^T + Y_i^T B_i^T & 0 & X_i U_i^T \\ Z_i A_{di}^T & 0 & 0 & 0 & Z_i A_{di}^T & 0 \\ 0 & Z_i A_{di}^T & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_i = \begin{bmatrix} X_i (E_{1i} + E_{2i})^T & 0 & X_i E_{1i}^T + Y_i^T E_{2i}^T & X_i E_{1i}^T + Y_i^T E_{2i}^T & 0 & Y_i^T M_{i0} J_i^{1/2} \\ 0 & Z_i E_{2i}^T & Z_i E_{2i}^T & 0 & 0 & 0 \\ Z_i E_{2i}^T & 0 & 0 & 0 & Z_i E_{2i}^T & 0 \end{bmatrix},$$

$$\Psi_i = \mathrm{diag}\Big\{-d^{-1}Z_i + \delta_1 H_i H_i^T + \delta_2 B_i J_i B_i^T, -d^2 R_i + \delta_1 H_i H_i^T, -R_i,$$
$$-d^{-1}\varepsilon_1 Z_i + \delta_1 H_i H_i^T + \delta_2 B_i J_i B_i^T, -d^{-1}\eta_1 Z_i + \delta_1 H_i H_i^T, -\left[d\rho_1(1+\varepsilon_1+\eta_1)+\rho_2\right]^{-1} I\Big\},$$

$$\Gamma_i = \mathrm{diag}\{-\delta_1 I, -\delta_1 I, -\delta_1 I, -\delta_1 I, -\delta_1 I, -\delta_2 I\},$$

$$\Omega_{ij} = \begin{bmatrix} \Omega_{ij1} & G_{ij}^T & W_{ij}^T A_{dj} \\ * & -G_{ij} - G_{ij}^T + 2\delta_2 E_{2j}^T E_{2j} & -G_{ij} \\ * & * & -\tau^{-1} Q_{ij} + \delta_3 E_{2j}^T E_{2j} \end{bmatrix},$$

$$\Omega_{ij1} = (A_j + A_{dj})^T P_{ij} + P_{ij}(A_j + A_{dj}) - (\beta - 1)P_{ij} + P_{ij} B_j Y_i X_i^{-1} + X_i^{-1} Y_i^T B_j^T P_{ij}$$
$$+ 3\delta_3 E_{1j}^T E_{1j} + \delta_4 P_{ij} B_j J_i B_j^T P_{ij},$$

$$\vartheta_{ij} = \begin{bmatrix} (A_j + B_j Y_i X_i^{-1})^T Q_{ij} & 0 & \tau\left(W_{ij}^T + P_{ij}\right) & (A_j + B_j Y_i X_i^{-1})^T Q_{ij} & 0 & U_j^T \\ A_{dj}^T Q_{ij} & 0 & 0 & 0 & A_{dj}^T Q_{ij} & 0 \\ 0 & A_{dj}^T S_{ij} & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi_{ij} = \begin{bmatrix} P_{ij} H_j & 0 & 0 & 0 & W_{ij}^T H_j \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \Delta_{ij} = \begin{bmatrix} 0 & 0 & Q_{ij} H_j & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{ij} H_j \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{ij} H_j & 0 \\ 0 & Q_{ij} H_j & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Psi_{ij} = \mathrm{diag}\Big\{-d^{-1} Q_{ij} + \delta_4 Q_{ij} B_j J_i B_j^T P_{ij}, -d^2 S_{ij}, -S_{ij}, -d^{-1}\varepsilon_2 Q_{ij} + \delta_4 Q_{ij} B_j J_i B_j^T Q_{ij}, -d^{-1}\eta_2 Q_{ij},$$
$$-\left[d\rho_3(1+\varepsilon_2+\eta_2)+\rho_4\right]^{-1} I\Big\},$$

$$\Gamma_{ij} = \mathrm{diag}\{-\delta_3 I, -\delta_3 I, -\delta_3 I, -\delta_3 I, -\delta_3 I, -\delta_4 I\}. \tag{3.48}$$

Proof. Consider the following inequalities (3.49) and (3.50):

$$T_i = \begin{bmatrix} \widehat{\Pi}_i & X_i & \widehat{A}_{di}Z_i & \widehat{\Xi}_i & 0 & 2dR_i & \widehat{\Xi}_i & 0 & X_iU_i^T \\ * & -2Z_i & -Z_i & Z_i\widehat{A}_{di}^T & 0 & 0 & 0 & Z_i\widehat{A}_{di}^T & 0 \\ * & * & -d^{-1}e^{-\alpha d}Z_i & 0 & Z_i\widehat{A}_{di}^T & 0 & 0 & 0 & 0 \\ * & * & * & -d^{-1}Z_i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -d^2R_i & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -R_i & 0 & 0 & 0 \\ * & * & * & * & * & * & -d^{-1}\varepsilon_1 Z_i & 0 & 0 \\ * & * & * & * & * & * & * & -d^{-1}\eta_1 Z_i & 0 \\ * & * & * & * & * & * & * & * & \Phi_1 \end{bmatrix} < 0, \tag{3.49}$$

$$T_{ij} = \begin{bmatrix} \widehat{\Lambda}_{ij} & G_{ij}^T & W_{ij}^T\widehat{A}_{dj} & \widehat{\Xi}_{ij} & 0 & d\left(W_{ij}^T + P_{ij}\right) & \widehat{\Xi}_{ij} & 0 & U_j^T \\ * & -G_{ij} - G_{ij}^T & -G_{ij} & \widehat{A}_{dj}^T Q_{ij} & 0 & 0 & 0 & \widehat{A}_{dj}^T Q_{ij} & 0 \\ * & * & -d^{-1}Q_{ij} & 0 & \widehat{A}_{dj}^T S_{ij} & 0 & 0 & 0 & 0 \\ * & * & * & -d^{-1}Q_{ij} & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -d^2 S_{ij} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -S_{ij} & 0 & 0 & 0 \\ * & * & * & * & * & * & -d^{-1}\varepsilon_2 Q_{ij} & 0 & 0 \\ * & * & * & * & * & * & * & -d^{-1}\eta_2 Q_{ij} & 0 \\ * & * & * & * & * & * & * & * & \Phi_2 \end{bmatrix} < 0, \tag{3.50}$$

where $\Phi_1 = -[d\rho_1(1+\varepsilon_1+\eta_1)]^{-1}I$, $\widehat{\Xi}_i = X_i\widehat{A}_i^T + Y_i^T M_i B_i^T$, $\widehat{\Pi}_i = (\widehat{A}_i + \widehat{A}_{di})X_i + X_i(\widehat{A}_i + \widehat{A}_{di})^T + (\alpha+1)X_i + B_iM_iY_i + Y_i^T M_i B_i^T$, $\Phi_2 = -[d\rho_2(1+\varepsilon_2+\eta_2)]^{-1}I$, $\widehat{\Xi}_{ij} = (\widehat{A}_j + B_jM_iY_iX_i^{-1})^T Q_{ij}$, $\widehat{\Lambda}_{ij} = (\widehat{A}_j + \widehat{A}_{dj})^T P_{ij} + P_{ij}(\widehat{A}_j + \widehat{A}_{dj}) - (\beta-1)P_{ij} + P_{ij}B_jM_iY_iX_i^{-1} + X_i^{-1}Y_i^T M_i B_j^T P_{ij}$.

Substituting (2.3) and (2.10) into (3.49), we can obtain

$$T_i = T_{1i} + T_{2i}, \tag{3.51}$$

where

$$T_{1i}=\begin{bmatrix}\Pi_i & X_i & A_{di}Z_i & \Xi_i & 0 & 2dR_i & \Xi_i & 0 & X_iU_i^T\\ * & -2Z_i & -Z_i & Z_iA_{di}^T & 0 & 0 & 0 & Z_iA_{di}^T & 0\\ * & * & -d^{-1}e^{-\alpha d}Z_i & 0 & Z_iA_{di}^T & 0 & 0 & 0 & 0\\ * & * & * & -d^{-1}Z_i & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & -d^2R_i & 0 & 0 & 0 & 0\\ * & * & * & * & * & -R_i & 0 & 0 & 0\\ * & * & * & * & * & * & -d^{-1}\varepsilon_1 Z_i & 0 & 0\\ * & * & * & * & * & * & * & -d^{-1}\eta_1 Z_i & 0\\ * & * & * & * & * & * & * & * & \Phi_1\end{bmatrix}, \tag{3.52}$$

$$T_{2i}=\begin{bmatrix}\Upsilon_i & 0 & \kappa_i & \phi_i^T & 0 & 0 & \phi_i^T & 0 & 0\\ * & 0 & 0 & \kappa_i^T & 0 & 0 & 0 & \kappa_i^T & 0\\ * & * & 0 & 0 & \kappa_i^T & 0 & 0 & 0 & 0\\ * & * & * & 0 & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & 0 & 0 & 0 & 0 & 0\\ * & * & * & * & * & 0 & 0 & 0 & 0\\ * & * & * & * & * & * & 0 & 0 & 0\\ * & * & * & * & * & * & * & 0 & 0\\ * & * & * & * & * & * & * & * & 0\end{bmatrix}, \tag{3.53}$$

where $\Pi_i=(A_i+A_{di})X_i+X_i(A_i+A_{di})^T+(\alpha+1)X_i+B_iM_{i0}Y_i+Y_i^TM_{i0}B_i^T$, $\Upsilon_i=H_iF_i(E_{1i}+E_{2i})X_i+X_i(E_{1i}+E_{2i})^TF_i^TH_i^T+B_iM_{i0}L_iY_i+Y_i^TL_iM_{i0}B_i^T$, $\kappa_i=H_iF_iE_{2i}Z_i$, $\phi_i=H_iF_iE_{1i}X_i+B_iM_{i0}L_iY_i$.

By Lemma 2.7, Lemma 2.8, and Schur complement lemma, we know that (3.49) is equivalent to (3.45). Similarly, substituting (2.3) into (3.50), we can obtain that (3.50) is equivalent to (3.46).

From Theorem 3.3, we know that Theorem 3.5 holds.

The proof is completed. □

Remark 3.6. In actual operation, the condition (3.27) or (3.47) is difficult to check. Let $\Delta_{\max}$ be a known positive scalar that represents the maximum delayed period, a simple condition of the average dwell time is proposed, that is to say, (3.27) or (3.47) can be reduced to the following condition:

$$\tau_a>\tau_a^*=\max\left\{\frac{\ln(\mu_1\mu_2)}{\lambda^*},\left(\frac{\beta+\lambda^*}{\alpha-\lambda^*}+1\right)\Delta_{\max}\right\},\quad 0<\lambda^*<\alpha. \tag{3.54}$$

Proof of (3.54): According to (3.27), we have

$$\begin{aligned}\frac{T^-(t_0,t)}{T^+(t_0,t)} \geq \frac{\beta+\lambda^*}{\alpha-\lambda^*} &\Longrightarrow \frac{(t-t_0)-T^+(t_0,t)}{T^+(t_0,t)} \geq \frac{\beta+\lambda^*}{\alpha-\lambda^*} \\ &\Longrightarrow t-t_0 \geq \frac{\beta+\lambda^*}{\alpha-\lambda^*}T^+(t_0,t)+T^+(t_0,t) \\ &\Longrightarrow t-t_0 \geq \left(\frac{\beta+\lambda^*}{\alpha-\lambda^*}+1\right)T^+(t_0,t).\end{aligned} \tag{3.55}$$

On the other hand

$$\begin{aligned}\left(\frac{\beta+\lambda^*}{\alpha-\lambda^*}+1\right)T^+(t_0,t) &\leq \left(\frac{\beta+\lambda^*}{\alpha-\lambda^*}+1\right)N_{\sigma(t)}(t_0,t)\Delta_{\max} \\ &\leq \left(\frac{\beta+\lambda^*}{\alpha-\lambda^*}+1\right)\frac{t-t_0}{\tau_a^*}\Delta_{\max}.\end{aligned} \tag{3.56}$$

Obviously, if the following inequality (3.57) is satisfied, then we have that (3.55) holds

$$t-t_0 \geq \left(\frac{\beta+\lambda^*}{\alpha-\lambda^*}+1\right)\frac{t-t_0}{\tau_a^*}\Delta_{\max}. \tag{3.57}$$

From (3.57), we have $\tau_a^* \geq ((\beta+\lambda^*)/(\alpha-\lambda^*)+1)\Delta_{\max}$.

The proof is completed.

4. Conclusion

In this paper, we have investigated the robust reliable stabilization problem for switched nonlinear systems with time-varying delays and delayed switching. The average dwell-time approach is utilized for stability analysis and controller design. Our future work will focus on extending the proposed design method to H_∞ performance analysis for switched nonlinear time-varying systems with delayed switching.

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