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Research Article

Applications of an Extended (G'/G)-Expansion Method to Find Exact Solutions of Nonlinear PDEs in Mathematical Physics

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We construct the traveling wave solutions of the (1+1)-dimensional modified Benjamin-Bona-Mahony equation, the (2+1)-dimensional typical breaking soliton equation, the (1+1)-dimensional classical Boussinesq equations, and the (2+1)-dimensional Broer-Kaup-Kuperschmidt equations by using an extended (G'/G)-expansion method, where G satisfies the second-order linear ordinary differential equation. By using this method, new exact solutions involving parameters, expressed by three types of functions which are hyperbolic, trigonometric and rational function solutions, are obtained. When the parameters are taken as special values, some solitary wave solutions are derived from the hyperbolic function solutions.

1. Introduction

The investigation of the traveling wave solutions of nonlinear partial differential equations (NPDEs) plays an important role in the study of nonlinear physical phenomena. In recent years, new exact solutions may help to find new phenomena. The exact solutions have been investigated by many authors (see, e.g., [1–27]) who are interested in nonlinear physical phenomena. Many powerful methods have been presented such as the homogeneous balance method [13], the tanh method [4, 15, 24], the inverse scattering transform [1], the exp-function expansion method [2, 6, 20], the Jacobi elliptic function expansion [17], the Backlund transform [8, 9], the generalized Riccati equation [18], the modified extended Fan sub-equation method [19], the truncated Painlevé expansion [27], and the auxiliary equation method [10, 11]. More recently, the (G'/G)-expansion method [14, 22, 23, 26] has been proposed to obtain traveling wave solutions. This method is firstly proposed by the Chinese mathematicians Wang et al. [14] for which the traveling wave solutions of the nonlinear evolution equations are obtained. This method has been extended to solve difference-differential equations [28, 29]. The improved (G'/G)-expansion method has been used in [21, 25]. Recently, Gou and Zhou [5] have obtained the exact traveling wave solutions of some nonlinear PDEs using an extended (G'/G)-expansion method.

In the present article, we use the extended (G'/G)-expansion method which is proposed in [5] to derive traveling wave solutions for some nonlinear PDEs in mathematical physics namely; the (1+1)-dimensional modified Benjamin-Bona-Mahony equation, the (2+1)-dimensional typical breaking soliton equation, the (1+1)-dimensional classical Boussinesq equations, and the (2+1)-dimensional Broer-Kaup-Kuperschmidt equations. The extended (G'/G)-expansion method used in this article can be applied to further equations such as difference-differential equations which can be done in forthcoming articles.

2. Description of an Extended (G'/G)-Expansion Method

Consider the nonlinear partial differential equation in the following form:

$$F(u, u_t, u_x, u_{tt}, u_{xt}, u_{xx}, \ldots) = 0, \tag{2.1}$$

where $u = u(x,t)$ is unknown functions, and F is a polynomial in $u(x,t)$ and its partial derivatives. In the following, we give the main steps for solving (2.1) using an extended (G'/G)-expansion method [5].

Step 1. The traveling wave variable

$$u(x,t) = u(\xi), \qquad \xi = x - Vt, \tag{2.2}$$

where V is a constant to be determined latter, permits us reducing (2.1) to an ODE in the form

$$P\left(u, -Vu', u', V^2u'', -Vu'', u'', \ldots\right) = 0, \tag{2.3}$$

where P is a polynomial in $u(\xi)$ and its total derivatives.

Step 2. Suppose the solution of (2.3) can be expressed in (G'/G) as follows:

$$u(\xi) = a_0 + \sum_{i=1}^{n} \left\{ a_i \left(\frac{G'}{G}\right)^i + b_i \left(\frac{G'}{G}\right)^{i-1} \sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)} \right\}, \tag{2.4}$$

where $G = G(\xi)$ satisfies the following second-order linear ODE:

$$G''(\xi) + \mu G(\xi) = 0, \tag{2.5}$$

while a_i, b_i $(i = 1, \ldots, n)$ and a_0 are constants to be determined, such that $\sigma = \pm 1$ and $\mu \neq 0$. The positive integer "n" can be determined by balancing the highest-order derivatives with the nonlinear terms appearing in (2.3).

Step 3. Substituting (2.4) into (2.3) and using (2.5), collecting all terms with the same powers of $(G'/G)^k$ and $(G'/G)^k\sqrt{\sigma(1+(1/\mu)(G'/G)^2)}$ together, and equating each coefficient of them to zero, yield a set of algebraic equations for a_0, a_i, b_i and V.

Step 4. Since the general solution of (2.5) has been well known for us, then substituting $a_i, b_i, V$and the general solution of (2.5) into (2.4), we have the traveling wave solutions of the nonlinear partial differential equation (2.1).

Remark 2.1. It is necessary to point out that by adding the term $(G'/G)^k\sqrt{\sigma(1+(1/\mu)(G'/G)^2)}$ into (2.4), the ansätz proposed here is more general than the ansätz in the original (G'/G)-expansion method [14]. Therefore, the extended (G'/G)-expansion method is more powerful than the original (G'/G)-expansion method [14] and some new types of traveling wave solutions and solitary wave solutions would be expected for some NPDEs. If we choose the parameters in (2.4) and (2.5) to take special values, the (G'/G)-expansion method can be recovered by our proposed method.

3. Applications

In this section, we will apply the extended (G'/G)-expansion method to some nonlinear PDEs in mathematical physics as follows.

3.1. Example 1: The (1+1)-Dimensional Modified Benjamin-Bona-Mahony Equation

We start with the following (1+1)-dimensional nonlinear dispersive modified Benjamin-Bona Mahony equation [20] written in the following form:

$$u_t + u_x - \alpha u^2 u_x + u_{xxx} = 0, \tag{3.1}$$

where α is a nonzero positive constant. This equation was first derived to describe an approximation for surface long waves in nonlinear dispersive media. It can also characterize the hydromagnetic waves in cold plasma, acoustic waves in inharmonic crystals and acoustic-gravity waves in compressible fluids. Yusufoglu [20] has used the Exp-function method to find the traveling wave solutions of (3.1). Let us now solve (3.1) by the proposed method. To this end, we see that the traveling wave variable (2.2) permits us converting (3.1) into the following ODE:

$$(1-V)u' - \alpha u^2 u' + u''' = 0. \tag{3.2}$$

Integrating (3.2) with respect to ξ once, we get

$$K + (1-V)u - \frac{1}{3}\alpha u^3 + u'' = 0, \tag{3.3}$$

where K is an integration constant. Considering the homogeneous balance between the highest-order derivatives and nonlinear terms in (3.3), we get $n = 1$. Hence we suppose that the solution $u(\xi)$ of (3.3) has the form

$$u(\xi) = a_0 + a_1\left(\frac{G'}{G}\right) + b_1\sqrt{\sigma\left(1 + \frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}, \tag{3.4}$$

where $G = G(\xi)$ satisfies (2.5). Substituting (3.4) along with (2.5) into (3.3), collecting all terms with the same powers of $(G'/G)^k$, $(G'/G)^k\sqrt{\sigma(1+(1/\mu)(G'/G)^2)}$, and setting them to zero, we have the following algebraic equations:

$$\begin{aligned}
&K + 3a_0(1-V) - \alpha a_0^3 - 3\sigma\alpha a_0 b_1^2 = 0,\\
&a_1(1-V) + 2\mu a_1 - \alpha a_1 {a_0}^2 - \sigma\alpha a_1 b_1^2 = 0,\\
&\mu\alpha a_0 a_1^2 + \sigma\alpha a_0 b_1^2 = 0,\\
&6\mu a_1 - \mu\alpha a_1^3 - 3\sigma\alpha a_1 b_1^2 = 0,\\
&3b_1(1+\mu-V) - 3\alpha b_1 a_0^2 - \sigma\alpha b_1^3 = 0,\\
&-3\mu\alpha b_1 a_1^2 + 6\mu b_1 - \sigma\alpha b_1^3 = 0,\\
&2\alpha a_0 a_1 b_1 = 0.
\end{aligned} \tag{3.5}$$

Solving these algebraic equations by Maple or Mathematica, we obtain the following results.

Case 1. One has

$$a_1 = \sigma\sqrt{\frac{6}{\alpha}}, \qquad V = 2\mu + 1, \qquad b_1 = a_0 = K = 0. \tag{3.6}$$

Case 2. One has

$$b_1 = \sigma\sqrt{\frac{6\mu}{\sigma\alpha}}, \qquad V = 1 - \mu, \qquad a_1 = a_0 = K = 0. \tag{3.7}$$

Case 3. One has

$$a_1 = \sigma\sqrt{\frac{3}{2\alpha}}, \qquad b_1 = \sigma\sqrt{\frac{3\mu}{2\sigma\alpha}}, \qquad V = \frac{1}{2}(2+\mu), \qquad a_0 = C = 0, \tag{3.8}$$

where $\sigma = \pm 1$. From (3.4) and the general solution of (2.5), we deduce the traveling wave solutions of (3.1) as follows.

When $\mu < 0$, then Case 1 gives the exact traveling wave solution:

$$u(\xi) = \sigma\sqrt{\frac{-6\mu}{\alpha}}\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right) + B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right) + B\sinh\left(\sqrt{-\mu}\xi\right)}\right), \tag{3.9}$$

where $\xi = x - (1 + 2\mu)t$ and A, B are arbitrary constants while $\sigma = \pm 1$. Case 2 gives the exact traveling wave solutions:

$$u(\xi) = \sigma\sqrt{\frac{6\mu}{\sigma\alpha}}\sqrt{\sigma\left[1 - \left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right) + B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right) + B\sinh\left(\sqrt{-\mu}\xi\right)}\right)^2\right]}, \tag{3.10}$$

where $\xi = x - (1 - \mu)t$. Case 3 gives the exact traveling wave solutions:

$$\begin{aligned} u(\xi) = \sigma\sqrt{\frac{3}{2\alpha}}\Bigg\{ & \sqrt{\frac{\mu}{\sigma}}\sqrt{\sigma\left[1 - \left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right) + B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right) + B\sinh\left(\sqrt{-\mu}\xi\right)}\right)^2\right]} \\ & + \sqrt{-\mu}\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right) + B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right) + B\sinh\left(\sqrt{-\mu}\xi\right)}\right)\Bigg\}, \end{aligned} \tag{3.11}$$

where $\xi = x - ((2 + \mu)/2)t$. When $\mu > 0$, then Case 1 gives the exact traveling wave solutions:

$$u(\xi) = \sigma\sqrt{\frac{6\mu}{\alpha}}\left(\frac{B\cos\left(\sqrt{\mu}\xi\right) - A\sin\left(\sqrt{\mu}\xi\right)}{A\cos\left(\sqrt{\mu}\xi\right) + B\sin\left(\sqrt{\mu}\xi\right)}\right). \tag{3.12}$$

Case 2 gives the exact traveling wave solutions

$$u(\xi) = \sigma\sqrt{\frac{6\mu}{\sigma\alpha}}\sqrt{\sigma\left[1 + \left(\frac{B\cos\left(\sqrt{\mu}\xi\right) - A\sin\left(\sqrt{\mu}\xi\right)}{A\cos\left(\sqrt{\mu}\xi\right) + B\sin\left(\sqrt{\mu}\xi\right)}\right)^2\right]}. \tag{3.13}$$

Case 3 gives the exact traveling wave solutions:

$$\begin{aligned} u(\xi) = \sigma\sqrt{\frac{3\mu}{2\alpha}}\Bigg\{ & \left(\frac{B\cos\left(\sqrt{\mu}\xi\right) - A\sin\left(\sqrt{\mu}\xi\right)}{A\cos\left(\sqrt{\mu}\xi\right) + B\sin\left(\sqrt{\mu}\xi\right)}\right) \\ & + \frac{1}{\sqrt{\sigma}}\sqrt{\sigma\left[1 + \left(\frac{B\cos\left(\sqrt{\mu}\xi\right) - A\sin\left(\sqrt{\mu}\xi\right)}{A\cos\left(\sqrt{\mu}\xi\right) + B\sin\left(\sqrt{\mu}\xi\right)}\right)^2\right]}\Bigg\}. \end{aligned} \tag{3.14}$$

In particular, we deduce from (3.9) that the solitary wave solutions of (3.1) are derived as follows.

If $A = 0, B \neq 0$ and $\mu < 0$, then we obtain

$$u(\xi) = \sigma\sqrt{\frac{-6\mu}{\alpha}}\coth\left(\sqrt{-\mu}\xi\right), \tag{3.15}$$

while if $A \neq 0, A^2 > B^2$, and $\mu < 0$, then we obtain

$$u(\xi) = \sigma\sqrt{\frac{-6\mu}{\alpha}}\tanh\left(\sqrt{-\mu}\xi + \xi_0\right), \tag{3.16}$$

where $\xi_0 = \tanh^{-1}(B/A)$. Similarly, we can find more solitary wave solutions of (3.1) using (3.10)-(3.11) but we omitted them for simplicity.

3.2. Example 2: The (2+1)-Dimensional Typical Breaking Soliton Equation

In this subsection, we study the following the (2+1)-dimensional typical breaking soliton equation [3] in the following form:

$$u_{xt} - 4u_x u_{xy} - 2u_{xx}u_y + u_{xxxy} = 0, \tag{3.17}$$

which was first introduced by Calogero and Degasperis [3]. Tian et al. [12] have reduced new families of soliton-like solutions via the generalized tanh method which are of important significance in explaining some physical phenomena. Mei and Zhang [7] have reduced more families of new exact solutions which contain soliton-like solutions and periodic solution based on a newly generally projective Riccati equation expansion method and its algorithm. Let us now solve (3.17) by the proposed method. To this end, we see that the traveling wave variable

$$u(x, y, t) = u(\xi), \quad \xi = x + y - Vt \tag{3.18}$$

permits us to convert (3.17) into the following ODE:

$$-Vu'' - 6u'u'' + u^{(4)} = 0. \tag{3.19}$$

Integrating (3.19) with respect to ξ once yields

$$K - Vu' - 3\left(u'\right)^2 + u''' = 0, \tag{3.20}$$

where K is an integration constant. Considering the homogeneous balance between the highest-order derivatives and nonlinear terms in (3.20) we deduce that the solution $u(\xi)$ of (3.20) has the same form of (3.4). Substituting (3.4) along with (2.5) into (3.20), collecting all terms with the same powers of $(G'/G)^k$, $(G'/G)^k\sqrt{\sigma(1+(1/\mu)(G'/G)^2)}$ and setting them to zero, we have the following algebraic equations:

$$\begin{gathered}
K+\mu a_1(V-2\mu-3\mu a_1)=0,\\
-3\sigma b_1^2+a_1(V-8\mu-6a_1)=0,\\
3\sigma b_1^2+3\mu a_1(a_1+2)=0,\\
b_1(V-5\mu-6\mu a_1)=0,\\
6b_1(a_1+1)=0.
\end{gathered}\tag{3.21}$$

Solving these algebraic equations by Maple or Mathematica, we obtain the following results.

Case 1. One has

$$a_1=-2,\qquad V=-4\mu,\qquad b_1=K=0,\tag{3.22}$$

Case 2. One has

$$a_1=-1,\qquad V=-\mu,\qquad b_1=\sigma\sqrt{\frac{\mu}{\sigma}},\qquad K=0.\tag{3.23}$$

From (3.4) and the general solution of (2.5), we deduce the traveling wave solutions of (3.17) as follows.

When $\mu<0$, then Case 1 gives the exact traveling wave solution:

$$u(\xi)=-2\sqrt{-\mu}\left(\frac{A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)+a_0,\tag{3.24}$$

where $\xi=x+4\mu t$. Case 2 gives the exact traveling wave solution:

$$\begin{aligned}
u(\xi)=\sigma\sqrt{\frac{\mu}{\sigma}}&\sqrt{\sigma\left[1-\left(\frac{A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)^2\right]}\\
&-\sqrt{-\mu}\left(\frac{A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)+a_0.
\end{aligned}\tag{3.25}$$

where $\xi=x-\mu t$. When $\mu>0$, then Case 1 gives the exact traveling wave solution:

$$u(\xi)=-2\sqrt{\mu}\left(\frac{B\cos(\sqrt{\mu}\xi)-A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi)+B\sin(\sqrt{\mu}\xi)}\right)+a_0.\tag{3.26}$$

Case 2 gives the exact traveling wave solution:

$$u(\xi) = \sqrt{\mu}\left\{ -\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right) + \frac{\sigma}{\sqrt{\sigma}}\sqrt{\sigma\left[1 + \left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2\right]}\right\} + a_0 \tag{3.27}$$

In particular, we deduce from (3.24) that the solitary wave solutions of (3.17) are derived as follows.

If $A = 0, B \neq 0$, and $\mu < 0$, then we obtain

$$u(\xi) = -2\sqrt{-\mu}\coth(\sqrt{-\mu}\xi) + a_0, \tag{3.28}$$

while if $A \neq 0, A^2 > B^2$, and $\mu < 0$, then we obtain

$$u(\xi) = -2\sqrt{-\mu}\tanh(\sqrt{-\mu}\xi + \xi_0) + a_0, \tag{3.29}$$

where $\xi_0 = \tanh^{-1}(B/A)$. Similarly, we can find more solitary solutions of (3.17) using (3.25) but we omitted them for simplicity.

3.3. Example 3: The (1+1)-Dimensional Classical Boussinesq Equations

In this subsection, we study the following (1+1)-dimensional classical Boussinesq equations [16]:

$$\begin{aligned} v_t + [(1+v)u]_x &= -\frac{1}{3}u_{xxx}, \\ u_t + uu_x + v_x &= 0. \end{aligned} \tag{3.30}$$

This system has been derived by Wu and Zhang [16] for modelling nonlinear and dispersive long gravity wave traveling in two horizontal directions on shallow water of uniform depth. Let us now solve (3.30) by the proposed method. To this end, we see that the traveling wave variables (2.2) permit us converting (3.30) into the following ODEs:

$$\begin{aligned} -Vv' + [(1+v)u]' + \frac{1}{3}u''' &= 0, \\ -Vu' + uu' + v' &= 0. \end{aligned} \tag{3.31}$$

Integrating (3.31) with respect to ξ once yields

$$K_1 - Vv + (1+v)u + \frac{1}{3}u'' = 0, \tag{3.32}$$

$$K_2 - Vu + \frac{1}{2}u^2 + v = 0, \tag{3.33}$$

where K_1 and K_2 are integration constants. Considering the homogeneous balance between highest order derivatives and nonlinear terms in Equations (3.32), (3.33) we deduce that the solution $u(\xi)$ which has the same form of (3.4) while, $v(\xi)$ has the following form:

$$v(\xi) = c_0 + c_1\left(\frac{G'}{G}\right) + d_1\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)} + c_2\left(\frac{G'}{G}\right)^2 + d_2\left(\frac{G'}{G}\right)\sqrt{\sigma\left(1+\frac{1}{\mu}\left(\frac{G'}{G}\right)^2\right)}. \tag{3.34}$$

Substituting (3.4) and (3.34) along with (2.5) into (3.32), collecting all terms with the same powers of $(G'/G)^k$, $(G'/G)^k\sqrt{\sigma(1+(1/\mu)(G'/G)^2)}$, and setting them to zero, we have the following algebraic equations:

$$\begin{gathered}
K_1 + a_0 + \sigma b_1 d_1 + c_0(a_0 - V) = 0, \\
3\sigma b_1 d_2 + a_1(3c_0 + 2\mu + 3) + 3c_1(a_0 - V) = 0, \\
\mu a_1 c_1 + \sigma b_1 d_1 + \mu c_2(a_0 - V) = 0, \\
3\sigma b_1 d_2 + \mu a_1(2 + 3c_2) = 0, \\
3d_1(a_0 - V) + b_1(3 + \mu + 3c_0) = 0, \\
d_2(a_0 - V) + a_1 d_1 + b_1 c_1 = 0, \\
b_1(3c_2 + 2) + 3a_1 d_2 = 0.
\end{gathered} \tag{3.35}$$

Similarly, substituting (3.4) and (3.34) along with (2.5) into (3.33), collecting all terms with the same powers of $(G'/G)^k$, $(G'/G)^k\sqrt{\sigma(1+(1/\mu)(G'/G)^2)}$ and setting them to zero, we have the following algebraic equations:

$$\begin{gathered}
2K_2 + 2c_0 + \sigma b_1^2 + a_0(a_0 - 2V) = 0, \\
c_1 + a_1(a_0 - V) = 0, \\
\sigma b_1^2 + 2\mu c_2 + \mu a_1^2 = 0 \\
d_1 + b_1(a_0 - V) = 0 \\
b_1 a_1 + d_2 = 0.
\end{gathered} \tag{3.36}$$

Solving these algebraic equations by Maple or Mathematica, we obtain the following results.

Case 1. One has

$$c_0 = \frac{-(2\mu+3)}{3}, \qquad c_2 = \frac{-2}{3}, \qquad a_1 = \frac{2\sigma}{\sqrt{3}}, \qquad V = a_0, \qquad c_1 = d_1 = b_1 = d_2 = 0,$$
$$K_1 = -a_0, \qquad K_2 = \frac{1}{6}\left(6 + 4\mu + 3a_0^2\right). \tag{3.37}$$

Case 2. One has

$$c_0 = \frac{-(\mu+3)}{3}, \qquad c_2 = \frac{-2}{3}, \qquad b_1 = 2\sigma\sqrt{\frac{\mu}{3\sigma}}, \qquad V = a_0, \qquad d_1 = d_2 = c_1 = a_1 = 0,$$
$$K_1 = -a_0, \qquad K_2 = \frac{1}{6}\left(6 - 2\mu + 3a_0^2\right). \tag{3.38}$$

Case 3. One has

$$c_0 = \frac{-(\mu+3)}{3}, \qquad c_2 = \frac{-1}{3}, \qquad b_1 = \sqrt{\frac{\mu}{3\sigma}},$$
$$d_2 = -\frac{\sigma}{3}\sqrt{\frac{\mu}{\sigma}}, \qquad a_1 = \frac{\sigma}{\sqrt{3}}, \qquad V = a_0, \qquad c_1 = d_1 = 0, \tag{3.39}$$
$$K_1 = -a_0, \qquad K_2 = \frac{1}{6}\left(6 + \mu + 3a_0^2\right).$$

where a_0 is an arbitrary constant. From (3.4), (3.34) and the general solution of (2.5), we deduce the traveling wave solutions of (3.30) as follows.

When $\mu < 0$, then Case 1 gives the exact traveling wave solution:

$$u(\xi) = 2\sigma\sqrt{\frac{-\mu}{3}}\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right) + B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right) + B\sinh\left(\sqrt{-\mu}\xi\right)}\right) + a_0,$$
$$v(\xi) = \frac{2\mu}{3}\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right) + B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right) + B\sinh\left(\sqrt{-\mu}\xi\right)}\right)^2 - \frac{(2\mu+3)}{3}, \tag{3.40}$$

Case 2 gives the exact traveling wave solution:

$$u(\xi) = 2\sigma\sqrt{\frac{\mu}{3\sigma}}\sqrt{\sigma\left[1-\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right)+B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right)+B\sinh\left(\sqrt{-\mu}\xi\right)}\right)^2\right]}+a_0,$$

$$v(\xi) = \frac{2\mu}{3}\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right)+B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right)+B\sinh\left(\sqrt{-\mu}\xi\right)}\right)^2-\frac{(\mu+3)}{3}. \tag{3.41}$$

Case 3 gives the exact traveling wave solution

$$u(\xi) = \sigma\sqrt{\frac{-\mu}{3}}\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right)+B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right)+B\sinh\left(\sqrt{-\mu}\xi\right)}\right)+\sqrt{\frac{\mu}{3\sigma}}\sqrt{\sigma\left[1-\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right)+B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right)+B\sinh\left(\sqrt{-\mu}\xi\right)}\right)^2\right]}+a_0, \tag{3.42}$$

$$\begin{aligned} v(\xi) = {}& -\frac{\sigma}{3}\sqrt{\frac{-\mu^2}{\sigma}}\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right)+B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right)+B\sinh\left(\sqrt{-\mu}\xi\right)}\right) \\ & \times\sqrt{\sigma\left[1-\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right)+B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right)+B\sinh\left(\sqrt{-\mu}\xi\right)}\right)^2\right]} \\ & +\frac{\mu}{3}\left(\frac{A\sinh\left(\sqrt{-\mu}\xi\right)+B\cosh\left(\sqrt{-\mu}\xi\right)}{A\cosh\left(\sqrt{-\mu}\xi\right)+B\sinh\left(\sqrt{-\mu}\xi\right)}\right)^2-\frac{(\mu+3)}{3}, \end{aligned} \tag{3.43}$$

where $\xi = x - a_0 t$. When $\mu > 0$, then Case 1 gives the exact traveling wave solution:

$$u(\xi) = 2\sigma\sqrt{\frac{\mu}{3}}\left(\frac{B\cos\left(\sqrt{\mu}\xi\right)-A\sin\left(\sqrt{\mu}\xi\right)}{A\cos\left(\sqrt{\mu}\xi\right)+B\sin\left(\sqrt{\mu}\xi\right)}\right)+a_0,$$

$$v(\xi) = \frac{-2\mu}{3}\left(\frac{B\cos\left(\sqrt{\mu}\xi\right)-A\sin\left(\sqrt{\mu}\xi\right)}{A\cos\left(\sqrt{\mu}\xi\right)+B\sin\left(\sqrt{\mu}\xi\right)}\right)^2-\frac{(2\mu+3)}{3}. \tag{3.44}$$

Case 2 gives the exact traveling wave solution

$$u(\xi) = 2\sigma\sqrt{\frac{\mu}{3\sigma}}\sqrt{\sigma\left[1+\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2\right]} + a_0,$$

$$v(\xi) = \frac{-2\mu}{3}\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2 - \frac{(\mu+3)}{3}. \tag{3.45}$$

Case 3 gives the exact traveling wave solution:

$$\begin{aligned}
u(\xi) &= \sqrt{\frac{\mu}{3}}\left\{\sigma\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)\right. \\
&\qquad \left. +\frac{1}{\sqrt{\sigma}}\sqrt{\sigma\left[1+\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2\right]}\right\} + a_0, \\
v(\xi) &= -\frac{\mu}{3}\left\{\frac{\sigma}{\sqrt{\sigma}}\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)\right. \\
&\qquad \times\sqrt{\sigma\left[1+\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2\right]} \\
&\qquad \left. +\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2\right\} - \frac{(\mu+3)}{3},
\end{aligned} \tag{3.46}$$

where $\xi = x - a_0 t$. In particular, we deduce from (3.40) that the solitary wave solutions of (3.30) are derived as follows:

If $A = 0, B \neq 0$ and $\mu < 0$, then we obtain

$$\begin{aligned}
u(\xi) &= 2\sigma\sqrt{\frac{-\mu}{3}}\coth(\sqrt{-\mu}\xi) + a_0, \\
v(\xi) &= \frac{2\mu}{3}\operatorname{csch}^2(\sqrt{-\mu}\xi) - 1;
\end{aligned} \tag{3.47}$$

if $A \neq 0$, $A^2 > B^2$, and $\mu < 0$, then we obtain

$$u(\xi) = 2\sigma\sqrt{\frac{\mu}{3}}\tanh\left(\sqrt{-\mu}\xi + \xi_0\right) + a_0,$$
$$v(\xi) = -\frac{2\mu}{3}\operatorname{sech}^2\left(\sqrt{-\mu}\xi + \xi_0\right) - 1, \tag{3.48}$$

where $\xi_0 = \tanh^{-1}(B/A)$. Similarly, we can find more solitary solutions of (3.30) using (3.41)-(3.43) but we omitted them for simplicity.

3.4. Example 4: The (2+1)-Dimensional Broer-Kaup-Kuperschmidt Equations

In this subsection, we study the following (2+1)-dimensional Broer-Kaup-Kuperschmidt equations [19]:

$$u_{yt} - u_{xxy} + 2(uu_x)_y + 2v_{xx} = 0,$$
$$v_t + v_{xx} + 2(uv)_x = 0. \tag{3.49}$$

This system has been widely applied to many branches of physics like plasma physics, fluid dynamics, nonlinear optics and so on. Yomba [19] has obtained new and more general solutions of (3.49) including a series of nontraveling wave and coefficient function solutions using the modified extended Fan subequation method. Let us now solve (3.49) by the proposed method. To this end, we see that the traveling wave variables (3.18) permit us converting (3.49) into the following ODEs:

$$-Vu'' - u''' + 2(uu')' + 2v'' = 0,$$
$$-Vv' + v'' + 2(uv)' = 0. \tag{3.50}$$

Integrating Equation (3.50) with respect to ξ once, yields

$$K_1 - Vu'' - u'' + 2uu' + 2v' = 0, \tag{3.51}$$

$$K_2 - Vv + v' + 2uv = 0, \tag{3.52}$$

where K_1 and K_2 are integration constants. Considering the homogeneous balance between the highest-order derivatives and nonlinear terms in (3.51) and (3.52), we deduce the solution $u(\xi)$ which has the same form of (3.4) while, $v(\xi)$ has the same form of (3.34). Substituting (3.4) and (3.34) along with (2.5) into (3.51), collecting all terms with the same powers of $(G'/G)^k$, $(G'/G)^k\sqrt{\sigma(1+1/\mu(G'/G)^2)}$, and setting them to zero, we have the following algebraic equations:

$$\begin{gathered}
K_1 - 2\mu c_1 + \mu a_1(V - 2a_0) = 0, \\
2\sigma b_1^2 + 2\mu\left(2c_2 + a_1 + a_1^2\right) = 0, \\
a_1(V - 2a_0) - 2c_1 = 0, \\
2\sigma b_1^2 + 4\mu c_2 + 2\mu a_1(1 + a_1) = 0, \\
2\mu d_2 + \mu b_1(1 + 2a_1) = 0, \\
b_1(V - 2a_0) - 2d_1 = 0, \\
b_1(2a_1 + 1) + 2d_2 = 0.
\end{gathered} \tag{3.53}$$

Similarly, substituting (3.4) and (3.34) along with (2.5) into (3.52), collecting all terms with the same powers of $(G'/G)^k$, $(G'/G)^k\sqrt{\sigma(1+1/\mu(G'/G)^2)}$, and setting them to zero, we have the following algebraic equations:

$$\begin{aligned}
&K_2 - \mu c_1 + 2\sigma b_1 d_1 + c_0(2a_0 - V) = 0, \\
&2\sigma b_1 d_2 + c_1(2a_0 - V) - 2\mu c_2 + 2c_0 a_1 = 0, \\
&2\sigma b_1 d_1 + \mu c_2(2a_0 - V) + \mu c_1(2a_1 - 1) = 0, \\
&2\sigma b_1 d_2 + \mu c_2(2a_1 - 1) = 0, \\
&2b_1 c_0 + d_1(2a_0 - V) - \mu d_2 = 0, \\
&2b_1 c_1 + d_2(2a_0 - V) + d_2(2a_1 - 1) = 0, \\
&2b_1 c_2 + d_2(2a_1 - 1) = 0.
\end{aligned} \tag{3.54}$$

Solving these algebraic equations by Maple or Mathematica, we obtain the following results.

Case 1. One has

$$c_0 = -\mu, \qquad c_2 = -1, \qquad a_1 = 1, \qquad V = 2a_0, \qquad K_1 = K_2 = d_1 = d_2 = c_1 = b_1 = 0. \tag{3.55}$$

Case 2. One has

$$\begin{gathered}
c_0 = -\frac{\mu}{4}, \qquad c_2 = -\frac{1}{2}, \qquad d_2 = \frac{\sigma}{2}\sqrt{\frac{\mu}{\sigma}}, \\
b_1 = -\sigma\sqrt{\frac{\mu}{\sigma}}, \qquad V = 2a_0, \qquad K_1 = K_2 = a_1 = d_1 = c_1 = 0.
\end{gathered} \tag{3.56}$$

Case 3. One has

$$c_0 = -\frac{\mu}{2}, \qquad c_2 = -\frac{1}{2}, \qquad a_1 = \frac{1}{2},$$
$$d_2 = \frac{\sigma}{2}\sqrt{\frac{\mu}{\sigma}}, \qquad b_1 = -\frac{\sigma}{2}\sqrt{\frac{\mu}{\sigma}}, \qquad V = 2a_0, \qquad K_1 = K_2 = d_1 = c_1 = 0, \tag{3.57}$$

where a_0 is an arbitrary constant. From (3.4), (3.34), and the general solution of (2.5), we deduce the traveling wave solutions of (3.49) as follows.

When $\mu < 0$, then Case 1 gives the exact traveling wave solution:

$$u(\xi) = \sqrt{-\mu}\left(\frac{A\sinh(\sqrt{-\mu}\xi) + B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi) + B\sinh(\sqrt{-\mu}\xi)}\right) + a_0,$$
$$v(\xi) = \mu\left(\frac{A\sinh(\sqrt{-\mu}\xi) + B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi) + B\sinh(\sqrt{-\mu}\xi)}\right)^2 - \mu. \tag{3.58}$$

Case 2 gives the exact traveling wave solution:

$$u(\xi) = -\sigma\sqrt{\frac{\mu}{\sigma}}\sqrt{\sigma\left[1 - \left(\frac{A\sinh(\sqrt{-\mu}\xi) + B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi) + B\sinh(\sqrt{-\mu}\xi)}\right)^2\right]} + a_0,$$
$$v(\xi) = \frac{\sigma}{2}\sqrt{\frac{-\mu^2}{\sigma}}\left(\frac{A\sinh(\sqrt{-\mu}\xi) + B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi) + B\sinh(\sqrt{-\mu}\xi)}\right)$$
$$\times\sqrt{\sigma\left[1 - \left(\frac{A\sinh(\sqrt{-\mu}\xi) + B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi) + B\sinh(\sqrt{-\mu}\xi)}\right)^2\right]} \tag{3.59}$$
$$+ \frac{\mu}{2}\left(\frac{A\sinh(\sqrt{-\mu}\xi) + B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi) + B\sinh(\sqrt{-\mu}\xi)}\right)^2 - \frac{\mu}{4}.$$

Case 3 gives the exact traveling wave solution

$$\begin{aligned}
u(\xi) &= \frac{\sqrt{-\mu}}{2}\left(\frac{A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)\\
&\quad-\frac{\sigma}{2}\sqrt{\frac{\mu}{\sigma}}\sqrt{\sigma\left[1-\left(\frac{A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)^2\right]}+a_0,\\
v(\xi) &= \frac{\sigma}{2}\sqrt{\frac{-\mu^2}{\sigma}}\left(\frac{A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)\\
&\quad\times\sqrt{\sigma\left[1-\left(\frac{A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)^2\right]}\\
&\quad+\frac{\mu}{2}\left(\frac{A\sinh(\sqrt{-\mu}\xi)+B\cosh(\sqrt{-\mu}\xi)}{A\cosh(\sqrt{-\mu}\xi)+B\sinh(\sqrt{-\mu}\xi)}\right)^2-\frac{\mu}{2},
\end{aligned}\tag{3.60}$$

where $\xi = x - 2a_0t$. When $\mu > 0$, then Case 1 gives the exact traveling wave solution:

$$\begin{aligned}
u(\xi) &= \sqrt{\mu}\left(\frac{B\cos(\sqrt{\mu}\xi)-A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi)+B\sin(\sqrt{\mu}\xi)}\right)+a_0,\\
v(\xi) &= -\mu\left(\frac{B\cos(\sqrt{\mu}\xi)-A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi)+B\sin(\sqrt{\mu}\xi)}\right)^2-\mu.
\end{aligned}\tag{3.61}$$

Case 2 gives the exact traveling wave solution:

$$\begin{aligned}
u(\xi) &= -\sigma\sqrt{\frac{\mu}{\sigma}}\sqrt{\sigma\left[1+\left(\frac{B\cos(\sqrt{\mu}\xi)-A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi)+B\sin(\sqrt{\mu}\xi)}\right)^2\right]}+a_0,\\
v(\xi) &= \frac{\mu}{2}\left\{\frac{\sigma}{\sqrt{\sigma}}\left(\frac{B\cos(\sqrt{\mu}\xi)-A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi)+B\sin(\sqrt{\mu}\xi)}\right)\sqrt{\sigma\left[1+\left(\frac{B\cos(\sqrt{\mu}\xi)-A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi)+B\sin(\sqrt{\mu}\xi)}\right)^2\right]}\right.\\
&\quad\left.-\left(\frac{B\cos(\sqrt{\mu}\xi)-A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi)+B\sin(\sqrt{\mu}\xi)}\right)^2-\frac{1}{2}\right\}.
\end{aligned}\tag{3.62}$$

Case 3 gives the exact traveling wave solution:

$$
\begin{aligned}
u(\xi) &= \frac{\sqrt{\mu}}{2}\left\{\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)\right. \\
&\qquad \left. -\frac{\sigma}{\sqrt{\sigma}}\sqrt{\sigma\left[1+\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2\right]}\right\} + a_0, \\
v(\xi) &= \frac{\mu}{2}\left\{\frac{\sigma}{\sqrt{\sigma}}\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)\right. \\
&\qquad \times\sqrt{\sigma\left[1+\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2\right]} \\
&\qquad \left. -\left(\frac{B\cos(\sqrt{\mu}\xi) - A\sin(\sqrt{\mu}\xi)}{A\cos(\sqrt{\mu}\xi) + B\sin(\sqrt{\mu}\xi)}\right)^2 - 1\right\},
\end{aligned}
\tag{3.63}
$$

where $\xi = x - 2a_0 t$. In particular, we deduce from (3.58) that the solitary wave solutions of (3.49) are derived as follows.

If $A = 0, B \neq 0$, and $\mu < 0$,then one obtain

$$
\begin{aligned}
u(\xi) &= \sqrt{-\mu}\coth(\sqrt{-\mu}\xi) + a_0, \\
v(\xi) &= \mu\,\mathrm{csch}^2(\sqrt{-\mu}\xi);
\end{aligned}
\tag{3.64}
$$

if $A \neq 0$, $A^2 > B^2$, and $\mu < 0$, then we obtain

$$
\begin{aligned}
u(\xi) &= \sqrt{-\mu}\tanh(\sqrt{-\mu}\xi + \xi_0) + a_0, \\
v(\xi) &= -\mu\,\mathrm{sech}^2(\sqrt{-\mu}\xi + \xi_0),
\end{aligned}
\tag{3.65}
$$

where $\xi_0 = \tanh^{-1}(B/A)$. Similarly, we can find more solitary solutions of (3.49) using (3.59)-(3.60) but we omitted them for simplicity.

4. Conclusion

In this paper, we have seen that three types of traveling wave solutions in terms of hyperbolic, trigonometric, and rational functions for the (1+1)-dimensional modified Benjamin-Bona-Mahony equation, the (2+1)-dimensional typical breaking soliton equation, the (1+1)-dimensional classical Boussinesq equations, and the (2+1)-dimensional Broer-Kaup-Kuperschmidt equations are successfully found out by using the extended (G'/G)-expansion method. The performance of this method is reliable, effective, and giving many new solutions to many other nonlinear PDEs. Finally, the solutions of the proposed nonlinear evolution equations in this paper have many potential applications in physics and engineering.

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References

[1] M. J. Ablowitz and P. A. Clarkson, *Solitons, Nonlinear Evolution Equations and Inverse Scattering*, Cambridge University Press, Cambridge, 1991.

[2] A. Bekir and A. Boz, "Exact solutions for nonlinear evolution equations using Exp-function method," *Physics Letters. A*, vol. 372, no. 10, pp. 1619–1625, 2008.

[3] F. Calogero and A. Degasperis, "Nonlinear evolution equations solvable by the inverse spectral transform. II," *Nuovo Cimento. B. Serie 11*, vol. 39, no. 1, pp. 1–54, 1977.

[4] E. G. Fan, "Extended tanh-function method and its applications to nonlinear equations," *Physics Letters. A*, vol. 277, no. 4-5, pp. 212–218, 2000.

[5] S. Guo and Y. Zhou, "The extended (G'/G)-expansion method and its applications to the Whitham-Broer-Kaup-like equations and coupled Hirota-Satsuma KdV equations," *Applied Mathematics and Computation*, vol. 215, no. 9, pp. 3214–3221, 2010.

[6] J.-H. He and X.-H. Wu, "Exp-function method for nonlinear wave equations," *Chaos, Solitons and Fractals*, vol. 30, no. 3, pp. 700–708, 2006.

[7] J.-Q. Mei and H.-Q. Zhang, "New types of exact solutions for a breaking soliton equation," *Chaos, Solitons and Fractals*, vol. 20, no. 4, pp. 771–777, 2004.

[8] M. R. Miura, *Backlund Transformation*, Springer, Berlin, Germany, 1978.

[9] C. Rogers and W. F. Shadwick, *Bäcklund Transformations and Their Applications*, vol. 161 of *Mathematics in Science and Engineering*, Academic Press, New York, NY, USA, 1982.

[10] Sirendareji, "A new auxiliary equation and exact traveling wave solutions of nonlinear equations," *Physics Letters. A*, vol. 356, pp. 124–130, 2006.

[11] Sirendaoreji, "Exact travelling wave solutions for four forms of nonlinear Klein-Gordon equations," *Physics Letters. A*, vol. 363, no. 5-6, pp. 440–447, 2007.

[12] B. Tian, K. Zhao, and Y.-T. Gao, "Symbolic computation in engineering: application to a breaking soliton equation," *International Journal of Engineering Science*, vol. 35, no. 10-11, pp. 1081–1083, 1997.

[13] M. L. Wang, "Solitary wave solutions for variant Boussinesq equations," *Physics Letters. A*, vol. 199, no. 3-4, pp. 169–172, 1995.

[14] M. L. Wang, X. Li, and J. Zhang, "The (G'/G)-expansion method and travelling wave solutions of nonlinear evolution equations in mathematical physics," *Physics Letters. A*, vol. 372, no. 4, pp. 417–423, 2008.

[15] A.-M. Wazwaz, "Compactons, solitons and periodic solutions for some forms of nonlinear Klein-Gordon equations," *Chaos, Solitons and Fractals*, vol. 28, no. 4, pp. 1005–1013, 2006.

[16] T. Y. Wu and J. E. Zhang, "On modeling nonlinear long waves," in *Mathematics Is for Solving Problems*, P. Cook, V. Roytburd, and M. Tulin, Eds., pp. 233–249, SIAM, Philadelphia, Pa, USA, 1996.

[17] Zhenya Yan, "Abundant families of Jacobi elliptic function solutions of the (G'/G)-dimensional integrable Davey-Stewartson-type equation via a new method," *Chaos, Solitons and Fractals*, vol. 18, no. 2, pp. 299–309, 2003.

[18] Z. Yan and H. Zhang, "New explicit solitary wave solutions and periodic wave solutions for Whitham-Broer-Kaup equation in shallow water," *Physics Letters. A*, vol. 285, no. 5-6, pp. 355–362, 2001.

[19] E. Yomba, "The modified extended Fan sub-equation method and its application to the $(2 + 1)$-dimensional Broer-Kaup-Kupershmidt equation," *Chaos, Solitons and Fractals*, vol. 27, no. 1, pp. 187–196, 2006.

[20] E. Yusufoglu, "New solitary solutions for the MBBM equations using Exp-function method," *Physics Letters. A*, vol. 372, pp. 442–446, 2008.

[21] Y.-B. Zhou and C. Li, "Application of modified (G'/G)-expansion method to traveling wave solutions for Whitham-Broer-Kaup-like equations," *Communications in Theoretical Physics*, vol. 51, no. 4, pp. 664–670, 2009.

[22] E. M. E. Zayed, "The (G'/G)-expansion method and its applications to some nonlinear evolution equations in the mathematical physics," *Journal of Applied Mathematics and Computing*, vol. 30, no. 1-2, pp. 89–103, 2009.

[23] E. M. E. Zayed and K. A. Gepreel, "The (G'/G)-expansion method for finding traveling wave solutions of nonlinear partial differential equations in mathematical physics," *Journal of Mathematical Physics*, vol. 50, no. 1, Article ID 013502, 12 pages, 2009.

[24] E. M. E. Zayed, H. A. Zedan, and K. A. Gepreel, "On the solitary wave solutions for nonlinear Euler equations," *Applicable Analysis*, vol. 83, no. 11, pp. 1101–1132, 2004.

[25] E. M. E. Zayed and A.-J. Shorog, "Applications of an improved (G'/G)-expansion method to nonlinear PDEs in mathematical physics," in *Proceedings of the International Conference of Numerical Analysis and Applied Mathematics (ICNAAM '09)*, vol. 1168 of *AIP Conference*, pp. 371–376, The American Institute of Physics, 2009.

[26] H. Zhang, "New application of the (G'/G)-expansion method," *Communications in Nonlinear Science and Numerical Simulation*, vol. 14, pp. 3220–3225, 2009.

[27] S.-L. Zhang, B. Wu, and S.-Y. Lou, "Painlevé analysis and special solutions of generalized Broer-Kaup equations," *Physics Letters. A*, vol. 300, no. 1, pp. 40–48, 2002.

[28] S. Zhang, L. Dong, J.-M. Ba, and Y.-N. Sun, "The (G'/G)-expansion method for nonlinear differential-difference equations," *Physics Letters. A*, vol. 373, no. 10, pp. 905–910, 2009.

[29] İ. Aslan, "Discrete exact solutions to some nonlinear differential-difference equations via the (G'/G)-expansion method," *Applied Mathematics and Computation*, vol. 215, no. 8, pp. 3140–3147, 2009.