Research Article

The Basic Differential Equations of Self-Anchored Cable-Stayed Suspension Bridge

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The static behavior of self-anchored cable-stayed suspension bridge under vertical load is described with the continuum method. Based on the partition generalized variation principle, considering the compression-bending coupling effect of the main girder and the tower, the large displacement incomplete generalized potential energy functional of three-span self-anchored cable-stayed suspension bridge is established. Then, the basic differential equations of self-anchored cable-stayed suspension bridge are derived through constraint variation. Taking a self-anchored cable-stayed suspension bridge with main span 100 m, for example, the results by the proposed analytic method agree with that of numerical analysis. Therefore, the basic differential equations proposed in this paper could be applied to the preliminary analysis of self-anchored cable-stayed suspension bridge. The equations also provide a theoretical basis to describe the static behavior of this type of bridge.

1. Introduction

Sea-crossing bridge projects in the 21st century are facing adverse natural conditions such as deep water foundations, strong winds, and so on. Either suspension bridge or cable-stayed bridge for large span bridge has shown shortcomings and inadequacies [1]. Therefore, the cable-stayed suspension bridge, which combines the advantages of cable-stayed bridge and suspension bridge to make up for their shortcomings, provides a suitable option for long-span bridge, particularly those sea-crossing bridge. There are some cable-stayed suspension bridges in the world, such as Albert Bridge in London, Brooklyn Bridge in New York, and so on [2]. Wujiang Bridge in China, with spans of 66 + 288 + 66 m is the longest modern cable-stayed suspension bridge. Nagisa Bridge, a single span footbridge in Japan, is a hybrid
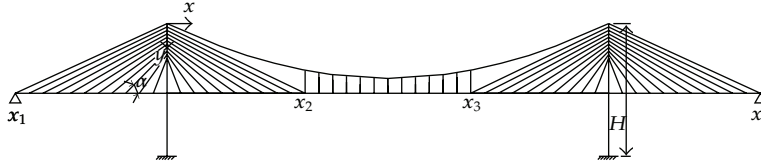
Figure 1: Illustration of a self-anchored cable-stayed suspension bridge.

structure of cable-stayed prestressed concrete bridge and steel suspension bridge. And the schemes of cable-stayed suspension bridge have been proposed many times, such as Turkey Izmit Bridge, Gibraltar Strait Bridge, Lingdingyang Bridge, and so on. However, due to lack of systematic study on such bridge type, the proposals are not implemented. At present, the schemes of cable-stayed suspension bridge just stay in the program design phase, and the proposed schemes all adopt the earth-anchored system with large anchor. Using a self-anchored system could not only save the anchor, but also possess the advantages of cable-stayed suspension bridge. The Jianshe Bridge in Dalian is the first self-anchored cable-stayed suspension bridge in the world [3]. And self-anchored cable-stayed suspension bridge proposals have been brought up for Dalian Bay Bridge, Dalian Jinzhou Bay Bridge, and Eastern Hubei Yangtze River Bridge.

The bridge structural behavior under static load is an important basis for its design. Ichiro Konishi gave a detailed elaboration on the static and dynamic basic differential equations of earth-anchored suspension bridge [4]. Based on the large-displacement partition generalized variation principle, the incomplete generalized potential energy functional of a three-span self-anchored cable-stayed suspension bridge is established with the continuum method. The differential equations of self-anchored cable-stayed suspension bridge are derived through constraint variation. The differential equations could analyze the structural behavior of self-anchored cable-stayed suspension bridge under vertical load and could provide a theoretical basis for static analysis of such type bridge.

2. The Establishment of the Large Displacement Partition Generalized Potential Energy Functional

Figure 1 shows a self-anchored cable-stayed suspension bridge. Based on the structural characteristics, the following items are assumed [4].

(1) All the materials conform with Hooke's Law.

(2) In operational bridge status, the dead load is distributed uniformly along the span, and the main cables are of parabolic shape.

(3) The main girder is a continuous beam with constant cross section, and vertical supports exist at the towers. Ignore the vertical curve and camber of the girder.

(4) The stay cables and the hangers are dense and could be compared to a homogeneous membrane with only axial resistance.

$q(x)$ is the dead load on the girder, while $p(x)$ the live load. Define the displacement and strain of various parts as follows: u_c, v_c are the longitudinal and vertical displacements of the main cable respectively, u_b, v_b are that of the girder, and u_t, v_t are that of the tower. And $\varepsilon_c, \varepsilon_h, \varepsilon_l$ are the strain increments of the main cable, the hanger, and the stay cable caused
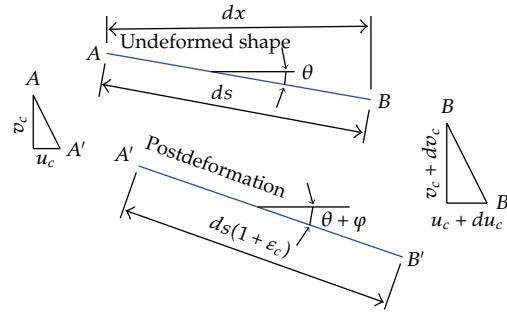
Figure 2: Deformation of the main cable element.

by the live load, respectively. In Figure 1, $[x_1, x_2] \cup [x_3, x_4]$ is the cable-stayed interval, while $[x_2, x_3]$ the suspension interval. Due to different boundary conditions between the cable-stayed interval and the suspension interval, derivation of each will be undertaken separately.

2.1. Establishment of Strain Energy of Components within the Cable-Stayed Interval

2.1.1. Establishment of Strain Energy of the Main Cable

Based on the geometric relationship shown in Figure 2, the following exists for the main cable

$$
\begin{aligned}
ds\cos\theta &= dx, \\
(1 + \varepsilon_c)ds\cos(\theta + \varphi) &= ds\,\cos\theta + du_c, \\
(1 + \varepsilon_c)ds\sin(\theta + \varphi) &= ds\,\sin\theta + dv_c,
\end{aligned}
\tag{2.1}
$$

where $\theta + \varphi$ is the inclination angle of the main cable after deformation and φ the change of the main cable inclination. Elimination of φ and element length ds leads to the cable strain increment given as

$$
\varepsilon_c = \sqrt{(\cos\theta + u_c'\cos\theta)^2 + (\sin\theta + v_c'\cos\theta)^2} - 1.
\tag{2.2}
$$

The strain energy of the main cable is [5]

$$
U_{c1} = \int_{x_1}^{x_2} \int_{\varepsilon_{c0}}^{\varepsilon_{c0}+\varepsilon_c} T\sec\theta\,d\varepsilon_c\,dx,
\tag{2.3}
$$

where ε_{c0} is the initial strain of the main cable and T is the cable tension.

2.1.2. Establishment of Strain Energy of the Stay Cables

According to the assumptions, the strain energy of the stay cables is [6]

$$U_l = \int_{x_1}^{x_2} \int_{\varepsilon_{l0}}^{\varepsilon_{l0}+\varepsilon_l} l(x)c(x)d\varepsilon_l dx,$$

(2.4)

where ε_{l0} is the initial strain of the stay cables, $l(x)$ the stay cable length, and $c(x)$ the tension of stay cables under dead and live loads.

2.1.3. Establishment of Strain Energy of the Stay Cables

Due to the action of the horizontal component force of the main cable, the girder is under compression and bending. The assumption that the structure is in an elastic stable state and shear deformation is neglected will lead to the expression of the axial strain [7]

$$\varepsilon_x = \frac{\partial u_b}{\partial x} - y\frac{\partial^2 v_b}{\partial x^2} + \frac{1}{2}\left(\frac{\partial v_b}{\partial x}\right)^2.$$

(2.5)

Then, the strain energy of the girder gives [8, 9]

$$U_{b1} = \frac{1}{2}\int_{x_1}^{x_2}\left[N_x\left(\frac{\partial u_b}{\partial x}\right) - M_z\left(\frac{\partial^2 v_b}{\partial x^2}\right) + N_x\left(\frac{\partial v_b}{\partial x}\right)^2\right]dx,$$

(2.6)

where N_x is the girder axial force and M_z is the girder bending moment. In (2.6), the first item is the strain energy caused by compression, the second that caused by bending, and the third that due to compression-bending coupling and could be neglected in linear structural analysis.

2.1.4. Establishment of Strain Energy of the Stay Cables

Based on the above derivation, strain energy of the tower gives [10, 11]

$$U_t = \frac{1}{2}\int_0^H\left[N_t\left(\frac{\partial v_t}{\partial y}\right) - M_t\left(\frac{\partial^2 u_t}{\partial y^2}\right) + N_t\left(\frac{\partial u_t}{\partial y}\right)^2\right]dy,$$

(2.7)

where N_t is the axial force of the tower and M_t is the bending moment. And due to the symmetry of the structure, the strain energy of the main cable, the tower, and the girder within the interval $[x_3, x_4]$ equals that within the interval $[x_1, x_2]$.

2.2. Establishment of Strain Energy of Components within the Suspension Interval

According to (2.3)–(2.7), the strain energy of the main cable, the girder, and the hanger is given as follows.

The strain energy of the main cable is

$$U_{c2} = \int_{x_2}^{x_3} \int_{\varepsilon_{c0}}^{\varepsilon_{c0}+\varepsilon_c} T \, d\varepsilon_c ds = \int_{x_2}^{x_3} \int_{\varepsilon_{c0}}^{\varepsilon_{c0}+\varepsilon_c} T \sec\theta d\varepsilon_c dx. \tag{2.8}$$

The strain energy of the girder is

$$U_{b2} = \frac{1}{2} \int_{x_2}^{x_3} \left[N_x \left(\frac{\partial u_b}{\partial x} \right) - M_z \left(\frac{\partial^2 v_b}{\partial x^2} \right) + N_x \left(\frac{\partial v_b}{\partial x} \right)^2 \right] dx. \tag{2.9}$$

The strain energy of the hanger is

$$U_h = \int_{x_2}^{x_3} \int_{\varepsilon_{h0}}^{\varepsilon_{h0}+\varepsilon_h} h(x)s(x) d\varepsilon_h dx, \tag{2.10}$$

where ε_{h0} is the initial strain of the hanger, $h(x)$ is the length of hanger and $s(x)$ is the tension of hanger under dead and live loads.

2.3. Establishment of the Potential Energy Functional

Based on the partition generalized variation principle, the potential energy functional of the system may be given as [12]

$$\prod = 2 \times (U_{c1} + U_l + U_{b1} + U_t) + U_{c2} + U_{b2} + U_h - \int_{x_1}^{x_4} \int_0^{v_b} q \, dv_b dx - \int_{x_1}^{x_4} \int_0^{v_b} p \, dv_b dx. \tag{2.11}$$

And the constraint conditions are as follows.

2.3.1. The Cable-Stayed Interval

(1) The deformation compatibility condition of the main cable is

$$G_{c1} = \sqrt{(\cos\theta + u_c' \cos\theta)^2 + (\sin\theta + v_c' \cos\theta)^2} - 1 - \varepsilon_c = 0. \tag{2.12}$$

(2) The deformation compatibility condition of the stay cable is

$$G_l = \frac{1}{l(x)} \sqrt{[l(x)\cos\alpha + u_b - u_t]^2 + [l(x)\sin\alpha + v_b - v_t]^2} - 1 - \varepsilon_l = 0. \tag{2.13}$$

2.3.2. The Suspension Interval

(1) The deformation compatibility condition of the main cable is the same as (2.12)

$$G_{c2} = \sqrt{(\cos\theta + u'_c \cos\theta)^2 + (\sin\theta + v'_c \cos\theta)^2} - 1 - \varepsilon_c = 0. \tag{2.14}$$

(2) The deformation compatibility condition of the hanger is

$$G_h = \frac{1}{h(x)}\sqrt{[h(x) + v_b - v_c]^2 + (u_b - u_c)^2} - 1 - \varepsilon_h = 0. \tag{2.15}$$

Then, the large-displacement partition incomplete generalized potential energy functional is

$$
\begin{aligned}
\prod{}^* &= \prod{} + \int_{x_1}^{x_2} \lambda_{c1} G_{c1} dx + \int_{x_1}^{x_2} \lambda_l G_l dx + \int_{x_2}^{x_3} \lambda_{c2} G_{c2} dx + \int_{x_2}^{x_3} \lambda_h G_h dx + \int_{x_3}^{x_4} \lambda_{c1} G_{c1} dx \\
&\quad + \int_{x_3}^{x_4} \lambda_l G_l dx + 2\int_0^H \lambda_l G_l dx \\
&= \prod{} + 2 \times \left(\int_{x_1}^{x_2} \lambda_{c1} G_{c1} dx + \int_{x_1}^{x_2} \lambda_l G_l dx + \int_0^H \lambda_l G_l dx \right) + \int_{x_2}^{x_3} \lambda_{c2} G_{c2} dx + \int_{x_2}^{x_3} \lambda_h G_h dx.
\end{aligned}
\tag{2.16}
$$

3. Establishment of the Basic Differential Equations of Self-Anchored Cable-Stayed Suspension Bridge

Perform variation of the generalized functional $\prod^*$ on $u_c, v_c,\ u_b,\ v_b,\ u_t,\ v_t,\ \varepsilon_c,\ \varepsilon_l,\ \varepsilon_h,\ \lambda_{c1},$ $\lambda_{c2},\ \lambda_l,$ and λ_h, respectively, and correspondent Euler equations could be achieved, that is, $\delta\prod^* = (\partial\prod^*/\partial\phi_i)\delta\phi_i = 0$ [13], where $\phi_i = u_c, v_c, u_b, v_b, u_t, v_t, \varepsilon_c, \varepsilon_l, \varepsilon_h, \lambda_{c1}, \lambda_{c2}, \lambda_l, \lambda_h$.

3.1. Basic Differential Equations of the Cable-Stayed Interval

For the cable-stayed interval $x \in [x_1, x_2] \cup [x_3, x_4]$, the balance equations are

$$T = C \quad (C \text{ represents the constant}),$$
$$c\cos(\alpha + \beta) - \frac{d}{dx}\left(N_x + \frac{1}{2}EA_b v_b^{'2}\right) = 0, \tag{3.1}$$

where $(\alpha + \beta)$ is the angle between the stay cable and the x-axis positive direction after deformation

$$\frac{d^2}{dx^2} M_z + \frac{d}{dx}(N_x v'_b) - c\sin(\alpha + \beta) + p + q = 0,$$

$$\frac{d^2}{dy^2} M_t + \frac{d}{dy}\left(N_t \frac{\partial u_t}{\partial y}\right) + c\cos(\alpha + \beta) = 0, \qquad (3.2)$$

$$\frac{d}{dy}\left[N_t + \frac{1}{2}EA_t\left(\frac{\partial u_t}{\partial y}\right)^2\right] + c\sin(\alpha + \beta) = 0.$$

3.2. Basic Differential Equations of the Suspension Interval

For the suspension interval $x \in [x_2, x_3]$, the balance equations are

$$\frac{d}{dx}\left[T\cos(\theta + \varphi)\right] + s(x)\frac{u_b - u_c}{\sqrt{[h(x) + v_b - v_c]^2 + (u_b - u_c)^2}} = 0, \qquad (3.3)$$

$$\frac{d}{dx}\left[T\sin(\theta + \varphi)\right] + s(x)\frac{h(x) + v_b - v_c}{\sqrt{[h(x) + v_b - v_c]^2 + (u_b - u_c)^2}} = 0, \qquad (3.4)$$

$$\frac{d}{dx}\left(N_x + \frac{1}{2}EA_b v'^2_b\right) - s(x)\frac{u_b - u_c}{\sqrt{[h(x) + v_b - v_c]^2 + (u_b - u_c)^2}} = 0, \qquad (3.5)$$

$$\frac{d^2}{dx^2} M_z + \frac{d}{dx}(N_x v'_b) + p + q - s(x)\frac{h(x) + v_b - v_c}{\sqrt{[h(x) + v_b - v_c]^2 + (u_b - u_c)^2}} = 0. \qquad (3.6)$$

The differential equations of either the cable-stayed interval or the suspension interval are quite complicated, and their solutions are difficult to reach. A convenient way to solve the equations is setting a family function of displacement for each parameter that meets the boundary conditions and approaches an approximate solution by utilizing the principle of stationary potential energy [14], but this is not the purpose of this paper. This paper intends to indicate that a relatively precise description of the static behavior of self-anchored cable-stayed suspension bridge under vertical load can be achieved through the continuum approach. And further assumptions are introduced to simplify the above equations.

4. Simplification of the Basic Differential Equations of Self-Anchored Cable-Stayed Suspension Bridge

4.1. Simplification of the Basic Differential Equations of the Cable-Stayed Interval

Based on the above-mentioned assumptions and excluding the impact of compression-bending coupling, the strain energy of the main girder and the tower become

$$
U_{b1} = \frac{1}{2} \int_{x_1}^{x_2} \left[N_x \left(\frac{\partial u_b}{\partial x} \right) - M_z \left(\frac{\partial^2 v_b}{\partial x^2} \right) \right] dx,
$$

$$
U_t = \frac{1}{2} \int_0^H \left[N_t \left(\frac{\partial v_t}{\partial y} \right) - M_t \left(\frac{\partial^2 u_t}{\partial y^2} \right) \right] dy.
$$

(4.1)

Then, the basic differential equations of the cable-stayed interval could be simplified into

$$
T = C \quad (C \text{ represents the constant}),
$$

$$
\frac{d}{dx} N_x = c \cos(\alpha + \beta),
$$

$$
\frac{d^2}{dx^2} M_z - c \sin(\alpha + \beta) + p + q = 0,
$$

$$
\frac{d^2}{dy^2} M_t + c \cos(\alpha + \beta) = 0,
$$

$$
\frac{d}{dy} N_t + c \sin(\alpha + \beta) = 0.
$$

(4.2)

It can be seen from the above equations that the action of stay cables on the main girder and the tower is equivalent to the action of distributed load, and the basic differential equations of the main girder and the tower accord with the equation of a classic beam.

4.2. Simplification of the Basic Differential Equations of the Suspension Interval

On the basis of the above assumptions, introduce some more as follows: excluding the elongation of the hangers and assuming the vertical displacement of the main girder equals that of the main cable.

Then, (3.5) can be simplified into $N_x + (1/2)EA_b v_b'^2 = C$ (C represents the constant). Omitting the higher-order terms leads to

$$
N_x = C.
$$

(4.3)

Equation (4.3) indicates that without considering the compression-bending coupling effect, the axial force in the main girder is constant and its value equals that of the horizontal component force of the main cable. And (3.6) can be simplified into

$$\frac{d^2}{dx^2}M_z + \frac{d}{dx}\left[N_x v_b' + T\sin(\theta + \varphi)\right] + p + q = 0. \tag{4.4}$$

Because the horizontal component force of the main cable accords $H = T\cos(\theta + \varphi)$ and the axial force of the main girder $N_x = -H$, the above equation could be transformed into

$$M'' + p + q + \frac{d}{dx}\left[H\tan(\theta + \varphi) - Hv_b'\right] = 0. \tag{4.5}$$

Introduction of (2.1) can transform (4.5) into $M'' + p + q + H(d/dx)(y' + v_c' - v_c') = 0$. Considering that $q = -H_q y''$ and $H = H_q + H_p$, where H_p represents the increment of the horizontal component force of the main cable caused by live load, then the above equation could be further transformed into $M'' + p + H_p y'' = 0$, integration of which leads to

$$M = M_p^0 - H_p y, \tag{4.6}$$

where M_p^0 is the bending moment of a simply supported beam with the same span under live load $p(x)$. It can be seen from (4.6) that the horizontal component force of the main cable caused by live load can reduce the live load bending moment of the main girder. Equation (4.6) is just the equilibrium differential equation of the deflection theory of self-anchored suspension bridge [15], and it is same as that of the elasticity theory of earth-anchored suspension bridge.

Equation (4.6) is an approximate equivalent of the linear equation of the elasticity theory. The additional bending moment of the girder caused by the enormous axial force offsets that caused by the live-load deflection when considering the compression-bending coupling effect. Therefore, the main girder of self-anchored cable-stayed suspension bridge can be analyzed utilizing the elasticity theory.

An equation of the compatibility condition is required since two unknowns H_p, v exist in (4.6), and according to the compatibility equation in the deflection theory of self-anchored suspension bridge, the specific form of the compatibility equation for the suspension interval is given as follows:

$$\frac{H_p}{E_c A_c}\int_{x_1}^{x_4}\frac{dx}{\cos^3\theta} \pm \alpha t\int_{x_1}^{x_4}\frac{dx}{\cos^3\theta} - \int_{x_1}^{x_4} y'v'dx = \frac{H_p L}{E_b A_b} \pm \alpha t L - 2\int_{x_1}^{x_2}\frac{N_x}{E_b A_b}dx, \tag{4.7}$$

where $\cos\theta = dx/ds$, $E_c A_c$, and $E_b A_b$, represent the axial stiffness of the main cable and the main girder, respectively, α is the temperature linear expansion coefficient, t, is the temperature variations, L, is the length of the main girder, and N_x is the axial force of the main girder within the cable-stayed interval. And the above equation indicates that the horizontal displacement at the anchor end of the main cable equals that at both ends of the main girder.

Table 1: Material and section properties of the structure components.

Components	E/MPa	A/m^2	Iz/m^4	Q/(KN/m)
Main cable	1.95×10^5	1.669×10^{-2}	0	1.40
Stay cables	1.95×10^5	3.502×10^{-3}	0	0.29
Hangers	1.95×10^5	2.117×10^{-3}	0	0.18
Main girder	3.45×10^4	9.02	3.06	234.52
Pylon	3.45×10^4	6.85	7.08	178.10
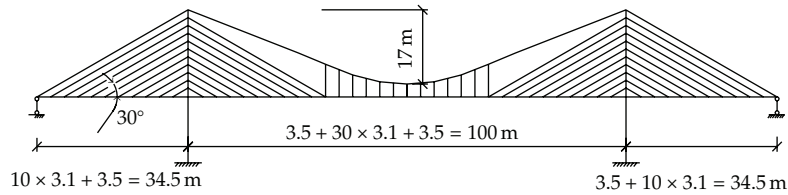
Figure 3: Example of a self-anchored cable-stayed suspension bridge (Unit: m).

Table 2: Analytic solution and numerical solution.

Location	Bending moment M/(KN·m)		
	Analytic solution	Numerical solution	Difference (%)
Middle of side span	−1488	−1328	−12
Junction between cable-stayed section and suspension section	1384	1442	−4
Middle of the main span	3460	3235	6.9

5. Example of Verification

Below is a simple example of verification of the above-derived differential equations. As shown in Figure 3, there is a self-anchored cable-stayed suspension bridge with span of 34.5 m + 100 m + 34.5 m. The spacing between both the stay cables and the hangers is 3.1 m, the inclination angles of the stay cables are all 30°, the sag of the main cable is 17 m, and a uniformly distributed load of $q = 20$ kN/m is imposed on the main girder. The material and sectional properties of the main components are shown in Table 1.

Note the continuity of the internal force and the displacement at the junction between the cable-stayed interval and the suspension one, solve the above equations referencing the solution approach [16], and compare the results with that of the nonlinear numerical solution. The comparison result is given as shown in Table 2.

It can be seen from Table 2 that the analytic solution is relatively quite close to the nonlinear numerical solution. The error is mainly due to the neglecting of the strain energy of hangers and the compression-bending coupling effect, and in particular, the error of the middle-span bending moment in side span reaches 12%. The result indicates that the analytic approach proposed in this paper can be applied to the preliminary analysis of simple self-anchored cable-stayed suspension bridge, but should not be used in design stage.

6. Conclusion

Based on the large displacement partition, incomplete generalized variation principle, and considering the compression-bending coupling effect of the main girder and the tower, the potential energy functional of a self-anchored cable-stayed suspension bridge is established with the continuum method. Then, the differential equations of that are derived, which describe the static behavior of self-anchored cable-stayed suspension bridge under vertical load. And the following could be seen from the basic differential equations.

(1) The axial force along the main cable within the cable-stayed interval is constant under vertical load. In addition, the action imposed on the main girder and the tower by the stay cables is equivalent to that by the distributed load, and the basic differential equations of the main girder and the tower accord with that of a classic beam.

(2) The additional bending moment caused by the enormous axial force imposed on the main girder by the main cable offsets the moment caused by the live load deflection when considering the compression-bending coupling effect. Therefore, the main girder of a self-anchored cable-stayed suspension bridge can be analyzed utilizing the elasticity theory.

(3) The analytic method proposed in this paper is mainly to provide a theoretical basis to describe the static behavior of self-anchored cable-stayed suspension bridge; the analytic solution is relatively close to the nonlinear numerical solution; therefore, the analytic approach can be applied to the preliminary analysis of simple self-anchored cable-stayed suspension bridge.

Nomenclature

$q(x)$: Dead load on the girder
$p(x)$: Live load
u_c: Longitudinal displacements of the main cable
v_c: Vertical displacements of the main cable
u_b: Longitudinal displacements of the girder
v_b: Vertical displacements of the girder
u_t: Longitudinal displacements of the tower
v_t: Vertical displacements of the tower
ε_c: Strain increments of the main cable
ε_h: Strain increments of the hanger
ε_l: Strain increments stay cable
$[x_1, x_2] \cup [x_3, x_4]$: Cable-stayed interval
$[x_2, x_3]$: Suspension interval
φ: Change of the main cable inclination
ε_{c0}: Initial strain of the main cable
T: Cable tension
ε_{l0}: Initial strain of the stay cables
$l(x)$: Stay cable length
$c(x)$: Tension of stay cables
N_x: Axial force of the girder
M_z: Bending moment of the girder

N_t: Axial force of the tower
M_t: Bending moment of the tower
ε_{h0}: Initial strain of the hanger
$h(x)$: Length of hanger
$s(x)$: Tension of hanger
U_c: Strain energy of the main cable
U_b: Strain energy of the girder
U_l: Strain energy of the stay cables
U_t: Strain energy of the tower
U_h: Strain energy of the hanger.

Acknowledgments

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