

Review Article

Robust State-Derivative Feedback LMI-Based Designs for Linear Descriptor Systems

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Techniques for stabilization of linear descriptor systems by state-derivative feedback are proposed. The methods are based on Linear Matrix Inequalities (LMIs) and assume that the plant is a controllable system with poles different from zero. They can include design constraints such as: decay rate, bounds on output peak and bounds on the state-derivative feedback matrix K , and can be applied in a class of uncertain systems subject to structural failures. These designs consider a broader class of plants than the related results available in the literature. The LMI can be efficiently solved using convex programming techniques. Numerical examples illustrate the efficiency of the proposed methods.

1. Introduction

The Linear Matrix Inequalities (LMIs) formulation has emerged recently as a useful tool for solving a great number of practical control problems [1–10]. Furthermore, LMI can be solved with polynomial convergence time, by convex optimization algorithms [1, 11–13].

Recently, LMI has been used for the study of descriptor systems [14–17]. Descriptor systems can be found in various applications, for instance, in electrical systems, or in robotics [18]. The proportional and derivative feedback ($u = Lx(t) - K\dot{x}(t)$, where $x(t)$ is the plant state vector) has been studied by many authors to design controllers in the following problems: stabilization and regularizability of linear descriptor systems [19, 20], feedback control of singular systems [21], nonlinear control with exact feedback linearization [22], \mathcal{H}_∞ -control of continuous-time systems with state delay [23], and design of PD observers [24]. In [18, 25] some properties of this type of feedback and its applications to pole placement were presented.

There exist few researches using only derivative feedback ($u = -K\dot{x}(t)$). In some practical problems the state-derivative signals are easier to obtain than the state signals, for instance, in the following applications: suppression of vibration in mechanical systems [26], control of car wheel suspension systems [27], vibration control of bridge cables [28], and vibration control of landing gear components [29]. The main sensors used in these problems are accelerometers. In this case, from the signals of the accelerometers it is possible to reconstruct the velocities with a good precision but not the displacements [26]. Defining the velocities and displacement as the state variables, then one has available for feedback only the state-derivative signals. Procedures for solving the pole-placement problem for linear systems using state-derivative feedback were proposed in [26, 30, 31]. In [28, 32] a Linear Quadratic Regulator (LQR) controller design scheme for standard state space systems was presented. The results were obtained in Reciprocal State Space (RSS) framework. Robust state-derivative feedback LMI-based designs for linear time-invariant systems were recently proposed in [33, 34]. These results considered only standard linear systems, and they can be applied to uncertain systems, with or without, structural failures.

Structural failures appear naturally in systems, for physical wear of the equipment, or for short circuit of electronic components. Recent researches on structural failures (or faults), have been presented in LMI framework [35–38].

In this paper, we will show that it is possible to extend the presented results in [33], for applications in a class of descriptor systems, subject to structural failures in the plant. The procedure can include some specifications: decay rate, bounds on output peak and bounds on the state-derivative feedback matrix K , which can make easier the practical implementation of the controllers. These methods allow new specifications, and also to consider a broader class of plants that the related results are available in the literature [19, 25, 31, 39]. Two examples illustrate the efficiency of the proposed method.

2. Statement of the Problem

Consider a controllable linear descriptor system described by

$$E\dot{x}(t) = Ax(t) + Bu(t), \quad (2.1)$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $E \in \mathbb{R}^{n \times n}$, $A \in \mathbb{R}^{n \times n}$, and $B \in \mathbb{R}^{n \times m}$. It is known that the stability problem for descriptor systems is more complicated than for standard systems, because it requires considering not only stability, but also regularity [15, 25]. In the next sections, LMI conditions for asymptotic stability of descriptor system (2.1) using state-derivative feedback, are proposed. The problem is defined as follows.

Problem 1. Find a constant matrix $K \in \mathbb{R}^{m \times n}$, such that the following conditions hold:

- (1) $(E + BK)$ has a full rank;
- (2) the closed-loop system (2.1) with the state-derivative feedback control

$$u(t) = -K\dot{x}(t), \quad (2.2)$$

is regular and asymptotically stable (in this work, a descriptor system is regular if it has uniqueness in the solutions and avoid impulsive responses).

Remark 2.1. In [25, 39] the authors assure that $(E + BK)$ has a full rank (nonsingular matrix) only if the following equation holds:

$$\text{rank}[E, B] = n. \quad (2.3)$$

Unfortunately, there exist several practical problems that not satisfy (2.3). In that way, the input control (2.2) can only be applied in descriptor systems (2.1), when (2.3) holds. Some authors have been using the state-derivative and state feedback ($u = Lx(t) - K\dot{x}(t)$) to solve (2.1), when (2.3) does not hold [18, 20]. However, usually these designs are more complex than the design procedures with only state or state-derivative feedback.

Assuming that $(E + BK)$ has a full rank, then from (2.2) it follows that (2.1) can be rewrite such as a standard linear system, given by

$$E\dot{x}(t) = Ax(t) - BK\dot{x}(t) \iff \dot{x}(t) = (E + BK)^{-1}Ax(t). \quad (2.4)$$

3. LMI-Based Stability Conditions for State-Derivative Feedback

Necessary and sufficient conditions for asymptotic stability of standard linear system (2.4) are proposed in the next theorems.

Theorem 3.1. *Assuming that (2.3) holds, the necessary and sufficient condition for the solution of Problem 1 is the existence of matrices $Q = Q'$, $Q \in \mathbb{R}^{n \times n}$ and $Y \in \mathbb{R}^{m \times n}$, such that,*

$$AQE' + EQA' + BYA' + AY'B' < 0, \quad (3.1)$$

$$Q > 0. \quad (3.2)$$

Furthermore, when (3.1) and (3.2) hold, then a state-derivative feedback matrix that solves Problem 1 can be given by

$$K = YQ^{-1}. \quad (3.3)$$

Proof. Observe that for any nonsymmetric matrix M ($M \neq M'$), $M \in \mathbb{R}^{n \times n}$, if $M + M' < 0$, then M has a full rank. Now, defining $Q = P^{-1}$ and $Y = KQ$, the following equations are equivalents:

$$AQE' + EQA' + BYA' + AY'B' = AQ(E + BK)' + (E + BK)QA' < 0 \quad (3.4)$$

$$\iff P(E + BK)^{-1}A + A'[(E + BK)^{-1}]'P < 0, \quad (3.5)$$

From (3.4) one has the matrix $(E + BK)QA'$ has full rank, and so, $(E + BK)$ also has a full rank, as required in Problem 1, and (3.5) was obtained after premultiplying by $P(E + BK)^{-1}$ and posmultiplying by $[(E + BK)^{-1}]'P$ in both sides of (3.4).

System (2.4) is globally asymptotically stable only if there exists $P = P' > 0$ (that is equivalent to $Q = Q' = P^{-1} > 0$) such that (3.4) or (3.5) holds. \square

Remark 3.2. Note that from (3.4) it follows that matrix A must have a full rank, and so, all its eigenvalues are different from zero. This condition was also considered in other papers [26, 28, 33] for linear systems.

Equations (3.1) and (3.2) are LMI. When (3.1) and (3.2) are feasible, they can be easily solved using available software, such as LMISol [40], that is a free software, or MATLAB [11]. The algorithms have polynomial time convergence.

Usually, only the stability of the control systems is insufficient to obtain a suitable performance. In the design of control systems, the specification of the decay rate can also be very useful.

3.1. Decay Rate in State-Derivative Feedback

Consider, for instance, the controlled system (2.4). According to [1], the decay rate is defined as the largest real constant γ , $\gamma > 0$, such that,

$$\lim_{t \rightarrow \infty} e^{\gamma t} \|x(t)\| = 0 \quad (3.6)$$

holds, for all trajectories $x(t)$, $t \geq 0$.

Theorem 3.3. *Assuming that (2.3) holds, the closed-loop system given by (2.4), in Problem 1, has decay rate greater or equal to γ if there exist matrices $Q = Q'$ and Y , where $Q \in \mathbb{R}^{n \times n}$ and $Y \in \mathbb{R}^{m \times n}$, such that:*

$$\begin{bmatrix} AQE' + EQA' + BYA' + AY'B' & EQ + BY \\ QE' + Y'B' & -\frac{Q}{2\gamma} \end{bmatrix} < 0, \quad (3.7)$$

$$Q > 0. \quad (3.8)$$

Furthermore, when (3.7) and (3.8) hold, then a state-derivative feedback matrix can be given by:

$$K = YQ^{-1}. \quad (3.9)$$

Proof. Stability corresponds to positive decay rate, $\gamma > 0$. One can use the quadratic Lyapunov function $V(x(t)) = x'(t)Px(t)$ to impose a lower bound on the decay rate with $\dot{V}(x(t)) < -2\gamma V(x(t))$, as described in [1]. Note that, from (2.4),

$$\begin{aligned} \dot{V}(x(t)) &= \dot{x}'(t)Px(t) + x'(t)P\dot{x}(t) \\ &= x'(t)A' \left[(E + BK)^{-1} \right]' Px(t) + x'(t)P(E + BK)^{-1} Ax(t). \end{aligned} \quad (3.10)$$

Then, from $\dot{V}(x(t)) < -2\gamma V(x(t))$ it follows that,

$$x'(t)A' \left[(E + BK)^{-1} \right]' Px(t) + x'(t)P(E + BK)^{-1} Ax(t) < -2\gamma x'(t)Px(t), \quad (3.11)$$

or

$$A'(P^{-1}E' + P^{-1}K'B')^{-1} + (EP^{-1} + BKP^{-1})^{-1}A < -2\gamma P. \quad (3.12)$$

After premultiplying by $(EP^{-1} + BKP^{-1})$ and posmultiplying by $(P^{-1}E' + P^{-1}K'B')$ in both sides of (3.12), observe that (3.12) holds if and only if

$$\begin{aligned} & (EP^{-1} + BKP^{-1})A' + A(P^{-1}E' + P^{-1}K'B') \\ & < (EP^{-1} + BKP^{-1})(-2\gamma P)(P^{-1}E' + P^{-1}K'B') \end{aligned} \quad (3.13)$$

and so

$$\begin{aligned} & - (EP^{-1} + BKP^{-1})A' - A(P^{-1}E' + P^{-1}K'B') \\ & - (-1)(EP^{-1} + BKP^{-1})(2\gamma P)(-1)(P^{-1}E' + P^{-1}K'B') > 0. \end{aligned} \quad (3.14)$$

Now, using the Schur complement [1], the equation above is equivalent to

$$\begin{bmatrix} -(EP^{-1} + BKP^{-1})A' - A(P^{-1}E' + P^{-1}K'B') & -(EP^{-1} + BKP^{-1}) \\ -(P^{-1}E' + P^{-1}K'B') & \frac{P^{-1}}{(2\gamma)} \end{bmatrix} > 0. \quad (3.15)$$

Therefore, defining $Q = P^{-1}$ and $Y = KP^{-1}$, then it follows the expression (3.7). If $P > 0$ then $Q > 0$, as specified in (3.8). So, when (3.7) and (3.8) hold, a state-derivative feedback matrix K is given by (3.9). \square

The next section shows that it is possible to extend the presented results, for the case where there exist polytopic uncertainties or structural failures in the plant. A fault-tolerant design is proposed.

4. Robust Stability Condition for State-Derivative Feedback

In this work, structural failure is defined as a permanent interruption of the system's ability to perform a required function under specified operating conditions [41]. Systems subject to structural failures can be described by uncertain polytopic systems.

Consider the linear time-invariant uncertain polytopic descriptor system, with or without structural failures, described as convex combinations of the polytope vertices:

$$\sum_{i=1}^{r_e} e_i E_i \dot{x}(t) = \sum_{j=1}^{r_a} a_j A_j x(t) + \sum_{k=1}^{r_b} b_k B_k u(t), \quad (4.1)$$

$$e_i \geq 0, \quad i = 1, \dots, r_e, \quad \sum_{i=1}^{r_e} e_i = 1,$$

$$a_j \geq 0, \quad j = 1, \dots, r_a, \quad \sum_{j=1}^{r_a} a_j = 1, \quad (4.2)$$

$$b_k \geq 0, \quad k = 1, \dots, r_b, \quad \sum_{k=1}^{r_b} b_k = 1,$$

where r_e, r_a , and r_b are the numbers of polytope vertices of E , A , and B , respectively. In (4.2), e_i, a_j , and b_k , are constant and unknown real numbers for all index i, j, k . The next theorem solves Problem 1, replacing system (2.1) by the uncertain system (4.1).

Theorem 4.1. *A sufficient condition for the solution of Problem 1 for the uncertain system (4.1) is the existence of matrices $Q = Q'$ and Y , where $Q \in \mathbb{R}^{n \times n}$ and $Y \in \mathbb{R}^{m \times n}$, such that,*

$$A_j Q E_i' + E_i Q A_j' + B_k Y A_j' + A_j Y' B_k' < 0, \quad (4.3)$$

$$Q > 0, \quad (4.4)$$

where $i = 1, 2, \dots, r_e, j = 1, 2, \dots, r_a$, and $k = 1, 2, \dots, r_b$. Furthermore, when (4.3) and (4.4) hold, then a state-derivative feedback matrix can be given by,

$$K = YQ^{-1}. \quad (4.5)$$

Proof. From (4.2) and (4.3) it follows that

$$\begin{aligned} & \sum_{i=1}^{r_e} e_i \sum_{j=1}^{r_a} a_j \sum_{k=1}^{r_b} b_k \left[A_j Q E_i' + E_i Q A_j' + B_k Y A_j' + A_j Y' B_k' \right] \\ &= \left(\sum_{j=1}^{r_a} a_j A_j \right) Q \left(\sum_{i=1}^{r_e} e_i E_i' \right) + \left(\sum_{i=1}^{r_e} e_i E_i \right) Q \left(\sum_{j=1}^{r_a} a_j A_j' \right) \\ &+ \left(\sum_{k=1}^{r_b} b_k B_k \right) Y \left(\sum_{j=1}^{r_a} a_j A_j' \right) + \left(\sum_{j=1}^{r_a} a_j A_j \right) Y' \left(\sum_{k=1}^{r_b} b_k B_k' \right) < 0. \end{aligned} \quad (4.6)$$

Therefore, condition (3.1) of Theorem 3.1 holds for the uncertain system (4.1), where $E = e_1 E_1 + \dots + e_{r_e} E_{r_e}$, $A = a_1 A_1 + \dots + a_{r_a} A_{r_a}$, and $B = b_1 B_1 + \dots + b_{r_b} B_{r_b}$. Now, conditions (4.4) and (4.5) are equivalent to conditions (3.2) and (3.3). Finally, from Theorem 3.1, the existence of matrices $Q = Q'$ and Y such that (4.3) and (4.4) hold is a sufficient condition for the solution of Problem 1. \square

Theorem 4.2. *A sufficient condition for the decay rate of the robust closed-loop system given by (2.2) and (4.1) to be greater or equal to γ is the existence of matrices $Q = Q'$ and Y , $Q \in \mathbb{R}^{n \times n}$, $Y \in \mathbb{R}^{m \times n}$, such that:*

$$\begin{bmatrix} A_j Q E'_i + E_i Q A'_j + B_k Y A'_j + A_j Y' B'_k & E_i Q + B_k Y \\ Q E'_i + Y' B'_k & -\frac{Q}{2\gamma} \end{bmatrix} < 0, \quad \forall i, j, \quad (4.7)$$

$$Q > 0.$$

Furthermore, when (4.7) hold, then a robust state-derivative feedback matrix can be given by

$$K = YQ^{-1}. \quad (4.8)$$

Proof. It follows directly from the proofs of Theorems 3.3 and 4.1. \square

Due to limitations imposed in the practical applications of control systems, many times it should be considered output constraints in the design.

5. Bounds on Output Peak

Consider that the output of the system (2.1) is given by

$$y(t) = Cx(t), \quad (5.1)$$

where $y(t) \in \mathbb{R}^p$ and $C \in \mathbb{R}^{p \times n}$. Assume that the initial condition of (2.1) and (5.1) is $x(0)$. If the feedback system (2.1), (2.2), and (5.1) is asymptotically stable, one can specify bounds on output peak as described in:

$$\max \|y(t)\|_2 = \max \sqrt{y'(t)y(t)} < \xi_0, \quad (5.2)$$

for $t \geq 0$, where ξ_0 is a known positive constant. From [1], (5.2) is satisfied when the following LMI holds:

$$\begin{bmatrix} 1 & x(0)' \\ x(0) & Q \end{bmatrix} > 0, \quad (5.3)$$

$$\begin{bmatrix} Q & QC' \\ CQ & \xi_0^2 I \end{bmatrix} > 0,$$

and the LMI that guarantees stability (Theorem 3.1 or Theorem 4.1), or stability and decay rate (Theorem 3.3 or Theorem 4.2).

An interesting method for specification of bounds on the state-derivative feedback matrix K was recently proposed in [33]. The result is presented below.

Lemma 5.1. *Given a constant $\mu_0 > 0$, then the specification of bounds on the state-derivative feedback matrix K can be described finding the minimum of β , $\beta > 0$, such that $KK' < \beta I / \mu_0^2$. The optimal value of β can be obtained by the solution of the following optimization problem:*

$$\begin{aligned} \min & \beta \\ \text{s.t.} & \begin{bmatrix} \beta I & Y \\ Y' & I \end{bmatrix} > 0, \\ & Q > \mu_0 I, \\ & (\text{Set of LMI}), \end{aligned} \tag{5.4}$$

where the Set of LMI can be equal to (3.1), (3.2) or (3.7), (3.8) or (4.3), (4.4) or (4.7), with or without the LMI (5.3).

Proof. See [33]. □

In the following section, Example 6.1 illustrates the efficiency of this optimization procedure that can reduce the practical difficulties in the implementation of the controllers.

6. Examples

The effectiveness of the proposed LMI designs is demonstrated by simulation results.

Example 6.1. A simple electrical circuit, can be represented by the linear descriptor system below [25]:

$$\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t), \tag{6.1}$$

where x_1 is the current and the x_2 is the potential of the capacitor.

Suppose the output of the system is given by $y(t) = x_1$. So it is a Single-Input/Single-Output (SISO) system, with $n = 2$, $m = 1$ and $p = 1$. Consider as specification an output peak bound $\xi_0 = 10$ and an initial condition equal to $x(0) = [1 \ 0]'$. Then, using the package "LMI control toolbox" from MATLAB [11] to solve the LMI (3.1) and (3.2) from Theorem 3.1, and (5.3), one feasible solution was obtained

$$\begin{aligned} Q &= \begin{bmatrix} 59.366 & -16.491 \\ -16.491 & 98.944 \end{bmatrix}, \\ Y &= [-98.944 \ -49.472]. \end{aligned} \tag{6.2}$$

A state-derivative feedback matrix was calculated using (3.3)

$$K = [-1.8932 \ -0.81553]. \tag{6.3}$$

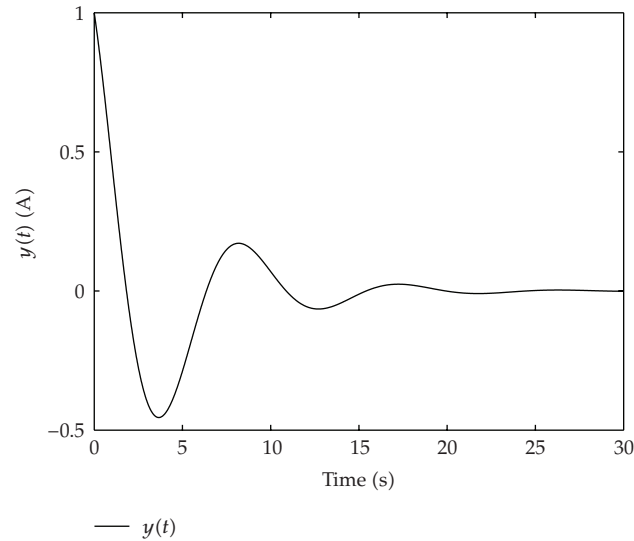


Figure 1: The response of the signal $y(t)$ of the controlled system (2.4).

Note that, as discussed before, the obtained solution K is such that $\det(E + BK) \neq 0$ (it is equal to 1.8932).

For the initial condition $x(0)$ given above, the simulation results of the controlled system are presented in Figure 1. From Figure 1, the settling time of the controlled system is approximately 25 seconds and $\max \sqrt{y'(t)y(t)}$ is equal to $1 < \xi_0 = 10$. The specification for the controlled system was satisfied using the designed controller. Note by Figure 1 that only the stability of the controlled system can be insufficient to obtain a suitable performance. Specifying a lower bound for the decay rate equal $\gamma = 2$, to obtain a faster transient response and using the LMI (3.7) and (3.8) from Theorem 3.3, and (5.3) from Section 5, one feasible solution was obtained

$$Q = \begin{bmatrix} 90.071 & -22.22 \\ -22.22 & 10.662 \end{bmatrix}, \quad (6.4)$$

$$Y = [5.4955 \quad -3.8158].$$

A state-derivative feedback matrix was calculated using (3.9)

$$K = [-0.056149 \quad -0.47492]. \quad (6.5)$$

For the solution (6.5) one has $\det(I + BK) = 0.056149$, and the simulation result of the controlled system for the same initial condition $x(0)$, is presented in Figure 2. Note that in Figure 2, the settling time was approximately equal to 1 second and $\max \sqrt{y'(t)y(t)}$ is equal to $1 < \xi_0 = 10$. Then, the specifications were satisfied by using the designed controller.

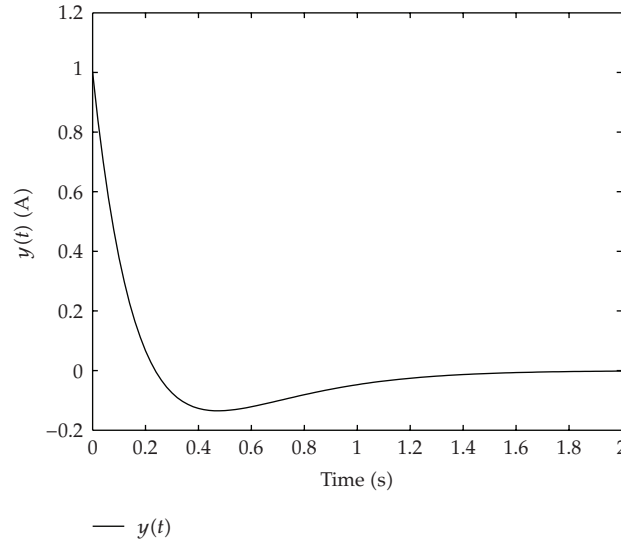


Figure 2: The response of the signal $y(t)$ of the controlled system (2.4), with bound on the decay rate.

Table 1

Stability	Stability with decay rate ($\gamma = 2$)
$Q = \begin{bmatrix} 99.846 & -5.7193 \\ -5.7193 & 1.3314 \end{bmatrix},$	$Q = \begin{bmatrix} 99.9 & -5.8266 \\ -5.8266 & 1.3436 \end{bmatrix},$
$Y = \begin{bmatrix} -0.0088332 & -0.076523 \end{bmatrix},$	$Y = \begin{bmatrix} -0.0096954 & -0.091559 \end{bmatrix},$
$K = \begin{bmatrix} -0.004484 & -0.076735 \end{bmatrix},$	$K = \begin{bmatrix} -0.0054497 & -0.091774 \end{bmatrix},$
$\beta = 0.005934.$	$\beta = 0.0084843.$

To facilitate the implementation of the controller, the specification of bounds on the state-derivative feedback matrix K can be done using the optimization procedure stated in Lemma 5.1, with $\mu_0 = 1$. The optimal values, obtained with the “LMI control toolbox” are given in Table 1.

Note that the absolute values of the entries of K are smaller than the obtained without optimization method, given in (6.3) and (6.5), respectively.

This procedure can also be applied to the control design of uncertain systems subject to failures.

Example 6.2. Consider the linear uncertain descriptor system represented by matrices:

$$E = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & e_{33} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad A = \begin{bmatrix} a_{11} & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 0 & 1 & 0 & -2 \end{bmatrix}, \quad (6.6)$$

where $0.8 \leq e_{33} \leq 1.2$ and $5.4 \leq a_{11} \leq 6.4$.

A fail in the actuator is described by:

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & b_{32} \\ 0 & 1 \end{bmatrix}, \quad (6.7)$$

where $b_{32} = 1$ without fail, or $b_{32} = 0$ with fail of the actuator. Then, the vertices of the polytope are given by triple: $(E_i, A_j, B_k) = \{(E_1, A_1, B_1), (E_1, A_1, B_2), (E_1, A_2, B_1), (E_1, A_2, B_2), (E_2, A_1, B_1), (E_2, A_1, B_2), (E_2, A_2, B_1), (E_2, A_2, B_2)\}$, where

$$\begin{aligned} E_1 &= \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0.8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, & E_2 &= \begin{bmatrix} 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 1.2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \\ A_1 &= \begin{bmatrix} 5.4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 0 & 1 & 0 & -2 \end{bmatrix}, & A_2 &= \begin{bmatrix} 6.4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ -1 & 0 & 3 & 0 \\ 0 & 1 & 0 & -2 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, & B_2 &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}. \end{aligned} \quad (6.8)$$

And the example was solved considering stability with decay rate. It was specified a lower bound for the decay rate equal to $\gamma = 2$, an output peak bound $\xi_0 = 10$, and an initial condition $x(0) = [0.3 \ 0.1 \ 0 \ 0]^T$. Using LMI control toolbox for solving the set of LMI (4.7) from Theorem 4.2 with (5.3), a feasible solution was the following:

$$\begin{aligned} Q &= \begin{bmatrix} 21.496 & -1.7143 & -24.031 & 5.9229 \\ -1.7143 & 5.2937 & -1.282 & -20.904 \\ -24.031 & -1.282 & 75.634 & 5.0044 \\ 5.9229 & -20.904 & 5.0044 & 268.7 \end{bmatrix}, \\ Y &= \begin{bmatrix} 43.512 & 3.359 & -135.59 & -7.6619 \\ 2.3436 & -7.8481 & 2.1942 & 18.933 \end{bmatrix}. \end{aligned} \quad (6.9)$$

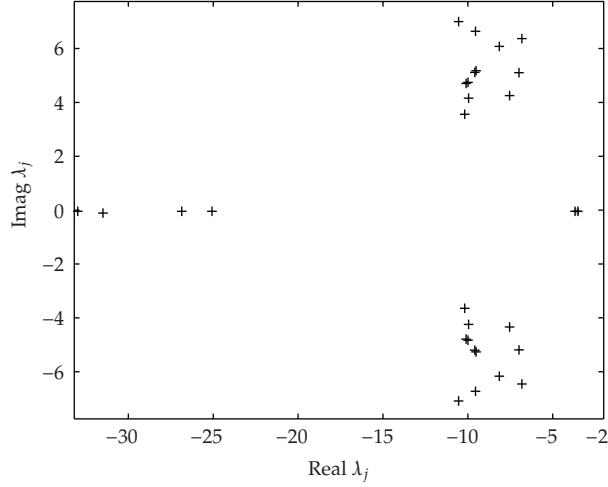


Figure 3: The eigenvalues location of the vertices from robust controlled uncertain system (4.1) and (2.2), subject to failures.

A robust state-derivative feedback matrix is obtained using (4.8)

$$K = \begin{bmatrix} 0.068019 & 0.34647 & -1.7672 & 0.029854 \\ -0.012215 & -1.7426 & -1.1708 \times 10^{-4} & -0.064834 \end{bmatrix}. \quad (6.10)$$

The locations in the s -plane of the eigenvalues, for the vertices (E_i, A_j, B_k) , of the robust controlled system, are plotted in Figure 3. There exist eight vertices, and four eigenvalues for each vertex.

Considering that the output system is

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad (6.11)$$

the responses of the controlled system with parameter $e_{33} = 0.8$, and $a_{11} = 6.4$ for uncertain matrices E and A respectively, are showed in Figure 4. Note that with (*dotted line*) or without (*solid line*) fail of the actuator the controlled system has fast transient responses.

Now, solving the optimization procedure stated in Lemma 5.1, with LMI (4.7), (5.3), and $\mu_0 = 1$, the optimal values, obtained with the “LMI control toolbox” were the following:

$$Q = \begin{bmatrix} 2.166 & -0.79427 & -1.9412 & 4.435 \\ -0.79427 & 1.7839 & 0.4143 & -10.039 \\ -1.9412 & 0.4143 & 7.6281 & 0.75171 \\ 4.435 & -10.039 & 0.75171 & 7.0245 \times 10^6 \end{bmatrix}, \quad (6.12)$$

$$Y = \begin{bmatrix} 3.4619 & -0.22593 & -12.759 & -1.4344 \\ 0.98266 & -2.662 & -1.5705 & 10.009 \end{bmatrix},$$

$$\beta = 178.26,$$

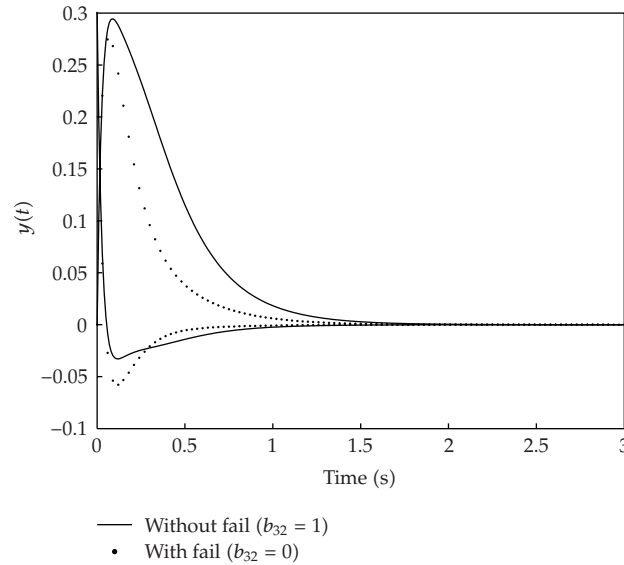


Figure 4: The response of the signal $y(t)$ of the controlled system, with and without fail of the actuator.

$$K = \begin{bmatrix} 0.28356 & 0.37604 & -1.6208 & 3.2763 \times 10^{-7} \\ -0.3028 & -1.5813 & -0.19705 & -6.2275 \times 10^{-7} \end{bmatrix}. \quad (6.13)$$

Note that some absolute values of the entries of K in (6.13) are greater than the obtained in first design, given in (6.10). However, the norm of matrix K obtained in first design is $\|K\| = 1.939$ and one obtained from optimization procedure is $\|K\| = 1.7655$. Therefore the optimization procedure was able to control problem with a smaller norm of the state-derivative feedback matrix K .

7. Conclusions

Necessary and sufficient stability conditions based on LMI for state-derivative feedback of linear descriptor systems, were proposed. We can include in the LMI-based control design, the specification of the decay rate, bounds on output peak, and bound on the state-derivative feedback matrix K . The plant can be linear time-invariant SISO or MIMO, and can also have polytopic uncertainties in its parameters or be subject to structural failures. In this case, one obtains a fault-tolerant design. Therefore, the new design methods allow a broader class of plants and performance specifications, than the related results available in the literature, for instance in [19, 25, 39]. The proposed methods are LMI-based designs that, when feasible, can be efficiently solved by convex programming techniques. Theoretical analysis and numerical simulations illustrate these results.

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