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Research Article

Less Conservative $\mathscr{H}_\infty$ Fuzzy Control for Discrete-Time Takagi-Sugeno Systems

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New analysis and control design conditions of discrete-time fuzzy systems are proposed. Using fuzzy Lyapunov's functions and introducing slack variables, less conservative conditions are obtained. The controller guarantees system stabilization and $\mathscr{H}_\infty$ performance. Numerical tests and a practical experiment in Chua's circuit are presented to show the effectiveness.

1. Introduction

Model-based fuzzy control is a widespread approach to deal with complex nonlinear dynamics [1]. Within this context, Takagi-Sugeno (TS) fuzzy model [2] is a landmark. It consists on fuzzy rules describing global (semiglobal) dynamics as linear models (locally valid) interpolated by membership functions.

From the modeling point of view, TS systems are known to be universal approximators [3, 4] and to possess a reduced number of rules, when compared with other fuzzy models [5]. Another interesting feature is the existence of a systematic procedure to obtain TS models from the nonlinear system equations, namely, the sector nonlinearity approach [3]. An advantage for control purposes is the possibility to use the Lyapunov stability theory and, simultaneously, to rely on tools from linear systems theory [3].

Methodologies based on Lyapunov's functions provide a straightforward way to describe stability and control design issues of TS systems by means of linear matrix inequalities (LMIs) [6], of which the solutions can be computed in polynomial-time by convex optimization techniques. There are several approaches to design fuzzy controllers that, besides stability, also guarantee some type of performance for the closed-loop nonlinear system as $\mathfrak{D}$-stability regions [7], constraints over input/output signals [3], and performance indexes, such as $\mathscr{H}_\infty$ and $\mathscr{H}_2$ norms [8–13].

In TS-based control, Lyapunov's function candidates are classified in three categories [1]: the common quadratic Lyapunov function (CQLF), the piecewise Lyapunov function (PLF), and the fuzzy Lyapunov function (FLF). Most efforts deal with sufficient conditions for the existence of a CQLF [3], a single quadratic function that guarantee stability for all fuzzy subsystems. However, as the number of rules increases, the CQLF turns out to be very conservative.

To keep obtaining solutions with the CQLF, many techniques were developed. Usually the underlaying strategy among them (see [14–16] and references therein) is the use of quadratic form relaxations (right-hand side slack matrix variables) into the LMI formulation. Extravariables improve the numeric behavior of LMI solvers, at the cost of higher computational time. Recently, sufficient and necessary conditions for the existence of CQLF were discussed in [17–19], reaching the limits of quadratic stability.

Nonetheless, a CQLF might not exist even for a stable TS system, as demonstrated in [20]. An well-established alternative to overcome this problem is the PLF [20–22], which consists of a finite combination of disjoint common quadratic Lyapunov's functions, each of them valid only into a compact domain.

The PLF is suitable whenever the TS model is not activating at each time the whole set of its linear models. Nevertheless, this assumption does not hold for many TS models. Another drawback is how to assess the behavior of the PLF at the boundaries of the partitions. Solutions range from considering boundary conditions [21] to introduce extra LMI constraints that guarantee continuity of the function across boundaries [20] and to use some methods ensuring that the function decreases when leaving one subspace for another closer to the equilibrium, relaxing the continuity assumption [22].

More recently, approaches based on the FLF [9, 23] were developed. The FLF is a fuzzy blending of multiple quadratic functions, in the same way the TS model is constructed. Unlike the PLFs, continuity is an inherent feature of the FLF. Furthermore, the quadratic functions used to construct an FLF do not need to be Lyapunov's functions by themselves, just its fuzzy combination does.

Thus, the FLF has been attracting much attention. For continuous-time fuzzy systems (CFS), recent results are given in [24–31]. As to discrete-time fuzzy systems (DFS) see [12, 13, 32, 33].

The $\mathscr{H}_\infty$ index is an adequate criterion to control and filter design in nonlinear systems with exogenous inputs with unknown spectral density and bounded energy (see [11, 34–37] and references therein for examples of practical applications) and computes the greatest ratio between the system output and the noisy input. Conservative conditions may not give a good tradeoff between disturbance attenuation level and a feasible controller, so the FLF is an interesting candidate [12, 13, 33].

This paper presents new sufficient conditions to $\mathscr{H}_\infty$ control for DFS in the TS form. To promote these improved conditions, three different strategies are employed. First, an FLF is adopted, since recent works reveal that this type of function is less conservative than the CQLF for TS systems. Then, a series of matrix transformations [38–40] are performed, allowing to obtain LMIs in which the controller gains are not directly dependent on the Lyapunov matrices, introducing extradegree of freedom for the optimization problem. Finally, [15] provides a successful approach to introduce slack matrix variables into the stabilization control, enhancing the numerical behavior of LMI solvers. This strategy was further extended to $\mathscr{H}_\infty$ control in [11] and is used in this paper. Numeric examples illustrate the improvement provided by merging these approaches. Smaller attenuation levels are achieved, and solutions are computed even for systems in which some other methods fail. A practical experiment using the Chuas's chaotic oscillator [41] is given to show the performance of the proposed methodology.

Notation 1. The notation is standard. Transpose of vectors and matrices are indicated by the superscript ($'$); the symbol ($\bullet$) denotes transposed terms in symmetric matrices; the sets $\{1,2,\ldots,r\}$ and $\{1,2,\ldots,s\}$ are indicated by $\mathcal{R}$ and $\mathcal{S}$, respectively; l_2 is the discrete Lebesgue space; $\|\cdot\|_2$ is the l_2 norm.

2. Preliminaries on TS Systems

Consider nonlinear discrete-time systems that can be expressed as Takagi-Sugeno (TS) fuzzy models [2], according to the following fuzzy rules:

$$R_i : \begin{cases} \text{IF} & q_k^1 \text{ IS } \mathcal{M}_1^i \text{ AND} \cdots \text{AND } q_k^s \text{ IS } \mathcal{M}_s^i, \\ & x_{k+1} = A_i x_k + B_i u_k + E_i w_k, \\ \text{THEN} & \\ & z_k = C_i x_k + D_i u + k + F_i w_k, \end{cases} \tag{2.1}$$

where $R_i,\ i \in \mathcal{R}$, denotes the ith fuzzy inference rule. In rule R_i, the fuzzy sets are given by $\mathcal{M}_j^i,\ j \in \mathcal{S}$; q_k^j are the premisse variables at instant k. $q_k \in \mathbb{R}^s$ is the premisse variables vector, stacking the premisse variables; $x_k \in \mathbb{R}^n$ is the state vector; $u_k \in \mathbb{R}^m$ is the control signal; $w_k \in \mathbb{R}^p$ is a disturbance input, belonging l_2; $z_k \in \mathbb{R}^q$ is the regulated output. A_i, B_i, C_i, D_i, E_i, and F_i are the local matrices of proper dimensions.

By using a standard fuzzy inference method, that is, using a singleton fuzzifier, product fuzzy inference, and center-average defuzzifier, the global inferred TS model is given by [1, 3]

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} h_i[q_k](A_i x_k + B_i u_k + E_i w_k), \\ z_k &= \sum_{i=1}^{r} h_i[q_k](C_i x_k + D_i u_k + F_i w_k), \end{aligned} \tag{2.2}$$

where $h_i[q_k]$ are weighting functions, denoting the normalized grade of membership of each rule, which satisfy

$$h_i[q_k] \geq 0 \quad i \in \mathcal{R}, \qquad \sum_{i=1}^{r} h_i[q_k] = 1. \tag{2.3}$$

To avoid clutter, assume $h_i[q_k] \triangleq h_i$ and $h_i[q_{k+1}] \triangleq h_i^+$.

2.1. Parallel Distributed Compensation

The control law adopted is given by the parallel distributed compensation (PDC) [3], where the control signal consists in a fuzzy combination of linear-state feedbacks, likewise the TS model

$$R_i : \begin{cases} \text{IF} & q_k^1 \text{ IS } \mathcal{M}_1^i \text{ AND} \cdots \text{AND } q_k^s \text{ IS } \mathcal{M}_s^i, \\ \text{THEN} & u_k = K_i x_k, \end{cases} \tag{2.4}$$

where K_i are the local gains.

Thus, the inferred nonlinear controller is given by

$$u_k = \sum_{i=1}^{r} h_i[q_k] K_i x_k, \tag{2.5}$$

where $h_i[q_k]$ are the same membership functions of (2.2).

Taking (2.5) into account, the closed-loop description for (2.2) is given by

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left(G_{ij} x_k + E_i w_k\right), \\ z_k &= \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \left(J_{ij} x_k + F_i w_k\right), \end{aligned} \tag{2.6}$$

where $G_{ij} \triangleq A_i + B_i K_j$ and $J_{ij} \triangleq C_i + D_i K_j$.

2.2. Fuzzy Lyapunov's Function

In order to obtain less conservative conditions, the following fuzzy Lyapunov function (FLF) [9] is adopted:

$$V_k = x'_k \left(\sum_{i=1}^{r} h_i P_i \right) x_k, \tag{2.7}$$

sharing the same membership functions of (2.6). It is interesting to note (see the discussion on Section II.B in [23]) that the locally valid functions $V_k = x'_k P_i x_k$ do not need to be Lyapunov's functions by themselves. Only its combination does, namely (2.7), representing an advantage when compared to the CQLF and the PLF.

3. Less Conservative Conditions

In this section, new sufficient conditions for the stabilization and analysis of (2.6) are developed.

3.1. $\mathscr{H}_\infty$ Performance

In addition to stabilization, the designed controller must attenuate exogenous entries into the regulated output. There are several ways to quantify the effect of w_k on z_k. In this paper, the $\mathscr{H}_\infty$-norm is adopted

$$\sup_{w_k} \frac{\|z\|_2}{\|w\|_2} \leq \gamma, \quad \forall z_k \neq 0. \tag{3.1}$$

The $\mathscr{H}_\infty$-norm computes the l_2-gain between the noisy input and the system output. In other words, the greatest ratio between the output energy and the exogenous input energy, being suitable when no information about the spectral density of the disturbances is known *a priori* [42].

There are two usual ways to apply the $\mathscr{H}_\infty$ performance: one goal is to find a controller that guarantees $\gamma_{\min}$, the minimum value of the $\mathscr{H}_\infty$-norm; on the other hand, it is possible to obtain a controller that stabilizes the system with a prescribed value γ or the $\mathscr{H}_\infty$-norm. In the following sections, both cases are addressed.

3.2. Control Design

The main result of this paper can be stated in the following theorem that provides a sufficient condition to obtain the gains of the fuzzy controller (2.5) that stabilizes the TS system (2.6) with the minimum guaranteed cost $\gamma_{\min}$.

Theorem 3.1. *The fuzzy controller* (2.5) *stabilizes the TS system in* (2.6)*, with a $\mathscr{H}_\infty$ guaranteed cost given by $\gamma_{\min} = \sqrt{\delta}$, if there exist symmetric matrices X_i, R_{ij}, T_{ijt}, and any matrices L, M_i, and S_{ijt} satisfying the following optimization problem:*

$$\begin{aligned} &\min_{\substack{X_i,R_{ij},T_{ijt},\\ L,M_i,S_{ijt},\delta}} \quad \delta \\ &\text{s.t.} \quad \Xi_t \prec 0, \quad X_i \succ 0, \quad (i,t \in \mathcal{R}), \\ &\qquad\ T_{ijt} \succeq 0, \quad (i > j,\ i,j,t \in \mathcal{R}), \end{aligned} \tag{3.2}$$

where

$$\Xi_t \triangleq \begin{bmatrix} Q_{11t} - Z_{1t} & \bullet & \cdots & \bullet \\ Q_{21t} + W_{21t} & Q_{22t} - Z_{2t} & \cdots & \bullet \\ \vdots & \vdots & \ddots & \vdots \\ Q_{r1t} + W_{r1t} & Q_{r2t} + W_{r2t} & \cdots & Q_{rrt} - Z_{rt} \end{bmatrix}, \tag{3.3}$$

$$Q_{ijt} \triangleq \begin{bmatrix} \frac{1}{2}(\Gamma_i + \Gamma_j) & \bullet & \bullet & \bullet \\ 0 & -\delta I & \bullet & \bullet \\ \frac{1}{2}(\Psi_{ij} + \Psi_{ji}) & \frac{1}{2}(E_i + E_j) & -X_t & \bullet \\ \frac{1}{2}(\Phi_{ij} + \Phi_{ji}) & \frac{1}{2}(F_i + F_j) & 0 & -I \end{bmatrix}, \tag{3.4}$$

$$\Gamma_i \triangleq X_i - L - L', \qquad W_{ijt} \triangleq V_{ijt} + T_{ijt} + S_{ijt} - S'_{ijt}, \tag{3.5}$$

$$\Psi_{ij} \triangleq A_i L + B_i M_j, \qquad \Phi_{ij} \triangleq C_i L + D_i M_j, \tag{3.6}$$

$$V_{ijt} = \begin{cases} \frac{1}{2} R_{ij}, & \text{if } i = t \text{ or } j = t, \\ 0, & \text{otherwise}, \end{cases} \qquad Z_{it} = \begin{cases} R_{it}, & \text{if } i < t, \\ R_{ti}, & \text{if } i > t, \\ 0, & \text{if } i = t. \end{cases} \tag{3.7}$$

Furthermore, the local gains are given by $K_i \triangleq M_i L^{-1}$.

Proof. The proof of Theorem 3.1 is given in the appendix. □

It is also desirable to design a stabilizing controller that provides a specific $\mathscr{H}_\infty$ guaranteed cost. In this case, the scalar γ should be provided beforehand and the next theorem should be applied.

Theorem 3.2. *Let $\delta > 0$ be a scalar given. The fuzzy controller* (2.5) *stabilizes the TS system in* (2.6), *with a prescribed $\mathscr{H}_\infty$ guaranteed cost given by $\gamma = \sqrt{\delta}$, if there exist symmetric matrices X_i, R_{ij}, T_{ijt}, and any matrices L, M_i, and S_{ijt} satisfying the following LMIs:*

$$\Xi_t \prec 0, \quad X_i \succ 0, \quad T_{ijt} \succeq 0, \quad (i > j,\ i, j, t \in \mathcal{R}), \tag{3.8}$$

where Ξ_t is given as in (3.3). *Furthermore, the local gains are given by $K_i \triangleq M_i L^{-1}$.*

Proof. See the appendix. □

3.3. Analysis

Another problem is the analysis of fuzzy system when the control gains are already given. To search for $\gamma_{\min}$ provided by a given controller, the next theorem should be used.

Theorem 3.3. *Let the gains K_i be given. The fuzzy controller* (2.5) *stabilizes the TS system in* (2.6), *with a $\mathscr{H}_\infty$ guaranteed cost given by $\gamma_{\min} = \sqrt{\delta}$ if there exist symmetric matrices X_i, R_{ij}, T_{ijt}, and any matrices L, and S_{ijt} satisfying the following optimization problem:*

$$\begin{aligned} \min_{\substack{X_i, R_{ij}, T_{ijt}, \\ L, S_{ijt}, \delta}} \quad & \delta \\ \text{s.t.} \quad & \Xi_t \prec 0, \quad X_i \succ 0, \quad (i, t \in \mathcal{R}), \\ & T_{ijt} \succeq 0, \quad (i > j,\ i, j, t \in \mathcal{R}), \end{aligned} \tag{3.9}$$

where Ξ_t is given as in (3.3) *and $\Psi_{ij} \triangleq (A_i + B_i K_j) L$ and $\Phi_{ij} \triangleq (C_i + D_i K_j) L$.*

Proof. See the appendix. □

Finally, it is possible to check by the next theorem if a given controller guarantees a prescribed $\mathscr{H}_\infty$ performance γ.

Theorem 3.4. *Let the scalar $\delta > 0$ and the gains K_i be given. The fuzzy controller* (2.5) *stabilizes the TS system in* (2.6)*, with a prescribed $\mathscr{H}_\infty$ guaranteed cost given by $\gamma = \sqrt{\delta}$ if there exist symmetric matrices X_i, R_{ij}, T_{ijt}, and any matrices L, and S_{ijt} satisfying the following LMIs:*

$$\Xi_t \prec 0, \quad X_i \succ 0, \quad T_{ijt} \succeq 0, \quad (i > j,\ i, j, t \in \mathcal{R}), \tag{3.10}$$

where Ξ_t is given as in (3.3) *and $\Psi_{ij} \triangleq (A_i + B_i K_j)L$ and $\Phi_{ij} \triangleq (C_i + D_i K_j)L$.*

Proof. See the appendix. □

4. Numeric Results

In this section, numerical examples illustrate the performance of the proposed approach in comparison to some methods presented in the literature. The tests were performed using SeDuMi [43] together with Yalmip [44] in Matlab 7.4.0.

Example 4.1. The following example is borrowed from [10]. Consider DFS as in (2.2) which the premisse variable is x_k^1. The local matrices are

$$\begin{gathered} A_1 = \begin{bmatrix} 1+a & -0.5 \\ 1 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} -1 & -0.5 \\ 1 & 0 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 1 \\ 1-b \end{bmatrix}, \qquad B_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix}, \qquad E_1 = \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0.5 \\ -0.1 \end{bmatrix}, \\ C_1 = \begin{bmatrix} 1 \\ 0.5 \end{bmatrix}', \qquad C_2 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}', \qquad \begin{matrix} D_1 = 1, & D_2 = 0.5, \\ F_1 = 0.4, & F_2 = 0.2. \end{matrix} \end{gathered} \tag{4.1}$$

The membership functions for this system are

$$h_1\left[x_k^1\right] = \frac{\left(1 - \sin\left(x_k^1\right)\right)}{2}, \qquad h_2\left[x_k^1\right] = \frac{\left(1 + \sin\left(x_k^1\right)\right)}{2}. \tag{4.2}$$

Let a PDC controller (2.5) be given with the following gains:

$$K_1 = \begin{bmatrix} -0.65 & 0.30 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} -0.87 & 0.11 \end{bmatrix}. \tag{4.3}$$

In this example, the objective is to determine the minimum $\mathscr{H}_\infty$ guaranteed cost $\gamma_{\min}$ achieved when the PDC controller (4.3) is employed. Table 1 shows the results obtained considering $b = 0$ and different fixed values for the parameter a. Note that the lower values of $\gamma_{\min}$ are found using Theorem 3.3.

Table 1: Comparison of $\gamma_{\min}$ by different strategies.

a	0.0	0.2	0.4	0.6
γ-[8, Theorem 2]	1.8141	2.0403	2.6983	9.6492
γ-[11, Theorem 1]	1.8107	2.0361	2.5716	5.9496
γ-[10, Theorem 1]	1.3244	1.5277	2.0395	5.5236
γ-Theorem 3.3	**1.3231**	**1.4667**	**1.8400**	**5.2001**

Table 2: $\gamma_{\min}$ computed for several values of β.

β	[13, Theorem 3]	[13, Theorem 4]	[11, Theorem 1]	Theorem 3.1
0.01000	**0.0167**	**0.0167**	**0.0167**	**0.0167**
0.10000	**0.0168**	0.0169	**0.0168**	**0.0168**
0.50000	0.0175	0.0192	0.0178	**0.0174**
1.00000	0.0200	0.3322	0.0269	**0.0200**
1.01459	0.0202	99.949	0.0277	**0.0202**
1.43200	8.7340	—	—	**0.0308**
1.45000	—	—	—	**0.0318**
1.50000	—	—	—	**0.0352**
1.75000	—	—	—	**0.1108**
1.80000	—	—	—	**0.2319**
1.84320	—	—	—	**10.153**
1.85000	—	—	—	**—**

Example 4.2. Consider another DFS with x_k^1 as premisse variable. The membership functions and local matrices are

$$
\begin{gathered}
h_1\left[x_k^1\right] = \frac{x_k^1 + \beta}{2\beta}, \qquad h_2\left[x_k^1\right] = 1 - h_1\left[x_k^1\right], \\
A_1 = \begin{bmatrix} 1 & -\beta \\ -1 & -0.5 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 1 & \beta \\ -1 & -0.5 \end{bmatrix}, \\
B_1 = \begin{bmatrix} 5+\beta \\ 2\beta \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 5-\beta \\ -2\beta \end{bmatrix}, \\
E_1 = E_2 = \begin{bmatrix} -0.03 & 0.01 \\ 0 & 0.01 \end{bmatrix}, \qquad C_1 = C_2 = \begin{bmatrix} -0.1 \\ -0.05 \end{bmatrix}', \\
D_1 = D_2 = 0.5, \qquad F_1 = F_2 = \begin{bmatrix} 0.01 & 0.01 \end{bmatrix}.
\end{gathered}
\tag{4.4}
$$

In this example, the controllers are designed to provide the minimum value of the $\mathcal{H}_\infty$ guaranteed cost, $\gamma_{\min}$. Table 2 summarizes $\gamma_{\min}$ using different approaches for several fixed values of the parameter β compared with Theorem 3.1. Note that for all strategies, $\gamma_{\min}$ increases as β increases. For $|\beta| < 0.5$ all approaches find very similar values for γ. When β is close to 1, the performance of [13, Theorem 4] deteriorates, whereas the remaining approaches calculate low values for γ. When β gets closer to 1.45, only the proposed approach and [13, Theorem 3] are feasible. For $\beta \geq 1.45$, only the proposed approach remains feasible.

Table 3: Gains obtained by Theorem 3.1 considering $\gamma = 0.5$.

Gain	Value
K_1	$[-1.0062\ \ -0.3178]$
K_2	$[-0.9730\ \ -0.2076]$

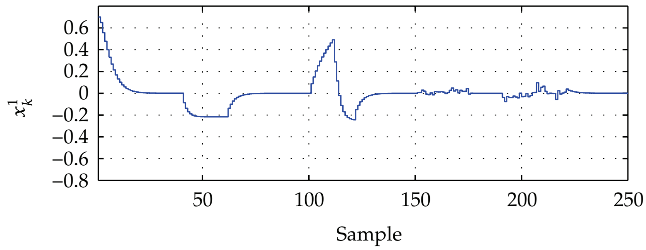

Figure 1: Closed-loop system, x_k^1 trajectory.

Clearly the proposed approach always provides the best signal attenuation since smaller values of γ are found, as shown in boldface in Table 2.

Example 4.3. Consider the same DFS described in Example 4.1. Theorem 3.2 is applied in order to design a controller that guarantees an attenuation level $\gamma = 0.5$ when $b = 0$ and $a = 0.5$. The gains obtained are given in Table 3.

Simulations were performed to show the response of the TS system under the designed PDC controller. A total of 250 iterations were considered and the disturbance signal applied is as follows:

$$w_k = \begin{cases} -0.25 & 40 \leq k \leq 60, \\ 0.25 & 100 \leq k \leq 110, \\ -0.30 & 110 < k \leq 120, \\ e_k^1 & 150 \leq k \leq 175, \\ e_k^2 & 190 \leq k \leq 220, \end{cases} \tag{4.5}$$

where e_k^1, e_k^2 are gaussian noises, with zero mean and standard deviations 0.05 and 0.10, respectively.

Assuming as initial conditions $x_0 = [0.7\ \ -0.2]'$ the trajectory described by the controlled systems is depicted in Figures 1 and 2. Note that the designed controller is able to stabilize the TS system after the initial conditions and also after the presence of an exogenous entry (both states converge asymptotically to zero).

Figure 3 reveals that the controller indeed reduces the effect of the noisy input into the regulated output. The computation revelas that $\|z_k\|_2 = 0.6406$ and that $\|w_k\|_2 = 1.7919$, resulting in an attenuation factor $\gamma = 0.3575$. Notice that even for a rather complicated disturbance signal as the one in (4.5), the attenuation provided by the controller was smaller than the upper bound prescribed. The control signal is shown in Figure 4.

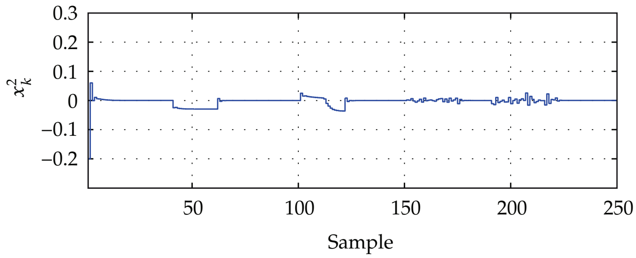

Figure 2: Closed-loop system, x_k^2 trajectory.

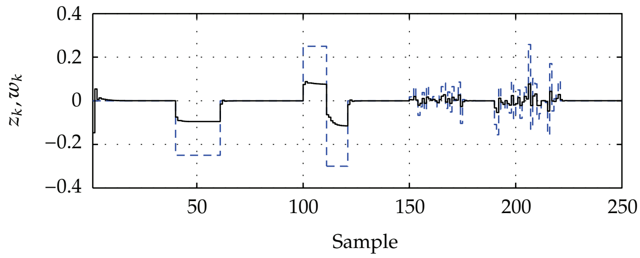

Figure 3: Disturbance signal (dashed) and system regulated output (continuous).

5. Experimental Results

This section presents practical control experiments using the proposed methodology into the Chua's chaotic oscillator [41, 45]. This system is implemented on a laboratory setup called PCCHUA, with a complete set of tools to perform discrete-time control and data acquisition. Constructive aspects and details can be found in [46].

5.1. TS Model of Chua's System

The Chua's chaotic oscillator is continuous-time system with three unstable fixed points, and its trajectory in the space state is confined to a double-scroll attractor [45]. The following equations describe the system dynamics:

$$\begin{aligned}
\dot{x}_1(t) &= \frac{1}{C_1}\left\{\frac{x_2(t)}{R} - \frac{x_1(t)}{R} - g(x_1(t))\right\},\\
\dot{x}_2(t) &= \frac{1}{C_2}\left\{\frac{x_1(t)}{R} - \frac{x_2(t)}{R} + x_3(t)\right\},\\
\dot{x}_3(t) &= -\frac{x_2(t)}{L} - \frac{R_0}{L}x_3(t),
\end{aligned} \tag{5.1}$$

where $g(x_1(t))$ is a nonlinear function given by

$$g_2x_1(t) + \frac{(g_1 - g_2)(|x_1(t) + E| - |x_1(t) - E|)}{2}, \tag{5.2}$$

with g_1 and g_2 being conductance values.

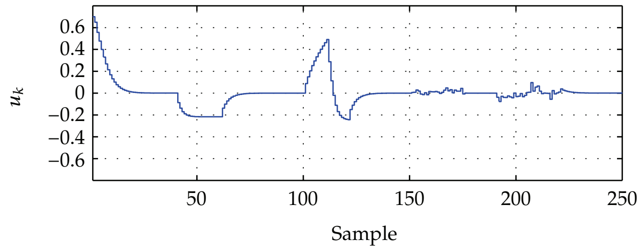

Figure 4: Control signal.

First, to use a discrete-time control strategy available in the PCCHUA framework, a discretized version of (5.1) is obtained by employing the methodology from [47]. Then, the sector nonlinearity approach [3] is applied to obtain a DFS. These modelling details can be found in [11].

A single-premisse variable is chosen: $x_k^1 \in [-d, d]$. The following fuzzy rules are able to exactly represent the dynamics of the discretized Chua's oscillator:

$$R_1 : \begin{cases} \text{IF} & x_k^1 \text{ IS } \mathcal{M}_1^1 \text{ (near 0)}, \\ \text{THEN} & \begin{aligned} x_{k+1} &= A_1 x_k + B_1 u_k + E_1 w_k, \\ y_k &= C_1 x_k + F_1 w_k, \end{aligned} \end{cases}$$
$$R_2 : \begin{cases} \text{IF} & x_k^1 \text{ IS } \mathcal{M}_1^2 \text{ (near} \pm d), \\ \text{THEN} & \begin{aligned} x_{k+1} &= A_2 x_k + B_2 u_k + E_2 w_k, \\ y_k &= C_2 x_k + F_2 w_k. \end{aligned} \end{cases} \tag{5.3}$$

The local matrices are given by

$$A_i = \begin{bmatrix} 1 - \dfrac{T}{RC_1} - \dfrac{Tg_i}{C_1} & \dfrac{T}{RC_1} & 0 \\ \dfrac{T}{RC_2} & 1 - \dfrac{T}{RC_2} & \dfrac{T}{C_2} \\ 0 & -\dfrac{T}{L} & 1 - \dfrac{TR_0}{L} \end{bmatrix},$$
$$B_i = \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}, \qquad E_i = \begin{bmatrix} 1 \times 10^{-3} \\ 0 \\ 0 \end{bmatrix}, \tag{5.4}$$
$$C_i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}', \qquad D_i = 0, \qquad F_i = 1 \times 10^{-4},$$

Table 4: DFS parameters.

Parameter	Value
C_1	$30.14\,\mu$F
C_2	$185.6\,\mu$F
L	52.28 H
R	1673 Ω
R_0	0 Ω
g_1	−0.801 mS
U	−0.365 mS
E	1. 74 V
d	6 V
T	10 ms

Table 5: Gains obtained by Theorem 3.1.

Gain	Value
K_1	[1.1667 0.2132 0.3935]
K_2	[1.0640 0.2132 0.3935]

where $g_2 \triangleq U + (g_1 - U)E/d$.

The membership functions are

$$h_1\left[x_k^1\right] = \begin{cases} \dfrac{-\alpha x_k^1 + E}{(1-\alpha)x_k^1}, & x_k^1 \geq E, \\ \dfrac{-\alpha x_k^1 - E}{(1-\alpha)x_k^1}, & x_k^1 \leq E, \\ 1, & \text{otherwise}, \end{cases}$$
$$h_2\left[x_k^1\right] = 1 - h_1\left[x_k^1\right], \tag{5.5}$$

where $\alpha \triangleq E/d$.

The parameters of the fuzzy model are given in Table 4.

5.2. Fuzzy Control of Chua's System

After obtaining a fuzzy model, it is possible to use Theorem 3.1 to design a PDC controller that guarantees asymptotically stability when $w_k = 0$ and that minimizes the $\mathscr{H}_\infty$ guaranteed cost between w_k and z_k. Since the PCCHUA contains a real Chua's circuit, the plant is subjected to noisy signals, quantization error, and random interferences, acting as disturbances inputs. Due to this reason, the proposed control scheme is suitable.

The gains shown in Table 5 are obtained after solving Theorem 3.1. These gains must be transformed to include the effect of the zero-order holder and to be compatible with the system actuators [36]. Therefore, the real gains are $\overline{K_i} = C_1 K_i / T$.

The elapsed time during this control experiment is 60 s. Only between 25 s and 35 s, the control action is performed. In the remaining time, the circuit runs freely. The system

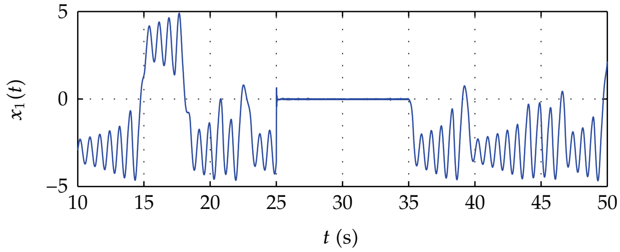

Figure 5: $x_1(t)$ trajectory.

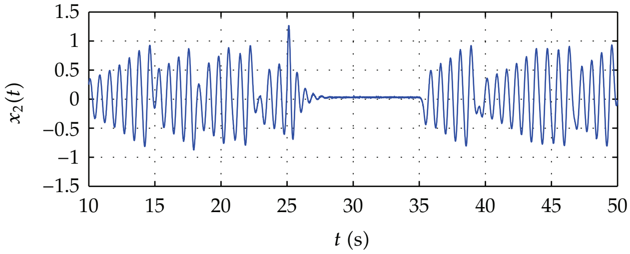

Figure 6: $x_2(t)$ trajectory.

trajectory is shown in Figures 5, 6, and 7. Notice that the fuzzy controller is able to stabilize the system at the origin. The convergence of $x_1(t)$ is faster than the convergence of the remaining states. Possible reasons are the fact that the control action is applied only on $x_1(t)$ and that the system output is given only by $x_1(t)$. Nonetheless, the design controller accomplishes its goals. Notice in Figure 5 that the presence of disturbances in the output is negligible.

The control signal is depicted in Figures 8 and 9. Figure 9 is a close view pointing out the zero-order hold characteristic of the control signal.

6. Conclusion

Different strategies to reduce numeric conservatism were combined with the fuzzy Lyapunov function to promote less conservative LMI conditions for $\mathscr{H}_\infty$ fuzzy control design and analysis. Some numerical results showed that the proposed approach outperforms recent strategies. Also a practical experiment in the Chua's oscillator was performed to show the effectiveness of the proposed approach.

Appendix

The following Lemmas are required in the development of the proof of Theorem 3.1.

Lemma A.1. *If $S > 0$, then*

$$A_i'SA_j + A_j'SA_i \le A_i'SA_i + A_j'SA_j. \tag{A.1}$$

Proof. Notice that

$$S > 0 \Longrightarrow (A_i - A_j)'S(A_i - A_j) \ge 0, \tag{A.2}$$

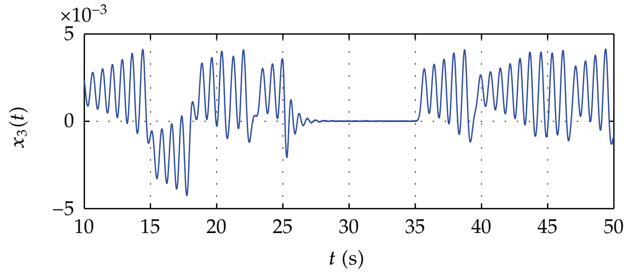

Figure 7: $x_3(t)$ trajectory.

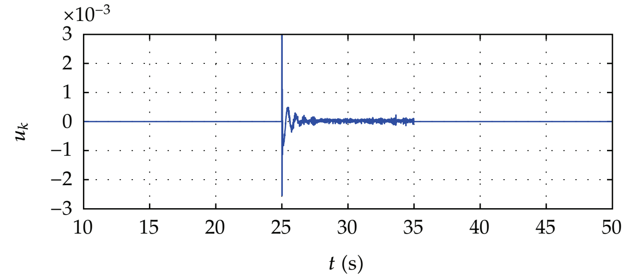

Figure 8: Control signal.

leading to

$$A_i' S A_i - A_i' S A_j - A_j' S A_i + A_j' S A_j \geq 0, \tag{A.3}$$

which completes the proof. □

Lemma A.2. *Assume that h_i ($i \in \mathcal{R}$) satisfy* (2.3) *and let R_{ij} ($i < j,\ i, j \in \mathcal{R}$) be symmetric matrices of appropriate dimension. Define*

$$H \triangleq \begin{bmatrix} H_{11} & \alpha_{12} R_{12} & \dots & \alpha_{1r} R_{1r} \\ \alpha_{12} R_{12} & H_{22} & \dots & \alpha_{2r} R_{2r} \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_{1r} R_{1r} & \alpha_{2r} R_{2r} & \dots & H_{rr} \end{bmatrix}, \tag{A.4}$$

where

$$\begin{gathered} H_{ii} \triangleq -\sum_{\substack{j=1 \\ j>i}}^{r} h_j R_{ij} - \sum_{\substack{j=1 \\ j<i}}^{r} h_j R_{ji}, \\ \alpha_{kl} \triangleq \frac{1}{2}(h_k + h_l). \end{gathered} \tag{A.5}$$

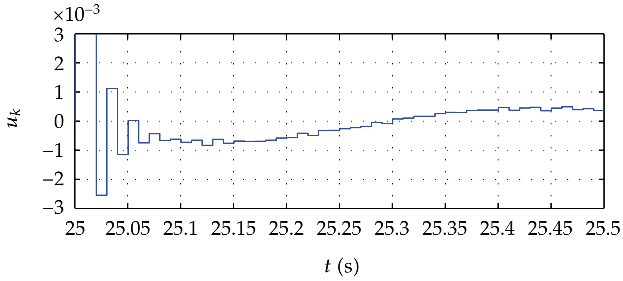

Figure 9: Control signal, close view.

Then,

$$[h_1 h_2 \cdots h_r] H [h_1 h_2 \cdots h_r]' = 0. \tag{A.6}$$

The proof of Lemma A.2 can be found in [15, Appendix A].

Lemma A.3. *If $W > 0$, then*

$$-SW^{-1}S' \preceq -(S' + S - W). \tag{A.7}$$

Proof. Since $W > 0$, there exists W^{-1}. Thus,

$$\begin{gathered} (W - S)W^{-1}(W - S)' \geq 0, \\ WW^{-1}W - WW^{-1}S' - SW^{-1}W + SW^{-1}S' \geq 0, \\ W - S' - S + SW^{-1}S' \geq 0, \end{gathered} \tag{A.8}$$

concluding the proof. □

Proof of Theorem 3.1. Consider the induced l_2-gain between the disturbance signal w_k and the system output z_k in (3.1) and the closed-loop TS system in (2.6). The stability and the $\mathcal{H}_\infty$ performance can be achieved if the following inequality holds [11, 36, 42]:

$$\mho = \Delta V_k + z_k' z_k - \gamma^2 w_k' w_k < 0, \tag{A.9}$$

where

$$\Delta V_k \triangleq x_{k+1}' \sum_{t=1}^{r} h_t^+ P_t x_{k+1} - x_k' \sum_{i=1}^{r} h_i P_i x_k, \tag{A.10}$$

is the increment of the fuzzy Lyapunov function candidate V_k, shown in (2.7).

According to (2.6), it follows that

$$\mho = \xi' \sum_{t=1}^{r} h_t^+ \sum_{i=1}^{r} \sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{v=1}^{r} h_i h_j h_l h_v \left(\Lambda_{ij}' \overline{P}_t \Lambda_{lv} - \widetilde{P}_i\right)\xi, \tag{A.11}$$

where $\xi \triangleq [x_k' w_k']$ and

$$\Lambda_{ab} \triangleq \begin{bmatrix} G_{ab} & E_a \\ J_{ab} & F_a \end{bmatrix}, \qquad \overline{P}_a \triangleq \begin{bmatrix} P_a & 0 \\ 0 & I \end{bmatrix}, \qquad \widetilde{P}_a \triangleq \begin{bmatrix} P_a & 0 \\ 0 & \gamma^2 I \end{bmatrix}. \tag{A.12}$$

Equation (A.11) is equivalent to

$$\begin{aligned} \mho = \xi' &\sum_{t=1}^{r} h_t^+ \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{v=1}^{r} h_i h_j h_l h_v \frac{1}{8} \\ &\times \left[(\Lambda_{ij}+\Lambda_{ji})'\overline{P}_t(\Lambda_{lv}+\Lambda_{vl}) + (\Lambda_{lv}+\Lambda_{vl})'\overline{P}_t(\Lambda_{ij}+\Lambda_{ji}) - 8\widetilde{P}_i\right]\xi. \end{aligned} \tag{A.13}$$

By applying Lemma A.1, one obtains

$$\begin{aligned} \mho &\le \xi' \sum_{t=1}^{r} h_t^+ \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r}\sum_{v=1}^{r} h_i h_j h_l h_v \frac{1}{8}\left[(\Lambda_{ij}+\Lambda_{ji})'\overline{P}_t(\Lambda_{ij}+\Lambda_{ji}) + (\Lambda_{lv}+\Lambda_{vl})'\overline{P}_t(\Lambda_{lv}+\Lambda_{vl}) - 8\widetilde{P}_i\right]\xi \\ &= \xi' \sum_{t=1}^{r} h_t^+ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i h_j \frac{1}{4}\left[(\Lambda_{ij}+\Lambda_{ji})'\overline{P}_t(\Lambda_{ij}+\Lambda_{ji}) - 4\widetilde{P}_i\right]\xi \\ &= \xi' \sum_{t=1}^{r} h_t^+ \left\{ \sum_{i=1}^{r} h_i^2 \left[\Lambda_{ii}'\overline{P}_t\Lambda_{ii} - \widetilde{P}_i\right] + \sum_{i<j}^{r} h_i h_j \left[(\Lambda_{ij}+\Lambda_{ji})'\overline{P}_t(\Lambda_{ij}+\Lambda_{ji}) - \widetilde{P}_i\right]\right\}\xi. \end{aligned} \tag{A.14}$$

Now define

$$\overline{\Lambda}_{abc} \triangleq \begin{bmatrix} \lambda_{abc}^{11} & \bullet \\ \lambda_{abc}^{21} & \lambda_{abc}^{22} \end{bmatrix}, \tag{A.15}$$

where

$$\begin{aligned} \lambda_{abc}^{11} &\triangleq G_{ab}' P_c G_{ab} - P_a + J_{ab}' J_{ab}, \\ \lambda_{abc}^{21} &\triangleq E_a' P_c G_{ab} + F_a' J_{ab}, \\ \lambda_{abc}^{22} &\triangleq -\gamma^2 I + E_a' P_c E_a + F_a' F_a, \end{aligned} \tag{A.16}$$

and rewrite (A.14) as

$$\mathcal{V} \leq \mathcal{N} = \xi' \sum_{t=1}^{r} h_t^+ \left[\sum_{i=1}^{r} h_i^2 \overline{\Lambda}_{iit} + 2 \sum_{i<j}^{r} h_i h_j \frac{\left(\overline{\Lambda}_{ijt} + \overline{\Lambda}_{jit} \right)}{2} \right] \xi. \tag{A.17}$$

For stability of (2.6) with a $\mathcal{H}_\infty$ guaranteed cost given by γ, it is sufficient that $\overline{\Lambda}_{iit} \prec 0$ and $\overline{\Lambda}_{ijt} \prec 0$ $(i < j,\ i, j, t \in \mathcal{R})$, since it follows that in (A.17), $\mathcal{N} < 0$, which implies that (A.9) holds.

At this point, the relaxation techniques from [11, 15, 38–40] are combined to promote the less conservative LMIs proposed in Theorem 3.1.

First, some matrix transformations are employed to allow the decoupling of the Lyapunov matrices from the system matrices. Apply Schur's complement to $\overline{\Lambda}_{abc}$, obtaining

$$\widetilde{\Lambda}_{abc} \triangleq \begin{bmatrix} -P_a & \bullet & \bullet & \bullet \\ 0 & -\gamma^2 I & \bullet & \bullet \\ P_c G_{ab} & P_c E_a & -P_c & \bullet \\ J_{ab} & F_a & 0 & -I \end{bmatrix}. \tag{A.18}$$

Define a slack matrix variable $L \in \mathbb{R}^{n \times n}$ (see [38–40] for details). Take (A.17), replace $\overline{\Lambda}_{abc}$ with $\widetilde{\Lambda}_{abc}$, and then apply the following transformations:

$$\overline{\mathcal{V}} \leq S_2 \left[S_1 (\mathcal{N}) S_1' \right] S_2', \tag{A.19}$$

where

$$S_1 \triangleq \operatorname{diag}\left\{ P_a^{-1}, I, P_c^{-1}, I \right\}, \qquad S_2 \triangleq \operatorname{diag}\{ L' P_a, I, I, I \},$$
$$\overline{\mathcal{V}} \triangleq S_2 S_1 \mathcal{V} S_1' S_2' \tag{A.20}$$

to obtain

$$\overline{\mathcal{V}} < N = \xi' \sum_{t=1}^{r} h_t^+ \left[\sum_{i=1}^{r} h_i^2 \widehat{\Lambda}_{iit} + 2 \sum_{i<j}^{r} h_i h_j \frac{\left(\widehat{\Lambda}_{ijt} + \widehat{\Lambda}_{jit} \right)}{2} \right] \xi, \tag{A.21}$$

such that

$$\widehat{\Lambda}_{abc} \triangleq \begin{bmatrix} -L' X_a^{-1} L & \bullet & \bullet & \bullet \\ 0 & -\gamma^2 I & \bullet & \bullet \\ G_{ab} L & E_a & -X_c & \bullet \\ J_{ab} L & F_a & 0 & -I \end{bmatrix}, \qquad X_a \triangleq P_a^{-1}. \tag{A.22}$$

Applying Lemma A.3 results in

$$\overline{\mathcal{N}} < M = \xi' \sum_{t=1}^{r} h_t^{+} \left[\sum_{i=1}^{r} h_i^2 \breve{\Lambda}_{iit} + 2 \sum_{i<j}^{r} h_i h_j \frac{(\breve{\Lambda}_{ijt} + \breve{\Lambda}_{jit})}{2} \right] \xi, \tag{A.23}$$

where

$$\breve{\Lambda}_{abc} \triangleq \begin{bmatrix} X_a - L' - L & \bullet & \bullet & \bullet \\ 0 & -\delta I & \bullet & \bullet \\ A_a L + B_a M_b & E_a & -X_c & \bullet \\ C_a L + D_a M_b & F_a & 0 & -I \end{bmatrix}, \tag{A.24}$$

and $M_a \triangleq K_a L$, $\delta \triangleq \gamma^2$ are linearizing change of variables.

Finally, the second relaxation technique is applied to add more slack matrix variables. Considering that $T_{ijt} = T'_{ijt} \geq 0$ $(i, j, t \in \mathcal{R})$ and using Lemma A.2, ones gets (A.25) as in the top of the next page

$$\begin{aligned}
\mathcal{M} &= \xi' \sum_{t=1}^{r} h_t^{+} \left[\sum_{i=1}^{r} h_i^2 \breve{\Lambda}_{iit} + 2 \sum_{i<j}^{r} h_i h_j \left(\frac{\breve{\Lambda}_{ijt} + \breve{\Lambda}_{jit}}{2} \right) \right] \xi \\
&\leq \xi' \sum_{t=1}^{r} h_t^{+} \left[\sum_{i=1}^{r} h_i^2 \breve{\Lambda}_{iit} + 2 \sum_{i<j}^{r} h_i h_j \left(\frac{\breve{\Lambda}_{ijt} + \breve{\Lambda}_{jit}}{2} + T_{ijt} + \overline{S}_{ijt} \right) \right] \xi \\
&= \xi' \sum_{t=1}^{r} h_t^{+} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_r \end{bmatrix}' \begin{bmatrix} \breve{\Lambda}_{11t} & \bullet & \cdots & \bullet \\ \overline{\mathrm{V}}_t & \breve{\Lambda}_{22t} & \cdots & \bullet \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\mathrm{V}}_{r1t} & \overline{\mathrm{V}}_{r2t} & \cdots & \breve{\Lambda}_{rrt} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_r \end{bmatrix} \xi \\
&= \xi' \sum_{t=1}^{r} h_t^{+} [h_1 h_2 \cdots h_r] \overline{\Xi}_t [h_1 h_2 \cdots h_r]' \xi
\end{aligned}$$

$$
\begin{aligned}
&= \xi' \sum_{t=1}^{r} h_t^{+} [h_1 h_2 \cdots h_r] \left(\overline{\Xi}_t + H\right) [h_1 h_2 \cdots h_r]' \xi \\
&= \xi' \sum_{t=1}^{r} h_t^{+} [h_1 h_2 \cdots h_r] \Xi_t [h_1 h_2 \cdots h_r] \xi.
\end{aligned}
\tag{A.25}
$$

where in (A.25), $\overline{\nabla}_t = (1/2)(\check{\Lambda}_{12t} + \check{\Lambda}_{21t}) + T_{21t} + \overline{S}_{21t}$, $\overline{\nabla}_{r1t} = (1/2)(\check{\Lambda}_{r1t} + \check{\Lambda}_{1rt}) + T_{r1t} + \overline{S}_{r1t}$, and $\overline{\nabla}_{r2t} = (1/2)(\check{\Lambda}_{r2t} + \check{\Lambda}_{2rt}) + T_{r2t} + \overline{S}_{r2t}$.

Matrices $\Xi_t, t \in \mathcal{R}$ are given as in (3.3). $\overline{S}_{ijt}$ are skew matrices which can be defined as the difference $S_{ijt} - S'_{ijt}$, with S_{ijt} being any matrices.

If the LMIs constraints given in (3.2) are satisfied, then $\mathcal{M} < 0$. Because of (A.23), (A.17), and (A.9), it also implies that the TS system (2.6) is asymptotically stable with $\mathcal{H}_\infty$ guaranteed cost δ. Furthermore, since the optimization problem in (3.2) is convex, the minimization of δ produces the minimum $\mathcal{H}_\infty$ disturbance attenuation level $\gamma_{\min}$, which concludes the proof. □

The proof of Theorem 3.2 follows the same steps as in Theorem 3.1. Because in Theorem 3.2 the value of the guaranteed cost is given beforehand, there is no need to impose the minimization constraint, since δ is not a variable anymore. Therefore, Theorem 3.2 is just a feasibility problem.

The proofs of Theorems 3.3 and 3.4 follow the same steps of the proof of Theorem 3.1 as well. However, since the gains are given, the linearizing variables M_i must be dropped out.

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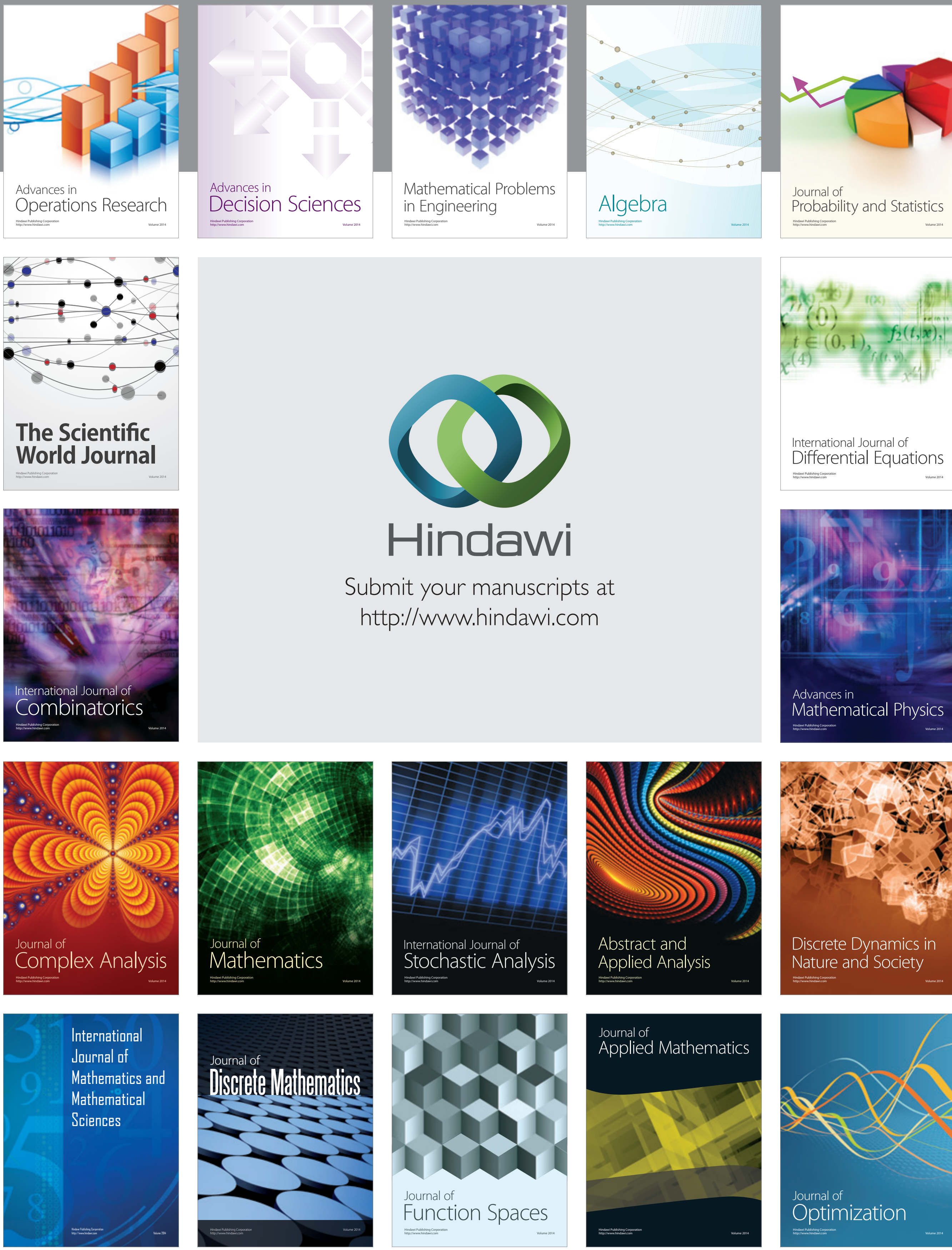

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