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Research Article

Hyperbolic, Trigonometric, and Rational Function Solutions of Hirota-Ramani Equation via (G'/G)-Expansion Method

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The (G'/G)-expansion method is proposed to construct the exact traveling solutions to Hirota-Ramani equation: $u_t - u_{xxt} + au_x(1-u_t) = 0$, where $a \neq 0$. Our work is motivated by the fact that the (G'/G)-expansion method provides not only more general forms of solutions but also periodic and solitary waves. If we set the parameters in the obtained wider set of solutions as special values, then some previously known solutions can be recovered. The method appears to be easier and faster by means of a symbolic computation system.

1. Introduction

Nonlinear evolution equations (NLEEs) have been the subject of study in various branches of mathematical-physical sciences such as physics, biology, and chemistry. The analytical solutions of such equations are of fundamental importance since a lot of mathematical-physical models are described by NLEEs. Among the possible solutions to NLEEs, certain special form solutions may depend only on a single combination of variables such as solitons. In mathematics and physics, a soliton is a self-reinforcing solitary wave, a wave packet or pulse, that maintains its shape while it travels at constant speed. Solitons are caused by a cancelation of nonlinear and dispersive effects in the medium. The term "dispersive effects" refers to a property of certain systems where the speed of the waves varies according to frequency. Solitons arise as the solutions of a widespread class of weakly nonlinear dispersive partial differential equations describing physical systems. The soliton phenomenon was first described by *John Scott Russell* (1808–1882) who observed a solitary wave in the Union

Canal in Scotland. He reproduced the phenomenon in a wave tank and named it the "Wave of Translation" [1]. Many exactly solvable models have soliton solutions, including the Korteweg-de Vries equation, the nonlinear Schrödinger equation, the coupled nonlinear Schrödinger equation, the sine-Gordon equation, and Gardner equation. The soliton solutions are typically obtained by means of the inverse scattering transform [2] and owe their stability to the integrability of the field equations. In the past years, many other powerful and direct methods have been developed to find special solutions of nonlinear evolution equations (NEE(s)), such as the Bcklund transformation [3], Hirota bilinear method [4], numerical methods [5], and the Wronskian determinant technique [6]. With the help of the computer software, many algebraic methods are proposed, such as tanh method [7], F-expanded method [8], homogeneous balance method [9], Jacobi elliptic function method [10], the Miura transformation [11], and some other new methods [12, 13].

Recently, the (G'/G)-expansion method, firstly introduced by Wang et al. [14], has become widely used to search for various exact solutions of NLEEs [14–18]. The value of the (G'/G)-expansion method is that one treats nonlinear problems by essentially linear methods. The method is based on the explicit linearization of NLEEs for traveling waves with a certain substitution which leads to a second-order differential equation with constant coefficients. Moreover, it transforms a nonlinear equation to a simple algebraic computation.

Our first interest in the present work is in implementing the (G'/G)-expansion method to stress its power in handling nonlinear equations, so that one can apply it to models of various types of nonlinearity. The next interest is in the determination of new exact traveling wave solutions for the Hirota-Ramani equation [11–13]:

$$u_t - u_{xxt} + au_x(1 - u_t) = 0, \tag{1.1}$$

where $a \neq 0$ is a real constant and $u(x,t)$ is the amplitude of the relevant wave mode. This equation was first introduced by Hirota and Ramani in [11]. Ji obtained some travelling soliton solutions of this equation by using Exp-function method [13]. This equation is completely integrable by the inverse scattering method. Equation (1.1) is studied in [11–13] where new kind of solutions were obtained. Hirota-Ramani equation is widely used in various branches of physics, and such as plasma physics, fluid physics, and quantum field theory. It also describes a variety of wave phenomena in plasma and solid state [11].

2. Description of the (G'/G)-Expansion Method

The objective of this section is to outline the use of the (G'/G)-expansion method for solving certain nonlinear partial differential equations (PDEs). Suppose we have a nonlinear PDE for $u(x,t)$, in the form

$$P(u, u_x, u_t, u_{xx}, u_{x,t}, u_{tt}, \ldots) = 0, \tag{2.1}$$

where P is a polynomial in its arguments, which includes nonlinear terms and the highest-order derivatives. The transformation $u(x,t) = U(\xi), \xi = kx + \omega t$, reduces (2.1) to the ordinary differential equation (ODE)

$$P\left(U, kU', \omega U', k^2U'', k\omega U'', \omega^2 U'', \ldots\right) = 0, \tag{2.2}$$

where $U = U(\xi)$, and prime denotes derivative with respect to ξ. We assume that the solution of (2.2) can be expressed by a polynomial in (G'/G) as follows:

$$U(\xi) = \sum_{i=1}^{m} \alpha_i \left(\frac{G'}{G}\right)^i + \alpha_0, \quad \alpha_m \neq 0, \tag{2.3}$$

where α_0, and α_i, are constants to be determined later, $G(\xi)$ satisfies a second-order linear ordinary differential equation (LODE):

$$\frac{d^2G(\xi)}{d\xi^2} + \lambda \frac{dG(\xi)}{d\xi} + \mu G(\xi) = 0, \tag{2.4}$$

where λ and μ are arbitrary constants. Using the general solutions of (2.4), we have

$$\frac{G'(\xi)}{G(\xi)} = \begin{cases} \dfrac{\sqrt{\lambda^2-4\mu}}{2}\left(\dfrac{C_1 \sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right) + C_2 \cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}{C_1 \cosh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right) + C_2 \sinh\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}\right) - \dfrac{\lambda}{2}, \\ \qquad \lambda^2 - 4\mu > 0, \\ \dfrac{\sqrt{4\mu-\lambda^2}}{2}\left(\dfrac{-C_1 \sin\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right) + C_2 \cos\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}{C_1 \cos\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right) + C_2 \sin\left(\left(\sqrt{\lambda^2-4\mu}/2\right)\xi\right)}\right) - \dfrac{\lambda}{2}, \\ \qquad \lambda^2 - 4\mu < 0, \end{cases} \tag{2.5}$$

and it follows, from (2.3) and (2.4), that

$$\begin{aligned} U' &= -\sum_{i=1}^{m} i\alpha_i \left(\left(\frac{G'}{G}\right)^{i+1} + \lambda\left(\frac{G'}{G}\right)^{i} + \mu\left(\frac{G'}{G}\right)^{i-1}\right), \\ U'' &= \sum_{i=1}^{m} i\alpha_i \left((i+1)\left(\frac{G'}{G}\right)^{i+2} + (2i+1)\lambda\left(\frac{G'}{G}\right)^{i+1} + i\left(\lambda^2 + 2\mu\right)\left(\frac{G'}{G}\right)^{i} \right. \\ &\qquad \left. + (2i-1)\lambda\mu\left(\frac{G'}{G}\right)^{i-1} + (i-1)\mu^2\left(\frac{G'}{G}\right)^{i-2}\right), \end{aligned} \tag{2.6}$$

and so on, here the prime denotes the derivative with respect to ξ.

To determine u explicitly, we take the following four steps.

Step 1. Determine the integer m by substituting (2.3) along with (2.4) into (2.2), and balancing the highest-order nonlinear term(s) and the highest-order partial derivative.

Step 2. Substitute (2.3), given the value of m determined in *Step 1*, along with (2.4) into (2.2) and collect all terms with the same order of (G'/G) together, the left-hand side of (2.2) is converted into a polynomial in (G'/G). Then set each coefficient of this polynomial to zero to derive a set of algebraic equations for $k, \omega, \lambda, \mu, \alpha_0$ and α_i for $i = 1, 2, \ldots, m$.

Step 3. Solve the system of algebraic equations obtained in *Step 2*, for $k, \omega, \lambda, \mu, \alpha_0$ and α_i by use of Maple.

Step 4. Use the results obtained in above steps to derive a series of fundamental solutions $u(\xi)$ of (2.2) depending on (G'/G); since the solutions of (2.4) have been well known for us, then we can obtain exact solutions of (2.1).

3. Application on Hirota-Ramani Equation

In this section, we will use our method to find solutions to Hirota–Ramani equation [10–12]:

$$u_t - u_{xxt} + a u_x (1 - u_t) = 0, \tag{3.1}$$

where $a \neq 0$. We would like to use our method to obtain more general exact solutions of (3.1) by assuming the solution in the following frame:

$$u = U(\xi), \quad \xi = kx + \omega t, \tag{3.2}$$

where k, ω are constants. We substitute (3.2) into (3.1) to obtain nonlinear ordinary differential equation

$$(\omega + ak)U' - k^2 \omega U''' - ak\omega (U')^2 = 0. \tag{3.3}$$

By setting $U' = V$, nonlinear ordinary differential equation (3.3) reduce to

$$(\omega + ak)V - k^2 \omega V'' - ak\omega V^2 = 0. \tag{3.4}$$

According to *Step 1*, we get $m + 2 = 2m$, hence $m = 2$. We then suppose that (3.4) has the following formal solutions:

$$V = \alpha_2 \left(\frac{G'}{G}\right)^2 + \alpha_1 \left(\frac{G'}{G}\right) + \alpha_0, \quad \alpha_2 \neq 0, \tag{3.5}$$

where α_2, α_1, and α_0, are unknown to be determined later.

Substituting (3.5) into (3.4) and collecting all terms with the same order of (G'/G), together, the left-hand sides of (3.4) are converted into a polynomial in (G'/G). Setting each coefficient of each polynomial to zero, we derive a set of algebraic equations for $\lambda, \mu, \alpha_0, \alpha_1$, and α_2, as follows:

$$\begin{aligned}
&\left(\frac{G'}{G}\right)^0 : \left(-2\omega\alpha_2\mu^2 - \omega\alpha_1\lambda\mu\right)k^2 - a\alpha_0(\omega\alpha_0 - 1)k + \omega\alpha_0 = 0,\\
&\left(\frac{G'}{G}\right)^1 : \left(-\omega\left(2\,\mu + \lambda^2\right)\alpha_1 - 6\omega\alpha_2\lambda\mu\right)k^2 - a(-1 + 2\,\omega\alpha_0)\alpha_1 k + \omega\alpha_1 = 0,\\
&\left(\frac{G'}{G}\right)^2 : \left(-3\omega\alpha_1\lambda - 4\omega\alpha_2\left(2\,\mu + \lambda^2\right)\right)k^2 - \left(a\alpha_2(-1 + 2\,\omega\alpha_0) + a\omega{\alpha_1}^2\right)k + \omega\alpha_2 = 0,\\
&\left(\frac{G'}{G}\right)^3 : (-10\omega\alpha_2\lambda - 2\omega\alpha_1)k^2 - 2ak\omega\alpha_2\alpha_1 = 0,\\
&\left(\frac{G'}{G}\right)^4 : -6k^2\omega\alpha_2 - ak\omega\alpha_2^2 = 0,
\end{aligned} \tag{3.6}$$

and solving by use of Maple, we get the following results.

Case 1.

$$\begin{aligned}
\lambda &= -\frac{1}{6}\frac{a\alpha_1}{k},\\
\mu &= \frac{1}{144}\frac{-36ak - 36\omega + a^2\omega\alpha_1^2}{k^2\omega},\\
\alpha_0 &= \frac{1}{24}\frac{-36ak - 36\omega + a^2\omega\alpha_1^2}{ak\omega},\\
\alpha_2 &= -\frac{6k}{a},
\end{aligned} \tag{3.7}$$

where k, ω, and α_1 are arbitrary constants. Therefore, substitute the above case in (3.5), and using the relationship $U(\xi) = \int V(\xi)d\xi$, we get

$$U = \int\left\{-\frac{6k}{a}\left(\frac{G'}{G}\right)^2 + \alpha_1\left(\frac{G'}{G}\right) + \frac{1}{24}\frac{-36ak - 36\omega + a^2\omega\alpha_1^2}{ak\omega}\right\}d\xi. \tag{3.8}$$

Substituting the general solutions (2.5) into (3.8), we obtain three types of traveling wave solutions of (3.1) in view of the positive, negative, or zero of $\lambda^2 - 4\mu$.

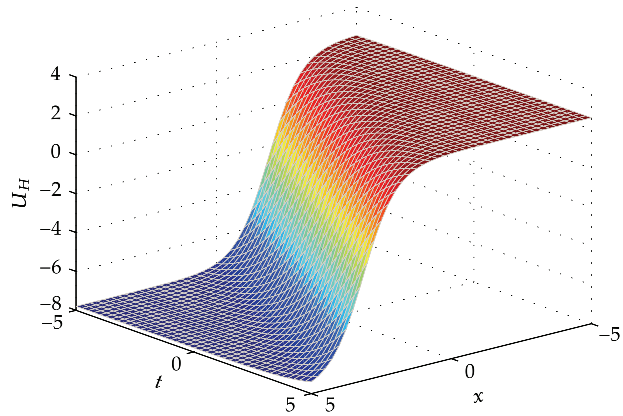

Figure 1: Hyperbolic function solution (3.9) of Hirota-Ramani equation, for $a = 1$, $k = 1$, $\omega = 1/2$, $C_1 = 2$, and $C_2 = 1$.

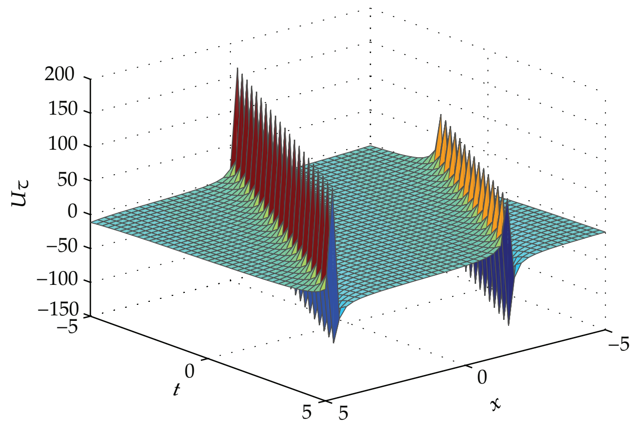

Figure 2: Trigonometric function solution (3.11) of Hirota-Ramani equation, for $a = 1$, $k = -1$, $\omega = 1/2$, $C_1 = 2$, and $C_2 = 1$.

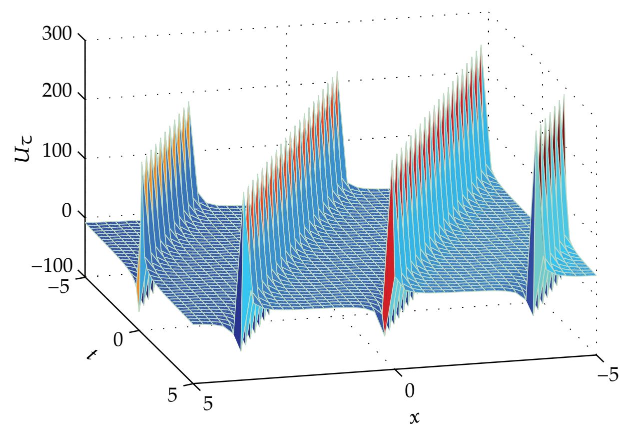

Figure 3: Trigonometric function solution (3.16) of Hirota-Ramani equation, for $a = -1$, $k = -1$, $\omega = 1/2$, $C_1 = 2$, and $C_2 = 1$.

When $\mathfrak{D}_1 = \lambda^2 - 4\mu = (ak+\omega)/k^2\omega > 0$, using the integration relationship (3.8), we obtain hyperbolic function solution $U_{\mathscr{H}}$, of Hirota-Ramani equation (3.1) as follows:

$$U_{\mathscr{H}}(\xi) = \frac{6(ak+\omega)\left(C_1^2 - C_2^2\right)}{ak\omega C_1\sqrt{\mathfrak{D}_1}} \times \frac{\sinh\left((1/4)\sqrt{\mathfrak{D}_1}\xi\right)\cosh\left((1/4)\sqrt{\mathfrak{D}_1}\xi\right)}{2C_2\sinh\left((1/4)\sqrt{\mathfrak{D}_1}\xi\right)\cosh\left((1/4)\sqrt{\mathfrak{D}_1}\xi\right) + 2C_1\cosh^2\left((1/4)\sqrt{\mathfrak{D}_1}\,\xi\right) - C_1}, \tag{3.9}$$

where $\xi = kx + \omega t$, and C_1, C_2, are arbitrary constants. This solution is shown in Figure 1 for $a = 1$, $k = 1$, $\omega = 1/2$, $C_1 = 2$, and $C_2 = 1$. It is easy to see that the hyperbolic solution (3.9) can be rewritten at $C_1^2 > C_2^2$, as follows:

$$u_{\mathscr{H}}(x,t) = \frac{3}{2}\frac{(ak+\omega)}{ak\omega\sqrt{\mathfrak{D}_1}}\left\{ 2\tanh\left(\frac{1}{2}\sqrt{\mathfrak{D}_1}\xi + \eta_{\mathscr{H}}\right) + \ln\left(\frac{\tanh\left((1/2)\sqrt{\mathfrak{D}_1}\xi + \eta_{\mathscr{H}}\right) - 1}{\tanh\left((1/2)\sqrt{\mathfrak{D}_1}\xi + \eta_{\mathscr{H}}\right) + 1}\right) + \sqrt{\mathfrak{D}_1}\,\xi \right\}, \tag{3.10a}$$

while at $C_1^2 < C_2^2$, one can obtain

$$u_{\mathscr{H}}(x,t) = \frac{3}{2}\frac{(ak+\omega)}{ak\omega\sqrt{\mathfrak{D}_1}}\left\{ 2\coth\left(\frac{1}{2}\sqrt{\mathfrak{D}_1}\xi + \eta_{\mathscr{H}}\right) + \ln\left(\frac{\coth\left((1/2)\sqrt{\mathfrak{D}_1}\xi + \eta_{\mathscr{H}}\right) - 1}{\coth\left((1/2)\sqrt{\mathfrak{D}_1}\xi + \eta_{\mathscr{H}}\right) + 1}\right) + \sqrt{\mathfrak{D}_1}\,\xi \right\}, \tag{3.10b}$$

where $\xi = kx + \omega t$, $\eta_{\mathscr{H}} = \tanh^{-1}(C_1/C_2)$, and k, ω, are arbitrary constants.

Now, when $\mathfrak{D}_1 = \lambda^2 - 4\mu = ((ak+\omega)/k^2\omega) < 0$, using the integration relationship (3.8), we obtain trigonometric function solution $U_{\mathcal{T}}$, of Hirota-Ramani equation (3.1) as follows:

$$U_{\mathcal{T}}(\xi) = -\frac{3(ak+\omega)\left(C_1^2 + C_2^2\right)}{ak\omega C_2\sqrt{-\mathfrak{D}_1}}\frac{1}{C_2\tan\left((1/2)\sqrt{-\mathfrak{D}_1}\xi\right) + C_1}, \tag{3.11}$$

where $\xi = kx + \omega t$, and C_1, C_2, are arbitrary constants. This solution is shown in Figure 2 for $a = 1$, $k = -1$, $\omega = 1/2$, $C_1 = 2$, and $C_2 = 1$. Similarity, it is easy to see that the trigonometric solution (3.11) can be rewritten at $C_1^2 > C_2^2$, and $C_1^2 < C_2^2$, as follows:

$$u_{\mathcal{T}}(x,t) = \frac{3(ak+\omega)}{ak\omega C_2\sqrt{-\mathfrak{D}_1}}\tan\left(\frac{1}{2}\sqrt{-\mathfrak{D}_1}\xi + \eta_{\mathcal{T}}\right), \tag{3.12a}$$

$$u_{\mathcal{T}}(x,t) = -\frac{3(ak+\omega)}{ak\omega C_2\sqrt{-\mathfrak{D}_1}}\cot\left(\frac{1}{2}\sqrt{-\mathfrak{D}_1}\xi + \eta_{\mathcal{T}}\right), \tag{3.12b}$$

respectively, where $\xi = kx + \omega t$, $\eta_{\mathcal{T}} = \tan^{-1}(C_1/C_2)$, and k, ω, are arbitrary constants.

Case 2.

$$\begin{aligned}
\lambda &= -\frac{1}{6}\frac{a\alpha_1}{k}, \\
\mu &= \frac{1}{144}\frac{36ak + 36\omega + a^2\omega\alpha_1^2}{k^2\omega}, \\
\alpha_0 &= -\frac{1}{24}\frac{12ak + 12\omega + a^2\omega\alpha_1^2}{ak\omega}, \\
\alpha_2 &= -\frac{6k}{a},
\end{aligned} \tag{3.13}$$

where k, ω and α_1 is an arbitrary constant. Similar on the previous case, substitute the above case in (3.5), and using the relationship $U(\xi) = \int V(\xi)d\xi$, we get

$$U = \int\left\{-\frac{6k}{a}\left(\frac{G'}{G}\right)^2 + \alpha_1\left(\frac{G'}{G}\right) - \frac{1}{24}\frac{12ak + 12\omega + a^2\omega\alpha_1^2}{ak\omega}\right\} d\xi, \tag{3.14}$$

then for $\mathfrak{D}_2 = \lambda^2 - 4\mu = -((ak+\omega)/k^2\omega) > 0$, the hyperbolic and for $\mathfrak{D}_2 = \lambda^2 - 4\mu = -((ak+\omega)/k^2\omega) < 0$, trigonometric types of traveling wave solutions of Hirota-Ramani equation (3.1), are obtained as follows:

$$\begin{aligned}
U_{\mathscr{H}}(\xi) = {} & \frac{2(ak+\omega)(C_2^2 - C_1^2)}{ak\omega\sqrt{\mathfrak{D}_2}} \\
& \times \left\{ \ln\left(\frac{\tanh\left((1/4)\sqrt{\mathfrak{D}_2}\xi\right) + 1}{\tanh\left((1/4)\sqrt{\mathfrak{D}_2}\ \xi\right) - 1}\right)\right. \\
& \left. + \frac{3\tanh\left((1/4)\sqrt{\mathfrak{D}_2}\xi\right)}{C_1^2\tanh^2\left((1/4)\sqrt{\mathfrak{D}_2}\xi\right) + 2C_1C_2\tanh\left((1/4)\sqrt{\mathfrak{D}_2}\ \xi\right) + C_1^2}\right\},
\end{aligned} \tag{3.15}$$

$$U_{\mathcal{T}}(\xi) = \frac{(ak+\omega)}{ak\omega C_2\sqrt{-\mathfrak{D}_2}}\left\{C_2\sqrt{-\mathfrak{D}_2}\xi + \frac{3(C_2^1 + C_2^2)}{C_2\tan\left((1/2)\sqrt{-\mathfrak{D}_2}\xi\right) + C_1}\right\}, \tag{3.16}$$

respectively, where $\xi = kx+\omega t$, and C_1, C_2, are arbitrary constants. The trigonometric function solution (3.16), for $a = -1$, $k = -1$, $\omega = 1/2$, $C_1 = 2$ and $C_2 = 1$ are shown in Figure 3. Similarly, to obtain some special forms of the solutions obtained above, we set $C_1^2 > C_2^2$, then hyperbolic and trigonometric function solutions (3.15)-(3.16) become

$$u_{\mathcal{H}}(x,t) = -\frac{1}{2}\frac{(ak+\omega)}{ak\omega\sqrt{\mathfrak{D}_2}}\left\{3\ln\left(\frac{\tanh\left((1/2)\sqrt{\mathfrak{D}_2}\xi+\eta_{\mathcal{H}}\right)-1}{\tanh\left((1/2)\sqrt{\mathfrak{D}_2}\xi+\eta_{\mathcal{H}}\right)+1}\right)\right.,$$
$$\left. +6\tanh\left(\frac{1}{2}\sqrt{\mathfrak{D}_2}\xi + \eta_{\mathcal{H}}\right)+\sqrt{\mathfrak{D}_2}\ \xi\right\}, \tag{3.17}$$
$$u_{\mathcal{T}}(x,t) = -\frac{1}{2}\frac{(ak+\omega)}{ak\omega\sqrt{-\mathfrak{D}_2}}\left\{-6\tan\left((1/2)\sqrt{-\mathfrak{D}_2}\xi + \eta_{\mathcal{T}}\right) + 6\eta_{\mathcal{T}}\right\},$$

while at $C_1^2 < C_2^2$, the hyperbolic and trigonometric function solutions (3.15)-(3.16) become

$$u_{\mathcal{H}}(x,t) = -\frac{1}{2}\frac{(ak+\omega)}{ak\omega\sqrt{\mathfrak{D}_2}}\left\{3\ln\left(\frac{\coth\left((1/2)\sqrt{\mathfrak{D}_2}\xi+\eta_{\mathcal{H}}\right)-1}{\coth\left((1/2)\sqrt{\mathfrak{D}_2}\xi+\eta_{\mathcal{H}}\right)+1}\right)\right.$$
$$\left. +6\coth\left(\frac{1}{2}\sqrt{\mathfrak{D}_2}\xi + \eta_{\mathcal{H}}\right)+\sqrt{\mathfrak{D}_2}\ \xi\right\}, \tag{3.18}$$
$$u_{\mathcal{T}}(x,t) = -\frac{1}{2}\frac{(ak+\omega)}{ak\omega\sqrt{-\mathfrak{D}_2}}\left\{6\cot\left(\frac{1}{2}\sqrt{-\mathfrak{D}_2}\xi + \eta_{\mathcal{T}}\right) - 3\pi + 6\eta_{\mathcal{T}}\right\},$$

respectively, where $\eta_{\mathcal{H}} = \tanh^{-1}(C_1/C_2)$, $\eta_{\mathcal{T}} = \tan^{-1}(C_1/C_2)$, k and ω are arbitrary constants.

3.1. Rational Solution

And finally, in both Cases 1 and 2, when $\mathfrak{D} = \lambda^2 - 4\mu = 0$, we obtain rational solution:

$$u_{\text{rat}}(x,t) = \frac{6kC_2}{a(C_1 + C_2(kx - akt))}, \tag{3.19}$$

where C_1, C_2, k are arbitrary constants. This solution is shown in Figure 4, for $a = 1$, $k = -1$, $\omega = 1/2$, $C_1 = 2$, and $C_2 = 1$.

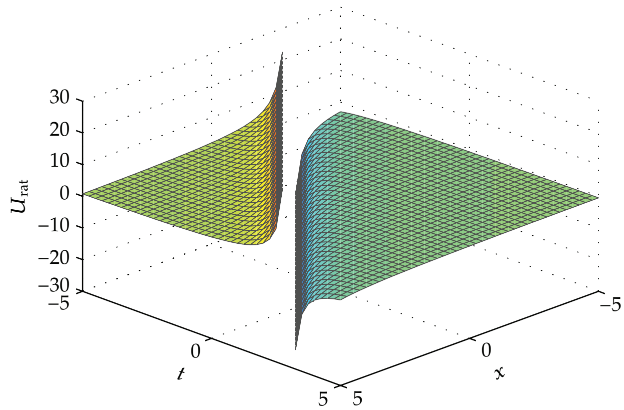

Figure 4: Rational function solution (3.19) of Hirota-Ramani equation, for $a = 1$, $k = -1$, $\omega = 1/2$, $C_1 = 2$, and $C_2 = 1$.

4. Conclusions

This study shows that the (G'/G)-expansion method is quite efficient and practically well suited for use in finding exact solutions for the Hirota-Ramani equation. Our solutions are in more general forms, and many known solutions to these equations are only special cases of them. With the aid of Maple, we have assured the correctness of the obtained solutions by putting them back into the original equation. We hope that they will be useful for further studies in applied sciences.

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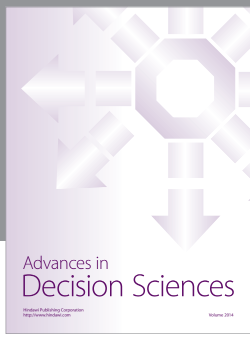

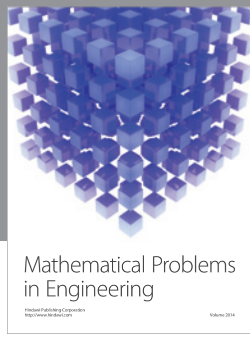

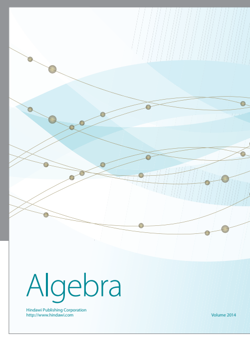

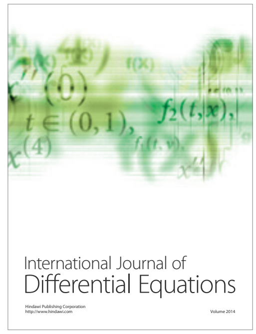

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