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Research Article

Mixed Variables-Attributes Test Plans for Single and Double Acceptance Sampling under Exponential Distribution

Yan Li, Xiaolong Pu, and Dongdong Xiang

School of Finance and Statistics, East China Normal University, no. 500 Dongchuan Road, Shanghai 200241, China

Correspondence should be addressed to Xiaolong Pu, xlpu@stat.ecnu.edu.cn

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The mixed variables-attributes test plans for single acceptance sampling are proposed to protect "good lots" from attributes aspect and to optimize sample sizes from variables aspect. For the single and double mixed plans, exact formulas of the operating characteristic and average sample number are developed for the exponential distribution. Numerical illustrations show that the mixed sampling plans have some advantages over the variables plans or attributes plans alone.

1. Introduction

In reality, especially in industry, acceptance sampling is applied to audit the percent nonconforming of the incoming materials, semimanufactured goods, finished products, and so on. Among the available plans for testing the percent nonconforming, attributes sampling plans are the most widely used plans in practice, due to the fact that they only require easy-to-collect and low-cost attributes data and produce reasonable judgments about the number of nonconforming items, as pointed out in [1].

Compared with attributes plans, variables acceptance procedures require less data under the same error probabilities and produce more accurate quality information. Hence, for costly and/or destructive inspections, variables plans are more attractive for practitioners. However, the necessary preprocessing of translating the test of the percent nonconforming into that of a variables characteristic would have the following problems [1, 2]: (i) It is generally more difficult and time consuming to measure the variables quality characteristic. (ii) The variables quality characteristic and its distribution should be specified. Since testing and estimating the percent nonconforming highly depend on the assumed distribution, unreliable conclusions may be made from the inappropriate distribution especially when the percent nonconforming is extremely small or large and is greatly inflected by the tail feature of the distribution. (iii) Under the same requirements on error probabilities, the protection for the two types of error probabilities under variables plans may be weaker than that of attributes plans. (iv) For variables plans, a lot may be rejected even if all of the sampled items are conforming, which violates the rule of attributes procedures and is always questioned by practitioners.

In fact, the authors found the above problem (iv) when applying a variables plan to the following application. To test the percent nonconforming p $(0 < p < 1)$ of a batch of bullets, a bullets sample is inspected by shooting at a circular objective with the center $(0, 0)$ and the radius $\sqrt{a}$ $(a > 0)$. A bullet is defined as conforming if it hits the objective under some circumstances, or as nonconforming otherwise. The inspection results include the variables data, that is, coordinates of the fall points, and the attributes data about whether the inspected bullets are nonconforming. According to specific needs, the test problem was set as

$$H_0 : p = p_0 = 0.1 \text{ versus } H_1 : p = p_1 = 0.4. \tag{1.1}$$

Under the binomial distribution, the attributes plan for single acceptance sampling (single attributes plan) with the error probabilities $\alpha = \beta = 0.15$ is $(n_a, c_a) = (11, 2)$, that is, to inspect 11 bullets and accept the null hypothesis in the test problem (1.1) when 2 or less bullets in the sample are nonconforming. The real error probabilities of the plan are $\alpha'_a = 0.0896$ and $\beta'_a = 0.1189$.

The variables plan for single acceptance sampling (single variables plan) is also accessible based on locations of the fall points. Suppose the coordinate of the fall point is (X, Y), where X and Y are independent and identically distributed (i.i.d.) with $N(0, \sigma^2)$ $(\sigma^2 > 0)$ according to the practitioners' experiences. Then, $(X^2 + Y^2)/\sigma^2$ conforms with $\chi^2(2)$ or Gamma(1,2). Equivalently, $R = X^2 + Y^2$ conforms with the exponential distribution $\text{Exp}(2\sigma^2)$ with $2\sigma^2$ as its mean. Hence, the percent nonconforming p and the variables parameter σ^2 have the following relationship:

$$p = P(R > a) = \exp\left\{-\frac{a}{2\sigma^2}\right\}, \tag{1.2}$$

or equivalently,

$$\sigma^2 = -\frac{a}{2\ln(p)}. \tag{1.3}$$

Taking $a = 1$ as an example, according to (1.3), the test problem (1.1) is equivalent to

$$H_0 : \sigma^2 = \sigma_0^2 = 0.2171 \text{ versus } H_1 : \sigma^2 = \sigma_1^2 = 0.5457. \tag{1.4}$$

For the test problem (1.4), setting $\{\overline{R}_{n_v} = \sum_{i=1}^{n_v} R_i/n_v > c_v\}$ as the rejection region, the single variables plan should be $(n_v, c_v) = (6, 0.6284)$ with the real error probabilities $\alpha'_v = \beta'_v = 0.1365$.

To save the inspection costs, we chose the variables scheme and obtained coordinates of the 6 sampled bullets, that is, $(-0.82, -0.51)$, $(0.59, -0.06)$, $(-0.74, 0.39)$, $(0.24, 0.24)$, $(-0.33, 0.93)$, and $(0.17, 0.88)$. Since $\overline{R}_{n_v} = \sum_{i=1}^{6} R_i/6 = 0.6460 > 0.6284$, the null hypothesis in (1.4) or in (1.1) was rejected. However, it is clear that all of the sampled bullets are conforming since $R_i \leq 1, \quad i = 1, \ldots, 6$. Thus, the judgement leads to some problems from attributes aspect as stated below.

(i) The basic rule of attributes plans is to accept a lot when none of the sampled items is nonconforming. Obviously, our test result is contradictory to this rule.

(ii) The producer can hardly be convinced about the decision of rejecting the lot when no nonconforming item is found. The producer may argue that the lot should be accepted even under the attributes plan $(\tilde{n}_a, \tilde{c}_a) = (6, 0)$, which guarantees the consumer's risk since $\beta' = (1 - 0.4)^6 = 0.0467 < \beta$, although it employs the producer's risk as large as $\alpha' = 1 - (1 - 0.1)^6 = 0.4686$. Further, for the hypothesis "$p = p_1 = 0.4$" with $\{d_{\tilde{n}_a} = \sum_{i=1}^{\tilde{n}_a} I_{(a,+\infty)}(R_i) \leq c\}$ ($I(\cdot)$ is an indicator function) as its form of rejection region, the P-value of the test result is $P_{p_1}(d_{\tilde{n}_a} \leq 0) = (1 - 0.4)^6 = 0.0467$. Therefore, the decision of rejecting the hypothesis "$p = p_1 = 0.4$" should be made with a quite sufficient evidence under the significance level $\gamma = 0.05$.

Motivated by this case and other similar problems in engineering (e.g., [3–8]), we try to find the answer about which one is more suitable for testing the percent nonconforming, the variables plan or the attributes plan. Scholars have different opinions and arguments in their papers. Some (e.g., [2]) support attributes plans due to their better protection for the two types of error probabilities. Some (e.g., [9, 10]) insist that variables plans be advantageous and acceptable according to the evaluation on some loss functions under superpopulations.

However, it is not necessarily a dichotomy scenario. Some scholars choose compromised schemes, that is, the mixed variables-attributes sampling plans, most of which (e.g., [11–15]) are double sampling procedures. For single acceptance sampling, Kao [16] proposes the mixed plan which permits to accept a lot when the number of nonconforming items is small and the conforming items in the sample are uniform under some variables characteristic. However, since it mainly aims at testing item variability and considers two independent quality characteristics for attributes and variables inspections, respectively, the mixed plan in [16] is inapplicable to our case. Anyway, the present mixed plans provide a way to find out a solution to our problem.

In Section 2, we propose the mixed variables-attributes plans for single acceptance sampling (single mixed plans). To make a comparison, we also obtain the existing mixed plans for double acceptance sampling (double mixed plans) with the derived performance formulas in Section 3. Summaries and further remarks are provided in Section 4. The appendix gives the related formula in evaluating performance of the mixed plans. Due to our interest in the bullets test problem, we only discuss the exponential distribution case, which is also vital for life testing according to [12].

2. Mixed Plans for Single Acceptance Sampling

To design the mixed variables-attributes schemes for single acceptance sampling, a key is to determine the form of the rejection region. For the bullets test problem in this paper, taking $A = \{\overline{R}_n = \sum_{i=1}^{n} R_i/n > c_r\}$ ($c_r \geq 0$) as the rejection region for variables sampling and $B = \{d_n = \sum_{i=1}^{n} I_{(a,+\infty)}(R_i) > c_d\}$ (c_d is an integer in $[0, n-1]$) as the counterpart for attributes sampling, we set the rejection region of the single mixed plans as $AB = \{\overline{R}_n > c_r\} \bigcap \{d_n > c_d\}$. Then, a single mixed plan contains 3 parameters, that is, the sample size n, the critical values c_r for variables measurement and c_d for attributes measurement.

Since $c_d \geq 0$, the single mixed plans can guarantee the rule of accepting a lot when no nonconforming item is found in the sample. Moreover, variables and attributes information can be integrated in the single mixed plans to obtain more convincible and applicable results.

However, the feature of discrete distributions (e.g., the binomial distribution) for the number of nonconforming items d_n impairs the flexibility of designing the plans. Therefore, sample sizes of the single mixed plans are larger than or equal to those of the single variables plans with the same requirements on error probabilities.

Next, we discuss how to determine the operating characteristic (OC) formula of the single mixed plan (n, c_r, c_d). With the rejection region AB, the probability of accepting the null hypothesis under the percent nonconforming p should be

$$L(p) = 1 - P_p(AB) = 1 - \sum_{k=c_d+1}^{n} P_p\left(\overline{R}_n > c_r, d_n = k\right). \tag{2.1}$$

As will be shown in the sequel, the OC formulas of the single mixed plans share the same difficulty as those of the double mixed plans in terms of the quantity $P(\overline{R}_n > c_r, d_n = k)$ in (2.1). According to [12], finding the OC formulas of the double mixed plans involves "severe technical difficulties" in two aspects. First, the random variable R_i is truncated as $R_i > a$ or $R_i \leq a$ $(i = 1, \ldots, n)$, which is specified by the number of nonconforming items d_n. Commonly, it is not readily to obtain the distributions of algebraic functions of truncated random variables such as $\overline{R}_n$. Second, the random variables in $\overline{R}_n$ are not identically distributed when $0 < d_n < n$.

Efforts have been made to derive the OC formulas of the double mixed plans. Savage [12] obtains the OC formulas for $d_n = 0$ and 1 based on the convolution for the exponential distribution. Although it is accessible in theory, the method is so complex that it is quite hard to derive the OC formulas for $d_n > 1$. For the double mixed plans under the normal distribution, Gregory and Resnikoff [11] provides the approximate OC formulas through numerical integration and asymptotic expansion, respectively. Thereafter, the exact OC formulas under the normal distribution are set up with a recursive form and tables about some related values are provided for applications in [13, 17–19].

In Theorem A.1 of the appendix, we derive an accurate formula for $P(\overline{R}_n > c_r, d_n = k)$ to obtain the OC curve of the mixed plans under the exponential distribution. It is simple in form and easy in operation, which is believed to further increase attractions of the mixed plans.

According to Theorem A.1 in the appendix, the probability of accepting the null hypothesis under the single mixed plans is

$$L(p) = 1 - \left\{ \sum_{k=c_d+1}^{n} \binom{n}{k} \sum_{i=0}^{n-k} (-1)^i \binom{n-k}{i} \left[1 - F_{\sigma_p}(nc_r - (k+i)a)\right] \exp\left\{ -\frac{(k+i)a}{2\sigma_p^2} \right\} \right\}, \tag{2.2}$$

where $F_{\sigma_p}(\cdot)$ is the cumulative distribution function (c.d.f.) of Gamma $(n, 2\sigma_p^2)$, and σ_p is obtained from (1.3).

Finally, we determine the parameters (n, c_r, c_d) with two proposed procedures here. One aims at protecting "good lots" from attributes aspect, and the other is to achieve optimization by minimizing the sample size.

2.1. Single Mixed Plan I: Protecting "Good Lots"

In Section 1, we point out the awkward problem of variables plans, that is, a "good lot" from the attributes aspect possibly being rejected. To solve the problem, the single mixed plan is designed to guarantee "good lots" to be accepted. This is called the single mixed plan I.

Then, the point is how to define a "good lot" from attributes aspect. According to the discussion in Section 1, we consider to find a proper region $\{d_n \le c_d\}$ for rejecting the hypothesis "$p = p_1$" with a sufficient evidence under the significance level γ. Then, when the number of nonconforming observations is not more than c_d, we have a sufficient evidence to conclude a "good lot" from attributes aspect. This provides a way to determine the critical value c_d of the single mixed plan I.

The sample size n of the single mixed plan I may be determined through searching in a computer program. In applications, the sample size n_v of the single variables scheme with the same error probabilities can be set as a starting value for searching n, since n must be larger than or equal to n_v as stated above.

As the most flexible value with continuous measurement, the critical value c_r should be determined according to requirements on the two error probabilities under some provided n and c_d.

The following steps provide a guidance to find the single mixed plan I.

Step 1. Find the sample size n_v of the single variables plan with the required error probabilities and set the initial sample size n as $n = n_v$.

Step 2. With the sample size n, determine the critical value c_d for "good lots" under the significance level γ.

Step 3. For the given n and c_d, search c_r through dichotomy and try to find a single mixed plan with the required two error probabilities. If the plan is not found, set $n = n + 1$ and go to Step 2.

To illustrate the procedure, we reconsider the bullets test problem in Section 1.

Example 2.1. In this example, we set up the single mixed plan I for the bullets test problem (1.1) under the two error probabilities $\alpha = \beta = 0.15$. Here, the significance level γ for defining "good lots" is set as 0.05. The steps are the following.

(i) According to the results in Section 1, the single variables plan is $(n_v, c_v) = (6, 0.6284)$. Thus, we set $n = 6$.

(ii) For $n = 6$, the region for rejecting "$p = p_1 = 0.4$" is $\{d_n \le 0\}$ under $\gamma = 0.05$. Therefore, we have $c_d = 0$.

(iii) After searching c_r through dichotomy in a computer program, we find that when $c_r = 0.6259$, the single mixed plan $(n, c_r, c_d) = (6, 0.6259, 0)$ makes $\alpha' = \beta' = 0.1368 < \alpha = \beta = 0.15$. Thus, the determined single mixed plan I is $(n, c_r, c_d) = (6, 0.6259, 0)$.

With the data of the 6 inspected bullets in Section 1, since $d_n = 0 = c_d$, the decision of accepting the null hypothesis in (1.1) should be made under the single mixed plan I $(n, c_r, c_d) = (6, 0.6259, 0)$.

More illustrations of the single mixed plan I are provided in Example 2.2.

Example 2.2. For the bullets test problem (1.1), let the two error probabilities be $\alpha = \beta = 0.10$ and 0.05, respectively. Also take $\gamma = 0.05$. Tables 2 and 3 list the single mixed plans I and their performance.

From Examples 2.1 and 2.2, it can be found that the sample sizes of the single mixed plans I are equal to or a little more than those of the single variables plans with the same controlled error probabilities. In theory, sample sizes of some single mixed plans I can be improved by determining the parameter c_d more flexibly. With the principal objective of minimizing the sample size, we propose the single mixed plan II.

2.2. Single Mixed Plan II: Minimizing Sample Size

To minimize the sample size, we develop the single mixed plan II in this section. The main idea is to obtain a plan with minimized sample size by searching, and to optimize the critical value c_d (by maximizing c_d) to protect "good lots".

To make the sample size as small as possible under controlled error probabilities and to protect "good lots" at the same time, the single mixed plan II should be determined according to the following steps.

Step 1. Obtain the sample size n_v of the single variables plan and set $n = n_v$.

Step 2. Set $c_d = 0$.

Step 3. With the provided n and c_d, find the critical value c_r that maximizes α' under the constraint $\alpha' \le \alpha$. If no c_r satisfying $\alpha' \le \alpha$ is found, set $n = n + 1$ and go to Step 2.

Step 4. With the n, c_d and c_r from Step 3, calculate β'. If $\beta' < \beta$, set $c_d = c_d + 1$ and then go to Step 3; If $\beta' = \beta$, go to Step 5; If $\beta' > \beta$ and $c_d > 0$, make $c_d = c_d - 1$ and go to Step 5; Otherwise, set $n = n + 1$ and go to Step 2.

Step 5. Readjust c_r under the determined n and c_d to obtain a more appropriate plan when necessary.

Example 2.1 (continued). Continue the bullets test problem (1.1). The following indicates the process of finding the single mixed plan II.

(i) According to the variables plan $(n_v, c_v) = (6, 0.6284)$, we make $n = 6$.

(ii) Set $c_d = 0$.

(iii) Under $n = 6$ and $c_d = 0$, the searched value is $c_r = 0.6121$ which makes $\alpha' = 0.15 = \alpha$.

(iv) For the mixed plan $(n, c_r, c_d) = (6, 0.6121, 0)$, we have $\beta' = 0.1277$. Since $\beta' < \beta$, set $c_d = 0 + 1 = 1$.

(v) Under $n = 6$ and $c_d = 1$, search c_r. The result shows that when $c_r = 0$, α' may reach its maximum value 0.1143.

Table 1: Plans for $p_0 = 0.1$, $p_1 = 0.4$, $\alpha = \beta = 0.15$.

Plans	$n(n_1)$	n_2	c_r	$c_d(c_1)$	c_2	α'	β'	ASN(p_0)	ASN(p_1)
SMI	6	—	0.6259	0	—	0.1368	0.1368	6	6
SMII	6	—	0.6259	0	—	0.1368	0.1368	6	6
SV	6	—	0.6284	—	—	0.1365	0.1365	6	6
SA	11	—	—	2	—	0.0896	0.1189	11	11
DIM	5	5	0.4802	—	0	0.1446	0.1446	6.7657	9.6376
DDM	4	4	0.4281	1	1	0.1480	0.1480	5.5699	5.6025
DV	3	3	0.2741	0.7937	0.6467	0.1492	0.1498	4.8484	3.9944
DA	5	6	0	2	2	0.1189	0.1382	6.9683	6.5552

Notations: SMI: single mixed plan I; SMII: single mixed plan II; SV: single variables plan; SA: single attributes plan; DIM: double independent mixed plan; DDM: double dependent mixed plan; DV:double variables plan; DA: double attributes plan.

Table 2: Plans for $p_0 = 0.1$, $p_1 = 0.4$, $\alpha = \beta = 0.10$.

Plans	$n(n_1)$	n_2	c_r	$c_d(c_1)$	c_2	α'	β'	ASN(p_0)	ASN(p_1)
SMI	9	—	0.6106	1	—	0.0972	0.0972	9	9
SMII	9	—	0.6106	1	—	0.0972	0.0972	9	9
SV	9	—	0.6404	—	—	0.0880	0.0880	9	9
SA	15	—	—	3	—	0.0556	0.0905	15	15
DIM	7	8	0.5813	—	0	0.0997	0.0997	8.4010	14.3251
DDM	8	3	0.6194	1	1	0.0994	0.0994	8.0789	8.1281
DV	5	5	0.4210	0.8043	0.6349	0.0926	0.0926	7.1051	6.3171
DA	7	8	0	3	3	0.0630	0.0970	10.9681	10.1353

(vi) For the plan $(n, c_r, c_d) = (6, 0, 1)$, we have $\beta' = 0.2333 > \beta$. Therefore, make $c_d = 1 - 1 = 0$.

(vii) With $n = 6$ and $c_d = 0$, we search c_r again and find that when $c_r = 0.6259$, $\alpha' = \beta' = 0.1368 < 0.15$. Thus, the determined single mixed plan II is $(n, c_r, c_d) = (6, 0.6259, 0)$, which is the same as the single mixed plan I.

Example 2.2 (continued). The single mixed plans II for $\alpha = \beta = 0.10$ and 0.05 are also listed in Tables 2 and 3.

In Examples 2.1 and 2.2, all of the sample sizes of the single mixed plans II are the same as those of the single variables plans. It indicates that the single mixed plans II are attractive and practical due to the comparative costs with the single variables plans and the ability of protecting "good lots" to some degree.

To conduct further comparisons, we provide the performance formulas of the double mixed plans under the exponential distribution in the next section.

3. Mixed Plans for Double Acceptance Sampling

The double mixed plans were proposed decades ago and then were developed and applied in practice (e.g., [11–15, 17–19]). There are two forms of double mixed plans: the independent mixed plans and dependent mixed plans.

Table 3: Plans for $p_0 = 0.1$, $p_1 = 0.4$, $\alpha = \beta = 0.05$.

Plans	$n(n_1)$	n_2	c_r	$c_d(c_1)$	c_2	α'	β'	ASN(p_0)	ASN(p_1)
SMI	15	—	0.6329	2	—	0.0441	0.0441	15	15
SMII	14	—	0.6475	1	—	0.0452	0.0452	14	14
SV	14	—	0.6490	—	—	0.0448	0.0448	14	14
SA	24	—	—	5	—	0.0277	0.0400	24	24
DIM	13	8	0.6059	—	0	0.0494	0.0494	13.6941	20.7346
DDM	12	7	0.5960	3	3	0.0497	0.0497	12.5804	13.3319
DV	8	8	0.5038	0.8238	0.6230	0.0498	0.0496	10.2071	9.8114
DA	12	9	1	5	4	0.0488	0.0465	15.0300	15.7673

3.1. Double Independent Mixed Plans

Under a double independent mixed plan (n_1, n_2, c_r, c_2), we should inspect n_1 items with variables measurement and accept the null hypothesis when $\overline{R}_{n_1} \le c_r$. If $\overline{R}_{n_1} > c_r$, another n_2 items should be inspected with attributes measurement. In the second stage, a decision is made based on the number of nonconforming items d_{n_2} in the n_2 items, that is, accepting H_0 when $d_{n_2} \le c_2$ or rejecting H_0 otherwise.

Obviously, the OC curve of the double independent mixed plan is

$$L(p) = P_p\left(\overline{R}_{n_1} \le c_r\right) + P_p\left(\overline{R}_{n_1} > c_r\right) P_p(d_{n_2} \le c_2). \tag{3.1}$$

And its ASN formula is

$$\mathrm{ASN}(p) = n_1 + n_2 P_p\left(\overline{R}_{n_1} > c_r\right). \tag{3.2}$$

The probabilities in (3.1) and (3.2) are easy to obtain because only one kind of measurement is considered in each of the stages, and the two stages are independent. Designing and selection of the double independent mixed plans for the normal distribution are discussed in [14, 15]. In the later examples, optimal double independent mixed plans, that is, plans with minimum ASN at some p, are obtained by searching in a computer program.

3.2. Double Dependent Mixed Plans

For a double dependent mixed plan $(n_1, n_2, c_r, c_1, c_2)$, n_1 items should be tested with variables and attributes measurements. In the first stage, we can accept H_0 if $\overline{R}_{n_1} \le c_r$ or reject H_0 if $d_{n_1} > c_1$ where d_{n_1} is the number of nonconforming items in the n_1 observations. Otherwise, another n_2 items should be inspected with attributes measurement. In the second stage, we should accept H_0 if $d_{n_1+n_2} \le c_2$ or reject H_0 otherwise, where $d_{n_1+n_2}$ is the total number of nonconforming items in the $n_1 + n_2$ observations.

For the bullets test problem, according to Theorem A.1 in the appendix, the OC curve of the double dependent mixed plan is

$$
\begin{aligned}
L(p) &= P_p\left(\overline{R}_{n_1} \le c_r\right) + P_p\left(\overline{R}_{n_1} > c_r, d_{n_1} \le c_1, d_{n_1+n_2} \le c_2\right) \\
&= P_p\left(\overline{R}_{n_1} \le c_r\right) + \sum_{l=0}^{c_1} P_p\left(\overline{R}_{n_1} > c_r, d_{n_1} = l\right) P_p\left(d_{n_2} \le c_2 - l\right) \\
&= F_{\sigma_p}(n_1 c_r) + \sum_{l=0}^{c_1} \binom{n_1}{l} \sum_{i=0}^{n_1-l} (-1)^i \binom{n_1 - l}{i} \left[1 - F_{\sigma_p}(n_1 c_r - (l+i)a)\right] \exp\left\{-\frac{(l+i)a}{2\sigma_p^2}\right\} \\
&\quad \times P_p\left(d_{n_2} \le c_2 - l\right),
\end{aligned}
\tag{3.3}
$$

where d_{n_2} is the number of nonconforming items in the second-stage n_2 observations.

The ASN formula of the double dependent mixed plan is

$$
\begin{aligned}
\mathrm{ASN}(p) &= n_1 + n_2 P_p\left(\overline{R}_{n_1} > c_r, d_{n_1} \le c_1\right) \\
&= n_1 + n_2 \sum_{l=0}^{c_1} \binom{n_1}{l} \sum_{i=0}^{n_1-l} (-1)^i \binom{n_1 - l}{i} \left[1 - F_{\sigma_p}(n_1 c_r - (l+i)a)\right] \exp\left\{-\frac{(l+i)a}{2\sigma_p^2}\right\}.
\end{aligned}
\tag{3.4}
$$

According to Schilling [20], a double dependent mixed plan in MIL-STD-414 is designed by combining the corresponding variables plan and attributes plan. In this procedure, n_1 should be much less than n_2, and c_1 is set equal to c_2. Although it can not find an exact double dependent mixed plan, this procedure is helpful to find an approximately matchable plan in a simple way. However, we find that such a design does not produce minimum ASN.

For the bullets test problems in Examples 2.1 and 2.2, we determine the double mixed plans with minimum $\mathrm{ASN}(p_0)$ and then possibly small $\mathrm{ASN}(p_1)$ by searching in a computer program. The plans and their performance are provided in Tables 1–3.

Further, the double variables plans and double attributes plans are found to make a comparison. Under the double variables plan, the null hypothesis is accepted when $\overline{R}_{n_1} \le c_r$ or is rejected when $\overline{R}_{n_1} \ge c_1$ at the first stage with a sample of size n_1. If $c_r < \overline{R}_{n_1} < c_1$, a second sample of size n_2 is needed and the null hypothesis is accepted when $\overline{R}_{n_1+n_2} \le c_2$ or is rejected otherwise. For the double attributes plan, the null hypothesis is accepted if $d_{n_1} \le c_r$ or is rejected if $d_{n_1} \ge c_1$ for the first-stage n_1 observations. If $c_r < d_{n_1} < c_1$, a second sample of size n_2 is inspected and the null hypothesis is accepted if $d_{n_1+n_2} \le c_2$ or is rejected otherwise.

To better analyze the performance of the mixed plans, the OC and ASN curves are illustrated in Figures 1, 2, 3, 4, 5, and 6. Interestingly, all of the plans have almost the same OC curves except the single attributes plans. Therefore, we provide the OC curves of the single mixed plans II (as a representative) and the single attributes plans in Figures 1–3. Figures 4-6 show the ASN curves of the double acceptance sampling plans.

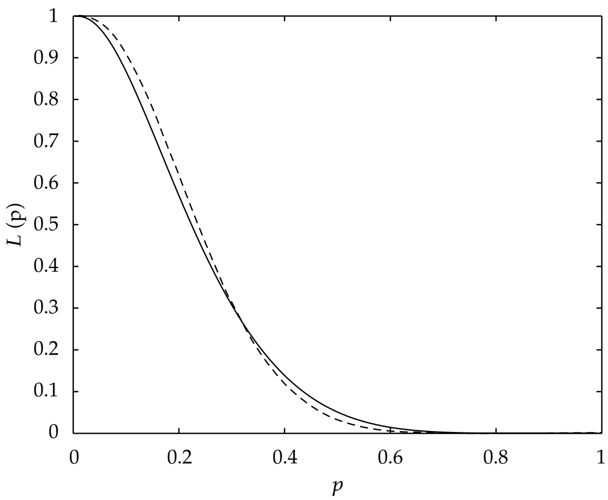

Figure 1: OC curves of plans for $p_0 = 0.1,\ p_1 = 0.4,\ \alpha = \beta = 0.15$ ("—": OC curve of the single mixed plan II; "- -": OC curve of the single attributes plan).

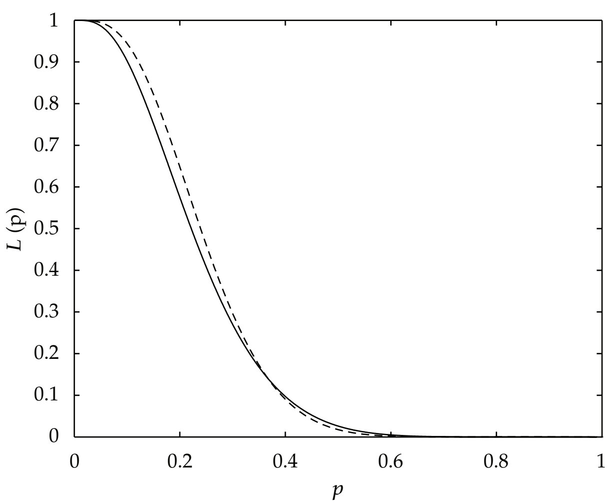

Figure 2: OC curves of plans for $p_0 = 0.1,\ p_1 = 0.4,\ \alpha = \beta = 0.10$ ("—": OC curve of the single mixed plan II; "- -": OC curve of the single attributes plan).

According to Figures 1–3, the single attributes plans perform a little stronger protection for producer and consumer's risks than other plans. Based on Figures 4–6, it is clear that the double dependent mixed plans are preferable in ASN than the double independent mixed plans. Especially, the ASN curves of the double independent mixed plans increase as p becomes large, while the double dependent mixed plans have small ASNs beyond (p_0, p_1). Moreover, the ASN curves of the double dependent mixed plans lie between those of the double variables plans and the double attributes plans in (p_0, p_1).

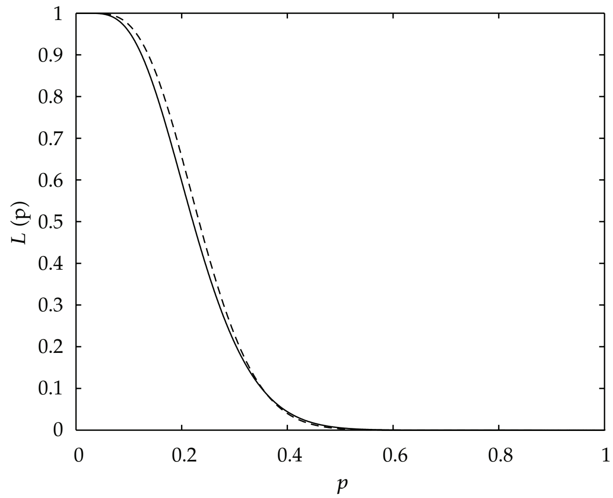

Figure 3: OC curves of plans for $p_0 = 0.1$, $p_1 = 0.4$, $\alpha = \beta = 0.05$ ("—": OC curve of the single mixed plan II; "- -": OC curve of the single attributes plan).

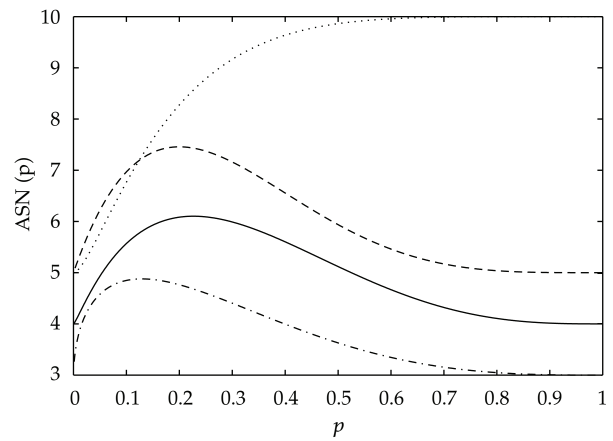

Figure 4: ASN curves of double plans for $p_0 = 0.1$, $p_1 = 0.4$, $\alpha = \beta = 0.15$ (":": ASN curve of the double independent mixed plan; "—": ASN curve of the double dependent mixed plan; "–·": ASN curve of the double variables plan; "- -": ASN curve of the double attributes plan).

4. Conclusions and Remarks

We have presented mixed plans for the single and double acceptance sampling problems. Our proposed methods combine certain strengths of the variables sampling plans and the attributes sampling plans. Numerical examples have shown that they perform well in various cases.

Compared to the variables sampling plans, the major advantages of our proposed mixed plans are as follows. (i) The mixed plans avoid the problem of rejecting a lot when no nonconforming item is inspected. (ii) Since attributes results are considered, the mixed plans can tolerate small quantity of outliers which depart from the "natural" form of the underlying distribution and then protect a generally good lot. Hence, the inference on the percent

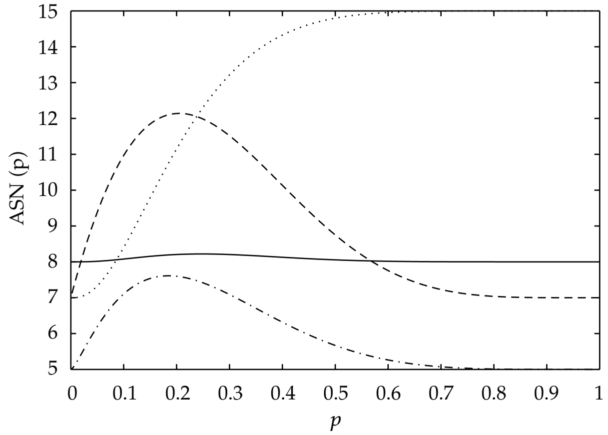

Figure 5: ASN curves of double plans for $p_0 = 0.1$, $p_1 = 0.4$, $\alpha = \beta = 0.10$ (":": ASN curve of the double independent mixed plan; "—": ASN curve of the double dependent mixed plan; "–·": ASN curve of the double variables plan; "- -": ASN curve of the double attributes plan).

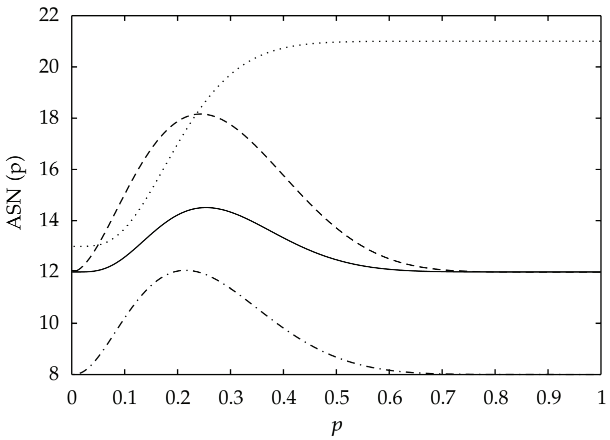

Figure 6: ASN curves of double plans for $p_0 = 0.1$, $p_1 = 0.4$, $\alpha = \beta = 0.05$ (":": ASN curve of the double independent mixed plan; "—": ASN curve of the double dependent mixed plan; "–·": ASN curve of the double variables plan; "- -": ASN curve of the double attributes plan).

nonconforming does not depend on the assumed distribution so much like the variables sampling. (iii) For rejected lots, the actual number of nonconforming items is helpful to convince producers psychologically.

Compared to the attributes sampling plans, the major advantages of our proposed mixed plans are as follows. (i) The mixed plans can reduce sample sizes compared with the attributes plans with the same error probabilities. (ii) The mixed plans contain a far more careful inspection and analysis based on variables measurement. (iii) Due to the variables feature, the mixed plans introduce more flexibility to obtain desired plans with controlled risks.

With our derived formulas for evaluating their exact performance, our proposed mixed plans should be useful for various applications, especially for those in which the related population distributions are exponential. Comparing the proposed single mixed plans with the double mixed plans, the former may be more convenient to use, but the latter would require less average sample sizes. When the test cost is our major concern and the quality is assured near p_0 or p_1, the double dependent mixed plans are recommended. Otherwise, we recommend to use the single mixed plans for testing the percent nonconforming conveniently and cheaply.

In our opinion, much future research is required to study the possibility to optimize the mixed plans, especially the double mixed plans. Further, the development of mixed plans for the sequential sampling cases is still an open problem and requires much future research as well.

Appendix

A Related Formula for Mixed Plans' Performance

Theorem A.1. *Suppose $R_1,\dots,R_n$ are i.i.d. random variables from the exponential distribution* $\mathrm{Exp}(2\sigma^2)$ *where $2\sigma^2$ is the expectation of R_i $(i = 1,\dots,n)$. Let $\overline{R}_n = \sum_{i=1}^{n} R_i/n$, $d_n = \sum_{i=1}^{n} I_{(a,+\infty)}(R_i)$. Then, for any integer l $(0 \le l \le n)$,*

$$P\left(\overline{R}_n > c_r, d_n = l\right) = \binom{n}{l}\sum_{i=0}^{n-l}(-1)^i\binom{n-l}{i}\left[1 - F_\sigma(nc_r - (l+i)a)\right]\exp\left\{-\frac{(l+i)a}{2\sigma^2}\right\}, \quad \text{(A.1)}$$

where $F_\sigma(\cdot)$ is the c.d.f. of Gamma $(n, 2\sigma^2)$.

Proof. Let $E_i = \{0 \le R_i \le a\}$, $i = 1,\dots,n$; $C_k = \{0 \le R_1,\dots,R_k \le a, R_{k+1},\dots,R_n > a, \overline{R}_n > c_r\}$, $k = 0,\dots,n$; $D_k = \{R_k,\dots,R_n > a, \overline{R}_n > c_r\}$, $k = 1,\dots,n$, $D_{n+1} = \{\overline{R}_n > c_r\}$.

Since $D_{k+1} = C_k + \bigcup_{i=1}^{k}(D_{k+1}E_i^c)$ $(k = 0,1,\dots,n)$, we have

$$\begin{aligned}
P(C_k) &= P(D_{k+1}) - P\left(\bigcup_{i=1}^{k}(D_{k+1}E_i^c)\right) \\
&= P(D_{k+1}) - \left[\sum_{i=1}^{k}P(D_{k+1}E_i^c) - \sum_{1\le i<j\le k}P\left(D_{k+1}E_i^cE_j^c\right) + \cdots + (-1)^{k-1}P\left(D_{k+1}E_1^c\cdots E_k^c\right)\right] \\
&= P(D_{k+1}) - \binom{k}{1}P(D_k) + \binom{k}{2}P(D_{k-1}) + \cdots + (-1)^kP(D_1) \\
&= \sum_{i=0}^{k}(-1)^i\binom{k}{i}P(D_{k+1-i}), \quad k = 0,1,\dots,n.
\end{aligned} \quad \text{(A.2)}$$

As R_i $(i = 1, \ldots, n)$ are i.i.d. $\sim \mathrm{Exp}(2\sigma^2)$, it is obvious that $R_i - a \mid R_i > a$ $(i = 1, \ldots, n)$ are i.i.d. $\sim \mathrm{Exp}(2\sigma^2)$. Therefore,

$$\begin{aligned}
P(D_k) &= P\left(R_k, \ldots, R_n > a, \overline{R}_n > c_r\right) \\
&= P\left(\overline{R}_n > c_r \mid R_k, \ldots, R_n > a\right) P(R_k, \ldots, R_n > a) \\
&= P(R_1 + \cdots + R_{k-1} + (R_k - a) + \cdots + (R_n - a) > nc_r - (n-k+1)a \mid R_k, \ldots, R_n > a) \\
&\quad \times P(R_k, \ldots, R_n > a) \\
&= [1 - F_\sigma(nc_r - (n-k+1)a)] \exp\left\{-\frac{(n-k+1)a}{2\sigma^2}\right\}, \quad k = 1, \ldots, n+1.
\end{aligned} \tag{A.3}$$

Then, for $0 \le l \le n$,

$$\begin{aligned}
P\left(\overline{R}_n > c_r, d_n = l\right) &= \binom{n}{l} P(C_{n-l}) \\
&= \binom{n}{l} \sum_{i=0}^{n-l} (-1)^i \binom{n-l}{i} P(D_{n-l+1-i}) \\
&= \binom{n}{l} \sum_{i=0}^{n-l} (-1)^i \binom{n-l}{i} [1 - F_\sigma(nc_r - (l+i)a)] \exp\left\{-\frac{(l+i)a}{2\sigma^2}\right\}.
\end{aligned} \tag{A.4}$$

The proof is completed. □

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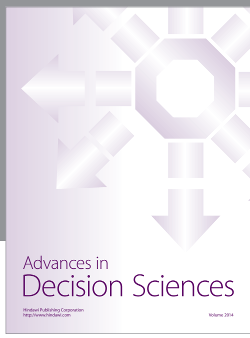

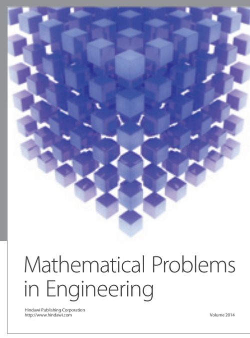

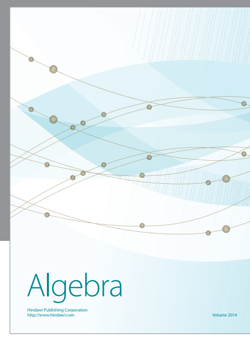

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