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Research Article

Fatigue Reliability Sensitivity Analysis of Complex Mechanical Components under Random Excitation

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Fatigue failure is the typical failure mode of mechanical components subjected to random load-time history. It is important to ensure that the mechanical components have an expected life with a high reliability. However, it is difficult to reduce the influence of factors that affect the fatigue reliability and thus a reliability sensitivity analysis is necessary. An approach of fatigue reliability sensitivity analysis of complex mechanical components under random excitation is presented. Firstly, load spectra are derived using a theoretical method. A design of experiment (DOE) is performed to study the stresses of dangerous points according to the change of design parameters of the mechanical component. By utilizing a Back-Propagation (BP) algorithm, the explicit function relation between stresses and design parameters is formulated and thus solves the problem of implicit limit state function. Based on the damage accumulation (DA) approach, the probability perturbation method, the fourth-moment method, the Edgeworth expansion is adopted to calculate the fatigue reliability and reliability-based sensitivity. The fatigue reliability sensitivity analysis of a train wheel is performed as an example. The results of reliability are compared with that obtained using Monte Carlo simulation. The reliability sensitivity of design parameters in the train wheel is analyzed.

1. Introduction

Mechanical components subjected to random excitation contain a lot of uncertain factors, such as external loads, material properties, and structure geometry. The case may consequently lead to an uncertainty of the fatigue life of the components. In order to ensure the safety and reliability of designed structures with an expected fatigue life, it is essential to take the uncertain factors into account during the process of design. Reliability sensitivity analysis refers to the partial derivative of the reliability with respect to basic random variables. It ranks the distribution parameters of the design variables and guides the reliability design; thus, it is important to assess the reliability sensitivity of the mechanical components. Besides, due to that the fatigue of mechanical components under random excitation is a process of damage accumulation, it is of interest to evaluate the variation of the reliability and reliability sensitivity of the mechanical components.

The methods of reliability design and reliability sensitivity design based on classical probability theories have been greatly developed in recent decades [1–7]. These publications have presented accurate and efficient computational reliability sensitivity methods. However, for complex mechanical structures, the limit-state function is usually available only in an implicit form; methods that require limit-state function gradients with respect to the basic variables, such as FORM and SORM, could not be well applied as their performance is affected. In such situations, the response surface method provides an efficacious tool for estimating the structure reliability. Another technique to obtain the approximate model of the limit-state function is artificial neural network [8].

In engineering practice, it is difficult to determine the distribution of random variables, and frequently, sufficient data are unavailable. As a result, the joint probability density or distribution function for reliability and reliability sensitivity analysis is difficult to obtain. However, the first few moments, such as the mean, variance, the third moment, and the fourth moment of the random variables can be evaluated with the available data.

This paper focuses on the fatigue reliability and the reliability sensitivity of mechanical components under random excitation. Based on the fatigue accumulation damage theory, reliability-based design theory, and sensitivity analysis approach, using the stochastic finite-element method, the design of experiment, artificial neural networks, the stochastic perturbation method, and the Edgeworth series, this paper proposes a practical and efficient computation framework to calculate the fatigue reliability and the reliability sensitivity of mechanical components under random excitation with arbitrary distribution parameters.

2. External Excitation Processing

The primary problem of conducting a fatigue analysis is to process the load applied on the component, including the deterministic and random loads. For random load processing, cycle-counting methods are commonly used to count stress cycles in stress-time histories. The rainflow cycle-counting method, which gives better counting results and approximate fatigue damage compared to practical conditions, is widely used [9].

The main procedure of random load processing using rainflow cycle-counting method is: measure the load-time history $\rightarrow$ compress the obtained load-time history $\rightarrow$ rainflow cycle-counting $\rightarrow$ compile load spectrum. Regularly, the load spectrum is divided into 8 load grades to exactly reflect the actual fatigue effect. However, it is usually hard to measure the actual load-time history for reasons such as high costs and lacking test conditions. Thus, a programmed load spectrum is often adopted for theoretical analysis. Conover et al. [10] suggest that the amplitude ratio of 8 stress grades could be 1.0, 0.95, 0.85, 0.725, 0.575, 0.425, 0.375, 0.125, which divides the load spectrum into 8 program segments. The load cycles of each load grade are equivalent to the frequency of each program segment. By combining 8 program segments together, a one-period load spectrum can be obtained.

3. Problem of Implicit Limit-State Function

Among the existing reliability calculation methods, an explicit limit-state function is required to analyse the reliability of components [11]. However, for big, complex components,

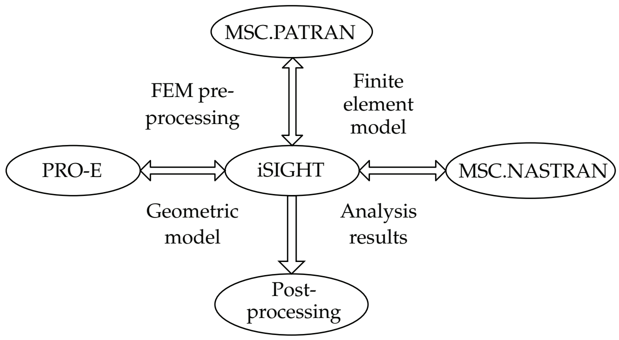

Figure 1: Schematic diagram of the integration process.

it converts to an implicit limit-state function problem which needs a finite-element analysis to obtain the data of responses such as stress amplitudes.

When conducting a reliability analysis, the uncertainties of random variables, such as structural dimensions, material properties, and external excitions, should be fully taken into consideration. Therefore, a three-dimensional model of the component should be parameterized and built. In this case, the model can be easily rebuilt by modifying driving parameters so that a stochastic finite element analysis can be carried out.

The purpose of stochastic finite element simulation is to obtain the mutative stress responses of the component under random load caused by uncertainties of random variables. For fatigue reliability analysis, the stress-time history of dangerous points could be obtained. In order to realize the simulation, an integrated process is performed to combine the three-dimensional modeling software and the finite element analysis software. Many tools for integration could be adopted, and here a multidisciptlinary design optimization software, iSIGHT [12], is utilized by preference. iSIGHT is an open and integrated platform; by using its process integration function, a typical design process may include some of the following tools: CAD software, CAE software, MATLAB, EXCEL, and so forth. In this paper, software such as PRO-E, MSC.PATRAN, and MSC.NASTRAN, and adopted, and Figure 1 is given as an example to illustrate the integration process using iSIGHT.

Besides, to conduct a reliability sensitivity analysis, an explicit function of stress with respect to design parameters is required, which can be gained using response surface method (RSM) or artificial neural networks (ANN) by fitting the samples of stress responses of dangerous points. The samples could be obtained by DOE, and many DOE methods could be adopted such as D-optimal design, orthogonal experiment design, uniform experimental design, Latin square design, and so forth [13]. iSIGHT provides uniform experimental design, central composite design, Latin hypercube sampling design, and so forth. Due to that the Latin hypercube sampling design get the samples randomly between the upper or lower limit of random variables, it cannot reflect the fact with inadequate sample number [14]. The uniform experimental design overcomes such a disadvantage and is thus used in this paper. For the selection of variation coefficients of random variables, larger ones which are set as 3σ according to the dimension tolerance are generally preferred for that as it is easier to simulate the stress variation under the condition of finite sample size. For the places where especially serious stress concentration occurs, small variation coefficients may lead to bigger errors with low accuracy of FEM calculation.

RSM and ANN are both effective techniques for estimating structure reliability by approximating real limit-state functions. Traditional RSM uses a quadratic function $B(\mathbf{X}) = \mathbf{X}^T A\mathbf{X}$ to adjust polynomials to the limit-state function. However, if the limit-state function is very complex with higher-order nonlinearity, the method cannot ensure a precise calculation result. As an alternative, ANN technique provides a new and practical tool to simulate the limit-state functions. Hecht-Nielsen [15] points that a three-layer feed forward ANN can fit a function with any required accuracy. Thus, a mostly used BP network is built here with three layers.

Considering that the dispersion of random samples would affect the test results, a different method should be used to get the test samples in the case that the generalization ability of neural network be weakened. Monte Carlo method is thus adopted as an alternative. Besides, because it is hard to determine the location of failure points, mean values are generally used for testing. Therefore, small variance of random samples generated by Monte Carlo method should be given to ensure that most sample points are in the neighborhood of the mean values.

4. Application of the Cumulative Damage Theory

Based on the S-N curve, the stress spectrum, and fatigue damage models, the cumulative damage of the component can be attained [16–19]. Miner's rule is expressed as follows.

A failure of structure is expected to occur if

$$D = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \cdots + \frac{n_i}{N_i} = \sum_1^i \frac{n_i}{N_i} \geq 1, \tag{4.1}$$

where i is the stress level, n_i is the frequency of stress level I, and N_i is the frequency to failure at a specified stress amplitude of stress level i, respectively. In this study, the critical cumulative damage value of D is chosen to be 1 in (4.1).

The explicit function, which is obtained through the DOE study and the sample fitting as described before, is integrated into the S-N curve of the material and then the Miner fatigue cumulative damage model. Assuming that the explicit function of the stress responses is $S_i = S_i(\mathbf{X})$, where $\mathbf{X}$ is the random variables of the model. By integrating S_i into the S-N curve, it gives the explicit function $N_i = N_i(\mathbf{X})$, noting that N_i herein is corresponding to the N_i in (4.1).

According to the S-N curve of the material and Miner's rule, the cumulative damage of dangerous points can be calculated as follows:

$$D_c(\mathbf{X}) = \sum_1^i \frac{n_i (S_i(\mathbf{X}))^m}{C}, \tag{4.2}$$

where i is the stress level of the stress spectrum, n_i and S_i are the frequency and stress amplitude of stress level i, m and C are constants for an amended expression form is employed in this paper -N curve.

Based on the cumulative damage (4.1), a limit-state function for fatigue reliability analysis can be built. Assuming that the action time corresponding to the applied load spectra is t_0, the fatigue life of the component to fail under random excitations can be expressed as

$$T(\mathbf{X}) = \frac{t_0}{D_c(\mathbf{X})} = \frac{t_0}{\sum_1^i (n_i (S_i(\mathbf{X}))^m / C)}. \tag{4.3}$$

Given the expected life T_0, the limit-state function for reliability and reliability sensitivity analysis is defined as

$$G(\mathbf{X}) = T(\mathbf{X}) - T_0. \tag{4.4}$$

5. Random Perturbation Method

The vector of basic random design parameters $\mathbf{X}$ and the limit-state function $G(\mathbf{X})$ are expanded as

$$\begin{gathered} \mathbf{X} = \mathbf{X}_d + \varepsilon \mathbf{X}_p, \\ G(\mathbf{X}) = G_d(\mathbf{X}) + \varepsilon G_p(\mathbf{X}), \end{gathered} \tag{5.1}$$

where ε is a small parameter. The subscript d represents the certain part of the random parameters, while the subscript p represents the random part, and the random parameters have a zero mean value. The value of the random part should be smaller than the value of the certain part. Both sides of (5.1) are evaluated about the mean value of random variables as follows:

$$\mathrm{E}(\mathbf{X}) = \mathrm{E}(\mathbf{X}_d) + \varepsilon \mathrm{E}(\mathbf{X}_p) = \mathbf{X}_d = \overline{\mathbf{X}}, \tag{5.2}$$

$$\mathrm{E}[G(\mathbf{X})] = \mathrm{E}[G_d(\mathbf{X})] + \varepsilon \mathrm{E}[G_p(\mathbf{X})] = G_d(\mathbf{X}) = \overline{G}(\mathbf{X}). \tag{5.3}$$

Similarly, according to the Kronecker algebra, both sides of (5.1) are evaluated about the variance, the third moment, and the fourth moment of the random variables as follows:

$$\mathrm{Var}(\mathbf{X}) = \mathrm{E}\left\{ [\mathbf{X} - \mathrm{E}(\mathbf{X})]^{[2]} \right\} = \varepsilon^2 \mathrm{E}\left[\mathbf{X}_p^{[2]}\right], \tag{5.4}$$

$$C_3(\mathbf{X}) = \mathrm{E}\left\{ [\mathbf{X} - \mathrm{E}(\mathbf{X})]^{[3]} \right\} = \varepsilon^3 \mathrm{E}\left[\mathbf{X}_p^{[3]}\right], \tag{5.5}$$

$$C_4(\mathbf{X}) = \mathrm{E}\left\{ [\mathbf{X} - \mathrm{E}(\mathbf{X})]^{[4]} \right\} = \varepsilon^4 \mathrm{E}\left[\mathbf{X}_p^{[4]}\right], \tag{5.6}$$

$$\mathrm{Var}[G(\mathbf{X})] = \mathrm{E}\left\{ [G(\mathbf{X}) - \mathrm{E}(G(\mathbf{X}))]^{[2]} \right\} = \varepsilon^2 \mathrm{E}\left\{ [G_p(\mathbf{X})]^{[2]} \right\}, \tag{5.7}$$

$$C_3[G(\mathbf{X})] = \mathrm{E}\left\{ [G(\mathbf{X}) - \mathrm{E}(G(\mathbf{X}))]^{[3]} \right\} = \varepsilon^3 \mathrm{E}\left\{ [G_p(\mathbf{X})]^{[3]} \right\}, \tag{5.8}$$

$$C_4[G(\mathbf{X})] = \mathrm{E}\left\{ [G(\mathbf{X}) - \mathrm{E}(G(\mathbf{X}))]^{[4]} \right\} = \varepsilon^4 \mathrm{E}\left\{ [G_p(\mathbf{X})]^{[4]} \right\}, \tag{5.9}$$

where the Kronecker power is $\mathbf{P}^{[k]} = \mathbf{P}\otimes\mathbf{P}\otimes\cdots\otimes\mathbf{P}$, and the symbol $\otimes$ represents the Kronecker product which is defined as $(\mathbf{A})_{p\times q} \otimes (\mathbf{B})_{s\times t} = [ai_j\mathbf{B}]_{ps\times qt}$.

By expanding the limit-state function $G_p(\mathbf{X})$ to first-order approximation in a Taylor series of vector-valued functions and matrix-valued functions at a point $\mathrm{E}(\mathbf{X}) = \mathbf{X}_d$, which is on the failure surface $G_p(\mathbf{X}_d) = 0$, the expression of $G_p(\mathbf{X})$ is given

$$G_p(\mathbf{X}) = \frac{\partial G_d(\mathbf{X})}{\partial \mathbf{X}^T}\mathbf{X}_p. \tag{5.10}$$

Substituting (5.10) into (5.7)–(5.9), we obtain

$$\sigma_G^2 = \mathrm{Var}[G(\mathbf{X})] = \varepsilon^2 \mathrm{E}\left[\left(\frac{\partial G_d(\mathbf{X})}{\partial (\mathbf{X})^T}\right)^{[2]} \mathbf{X}_p^{[2]}\right] = \left(\frac{\partial G_d(\mathbf{X})}{\partial (\mathbf{X})^T}\right)^{[2]} \mathrm{Var}(\mathbf{X}), \tag{5.11}$$

$$\theta_G = C_3[G(\mathbf{X})] = \varepsilon^3 \mathrm{E}\left[\left(\frac{\partial G_d(\mathbf{X})}{\partial (\mathbf{X})^T}\right)^{[3]} \mathbf{X}_p^{[3]}\right] = \left(\frac{\partial G_d(\mathbf{X})}{\partial (\mathbf{X})^T}\right)^{[3]} C_3(\mathbf{X}), \tag{5.12}$$

$$\eta_G = C_4[G(\mathbf{X})] = \varepsilon^4 \mathrm{E}\left[\left(\frac{\partial G_d(\mathbf{X})}{\partial (\mathbf{X})^T}\right)^{[4]} \mathbf{X}_p^{[4]}\right] = \left(\frac{\partial G_d(\mathbf{X})}{\partial (\mathbf{X})^T}\right)^{[4]} C_4(\mathbf{X}), \tag{5.13}$$

where $\mathrm{Var}(\mathbf{X})$ is the variance matrix and $\mathbf{C}_3(\mathbf{X})$ and $\mathbf{C}_4(\mathbf{X})$ are the third and the fourth central moments matrix, respectively. σ_G^2, θ_G and η_G are the variance, the third, and the fourth central moments of the limit-state function $G(\mathbf{X})$, respectively.

As the random variables-vector follows normal distribution, the corresponding reliability index based on the first two moments of the limit-state function can be obtained. According to (5.3) and (5.7), the reliability index is as follows:

$$\beta_{2M} = \frac{\mu_G}{\sigma_G} = \frac{E[G(\mathbf{X})]}{\sqrt{\mathrm{Var}[G(\mathbf{X})]}}. \tag{5.14}$$

As a matter of fact, the acquired statistical data in engineering practice may only be sufficient to evaluate the first few moments such as mean, variance, the third moment, and the fourth moment of the random variables. In this case, the reliability index can be obtained [20, 21] by using the perturbation method, and the unknown probability distribution of the state function can be approximated as standard normal distribution by using the Edgeworth expansion. Thus, the reliability and the reliability sensitivity of mechanical components with arbitrary distribution variables can be obtained. According to the Edgeworth expansion [22],the probability distribution function of a standardized variable with arbitrary distribution is approximately expressed as the standard normal distribution function as follows:

$$F(y) = \Phi(y) - \varphi(y)\left[\frac{1}{6}\frac{\theta_G}{\sigma_G^3}H_2(y) + \frac{1}{24}\left(\frac{\eta_G}{\sigma_G^4} - 3\right)H_3(y) + \frac{1}{72}\left(\frac{\theta_G}{\sigma_G^3}\right)^2 H_5(y) + \cdots\right], \tag{5.15}$$

where $\Phi(\,)$ is the cumulative distribution function of a standard normal random variable, $\varphi(\,)$ is the standard normal probability density function, and $H_{i-1}(\,)$ is the Hermite polynomial

$$\begin{aligned} H_{j+1}(y) &= yH_j(y) - jH_{j-1}(y), \\ H_0(y) &= 1, \qquad H_1(y) = y. \end{aligned} \tag{5.16}$$

Thus, the failure probability P_{4M} is represented as

$$\begin{aligned} P_{4M} &= \Phi(-\beta_{2M}) - \varphi(-\beta_{2M}) \\ &\quad\times\left[\frac{1}{6}\frac{\theta_G}{\sigma_G^3}H_2(-\beta_{2M}) + \frac{1}{24}\left(\frac{\eta_G}{\sigma_G^4} - 3\right)H_3(-\beta_{2M}) + \frac{1}{72}\left(\frac{\theta_G}{\sigma_G^3}\right)^2 H_5(-\beta_{2M}) + \cdots\right], \\ \beta_{4M} &= -\Phi^{-1}(P_{4M}), \\ R(\beta_{4M}) &= P(G(\mathbf{X}) \geq 0) = 1 - P(-\beta_{2M}). \end{aligned} \tag{5.17}$$

If reliability $R > 1$ appears, an amended expression form is employed in this paper

$$R^* = R(\beta_{2M}) - \frac{R(\beta_{2M}) - \Phi(\beta_{2M})}{\{1 + [R(\beta_{2M}) - \Phi(\beta_{2M})]\beta_{2M}\}^{\beta_{2M}}}. \tag{5.18}$$

6. Reliability Sensitivity Analysis

The reliability sensitivity with respect to the mean value of random parameters is approximately derived as follows:

$$\frac{\mathrm{D}R}{D\overline{\mathbf{X}}^T} = \frac{\partial R(\beta_{4M})}{\partial\beta_{2M}}\frac{\partial\beta_{2M}}{\partial\mu_G}\frac{\partial\mu_G}{\partial\overline{\mathbf{X}}^T}, \tag{6.1}$$

where

$$\frac{\partial R(\beta_{4M})}{\partial \beta_{2M}} = \varphi(-\beta_{2M}) \times \left\{ 1 - \beta_{2M} \left[\frac{1}{6}\frac{\theta_G}{\sigma_G^3} H_2(-\beta_{2M}) + \frac{1}{24}\left(\frac{\eta_G}{\sigma_G^4} - 3\right) H_3(-\beta_{2M}) + \frac{1}{72}\left(\frac{\theta_G}{\sigma_G^3}\right)^2 H_5(-\beta_{2M}) \right] - \left[\frac{1}{3}\frac{\theta_G}{\sigma_G^3} H_1(-\beta_{2M}) + \frac{1}{8}\left(\frac{\eta_G}{\sigma_G^4} - 3\right) H_2(-\beta_{2M}) + \frac{5}{72}\left(\frac{\theta_G}{\sigma_G^3}\right)^2 H_4(-\beta_{2M}) \right] \right\},$$

$$\frac{\partial \beta_{2M}}{\partial \mu_G} = \frac{1}{\sqrt{\left(\partial G_d(\mathbf{X})/\partial(\mathbf{X})^T\right)^{[2]} \operatorname{Var}(\mathbf{X})}},$$

$$\frac{\partial \mu_G}{\partial \mathbf{X}^T} = \left[\frac{\partial G\left(\overline{\mathbf{X}}\right)}{\partial X_1} \frac{\partial G\left(\overline{\mathbf{X}}\right)}{\partial X_2} \cdots \frac{\partial G\left(\overline{\mathbf{X}}\right)}{\partial X_n} \right]. \tag{6.2}$$

As reliability $R > 1$ appears, sensitivity of reliability index β is described as follows:

$$\frac{\partial R^*}{\partial \beta_{2M}} = \frac{\partial R}{\partial \beta_{2M}} + \left[\frac{\partial R}{\partial \beta_{2M}} - \varphi(\beta_{2M}) \right] \frac{\beta_{2M}(\beta_{2M} - 1)\mathcal{A} - 1}{\{1 + \mathcal{A}\beta_{2M}\}^{\beta_{2M}+1}} + \frac{\mathcal{A}(\{1 + \mathcal{A}\beta_{2M}\} \ln\{1 + \mathcal{A}\beta_{2M}\} + \mathcal{A}\beta_{2M})}{\{1 + \mathcal{A}\beta_{2M}\}^{\beta_{2M}+1}}, \tag{6.3}$$

where $\mathcal{A}$ denotes $[R(\beta_{2M}) - \Phi(\beta_{2M})]$. Substituting (6.2)-(6.3) into (6.1), the reliability sensitivity of random variables $DR/D(\overline{\mathbf{X}})^T$ can be obtained.

7. Numerical Example

With the development of railway heavy haul transport and high speed of trains, the fatigue reliability of wheels appears to be more important than ever before. To ensure the traffic safety of trains, it requires a higher demand on the reliability and service life of train wheels.

Train wheels are typical complex mechanical components under random excitation. In order to estimate the fatigue reliability, it is necessary to find the dangerous areas of the wheel, which need a finite element analysis under working conditions. Considering the practical working condition of wheels, according to UIC code [23], a model of wheelset is required for further study. The purpose here is to estimate the impact of parameters with uncertainties on fatigue reliability, thus, a 3-D parametric model of train wheelset is built in PRO-E firstly, and then imported to MSC. PATRAN for finite element modeling. Since geometrical shape,

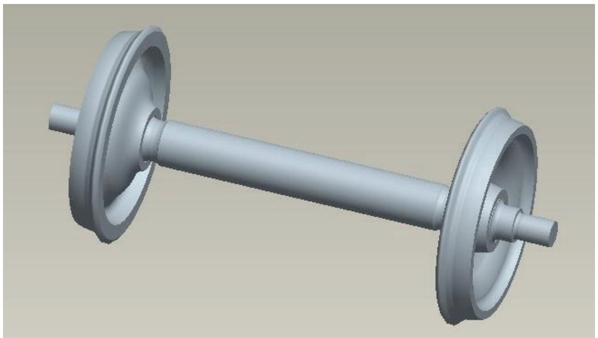

Figure 2: Parametric model of the wheelset.

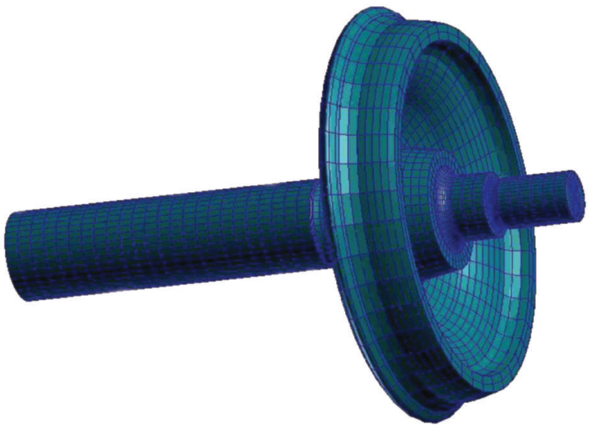

Figure 3: Finite-element model of the wheelset.

load, and boundary conditions are symmetrical, a half-model is used as the effective model for FE analysis. The 3-D model and the finite element model are illustrated in Figures 2 and 3. Through the literature review [19], the most dangerous areas are often at lateral position and web plate of the wheel. Accordingly, a few parameters are defined on the wheel model and shown in Figure 4.

According to UIC code, the load that wheels are subjected to can be classified into three load conditions:

(1) straight track condition: vertical dynamic load (P_1) + interference Δ + angular velocity with the maximum running speed;

(2) curve track condition: vertical dynamic load (P_2) + lateral dynamic load (H_2) + interference Δ + angular velocity with the maximum running speed;

(3) railroad switch condition: vertical dynamic load (P_3) + lateral dynamic load (H_3) + interference Δ + angular velocity with the maximum running speed.

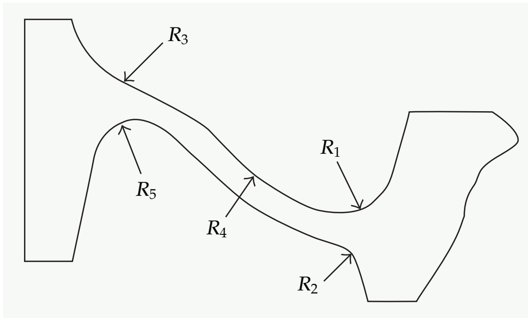

Figure 4: Schematic diagram of wheel parameters.

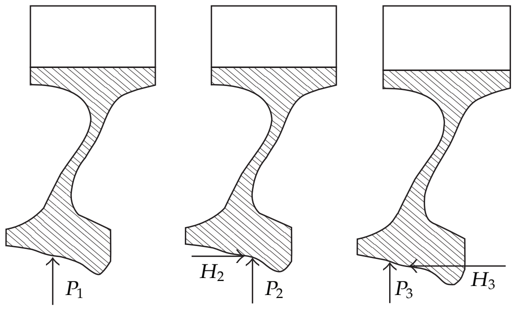

Figure 5: Force conditions of each load case.

The relationship between static load and dynamic load is as follow:

$$\begin{aligned} P_j &= 1.25P_0 \quad (j = 1, 2, 3), \\ H_2 &= \alpha P_0, \\ H_3 &= 0.6\alpha P_0, \end{aligned} \tag{7.1}$$

where P_0 is the vertical static load, $\alpha = 0.7$ for a directive wheel and $\alpha = 0.6$ for a nondirective wheel.

KDQ type wheels are analyzed here, the material is CL50A, the diameter of the wheel is 840 mm, density is 7.85×10^3 kg/m^3, elastic modulus is 2.1×10^5 MPa, Poisson's ratio is 0.3, and the maximum axle load P is 18500 kg. Thus, according to the rules expressed above, the maximum vertical static load of a wheel can be calculated as $P_0 = P/2 = 9250$ kg $= 90650$ N, and thus the maximum vertical dynamic load $P_j = P_0 \times 1.25 = 113312.5$ N, the maximum transverse dynamic load $H_2 = 0.6 \times P_0 = 54390$ N, $H_3 = 0.6 \times 0.6 \times P_0 = 6.66\ t = 32634$ N. The force conditions of the wheel are shown in Figure 5. Considering the actual train line, rail-wheel vertical force and lateral force are main external excitation.

As the actual load spectrum is acquired through a complex measurement on wheels, which is hard to realize, an empirical method [24] is adopted here. From the literature, the ratio of load and that of cyclic number of each grade are shown in Table 1. Based on the UIC code, an approximate load spectrum is derived, which is shown in Tables 2 and 3. Note that the frequency of both lateral force and vertical force are the same according to the method.

Table 1: Ratio of load and cyclic number of each load grade.

Load grade (i)	Radio of loads (F_i/F_1)	Radio of cycles (n_i/n_1)
1	1	1
2	0.95	106
3	0.85	6834
4	0.725	174,620
5	0.575	611,563
6	0.425	201,936
7	0.275	4927
8	0.125	13

Table 2: Load spectrum of lateral force.

Load grade (i)	Load amplitude (N)	Frequency
1	5.439e4	1.0500e2
2	5.167e4	1.1130e4
3	4.623e4	7.1757e5
4	3.943e4	1.8335e7
5	3.127e4	6.4214e7
6	2.312e4	2.1203e7
7	1.496e4	5.1734e5
8	6.799e3	1.3650e3

Table 3: Load spectrum of vertical force.

Load grade (i)	Load amplitude (N)	Frequency
1	1.133e5	1.0500e2
2	1.077e5	1.1130e4
3	9.632e4	7.1757e5
4	8.215e4	1.8335e7
5	6.516e4	6.4214e7
6	4.816e4	2.1203e7
7	3.116e4	5.1734e5
8	1.416e4	1.3650e3

Then, the load spectra are applied on the FE model of the wheelset. Consequently, the stress spectrum of the critical point relevant to the load spectra is obtained, which is shown in Table 4.

The reliability analysis which is based on the gradient algorithm requires an explicit function expression. For the complex structure and working condition of the wheel, it is hard to obtain the required algebraic equation. In this paper, a design of experiment is utilized firstly to obtain samples of mutative parameters and corresponding stresses of the dangerous point. Furthermore, the samples are imported into an artificial neural network (ANN), after which the explicit function expression needed could be obtained. By using iSIGHT, the DOE is performed by integrating PRO-E and MSC. PATRAN (MSC. NASTRAN).

Table 4: Stress spectrum of dangerous position.

Stress grade (i)	Stress amplitude (MPa)	Frequency
1	173	1.0500e2
2	166	1.1130e4
3	158	7.1757e5
4	142	1.8335e7
5	124	6.4214e7
6	96.7	2.1203e7
7	72.6	5.1734e5
8	61.4	1.3650e3

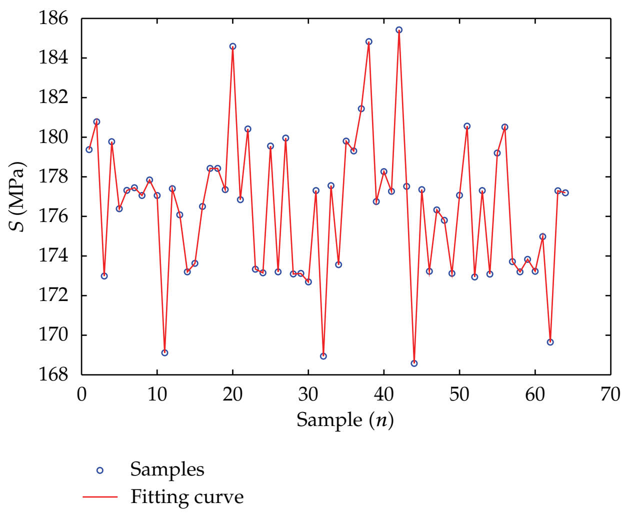

Figure 6: Fitting curve of the first stress grade.

Considering the number of design parameters, the factor level of structure parameters is defined. Each load level is analyzed by FEM, and corresponding data are obtained which are used as samples for ANN training.

In order to determine the function relation between stresses of the dangerous point and design parameters, a BP network with single hidden layer is modeled. All the data obtained from the DOE are normalized for improving the stability and shorten the training time of the BP network. According to Miner's rule, the damage occurs only when the stress is larger than the fatigue limit; hence, according to the stress spectra obtained, the first five levels of the stress spectrum are selected as inputs of the BP network for training. Figures 6, 7, 8, 9, and 10 illustrate the fitting curve of each stress level, and Figure 11 shows the relative error of the fitting curve. Function expressions of each stress level can be expressed simply as follows:

$$S_i = f_i(R_1, R_2, R_3, R_4, R_5), \quad i = 1, 2, \ldots, 5. \tag{7.2}$$

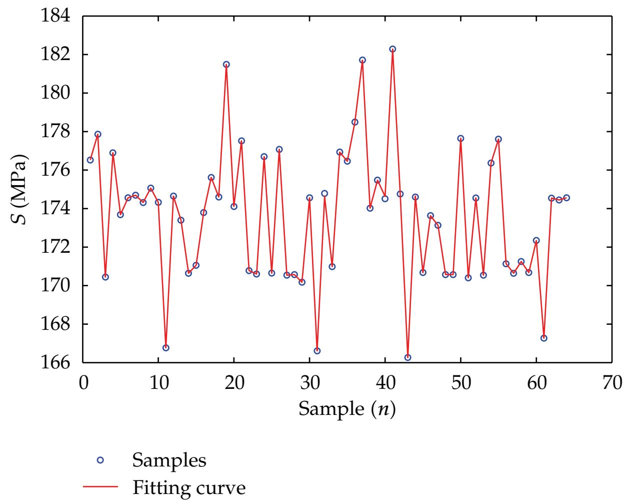

Figure 7: Fitting curve of the second stress grade.

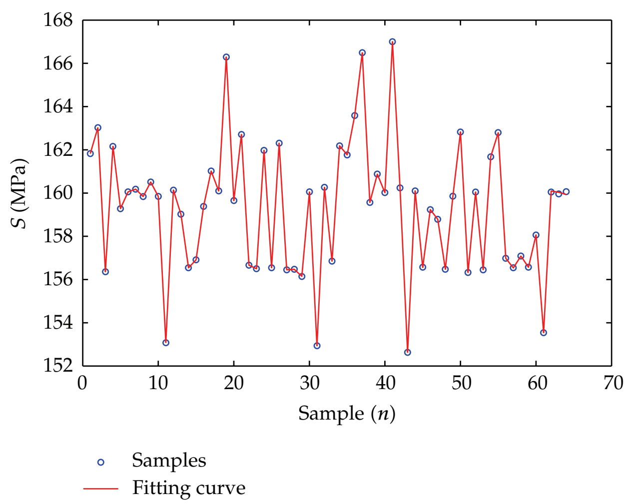

Figure 8: Fitting curve of the third stress grade.

Assuming that h_0 is the distance that train could reach under applied load spectra, the total distance can be expressed as

$$H = \frac{h_0}{D_c} = \frac{h_0}{\sum_1^i \left(n_i (S_i)^m / C\right)}. \tag{7.3}$$

Note that S_i is a function with respect to design parameters R_i; thus, we can obtain the function of total distance H as

$$H = g(R_1, R_2, R_3, R_4, R_5). \tag{7.4}$$

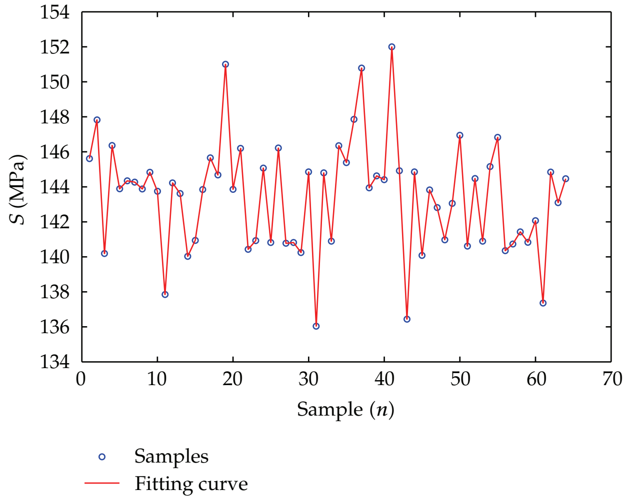

Figure 9: Fitting curve of the fourth stress grade.

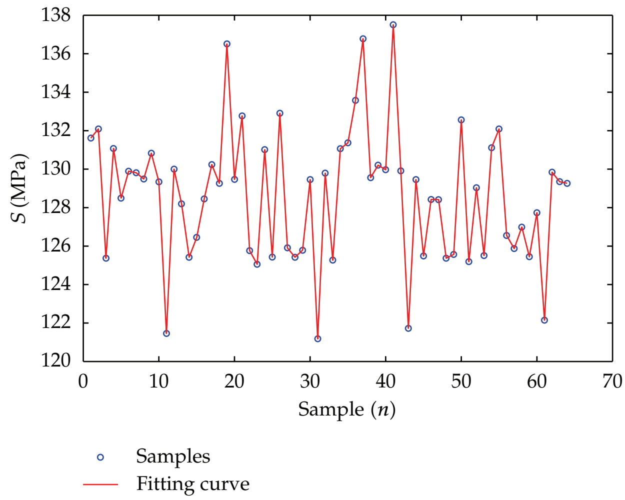

Figure 10: Fitting curve of the fifth stress grade.

Thus, the limit-state function of fatigue life of the wheel is defined as

$$G(\mathbf{X}) = H - H_0, \tag{7.5}$$

where H_0 is the expected distance that the train could reach.

Given the first four moments of the random parameters, the statistical characteristic of the limit-state function $G(\mathbf{X})$ can be obtained. According to the random perturbation method, the first four moments of $G(\mathbf{X})$, which are μ_G, σ_G^2, θ_G, and η_G, are calculated, respectively, with (5.3), (5.11), (5.12), and (5.13). Based on (5.14) and (5.15), the approximation distribution of $G(\mathbf{X})$ is derived. Then, according to (5.16)–(6.3), the reliability and reliability sensitivity with respect to design parameters R_i ($i = 1, 2, \ldots, 5$) could be obtained. For comparison,

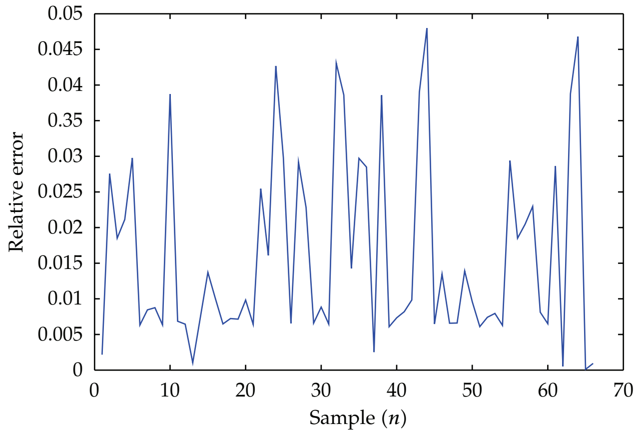

Figure 11: Relative error of the fitting function.

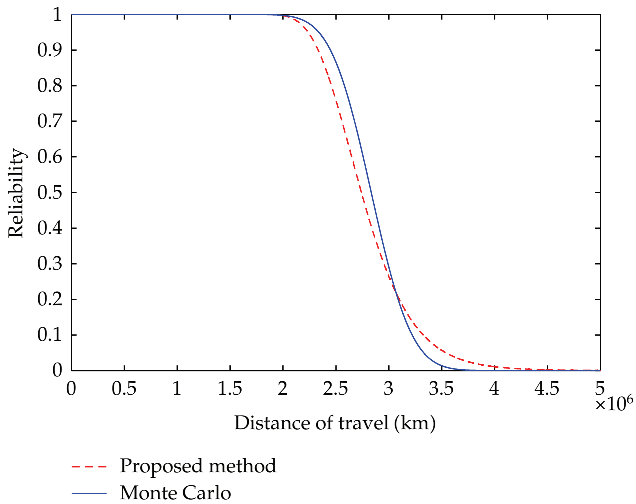

Figure 12: Fatigue reliability of the wheel.

a Monte Carlo simulation is necessary. By comparing the results obtained above, these are well matched with those obtained by MCS, which is shown in Figure 12. Moreover, from the figure, the result obtained by MCS is conservative for high reliability but not for low reliability.

The reliability-based sensitivity curve of design parameters of the wheel is illustrated in Figure 13 and the sensitivities of different design parameters are compared. According to Figure 13, the reliability sensitivities of R_1 and R_3 are negative values, that is, the fatigue reliability of the wheel reduces with increasing values of R_1 and R_3. Similarly, the reliability sensitivities of R_2, R_4, and R_5 are positive numbers; that is, the fatigue reliability of the wheel increases as R_2, R_4, and R_5 increases. In other words, with the results obtained, it follows that the reliability is very sensitive to R_1 and R_2, moderate sensitive to R_3 and R_4, and little sensitive to R_5. The results of the reliability sensitivities are largely in accord with the practical operation conditions.

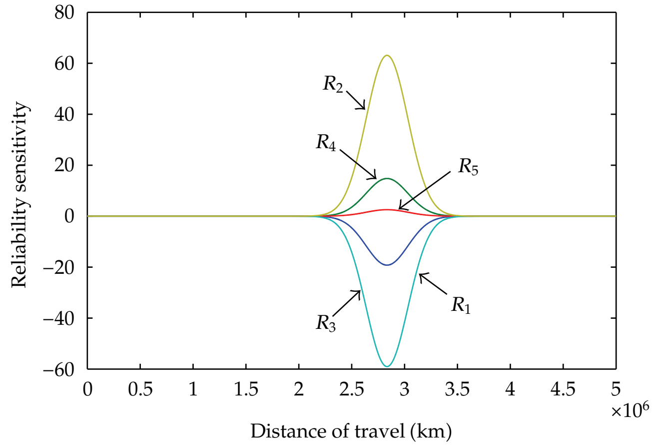

Figure 13: Curves of reliability-based sensitivities.

8. Conclusions

An approach to fatigue reliability sensitivity analysis of complex mechanical components under random excitation is presented in this paper. The problem of implicit limit-state function caused by the complexity of the components is solved by DOE and ANN. The corresponding explicit function of the response about random variables is obtained. By combining damage accumulative approach, the limit-state function of fatigue life of the component is derived. Furthermore, the fatigue reliability and the reliability-based sensitivity can be calculated with the algorithm described in the paper. A train wheel is taken as an example, which indicates that the approach proposed is an effective way of solving similar problems.

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