Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2011, Article ID 705346, 12 pages
doi:10.1155/2011/705346

Research Article

Design of Switching Multiobjective Controller: A New Approach

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Received 11 February 2011; Revised 9 June 2011; Accepted 29 August 2011

Academic Editor: John Burns

Design of switching H_2/H_∞ output-feedback controller for discrete-time LTI systems with state-multiplicative noise is considered. The closed loop system achieves a minimum bound on the stochastic H_2 performance level, while satisfying the H_∞ performance. The proposed controller is based on a fuzzy supervisor which manages the combination of two separate H_2 and H_∞ controllers. A convex formulation of the two controllers leads to a structure which benefits from the advantages of both controllers to ensure a good performance in both the transient phase (H_2 controller) and the steady phase (H_∞ controller). The stability analysis uses the Lyapunov technique, inspired from switching system theory, to prove that the closed loop system with the proposed controller structure remains globally stable despite the configuration changing.

1. Introduction

Systems with stochastic nature have received much attention in the last decade, mainly in the H_∞ control theory framework. Solutions to various control and estimation problems that ensure the worst case performance bound in the H_∞ sense have been derived, in both, the continuous-time framework and the discrete-time counterpart. The modeling of parameter system uncertainties as white-noise processes in a linear setting is encountered in many areas of applications such as nuclear fission, heat transfer, population models, and immunology. In control theory, such models are encountered in gain scheduling when the scheduling parameters are corrupted with measurement noise [1–3]. Following the researches done in 1960s and 1970s, where the main issues were stability and control of continuous-time state multiplicative systems in the stochastic H_2 framework (see [4] and the references therein), researches in the last decade have focused on the H_∞ control setting. Thus, the continuous time stochastic state-multiplicative bounded real lemma (BRL) and the discrete-time counterpart were obtained. In [5], a discrete-time stochastic estimation for a guidance-motivated tracking problem was solved, and its results were shown to achieve better results than those achieved by the Kalman-filter. A parameter dependent approach for designing static output-feedback controller for linear time-invariant systems with state-multiplicative noise is introduced in [2], which achieve a minimum bound on either the stochastic H_2 or the H_∞ performance level. But previous studies paid less attention to mixed H_2/H_∞ control of these systems because of their complexity [6].

Combination of different techniques to obtain the different performances is widely used today [4, 7, 8]. This method results in hybrid dynamical systems which include continuous and discrete dynamics and a mechanics (supervisor) managing the interaction between these dynamics [9].

In an actual engineering control problem, various conflicting requirements such as disturbance rejection and robustness to changing conditions and plant uncertainties have to be satisfied. General multiobjective control problems are difficult and remain mostly open up to now. By multiobjective control, we mean synthesis problems with a mix of performances. The mixed H_2/H_∞ control is an important robust control method and has been studied by many researchers. The mixed H_2/H_∞ control is concerned with the design of a controller that minimizes the H_2 performance of the system with respect to some input noises while it guarantees certain worst case performance with respect to other external disturbances. Compared with the sole H_∞ control, the mixed H_2/H_∞ control is more attractive in engineering. Since the former is a worst-case design which tends to be conservative, whereas the later minimizes the average performance with a guaranteed worse-case performance. For the general multiobjective control problem, the usual approach is to design one controller and to force all Lyapunov matrices used to test the several design specifications to be the same. These constraints offer a computationally efficient solution to the control problems with multiple objectives and are thoroughly investigated through the previous works [10]. Designing one controller and specifying the closed-loop objectives in terms of a common Lyapunov function is the core of the Lyapunov-shaping paradigm and constitutes an important source of conservatism [10, 11].

In order to release these constraints and let several Lyapunov matrices be simultaneously considered in the Multiobjective control synthesis problem, the switching approach is advocated in this paper. Extensive benefits are obtained from this feature in order to offer a less conservative controller parameterization in the multiobjective control synthesis problems. While the new switching controllers release the constraints on the Lyapunov instrumental variable, this is obtained at the expense of designing a controller for each specification and adding a simple constraint to ensure the stability. The number of free parameters is significantly increased as compared with the available techniques. Finally, it can be easily verified that the results obtained with the standard parameterization are always encompassed by the presented switching formulations. In [12, 13], a new framework of switching controller structure for Multiobjective control (mix H_2/H_∞) of singular perturbation systems has been developed that has a better performance rather than conventional controller structure. In the present paper, the switching H_2 and the H_∞ static control problems for discrete-time linear systems are solved that contain stochastic white-noise parameter uncertainties in the matrices of the state-space model that describe the system. The simple design methods of [2] are applied to derive the static output-feedback gains that satisfy the prescribed H_2 and H_∞ performance criteria, separately.

A fuzzy supervisor is proposed for hybrid combination of H_2 and H_∞ controllers to use their advantages and to ensure the required performances and the stability of the closed loop system.

The main contribution of the presented work is combining H_2 and H_∞ controllers using a supervisor, which manages the gradual transition from one controller to another. This method is applied to use the advantages of each controller. The gradual transition attenuates the uncontrollability and instability problems related to the abrupt switch. The control signal is obtained via a weighted sum of the two signals given by the H_2 and the H_∞ controllers. This weighted sum is managed thanks to a fuzzy supervisor, which is adapted to obtain the desired closed loop system performances by benefiting from the advantage of the H_2 in the approaching phase, minimizing the energy of impulse response and the ability of the H_∞ control to eliminate the chattering and to guarantee the system robustness. So, the H_2 mainly acts in the transient phase providing a fast dynamic response and enlarging the stability limits of the system, while the H_∞ control acts mainly in the steady state to reduce chattering and to maintain the tracking performances. Furthermore, the global stability of the system even if the system switches from one configuration to another (transient to steady state and vice versa) is guaranteed.

The structure of the paper is as follows. Section 2 presents the system definition and the controllers used. In Section 3, the fuzzy supervisor and the proposed control law are described. Stability analysis is demonstrated in Section 4. The design procedure is explained in Section 5 and an example is given to illustrate the efficiency of the proposed method, followed by conclusions in Section 6.

Notation. Throughout the paper, the superscript "T" stands for matrix transposition, R^n denotes the n dimensional Euclidean space, $R^{n\times m}$ is the set of all $n \times m$ real matrices, N is the set of natural numbers and the notation $P > 0$, (resp., $P \geq 0$) for $P \in R^{n\times n}$ means that P is symmetric and positive definite (resp., semipositive definite). The variables $\{\zeta_k\}$ and $\{\nu_k\}$ are zero-mean real scalar white-noise sequences that satisfy $E\{\nu_k \nu_j\} = \delta_{kj}$, $E\{\zeta_k \zeta_j\} = \delta_{kj}$, $E\{\zeta_k \nu_j\} = \delta_{kj}$, for all $k, j \geq 0$. By $L^2(\Omega, R^n)$, the space of square-summable R^n-valued functions on the probability space $(\Omega, \mathbb{F}, P)$ is denoted, where Ω is the sample space, $\mathbb{F}$ is a σ algebra of a subset of Ω called events, and P is the probability measure on $\mathbb{F}$. By $(\mathbb{F}_k)_{k\in N}$, an increasing family of σ-algebras $\mathbb{F}_k \subset \mathbb{F}$ is denoted which is generated by $\nu_j,\ \zeta_j,\ j \leq k-1$. $\widetilde{l}^2(N; R^n)$ is the space of non-anticipative stochastic processes $\{f_k\} = \{f_k\}_{k\in[0,\infty]} \in \widetilde{l}^2(N; R^n)$ in R^n with respect to $(\mathbb{F}_k)_{k\in[0,\infty]}$ satisfying

$$\|f_k\|_{\widetilde{l}^2}^2 = E\left\{\sum_0^\infty \|f_k\|^2\right\} = \sum_0^\infty E\left\{\|f_k\|^2\right\} < \infty, \tag{1.1}$$

where $\|\cdot\|$ is the standard Euclidean norm. By δ_{ij}, the Kronecker delta function is denoted.

2. Problem Statement

The following linear system is considered:

$$\begin{aligned} x_{k+1} &= (A + D\nu_k)x_k + B_1\omega_k + (B_2 + G\zeta_k)u_k, \quad x_0 = 0, \\ y_k &= C_2 x_k + D_{21} n_k, \end{aligned} \tag{2.1}$$

with the objective vector

$$z_k = C_1 x_k + D_{12} u_k, \tag{2.2}$$

where $\{x_k\} \in R^n$ is the system state vector, $\{\omega_k\} \in R^q$ is the exogenous disturbance signal, $\{n_k\} \in R^p$ is the measurement noise sequence, $\{u_k\} \in R^l$ is the control input, $\{y_k\} \in R^m$ is the measured output, and $\{z_k\} \in R^r \subset R^n$ is the state combination (objective function signal) to be regulated. The state-multiplicative white-noise sequences are defined in the notation subsection. The matrices in (2.1) and (2.2) are assumed to be constant matrices of appropriate dimensions.

In each stage, a constant output-feedback controller

$$u_k = K y_k \tag{2.3}$$

is sought to achieve a certain performance requirement. The following performance criterion is treated.

The stochastic H_2/H_∞ control problem: $\{\omega_k\}$, $\{n_k\}$ are realizations of a unit variance, stationary, white-noise sequences that are uncorrelated with $\{v_k\}, \{\zeta_k\}$. The exogenous disturbance signal is energy bounded; the following performance index should be minimized, which is useful to handle stochastic aspects such as measurement noises or random disturbances:

$$J_2 = \underset{\omega,n}{E}\left\{ \|z_k\|^2_{\tilde{l}_2} \right\}, \tag{2.4}$$

while for a prescribed scalar $\gamma > 0$ and for all nonzero $\{\omega_k\} \in R^q$, $\{n_k\} \in R^p$, guarantees that $J_\infty < 0$ where

$$J_\infty = \|z_k\|^2_{\tilde{l}_2} - \gamma^2 \left[\|\omega_k\|^2_{\tilde{l}_2} + \|n_{k+1}\|^2_{\tilde{l}_2} \right]. \tag{2.5}$$

That is useful for disturbance rejection, reference tracking, low-energy consumption, bandwidth limitation, low steady state control error, and robust stability. H_∞ norm measures the system input-output gain for finite energy or finite rms input signals.

Interconnection of (2.1) and (2.2) is denoted by the Redheffer star product to include y_k. The augmented state vector $\xi_k = \operatorname{col}\{x_k, y_k\}$ is defined, and the following representation is obtained to the closed loop system:

$$\begin{gathered} \xi_{k+1} = \widetilde{A}\xi_k + \widetilde{B}\widetilde{\omega}_k + \widetilde{D}\xi_k v_k + \widetilde{G}\xi_k \zeta_k, \quad \xi_0 = 0, \\ z_k = \widetilde{C}\xi_k, \end{gathered} \tag{2.6}$$

where

$$\tilde{\omega}_k = \begin{bmatrix} \omega_k \\ n_{k+1} \end{bmatrix}, \quad \tilde{A} = \begin{bmatrix} A & B_2K \\ C_2A & C_2B_2K \end{bmatrix}, \quad \tilde{D} = \begin{bmatrix} D & 0 \\ C_2D & 0 \end{bmatrix},$$
$$\tilde{G} = \begin{bmatrix} 0 & GK \\ 0 & C_2GK \end{bmatrix}, \quad \tilde{B} = \begin{bmatrix} B_1 & 0 \\ C_2B_1 & D_{21} \end{bmatrix}, \quad \tilde{C} = \begin{bmatrix} C_1 & D_{12}K \end{bmatrix}. \tag{2.7}$$

The Lyapunov functions $V_i = \xi^T \tilde{P}_i \xi$, $i = 2, \infty$ are considered with:

$$\tilde{P}_i = \begin{bmatrix} P_i & -\beta^{-1}P_iC_2^T \\ \beta^{-1}C_2P & \hat{P}_i \end{bmatrix}, \quad \begin{matrix} \hat{P}_i \in R^{m\times m} \\ P_i \in R^{n\times n} \end{matrix}, \; \hat{P}_i > 0. \tag{2.8}$$

While the parameter β is an important positive scalar tuning parameter and

$$\widetilde{Q_i} = \tilde{P}_i^{-1} = \begin{bmatrix} Q_i & C_2^T\widehat{Q_i} \\ \widehat{Q_i}C_2 & \beta\widehat{Q_i} \end{bmatrix}, \quad \begin{matrix} Q_i \in R^{n\times n} \\ \widehat{Q_i} \in R^{m\times m} \end{matrix}. \tag{2.9}$$

Lemma 2.1 (the stochastic H_2 control). *Consider systems* (2.1), (2.2)*. The output-feedback control law* (2.3) *achieves a prescribed H_2-norm bound $\delta > 0$, if there exist $Q \in R^{n\times n}$, $\hat{Q} \in R^{m\times m}$, $Y \in R^{l\times m}$, and $H \in R^{(q+p)\times(q+p)}$ that, for some tuning scalar $\beta > 0$, the following LMIs are satisfied: [2]*

$$\tilde{\Gamma} = \begin{bmatrix} -Q & -C_2^T\hat{Q} & \tilde{\Gamma}_{13} & \tilde{\Gamma}_{14} & 0 & 0 & 0 & 0 & 0 \\ * & -\beta\hat{Q} & \tilde{\Gamma}_{23} & \tilde{\Gamma}_{24} & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q & -C_2^T\hat{Q} & \tilde{\Gamma}_{35} & QD^T & \tilde{\Gamma}_{37} & C_2^TY^TG^T & \tilde{\Gamma}_{39} \\ * & * & * & -\beta\hat{Q} & \tilde{\Gamma}_{45} & \hat{Q}C_2D^T & \tilde{\Gamma}_{47} & \beta Y^TG^T & \tilde{\Gamma}_{49} \\ * & * & * & * & -I_r & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Q & -C_2^T\hat{Q} & 0 & 0 \\ * & * & * & * & * & * & -\beta\hat{Q} & 0 & 0 \\ * & * & * & * & * & * & * & -Q & -C_2^T\hat{Q} \\ * & * & * & * & * & * & * & * & -\beta\hat{Q} \end{bmatrix}, \tag{2.10}$$

$$\tilde{\Gamma} < 0, \quad \begin{bmatrix} H_{11} & H_{12} & B_1^T & B_1^TC_2^T \\ * & H_{22} & 0 & D_{21}^T \\ * & * & Q & C_2^T\hat{Q} \\ * & * & * & \beta\hat{Q} \end{bmatrix} > 0, \quad \operatorname{trace}(H) < \delta^2, \tag{2.11}$$

where $H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}$,

$$\begin{gathered}
\tilde{\Gamma}_{37} = QD^T C_2^T, \qquad \tilde{\Gamma}_{23} = \left[C_2 AQ + C_2 B_2 Y C_2 - \hat{Q} C_2\right] + \hat{Q} C_2, \\
\tilde{\Gamma}_{13} = AQ + B_2 Y C_2, \qquad \tilde{\Gamma}_{39} = C_2^T Y^T G^T C_2^T, \qquad \tilde{\Gamma}_{47} = \hat{Q} C_2 D^T C_2^T, \\
\tilde{\Gamma}_{24} = \left[C_2 A C_2^T + \beta C_2 B_2 Y - \beta \hat{Q}\right] + \beta \hat{Q}, \\
\tilde{\Gamma}_{45} = \beta Y^T D_{12}^T + \hat{Q} C_2 C_1^T, \qquad \tilde{\Gamma}_{49} = \beta Y^T G^T C_2^T, \\
\tilde{\Gamma}_{14} = \beta B_2 Y + A C_2^T \hat{Q}, \qquad \tilde{\Gamma}_{35} = Q C_1^T + C_2^T Y^T D_{12}^T.
\end{gathered} \tag{2.12}$$

If a solution to the latter LMIs exists, the gain matrix K that stabilizes the system and achieves the required performance is given by $K_2 = Y\hat{Q}^{-1}$.

Lemma 2.2 (the stochastic H_∞ problem). *Consider the system of* (2.1), (2.2). *The control law* (2.3) *achieves a prescribed H_∞-norm bound $\gamma > 0$, if there exist, $Q \in R^{n\times n}$, $\hat{Q} \in R^{m\times m}$, $Y \in R^{l\times m}$ that, for some scalar $\beta > 0$, the following LMI is satisfied: [2]*

$$\begin{bmatrix} \tilde{\Gamma} & \begin{bmatrix} B_1 & 0 \\ C_2 B_1 & D_{21} \\ 0 & 0 \end{bmatrix} \\ * & -\gamma^2 I_{q+p} \end{bmatrix} < 0, \tag{2.13}$$

where $\tilde{\Gamma}$ is defined in (2.11) *and $K_\infty = Y\hat{Q}^{-1}$.*

3. Fuzzy Supervisor

H_2 control provides a fast dynamic response, a stable control system, and a simple implementation. Conversely, this control strategy has some drawbacks that appear in the steady state. The H_∞ techniques are alternatives that can guarantee the robustness and the global stability. In order to take advantage of both controllers, H_2 during the transient time and H_∞ control during the steady state, their control actions are combined by means of a weighting factor, $\alpha \in [\,0 \;\; 1\,]$, representing the output of a fuzzy logic supervisor that takes the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \ldots, e^{n-1}$ as inputs. The global control scheme of the proposed approach is illustrated in Figure 1.

The fuzzy system is constructed from a collection of fuzzy rules whose jth component can be given in the form

$$\text{If } e \text{ is } H_1^j \text{ And} \ldots \text{And } e^{n-1} \text{ is } H_n^j \text{ Then } \alpha = \alpha_j, \tag{3.1}$$

where H_i^j is a fuzzy set and α_j is a singleton.

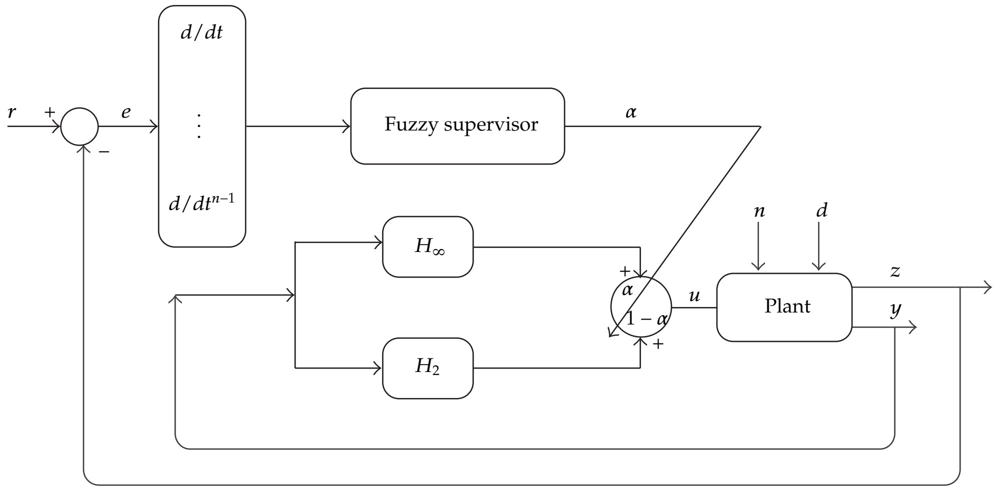


Figure 1: The control scheme of the proposed method.

It is easy to see that it can be considered as a fuzzy rule of a Takagi-Sugeno fuzzy system. The fuzzy implication uses the product operation rule. The connective AND is implemented by means of the minimum operation, whereas fuzzy rules are combined by algebraic addition. Defuzzification is performed using the centroid method, which generates the gravity centre of the membership function of the output set. Since the membership functions that define the linguistic terms of the output variable are singletons, the output of the fuzzy system is given by

$$\alpha = \frac{\sum_{i=1}^{m} \alpha_i \prod_{j=1}^{n} \mu_i^j}{\sum_{i=1}^{m} \prod_{j=1}^{n} \mu_i^j}, \tag{3.2}$$

where μ_i^j is the degree of membership of H_i^j and m is the number of fuzzy rules used.

The objective of this fuzzy supervisor is to determine the weighting factor, α, which gives the participation rate of each control signal. Indeed, when the norm of the tracking error e and its time derivatives $\dot{e}, \ddot{e}, \ldots, e^{n-1}$ are small, the plant is governed by the H_∞ controller $(\alpha = 1)$. Conversely, if the error and its derivatives are large, the plant is governed by the H_2 $(\alpha = 0)$. The control action, u, is determined by

$$u = (1-\alpha) u_{H_2} + \alpha u_{H_\infty}. \tag{3.3}$$

Remark 3.1. In the case of a large rule base, some techniques can be employed to significantly reduce the number of rules activated at each sampled time by using the system position in the state space. Indeed, it is demonstrated that using a strict triangular partitioning allows guaranteeing that, at each sampling time, each input variable is described with two linguistic terms at the most [6]. Thus, the output generated by the fuzzy system with n inputs is then reduced to that produced by the subsystem composed of the 2^n fired rules.

4. Stability Analysis

The theorem of Essounbouli et al. [7] is used to prove the global stability of the system governed by the control law (3.3). Similar to [7], using H_2 and H_∞ controls, this theorem is rewritten as follows.

Theorem 4.1. *Consider a combined fuzzy logic control system as described in this work. If*

(1) *there exists a positive definite, continuously differentiable, and radially unbounded scalar function V for each subsystem;*

(2) *every fuzzy subsystem gives a negative definite $\dot{V}$ in its active region;*

(3) *the weighted sum defuzzification method is used, such that for any output u*

$$\min(u_{H_2}, u_{H_\infty}) \leq u \leq \max(u_{H_2}, u_{H_\infty}). \tag{4.1}$$

Then the resulting control u, given by (3.3), *guarantees the global stability of the closed loop system.*

Proof. Satisfying the two first conditions guarantees the existence of a Lyapunov function in the active region which is a sufficient condition for ensuring the asymptotic stability of the system during the transition from the H_2 control to the H_∞ one. Consider the Lyapunov function $V_2 = \xi^T P_2 \xi$ where $P_2 = P$ is a positive definite matrix and the solution of (2.11) and we have $\lambda_{\min}(P_2)\xi^T\xi \leq \xi^T P_2 \xi$, where $\lambda_{\min}(P_2)$ is the minimal eigenvalue of P_2. In Lemma 2.1, it was shown that the synthesized H_2 control ensures the decrease of the Lyapunov function V_2. Consider the Lyapunov function $V_\infty = \xi^T P_\infty \xi$ where $P_\infty = P$ is a positive definite matrix and the solution of (2.13) and we have $\xi^T P_\infty \xi \leq \lambda_{\max}(P_\infty)\xi^T\xi$, where $\lambda_{\max}(P_\infty)$ is the maximal eigenvalue of P_∞. In Lemma 2.2, it was shown that the synthesized H_∞ control ensures the decrease of the Lyapunov function V_∞.

To satisfy the second condition of the theorem, it is enough to choose P_2, P_∞ such that

$$\lambda_{\max}(P_\infty) \leq \lambda_{\min}(P_2). \tag{4.2}$$

This condition guarantees that in the neighborhood of the steady state (H_∞ control), the value of the Lyapunov function V_2 is greater than that of V_∞. To guarantee the third condition, the balancing term α takes its values in the interval [0 1]. Consequently, the three conditions of the above theorem are satisfied and the global stability of the system is guaranteed. So, The Problem formulation (switching H_2/H_∞ control) will be as:

$$\begin{array}{lll} \text{minimize} & J_2 & (2.11) \longrightarrow K_2 \\ \text{subject to:} & \left\{ \begin{array}{l} J_\infty < 0 \\ \lambda_{\max}(P_\infty) \leq \lambda_{\min}(P_2) \end{array} \right. & \begin{array}{l} (2.13) \longrightarrow K_\infty \\ (4.2) \end{array} \end{array} \tag{4.3}$$

P_2 and P_∞, which influence the stability and relation (4.2) are dependent on β. Genetic algorithm is used to find optimal β that solve the above problem. □

Remark 4.2. It should be noted that the proof of stability in this case is similar to those used for switching system theory [7, 11]. Indeed, the energy of the system corresponding to the

H_∞ controller is less than that for H_2, guaranteeing the stability of the closed loop system during the transition from H_2 to H_∞. In the event of large external disturbance, which forces the system back to a transient phase, the proposed controller adjusts the weighting factor in a way that the system remains stable in the new configuration until returning to the steady state, which implies a new variation of the control signal.

5. Design Procedure

In order to minimize the online computing time of the proposed method and to simplify its real time implementation, the design procedure implies an offline processing step and an online step during control execution. In the offline step, the gains and β are defined in order to satisfy (4.3). The supervisor design is essentially based on the available information of the process under study. Indeed, when a sufficient amount of information is available, it becomes possible to reduce the number of inputs and the fuzzy rules.

To construct the fuzzy supervisor, firstly, the fuzzy sets are defined for each input (the error and its derivatives) and output; then, the rule base is elaborated. For the online step, the error vector is computed and then is injected in the supervisor to determine the value of α to apply the global control signal.

Example 5.1. To demonstrate the solvability of the various LMIs, simplicity and low conservatives of the proposed method, a third-order, two-output, one-input example is considered and a switching output feedback controllers is sought

$$A = \begin{bmatrix} 0.9813 & 0.342 & 1.3986 \\ 0.0052 & 0.984 & -0.1656 \\ 0 & 0 & 0.5488 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.4 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 0.0198 & 0.0034 & 0.0156 \\ 0.0001 & 0.0198 & -0.0018 \\ 0 & 0 & 0.015 \end{bmatrix}, \qquad D_{12} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad D_{21} = 0, \tag{5.1}$$
$$B_2 = \begin{bmatrix} -1.47 \\ -0.0604 \\ 0.4512 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad G = 0.$$

Based on the proposed control scheme, the following results are obtained:

applying the genetic algorithm and solving (4.3) a minimum H_2-norm bound of $\delta = 0.0449$ and a minimum value of $\gamma = 0.8916$ is obtained for $\beta = 2.4$. The corresponding controllers are $K_2 = \begin{bmatrix} 0.3469 & 0.6216 \end{bmatrix}$ and $K_\infty = \begin{bmatrix} 0.3567 & 1.2622 \end{bmatrix}$.

Consider multiobjective

$$\begin{array}{ll} \text{minimize} & J_2 \\ \text{subject to:} & J_\infty < 0 \text{ for } \gamma = 0.8916 \end{array} \tag{5.2}$$

Table 1: The simulation result.

Method	H_2 norm	H_∞ norm
Conventional method [2]	0.0528	0.8916
Proposed switching method	0.0449	0.8916

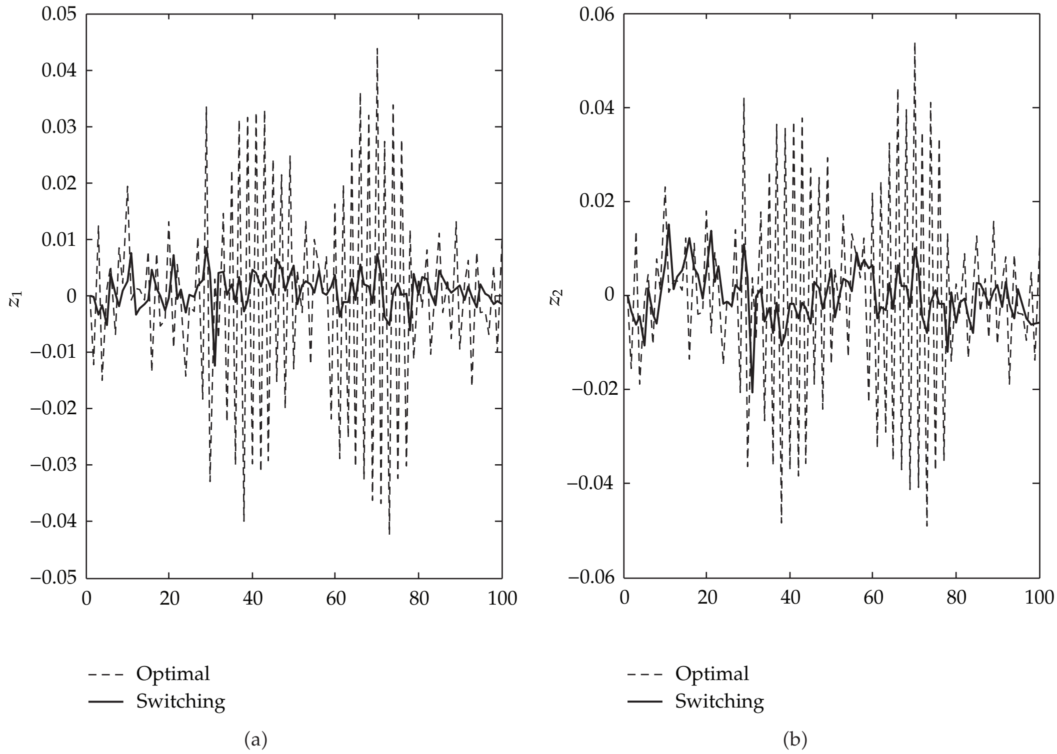


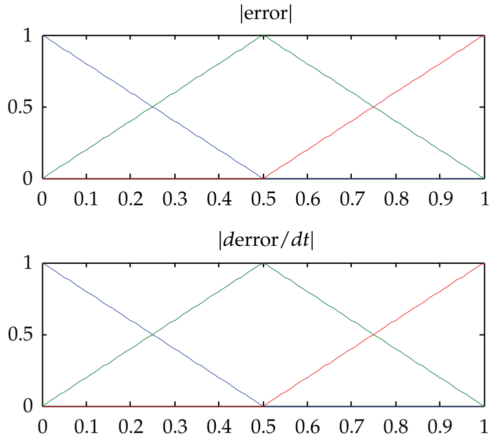
Figure 2: The output response of Example 5.1.

Solving (5.2) based on the Lyapunov-shaping paradigm that is given in [2] yields $\delta = 0.0528$ as the best constrained H_2 performance, which is 17.69% higher than the optimal value 0.0449 due to the use of one controller for each objective and different Lyapunov functions. We can summarize the results in Table 1.

According to Table 1, we can see that by a similar constraint on H_∞ norm of closed loop system, by using our proposed method H_2 norm of closed loop system will be 17.69% lower than the conventional Multiobjective method [2], that it is satisfactory.

The output response of Example 5.1 by using both conventional and our proposed switching method has been depicted in Figure 2. The solid line graph is our proposed method response, and the dash line graph is output response of conventional optimal method. As we see, it is clear that output regulation of our proposed method is much better than the conventional method, that it is satisfactory.

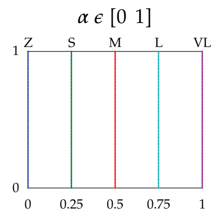
The fuzzy supervisor is constructed by using three fuzzy sets zero, medium, and large for the tracking error and its time derivative. The corresponding membership functions are triangular, as shown in Figure 3. For the output, five singletons are selected: very large (VL), large (L), medium (M), small (S), and zero (Z), corresponding to 1, 0.75, 0.5, 0.25, and 0,



(a)

		|derror/*dt*|		
		Z	M	L
|error|	Z	VL	L	M
	M	S	S	Z
	L	Z	Z	Z

(b)



(c)

Figure 3: The structure of the proposed fuzzy supervisor.

respectively. The fuzzy rule base is depicted in Figure 2. Rules are defined by a table; for example, a rule in the table can be stated as follows: "if the norm of the error is medium and the norm of the error derivative is large, then α is zero."

Results show that H_2 and the combined controller provide a fast dynamic response compared to H_∞ and that H_∞ and the combined controller provide a smooth variation of the control signal. Hence, the proposed control set-up benefits from the advantages of both H_∞ and H_2, in terms of tracking performance and the robustness to external perturbations, which is ensured by H_∞ control in the steady state (The fuzzy supervisor favors H_∞ to reach the steady state with a fast dynamic). As it is shown, the conservatism introduced by means of the proposed methodology is significantly decreased in comparison with the Lyapunov-shaping methods which oblige the designer to employ a common Lyapunov matrix for all the performance criteria and design one controller that satisfy all the objectives. The applied control signal forces the system to remain stable and attain the desired trajectory. Thus, an intermediate dynamics whose advantage is to have a compromise between the settling time and the actuator solicitations is obtained. Comparing the results shows that the proposed controller ensures a good convergence towards the desired trajectory. The conditions of Theorem 4.1. are satisfied, and the system global stability is guaranteed despite the configuration changing.

6. Conclusions

A convex programming method is presented which provides an efficient design of switching robust static output-feedback controllers for linear systems with state multiplicative noise. Sufficient conditions are derived for the existence of switching controller that stabilizes the system and achieves a prescribed bound on its performance. The stochastic H_2/H_∞ performance criterion is considered.

In this work, a hybrid robust controller is developed. The main idea is the use of a fuzzy supervisor to manage efficiently the action of two controllers based on H_2 and H_∞, in a way that the system remains stable with good performance and low conservatives despite the plant switching from one mode to a new one. Furthermore, this structure allows us to take the advantage of both controllers and to efficiently eliminate their drawbacks. Simulation results showed the efficiency and the design simplicity of the proposed approach. Indeed, the H_2 provides good performances in the transient state (a fast dynamic response, enlarged stability limits of the system), while the H_∞ control acts mainly in the steady state (reduces chattering and the effect of the external disturbances). This work can be generalized to multiple controllers, more than two, managed by the same fuzzy supervisor. Indeed, the structure of the fuzzy supervisor allows partitioning the state into different substates. An adequate controller can be defined for each substate to ensure the desired performances. The rule base of the fuzzy supervisor will be reconstructed so that the premise part defines the subspace and the conclusion part defines the corresponding control law and the applied control signal will be a weighted sum of all the controllers used.

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