Research Article

Dual Solutions on Thermosolutal Marangoni Forced Convection Boundary Layer with Suction and Injection

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This paper considers the extended problem of the thermosolutal Marangoni forced convection boundary layer by Pop et al. (2001) when the wall is permeable, namely, there is a suction or injection effect. The governing system of partial differential equations is transformed into a system of ordinary differential equations, and the transformed equations are solved numerically using the shooting method. The effects of suction or injection parameter f_0 on the velocity, temperature, and concentration profiles are illustrated and presented in tables and figures. It is shown that dual solutions exist for the similarity parameter β less than 0.5.

1. Introduction

Thermosolutal Marangoni forced convection boundary layer flow refers to the thermal and solutal concentration in Marangoni forced convection boundary layer flow due to the surface tension gradients. The study of Marangoni convection has attracted the interest of many researchers in recent years. This is mainly because of its vast contributions in the industrial field especially in the art work of dyeing on the ground (Kuroda [1]) and in the field of crystal growth (Arafune and Hirata [2]). Some of the relevant works are done by Christopher and Wang [3] who studied the effects of Prandtl number on the Marangoni convection over a flat plate. They have also presented a similarity solution for Marangoni flow for both the momentum and the energy equations assuming a developing boundary

layer along a surface. Besides, Chamkha et al. [4] have dealt with a steady coupled dissipative layer, called Marangoni mixed convection boundary layer. The mixed convection boundary layer is generated when besides the Marangoni effects there are also buoyancy effects due to the gravitational and external pressure gradient effects. Furthermore, in the following papers, numerical solutions on Marangoni boundary layers in various geometries were discovered, analyzed, and discussed, for instance, papers by Golia and Viviani [5], Dressler and Sivakumaran [6], Al-Mudhaf and Chamkha [7], and Magyari and Chamkha [8]. An excellent paper on Marangoni boundary layer flow along the interface of the immiscible nanofluid by Arifin et al. [9] has been recently published. They show that the results indicate that dual solutions exist when $\beta < 0.5$. The paper complements also the work by Golia and Viviani [5] concerning the dual solutions in the case of adverse pressure gradient.

Investigations into the effects of suction and injection on the boundary layer flow have generated the interest of many researchers nowadays. Pop and Watanabe [10] analyzed the effects of uniform suction or injection on the boundary layer flow and heat transfer on a continuous moving permeable surface. Meanwhile, Hamza [11] obtained the similarity solution for a flow between two parallel plates (rectangular or circular) approaching or receding from each other with suction or injection at the porous plate. Shojaefard et al. [12] investigated the numerical study concerning flow control by suction and injection on a subsonic airfoil. They discovered that the surface suction can significantly increase the lift coefficient; meanwhile, the injection decreases the skin friction.

The existence of dual solutions or second solutions has aroused a lot of interest of many authors. We mention here some of the papers that discovered the dual solution in their problems. For example, papers by de Hoog et al. [13], Ingham [14], Ramachandran et al. [15], Xu and Liao [16] and Ishak et al. [17–19]. Ridha [20] considered the dual solutions of two coupled third-degree nonlinear ordinary differential equations associated with the incompressible viscous laminar flow along a corner. Further, Xu and Liao [16], obtained the dual solutions of boundary layer flow over an upstream moving plate using the homotopy analysis method (HAM). Ishak et al. [19] discovered the dual solution of the classical Blasius problem. In their study, they considered the boundary layer flow over a static flat plate and introduced a new parameter, namely, the velocity ratio parameter to analyze the case when both the flat plate and the free stream are in moving situations. Dual solutions are found to exist when the plate and the free stream move in the opposite directions.

In this paper, we extend the problem by Pop et al. [21] by taking the effects of suction or injection for the boundary layer flow. We would also like to investigate whether the dual solutions exist in the above problem. To validate our findings, we have compared the present results with those of Pop et al. [21] for the case of impermeable surface. It is found that the results are in very good agreement. The effects of suction and injection parameter on the surface velocity, temperature, and concentration as well as on velocity, temperature, and concentration profiles are discussed and illustrated in tables and figures.

2. Mathematical Formulation

We consider the steady two-dimensional flow along the interface S of two Newtonian immiscible fluids, where x and y are the axes of a Cartesian coordinate system. We also assume that the temperature and concentration at the interface are $T_s(x)$ and $C_s(x)$, respectively. Besides, the surface is assumed to be permeable so as to allow for possible suction or injection at the wall (Al-Mudhaf and Chamkha [7]). Under the usual boundary-

layer approximations, the basic governing equations are (see Pop et al. [21])

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + v\frac{\partial^2 u}{\partial y^2},$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2},$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2},$$

(2.1)

with the boundary conditions,

$$v = v_w, \quad T = T_s(x), \quad C = C_s(x), \quad \mu\frac{\partial u}{\partial y} = \sigma_T\frac{\partial T}{\partial x} + \sigma_C\frac{\partial C}{\partial x} \quad \text{on } y = 0, \qquad (2.2)$$

$$u \longrightarrow u_e(x), \quad T \longrightarrow T_m \quad C \longrightarrow C_m \quad \text{as } y \longrightarrow \infty, \qquad (2.3)$$

where u and v are the velocity components along x and y axes, T is the fluid temperature, C is the solutal concentration, $u_e(x)$ is the external velocity, α is the thermal diffusivity, v is the kinematic viscosity, D is the mass diffusivity, and v_w is the constant suction ($v_w > 0$) or injection ($v_w < 0$) velocity, μ is the dynamic viscosity, and σ_T and σ_C are the rates of change of surface tension with temperature and solute concentration, respectively. As mentioned by Pop et al. [21], the forth condition of (2.2) represents the Marangoni coupling conditions at the interface (balance of the surface tangential momentum), having considered for the surface tension the linear relation given by

$$\sigma = \sigma_m - \sigma_T(T - T_m) - \sigma_C(C - C_m), \quad \sigma_T = -\frac{\partial\sigma}{\partial T}, \ \sigma_C = -\frac{\partial\sigma}{\partial C}. \qquad (2.4)$$

Following Pop et al. [21], we define the following nondimensional variables:

$$x = L_0 + XL, \qquad y = \delta LY, \qquad u = U_cU, \qquad v = \delta U_cV,$$

$$T = T_m + t\Delta T, \qquad C = C_m + \phi\Delta C, \qquad u_e(x) = U_cU_e(x), \qquad v_w = U_cL\delta^2V_w,$$

(2.5)

where L_0 locates the origin of the curvilinear abscissa x, L is the extension of the relevant interface S, and ΔT and ΔC are positive increments of temperature and solute concentration linked to the temperature and solute concentration gradients imposed on the interface,

respectively. Further, δ is a scale factor in the direction normal to the interface, and U_c is the reference velocity which is defined as $\delta = \mathrm{Re}^{-1/3}$ and $U_c = \nu/(L\delta^2)$ with $\mathrm{Re} = \sigma_T \Delta T L / \nu \mu$ being the Reynolds number. Substituting (2.5) into (2.1), we obtain the following nondimensional equations:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0,$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = U_e \frac{dU_e}{dX} + \frac{\partial^2 U}{\partial Y^2},$$

$$U\frac{\partial t}{\partial X} + V\frac{\partial t}{\partial Y} = \frac{1}{P_r}\frac{\partial^2 t}{\partial Y^2}, \tag{2.6}$$

$$U\frac{\partial \phi}{\partial X} + V\frac{\partial \phi}{\partial Y} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial Y^2},$$

and the boundary conditions (2.2) and (2.3) reduce to

$$V = V_w, \quad t = t_s(X), \quad \phi = \phi_s(X), \quad \frac{\partial U}{\partial Y} = \frac{\partial t}{\partial X} + \varepsilon\frac{\partial \phi}{\partial X} \quad \text{on } Y = 0,$$

$$U \longrightarrow U_e(X), \quad t \longrightarrow 0, \quad \phi \longrightarrow 0 \quad \text{as } Y \longrightarrow \infty, \tag{2.7}$$

where Pr is the Prandtl number, Sc is the Schmidt number, and ε is the Marangoni parameter and is defined as

$$\varepsilon = \frac{Ma_C}{Ma_T} = \frac{\sigma_C \Delta C}{\sigma_T \Delta T}, \tag{2.8}$$

with $Ma_C = \sigma_C \Delta C L / \mu\alpha$ and $Ma_T = \sigma_T \Delta T L / \mu\alpha$ being the solutal Marangoni number and thermal Marangoni number, respectively. It is also noted that positive V_w is for fluid suction and negative for fluid injection at the wall. Equations (2.6) can be transformed into the corresponding ordinary differential equations by the following transformations (see Pop et al. [21]):

$$U = u_0 X^{(2\beta-1)/3} f'(\eta), \qquad V = \frac{1}{3} u_0 l_0 x^{(\beta-2)/3}\left((2-\beta)\eta f'(\eta) - (1+\beta)f(\eta)\right),$$

$$t = -t_0 X^\beta \theta(\eta), \qquad \phi = -c_0 X^\beta h(\eta), \qquad U_e = u_0 X^{(2\beta-1)/3}, \qquad \eta = \frac{Y}{l_0 x^{(2-\beta)/3}}, \tag{2.9}$$

where $f'(\eta)$, $\theta(\eta)$, and $h(\eta)$ represent the velocity, temperature, and concentration profiles in the similarity plane and η being the similarity variable. The constant scale factors u_0, t_0, c_0, and l_0 are chosen in order to simplify the equations, and these must satisfy the following conditions:

$$u_0 l_0^2 = \frac{3}{\beta+1}, \qquad \frac{t_0 l_0}{u_0} = \frac{1}{\beta}, \qquad \frac{c_0}{t_0} = 1. \tag{2.10}$$

If we take $t_0 = 1$ or $c_0 = 1$ (due to arbitrariness of the temperature difference ΔT or the solutal concentration difference ΔC), then l_0 and u_0 are uniquely determined as

$$l_0 = \left(\frac{3}{1+\beta}\right)^{1/3} \beta^{-1/3}, \qquad u_0 = \left(\frac{3}{1+\beta}\right)^{1/3} \beta^{2/3}. \tag{2.11}$$

The transformed ordinary differential equations are

$$f''' + ff'' + \frac{2\beta - 1}{\beta + 1}\left(1 - f'^2\right) = 0,$$

$$\frac{1}{\text{Pr}}\theta'' + f\theta' - \frac{3\beta}{1+\beta}f'\theta = 0, \tag{2.12}$$

$$\frac{1}{\text{Sc}}h'' + fh' - \frac{3\beta}{1+\beta}f'h = 0,$$

along with the boundary conditions

$$f(0) = f_0, \qquad f''(0) = -1 - \varepsilon, \qquad \theta(0) = 1, \qquad h(0) = 1,$$

$$f'(\infty) = 1, \qquad \theta(\infty) = 0, \qquad h(\infty) = 0, \tag{2.13}$$

where $f_0(>0)$ is the constant suction parameter and $f_0(<0)$ is the constant injection parameter.

3. Results and Discussion

The system of transformed governing equations (2.12) along with the boundary equations (2.13) is solved numerically using the shooting method. Tables 1, 2, and 3 illustrate the influence of the suction and injection parameter $f_0 = 1, 0$, and -1 on the surface velocity, $f'(0)$, surface temperature, $-\theta'(0)$, and surface concentration, $-h'(0)$ for different values of the similarity parameter, β and different values of the Schmidt number, Sc in the case of Marangoni parameter, $\varepsilon = 0, 1$, and -1. Further, the Prandtl number, Pr, is taken to be Pr = 0.7 corresponding to air and the Schmidt number; Sc has the following values: Sc = 0.22 (hydrogen), 0.6 (water), 0.75 (oxygen), and 0.78 (ammonia). It should be noticed that the results given in the parentheses ( ) are the second (dual) solutions. Results obtained by Pop et al. [21] for the case of impermeable surface are also included in these tables. It is clearly seen that the results agree well in all the three tables.

Figures 1, 2, and 3 present the variations of the surface velocity, temperature, and concentration with β, respectively, when Pr = 0.7, Sc = 0.78 in the case of $\varepsilon = 0$ (Marangoni effect neglected) with the effect of the parameter f_0. The dashed line refers to the second solution of the problem. From these figures, we can see that the second solutions exist for $\beta < 0.5$. This discovery is consistent with the statement given in Pop et al. [21], that for $\beta < 0.5$, the solutions are not unique. Not only that it can be observed from the figures but the effects of suction or injection parameter can be clearly seen when $\beta > 0.4$. The imposition of suction ($f_0 > 0$) at the surface has the tendency to reduce the velocity but increase the temperature and concentration gradients along the interface. Meanwhile, the opposite results are observed

Table 1: Values of $f'(0)$, $-\theta'(0)$, and $-h'(0)$ for $\varepsilon = 0$ (absent Marangoni effect) and $f_0 = 0$ (impermeable surface) for different values of β when $\mathrm{Pr} = 0.7$.

β	f_0	Source	$f'(0)$	$-\theta'(0)$	$-h'(0)$	$f'(0)$	$-\theta'(0)$	$-h'(0)$
0.1	−1	Present	20.42222 (7.52155)	2.73049 (1.39352)	1.33457 (0.55567)	2.50450 (1.25965)	2.83556 (1.45391)	2.89636 (1.48832)
	0	Present	5.63217 (1.13296)	1.66616 (0.63301)	0.81816 (0.34081)	1.52112 (0.57999)	1.73479 (0.65856)	1.77485 (0.67361)
	1	Present	2.18587 (0.32671)	1.51789 (0.81825)	0.69038 (0.36368)	1.36228 (0.72258)	1.59367 (0.86650)	1.63858 (0.89559)
0.2	−1	Present	6.88979 (3.22734)	1.70167 (0.89556)	0.91512 (0.42147)	1.57757 (0.81507)	1.75892 (0.93265)	1.79192 (0.95395)
	0	Present	3.14160 (0.96668)	1.40729 (0.58243)	0.72749 (0.30936)	1.29013 (0.53105)	1.46279 (0.60744)	1.49521 (0.62224)
	1	Present	1.87556 (0.23194)	1.54931 (0.65695)	0.72307 (0.29068)	1.39337 (0.57422)	1.62523 (0.69967)	1.67021 (0.72573)
0.3	−1	Present	4.09080 (2.49526)	1.38372 (0.82414)	0.78472 (0.39756)	1.29037 (0.75041)	1.42662 (0.85849)	1.45130 (0.87834)
	0	Present	2.44523 (0.97452)	1.34800 (0.54113)	0.71088 (0.26932)	1.23812 (0.48650)	1.40007 (0.56816)	1.43051 (0.58430)
	1	Present	1.73727 (0.24512)	1.59029 (0.43442)	0.75025 (0.20410)	1.43243 (0.37519)	1.66711 (0.46669)	1.71261 (0.48694)
0.4	−1	Present	3.09833 (2.2474)	1.26054 (0.14478)	0.73410 (0.36768)	1.17886 (0.56070)	1.29804 (7.84348)	1.31959 (1.83203)
	0	Present	2.14605 (1.08908)	1.34240 (0.35840)	0.71485 (0.22042)	1.23425 (0.35627)	1.39369 (0.34761)	1.42366 (0.33547)
	1	Present	1.65874 (0.34764)	1.62937 (−1.28801)	0.77466 (0.07600)	1.46934 (−0.48622)	1.70719 (−2.35420)	1.75326 (−3.77805)
0.5	−1	Present	2.63924	1.21201	0.71510	1.13497	1.24737	1.26770
	0	Pop et al. [21]	1.98426	1.35416	0.72496	1.24583	1.40553	1.43556
	0	Present	1.98427	1.35417	0.72506	1.24583	1.40553	1.43556
	1	Present	1.60781	1.66444	0.79602	1.50233	1.74322	1.78986

Table 1: Continued.

β	f_0	Pop et al. [21]						Present					
		$f'(0)$	$-\theta'(0)$	$-h'(0)$				$f'(0)$	$-\theta'(0)$	$-h'(0)$			
				0.22	0.60	0.75	0.78			0.22	0.60	0.75	0.78
1.0	−1							1.98027	1.21194	0.72270	1.13609	1.24683	1.26692
	0	1.69687	1.44233	0.77894	1.32869	1.49624	1.52774	1.69687	1.44233	0.77786	1.32866	1.49623	1.52774
	1							1.49425	1.78801	0.86768	1.61811	1.87042	1.91915
2.0	−1							1.74213	1.29145	0.77020	1.21009	1.32898	1.35062
	0	1.56595	1.55062	0.84232	1.42944	1.60807	1.64166	1.56595	1.55061	0.84097	1.42942	1.60807	1.64166
	1							1.42871	1.90694	0.93762	1.72918	1.99303	2.04390
5.0	−1							1.61621	1.38658	0.82498	1.29840	1.42734	1.45087
	0	1.48856	1.65944	0.90415	1.53047	1.72058	1.75632	1.48857	1.65943	0.90322	1.53045	1.72057	1.75631
	1							1.38478	2.01946	1.00309	1.83407	2.10912	2.16207
Sc				0.22	0.60	0.75	0.78			0.22	0.60	0.75	0.78

Table 2: Values of $f'(0)$, $-\theta'(0)$, and $-h'(0)$ for $\varepsilon = 1$ and $f_0 = 0$ (impermeable surface) for different values of β when Pr = 0.7.

β	f_0	Pop et al. [21]						Present					
		$f'(0)$	$-\theta'(0)$				$-h'(0)$	$f'(0)$	$-\theta'(0)$				$-h'(0)$
0.1	−1							21.97653 (10.27675)	2.83813 (1.70953)	1.38014 (0.66173)	2.60161 (1.54850)	2.94816 (1.78238)	3.01186 (1.82402)
	0							7.19144 (2.24508)	1.85863 (0.80540)	0.89765 (0.36166)	1.69425 (0.71946)	1.93639 (0.84714)	1.98180 (0.87179)
	1							3.12331 (0.85654)	1.67897 (0.87403)	0.75490 (0.35695)	1.50685 (0.76213)	1.76256 (0.93064)	1.81201 (0.96479)
0.2	−1							8.28421 (4.98050)	1.87820 (1.20322)	0.98806 (0.50472)	1.73707 (1.08332)	1.94347 (1.25792)	1.98114 (1.28916)
	0							4.29569 (2.03295)	1.61240 (0.81721)	0.80832 (0.35273)	1.47475 (0.72457)	1.67759 (0.86250)	1.71568 (0.88933)
	1							2.61520 (0.83925)	1.69789 (0.78169)	0.77746 (0.30148)	1.52666 (0.67010)	1.78105 (0.83928)	1.83026 (0.87434)
0.3	−1							5.32651 (3.83896)	1.59751 (1.08646)	0.87461 (0.46868)	1.48397 (0.97392)	1.64991 (1.13936)	1.68012 (1.16998)
	0							3.39676 (1.97392)	1.55082 (0.80880)	0.79147 (0.32899)	1.42074 (0.70805)	1.61248 (0.85897)	1.64851 (0.88895)
	1							2.37773 (0.89485)	1.73187 (0.60914)	0.80281 (0.22770)	1.55938 (0.50839)	1.81562 (0.66417)	1.86517 (0.69865)
0.4	−1							4.18935 (3.36425)	1.48697 (1.27934)	0.83076 (0.40355)	1.38419 (1.31977)	1.53439 (1.29417)	1.56172 (1.30698)
	0							2.97529 (1.98120)	1.53913 (2.00757)	0.79355 (0.26050)	1.41135 (−2.95626)	1.59972 (1.64605)	1.63513 (1.55295)
	1							2.23941 (1.00247)	1.76664 (2.39461)	0.82624 (0.08520)	1.59238 (10.44973)	1.85121 (2.02516)	1.90123 (1.90673)
0.5	−1							3.61818	1.43979	0.81724	1.34168	1.48506	1.51116
	0	2.73362	1.54507	0.80616	1.41769	1.60549	1.64081	2.73363	1.54508	0.80624	1.41769	1.60549	1.64082
	1							2.14837	1.79888	0.85103	1.62283	1.88430	1.93481

Table 2: Continued.

β	f_0	Pop et al. [21]					Present				
		$f'(0)$	$-\theta'(0)$	$-h'(0)$			$f'(0)$	$-\theta'(0)$	$-h'(0)$		
1.0	−1						2.70104	1.42520	0.82117	1.32957	1.46943
	0	2.27460	1.61711	0.85436	1.48585	1.71581	2.27460	1.61711	0.85442	1.48585	1.67939
	1						1.94189	1.91652	0.92221	1.73321	2.00533
2.0	−1						2.32953	1.49149	0.86159	1.39128	1.53794
	0	2.05196	1.71597	0.91272	1.57799	1.81972	2.05197	1.71596	0.91276	1.57799	1.81972
	1						1.82082	2.03278	0.99058	1.84182	2.17974
5.0	−1						2.12308	1.57802	0.91143	1.47155	1.65606
	0	1.91654	1.81917	0.97201	1.67387	1.88812	1.91654	1.81917	0.97203	1.67387	1.92843
	1						1.73902	2.14410	1.05523	1.94559	2.29666
Sc			0.22	0.60	0.75	0.78		0.22	0.60	0.75	0.78

Table 3: Values of $f'(0)$, $-\theta'(0)$, and $-h'(0)$ for $\varepsilon = -1$ and $f_0 = 0$ (impermeable surface) for different values of β when $\mathrm{Pr} = 0.7$.

β	f_0	Pop et al. [21] $f'(0)$	$-\theta'(0)$	$-h'(0)$				Present $f'(0)$	$-\theta'(0)$	$-h'(0)$			
0.1	−1							1 (18.69400)	0.42585 (2.60605)	0.31791 (1.28181)	0.41446 (2.39223)	0.43041 (2.70538)	0.43284 (2.76284)
	0							1 (3.43343)	0.78611 (1.34348)	0.44073 (0.68254)	0.72780 (1.23088)	0.81370 (1.39675)	0.82982 (1.42786)
	1							1 (0.02861)	1.26652 (0.88734)	0.58489 (0.41760)	1.13660 (0.79323)	1.33015 (0.93423)	1.36796 (0.96233)
0.2	−1							1 (4.97600)	0.52173 (1.42764)	0.37065 (0.79313)	0.50312 (1.32976)	0.52969 (1.47255)	0.53409 (1.49836)
	0							1 (1.00014)	0.87518 (0.87524)	0.49066 (0.49068)	0.81026 (0.81033)	0.90590 (0.90596)	0.92384 (0.92390)
	1							1 (−0.28079)	1.34283 (0.63887)	0.63095 (0.31836)	1.20818 (0.57116)	1.40866 (0.67331)	1.44774 (0.69417)
0.3	−1							1 (1.91347)	0.59576 (0.90172)	0.41163 (0.56117)	0.57163 (0.85234)	0.60631 (0.92399)	0.61223 (0.93668)
	0							1 (0.24145)	0.94510 (0.56101)	0.52985 (0.35365)	0.87499 (0.52701)	0.97827 (0.57702)	0.99765 (0.58635)
	1							1 (−0.40926)	1.40391 (0.34253)	0.66752 (0.20927)	1.26538 (0.31198)	1.47153 (0.35879)	1.51165 (0.36889)
0.4	−1							1 (0.69584)	0.65507 (0.51595)	0.44459 (0.37337)	0.62654 (0.49813)	0.66768 (0.52364)	0.67480 (0.52792)
	0							1 (−0.08927)	1.00170 (0.25272)	0.56157 (0.21647)	0.92739 (0.24983)	1.03686 (0.25371)	1.05739 (0.25419)
	1							1 (−0.43782)	1.45403 (−0.21212)	0.69734 (0.07165)	1.31228 (−0.12659)	1.52315 (−0.26250)	1.56413 (−0.29559)
0.5	−1							1	0.70405	0.48098	0.67213	0.71830	0.72637
	0	1	1.04866	0.59509	0.97098	1.08544	1.10693	1	1.04866	0.59522	0.97098	1.08544	1.10692
	1							1	1.49598	0.72752	1.35154	1.56636	1.60807

Table 3: Continued.

β	f_0	Pop et al. [21]						Present					
		$f'(0)$	$-\theta'(0)$	$-h'(0)$				$f'(0)$	$-\theta'(0)$	$-h'(0)$			
1.0	−1							1	0.85950	0.56415	0.81603	0.87916	0.89038
	0	1	1.19977	0.67691	1.11081	1.24187	1.26646	1	1.19976	0.67700	1.11081	1.24186	1.26645
	1							1	1.63339	0.80629	1.47984	1.70800	1.75217
2.0	−1							1	0.99728	0.63936	0.94367	1.02170	1.03569
	0	1	1.33513	0.75117	1.23611	1.38198	1.40935	1	1.33512	0.75122	1.23611	1.38198	1.40935
	1							1	1.75885	0.87824	1.59682	1.83743	1.88389
5.0	−1							1	1.12221	0.70834	1.05945	1.15093	1.16742
	0	1	1.45865	0.81945	1.35046	1.50985	1.53975	1	1.45865	0.81949	1.35045	1.50984	1.53974
	1							1	1.87489	0.94473	1.70489	1.95719	2.00582
Sc				0.22	0.60	0.75	0.78			0.22	0.60	0.75	0.78
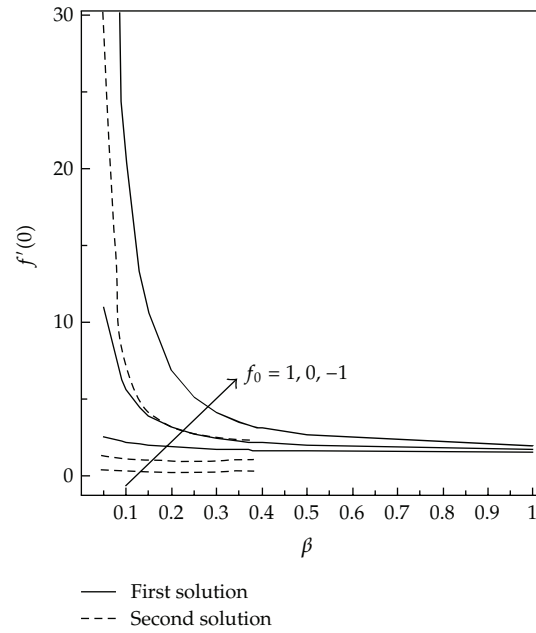
Figure 1: Variation of the surface velocity with β when Pr = 0.7, Sc = 0.78, and $\varepsilon = 0$.
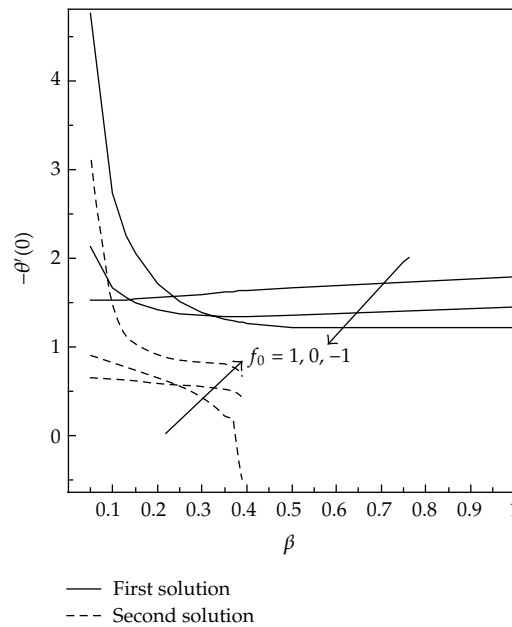
Figure 2: Variation of the surface temperature with β when Pr = 0.7, Sc = 0.78, and $\varepsilon = 0$.

for the case of surface injection ($f_0 < 0$), an increase in the surface velocity and decrease in the surface temperature and concentration gradients. Hence, consequently, the interface heat transfer and the interface mass transfer will decrease as well.
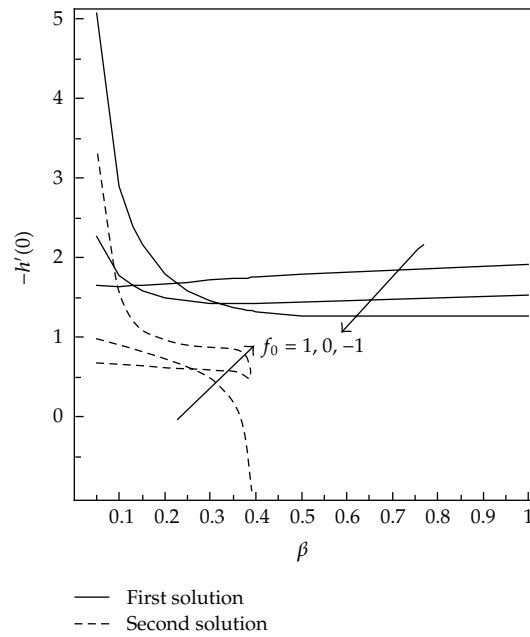
Figure 3: Variation of the surface concentration with β when Pr $= 0.7$, Sc $= 0.78$, and $\varepsilon = 0$.
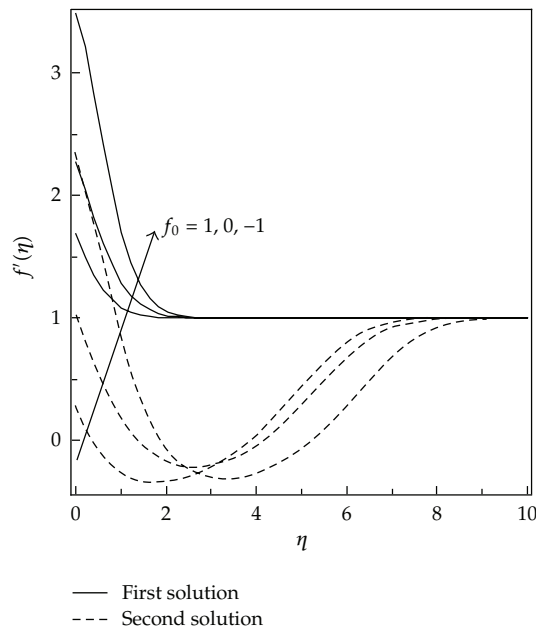
Figure 4: Effects of f_0 on the velocity profiles when Pr $= 0.7$, Sc $= 0.78$, $\beta = 0.35$, and $\varepsilon = 0$.

Figures 4 to 12 illustrate the effects of parameter f_0 on the velocity $f'(\eta)$, temperature $\theta(\eta)$, and concentration $h(\eta)$ profiles when Pr $= 0.7$ (air), Sc $= 0.78$ (ammonia), and $\beta = 0.35$ in the case of $\varepsilon = 0, 1$, and -1. The influence of f_0 on the velocity profiles is depicted in Figures 4, 5, and 6. The figures show that the suction parameter decreases the velocity profiles, and the
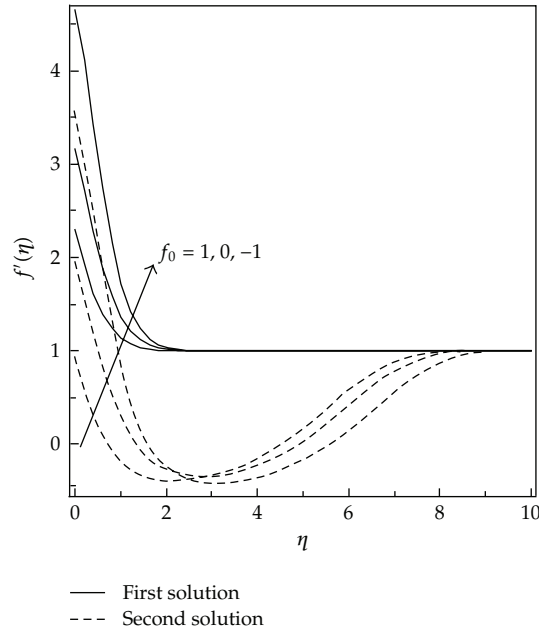
Figure 5: Effects of f_0 on the velocity profiles when Pr = 0.7, Sc = 0.78, β = 0.35, and ε = 1.
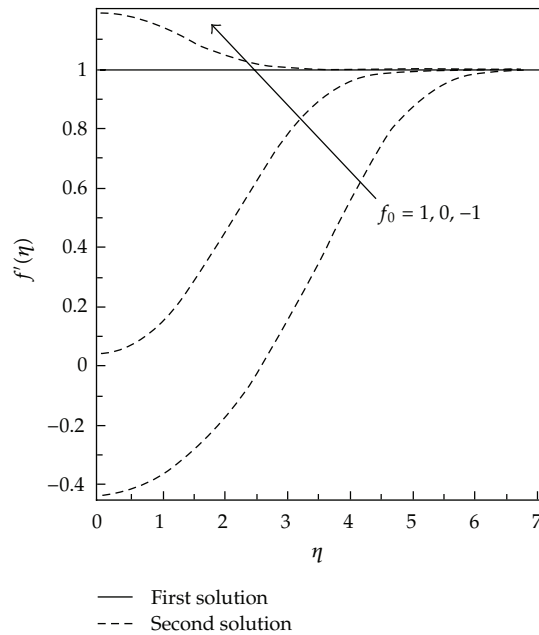
Figure 6: Effects of f_0 on the velocity profiles when Pr = 0.7, Sc = 0.78, β = 0.35, and ε = − 1.

injection parameter increases the velocity profiles for the case of the Marangoni parameter ε = 0 and 1. Further, in Figures 7, 8, and 9, we can see the effects of f_0 on the temperature profiles. The figures indicate that for ε = 0, 1, and −1, imposition of the suction parameter tends to
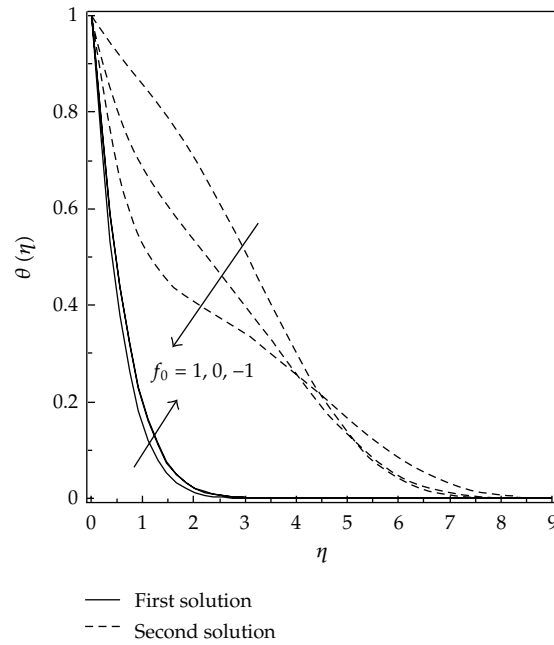
Figure 7: Effects of f_0 on the temperature profiles when Pr = 0.7, Sc = 0.78, β = 0.35, and ε = 0.
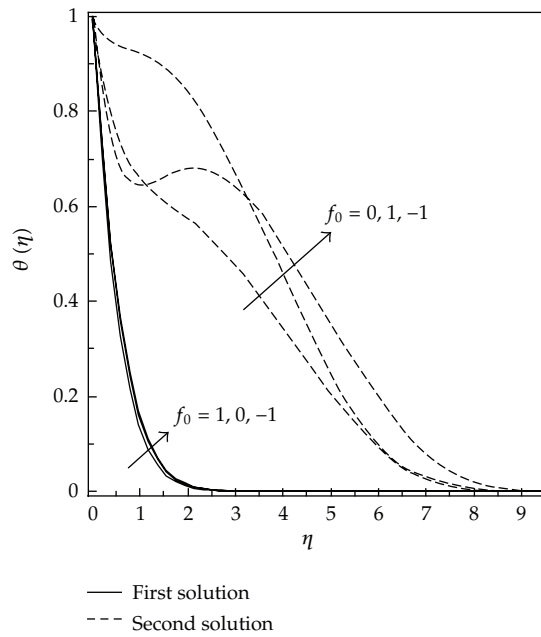
Figure 8: Effects of f_0 on the temperature profiles when Pr = 0.7, Sc = 0.78, β = 0.35, and ε = 1.

reduce the temperature profiles, whereas the injection parameter increases the profiles. It should be noticed that the second solutions in these cases have positive values. Besides, it can also be seen that reduction in Marangoni parameter ε causes the dual solutions to be
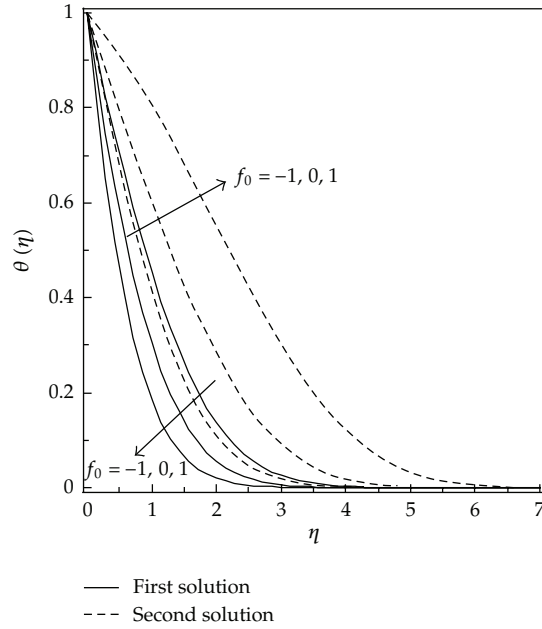
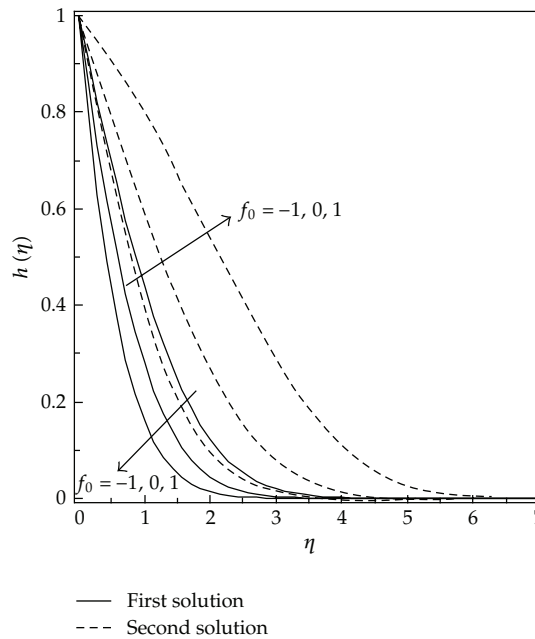**Figure 9:** Effects of f_0 on the temperature profiles when Pr = 0.7, Sc = 0.78, β = 0.35, and $\varepsilon = -1$.
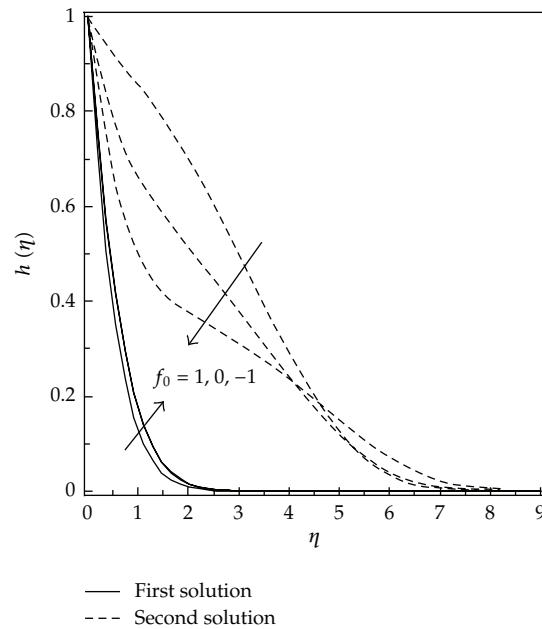
Figure 10: Effects of f_0 on the concentration profiles when Pr = 0.7, Sc = 0.78, β = 0.35, and $\varepsilon = 0$.

more stable. On the other hand, the effects of suction or injection parameter on concentration profiles $h(\eta)$ are shown in Figures 10, 11, and 12 and are similar to the concentration profiles.
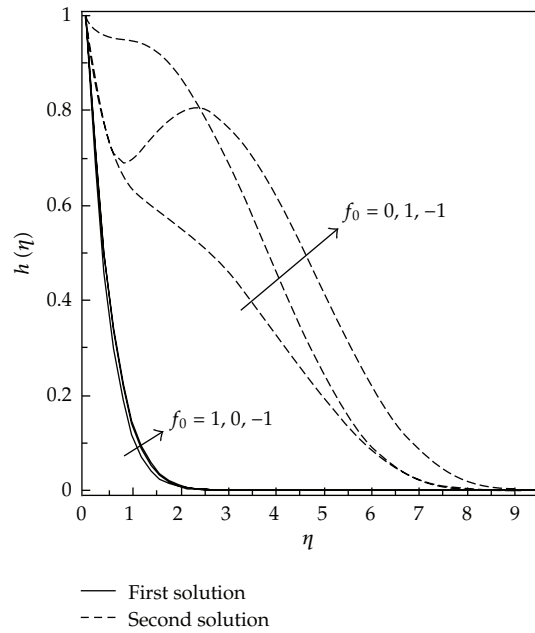
Figure 11: Effects of f_0 on the concentration profiles when $Pr = 0.7$, $Sc = 0.78$, $\beta = 0.35$, and $\varepsilon = 1$.

Figure 12: Effects of f_0 on the concentration profiles when $Pr = 0.7$, $Sc = 0.78$, $\beta = 0.35$, and $\varepsilon = -1$.

4. Conclusions

The present work deals with the thermosolutal Marangoni forced convection boundary layer flow as considered by Pop et al. [21]. We extended the paper by taking into consideration

the effects of suction or injection. On the other hand, we also studied the existence of dual similarity solutions in the present problem. Further, the governing equations are transformed into ordinary differential equations and are then solved numerically using the shooting method. The effects of suction or injection parameter on the flow and heat transfer characteristics are studied. In general, imposition of suction is to decrease the velocity, temperature, and concentration profiles, whereas injection shows the opposite effects. On the other hand, the second (dual) solutions are discovered to exist when similarity parameter $\beta < 0.5$.
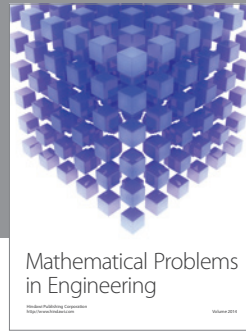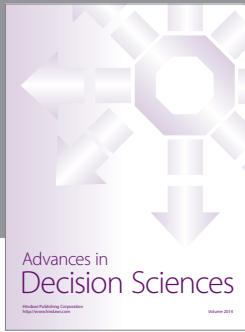
Acknowledgment

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