

## Research Article

# ***P*th Moment Exponential Stability of Impulsive Stochastic Neural Networks with Mixed Delays**

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This paper investigates the problem of  $p$ th moment exponential stability for a class of stochastic neural networks with time-varying delays and distributed delays under nonlinear impulsive perturbations. By means of Lyapunov functionals, stochastic analysis and differential inequality technique, criteria on  $p$ th moment exponential stability of this model are derived. The results of this paper are completely new and complement and improve some of the previously known results (Stamova and Ilarionov (2010), Zhang et al. (2005), Li (2010), Ahmed and Stamova (2008), Huang et al. (2008), Huang et al. (2008), and Stamova (2009)). An example is employed to illustrate our feasible results.

## **1. Introduction**

The dynamics of neural networks have drawn considerable attention in recent years due to their extensive applications in many fields such as image processing, associative memories, classification of patterns, and optimization. Since the integration and communication delays are unavoidably encountered in biological and artificial neural systems, it may result in oscillation and instability. The stability analysis of delayed neural networks has been extensively investigated by many researchers, for instance, see [1–30].

In real nervous systems, there are many stochastic perturbations that affect the stability of neural networks. The result in Mao [24] suggested that one neural network could be stabilized or destabilized by certain stochastic inputs. It implies that the stability analysis of stochastic neural networks has primary significance in the design and applications of neural networks, such as [7, 12–16, 18, 20, 22–24, 26, 27, 30].

On the other hand, it is noteworthy that the state of electronic networks is often subjected to some phenomenon or other sudden noises. On that account, the electronic networks will experience some abrupt changes at certain instants that in turn affect dynamical behaviors of the systems [5, 6, 17–23, 28, 29]. Therefore, it is necessary to take both stochastic effects and impulsive perturbations into account on dynamical behaviors of delayed neural networks [18, 20, 22, 23].

Very recently, Li et al. [22] have employed the properties of M-cone and inequality technique to investigate the mean square exponential stability of impulsive stochastic neural networks with bounded delays. Wu et al. [23] studied the exponential stability of the equilibrium point of bounded discrete-time delayed dynamic systems with linear impulsive effects by using Razumikhin theorems. To the best of authors' knowledge, however, few authors have considered the  $p$ th moment exponential stability of impulsive stochastic neural networks with mixed delays.

Motivated by the discussions above, our object in this paper is to present the sufficient conditions ensuring  $p$ th moment exponential stability for a class of stochastic neural networks with time-varying delays and distributed delays under nonlinear impulsive perturbations by virtue of Lyapunov method, inequality technique and Itô formula. The results obtained in this paper generalize and improve some of the existing results [5, 8, 18, 19, 26–28]. The effectiveness and feasibility of the developed results have been shown by a numerical example.

## 2. Model Description and Preliminaries

Let  $\mathbb{R}$  denote the set of real numbers,  $\mathbb{R}^n$  the  $n$ -dimensional real space equipped with the Euclidean norm  $|\cdot|$ ,  $\mathbb{Z}^+$  the set of nonnegative integral numbers.  $E(\cdot)$  stands for the mathematical expectation operator.  $\mathcal{L}$  denotes the well-known  $\mathcal{L}$ -operator given by the Itô formula.  $w(t) = (w_1(t), \dots, w_m(t))$  is  $m$ -dimensional Brownian motion defined on a complete probability space  $(\Omega, \mathcal{F}, P)$  with a natural filtration  $\{\mathcal{F}_t\}_{t \geq 0}$  generated by  $\{w(s) : 0 \leq s \leq t\}$ , where we associate  $\Omega$  with the canonical space generated by  $w(t)$  and denote by  $\mathcal{F}$  the associated  $\sigma$ -algebra generated by  $w(t)$  with the probability measure  $P$ . Let  $\sigma(t, x, y) = (\sigma_{il}(t, x_i, y_i))_{n \times m} \in \mathbb{R}^{n \times m}$ , and  $\sigma_i(t, x_i, y_i)$  be  $i$ th row vector of  $\sigma(t, x, y)$ .

In [5, 6], the researchers investigated the following impulsive neural networks with time-varying delays:

$$\begin{aligned} \dot{x}_i(t) &= -a_i(x_i(t)) + \sum_{j=1}^n b_{ij} f_j(x_j(t)) + \sum_{j=1}^n c_{ij} g_j(x_j(t - \tau_j(t))) + I_i, \quad t \neq t_k, \\ x_i(t_k) &= p_{ik}(x(t_k^-)), \quad k \in \mathbb{Z}^+, i \in \Lambda. \end{aligned} \quad (2.1)$$

The authors in [7, 26, 27] studied the stochastic recurrent neural networks with time-varying delays:

$$\begin{aligned} dx_i(t) &= \left( -a_i(x_i(t)) + \sum_{j=1}^n b_{ij} f_j(x_j(t)) + \sum_{j=1}^n c_{ij} g_j(x_j(t - \tau_j(t))) + I_i \right) dt \\ &+ \sum_{l=1}^m \sigma_{il}(t, x_i(t), x_i(t - \tau_i(t))) dw_l(t). \end{aligned} \quad (2.2)$$

In this paper, we will study the generalized stochastically perturbed neural network model with time-varying delays and distributed delays under nonlinear impulses defined by the state equations:

$$\begin{aligned}
 dx_i(t) = & \left[ -a_i(x_i(t)) + \sum_{j=1}^n b_{ij} f_j(x_j(t)) + \sum_{j=1}^n c_{ij} g_j(x_j(t - \tau_j(t))) \right. \\
 & \left. + \sum_{j=1}^n d_{ij} \int_{-\infty}^t K_{ij}(t-s) \cdot h_j(x_j(s)) ds + I_i \right] dt \\
 & + \sum_{l=1}^m \sigma_{il}(t, x_i(t), x_i(t - \tau_i(t))) d\omega_l(t), \quad t \neq t_k, \\
 x_i(t_k) = & p_{ik}(x(t_k^-)), \quad k \in \mathbb{Z}^+, i \in \Lambda,
 \end{aligned} \tag{2.3}$$

where  $\Lambda = \{1, 2, \dots, n\}$ , the time sequence  $\{t_k\}$  satisfies  $0 = t_0 < t_1 < t_2 < \dots < t_k < t_{k+1} \dots$ ,  $\lim_{k \rightarrow \infty} t_k = \infty$ ;  $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$  and  $x_i(t)$  corresponds to the state of the  $i$ th unit at time  $t$ ;  $b_{ij}$ ,  $c_{ij}$ , and  $d_{ij}$  denote the constant connection weight;  $\tau_j(t)$  is the time-varying transmission delay and satisfies  $0 \leq \tau_j(t) \leq \tau_j$ ,  $0 < \eta_j = \inf_{t \in \mathbb{R}^+} \{1 - \dot{\tau}_j(t)\}$ , for  $j \in \Lambda$ .  $f_j(\cdot)$ ,  $g_j(\cdot)$ ,  $h_j(\cdot)$  denote the activation functions of the  $j$ th neuron; the delay kernel  $K_{ij}(\cdot)$  is the real-valued nonnegative piecewise continuous functions defined on  $[0, \infty)$ ;  $n$  corresponds to the numbers of units in a neural network;  $I_i$  denotes the external bias on the  $i$ th unit;  $p_{ik}(x(t_k^-))$  represents the abrupt change of the state  $x_i(t)$  at the impulsive moment  $t_k$ .

System (2.3) is supplemented with initial condition given by

$$x_i(s) = \varphi_i(s), \quad s \in (-\infty, 0], i \in \Lambda, \tag{2.4}$$

where  $\varphi(s) = (\varphi_1(s), \varphi_2(s), \dots, \varphi_n(s))^T \in \text{PCB}_{\mathcal{F}_0}^b((-\infty, 0], \mathbb{R}^n) \doteq \text{BPCB}_{\mathcal{F}_0}^b$ . Denote by  $\text{PCB}_{\mathcal{F}_0}^b$  the family of all bounded  $\mathcal{F}_0$ -measurable,  $\text{PC}((-\infty, 0], \mathbb{R}^n)$ -value random variables  $\varphi$ , satisfying  $\sup_{\theta \in (-\infty, 0]} E|\varphi(\theta)|^p < \infty$ , where  $\text{PC}((-\infty, 0], \mathbb{R}^n) = \{\varphi : [-\infty, 0] \rightarrow \mathbb{R}^n\}$  is continuous everywhere except at finite number of points  $t_k$ , at which  $\varphi(t_k^+)$  and  $\varphi(t_k^-)$  exist and  $\varphi(t_k^+) = \varphi(t_k^-)$ .

The norms are defined by the following norms, respectively:

$$\|\varphi\|_p = \sup_{s \in (-\infty, 0]} \left( \sum_{i=1}^n |\varphi_i(s)|^p \right)^{1/p}, \quad \|x\|_p = \left( \sum_{i=1}^n |x_i|^p \right)^{1/p}. \tag{2.5}$$

Throughout this paper, the following standard hypothesis are needed.

(H1) Functions  $a_i(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$  are continuous and monotone increasing, that is, there exist real numbers  $a_i > 0$  such that

$$\frac{a_i(u) - a_i(v)}{u - v} \geq a_i, \tag{2.6}$$

for all  $u, v \in \mathbb{R}$ ,  $u \neq v$ ,  $i \in \Lambda$ .

(H2) Functions  $f_j$ ,  $g_j$ , and  $h_j$  are Lipschitz-continuous on  $\mathbb{R}$  with Lipschitz constants  $L_j^f > 0$ ,  $L_j^g > 0$ , and  $L_j^h > 0$ , respectively. That is,

$$|f_j(u) - f_j(v)| \leq L_j^f |u - v|, \quad |g_j(u) - g_j(v)| \leq L_j^g |u - v|, \quad |h_j(u) - h_j(v)| \leq L_j^h |u - v|, \quad (2.7)$$

for all  $u, v \in \mathbb{R}$ ,  $i \in \Lambda$ .

(H3) The delay kernels  $K_{ij} : [0, \infty) \rightarrow \mathbb{R}^+$  satisfy

$$K_{ij}(s) \leq \mathcal{K}(s) \quad \forall i, j \in \Lambda, \quad s \in [0, \infty), \quad \int_0^\infty \mathcal{K}(s) e^{\mu_0 s} ds < \infty, \quad (2.8)$$

where  $\mathcal{K}(s) : [0, \infty) \rightarrow \mathbb{R}^+$  is continuous and integrable, and the constant  $\mu_0$  denotes some positive number.

(H4) There exist nonnegative constants  $e_i, l_i$  such that

$$[\sigma_i(t, u', v') - \sigma_i(t, u, v)] [\sigma_i(t, u', v') - \sigma_i(t, u, v)]^T \leq e_i |u' - u|^2 + l_i |v' - v|^2, \quad (2.9)$$

for all  $u, v, u', v' \in \mathbb{R}$ ,  $i \in \Lambda$ .

(H5) There exist nonnegative matrixes  $P_k = (p_{ij}^k)_{n \times n}$  such that

$$|p_{ik}(u_1, u_2, \dots, u_n) - p_{ik}(v_1, v_2, \dots, v_n)|^p \leq \sum_{j=1}^n p_{ij}^k |u_j - v_j|^p, \quad (2.10)$$

for any  $(u_1, u_2, \dots, u_n)^T, (v_1, v_2, \dots, v_n)^T \in \mathbb{R}^n$ , where  $p \geq 1$  is an integer.

We end this section by introducing three definitions.

*Definition 2.1.* A constant vector  $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T \in \mathbb{R}^n$  is said to be an equilibrium point of system (2.3) if  $x^*$  is governed by the algebraic system

$$a_i(x_i^*) = \sum_{j=1}^n b_{ij} f_j(x_j^*) + \sum_{j=1}^n c_{ij} g_j(x_j^*) + \sum_{j=1}^n d_{ij} \int_{-\infty}^t K_{ij}(t-s) h_j(x_j^*) ds + I_i, \quad (2.11)$$

where it is assumed that impulse functions  $p_{ik}(\cdot)$  satisfy  $p_{ik}(x_1^*, x_2^*, \dots, x_n^*) = x_i^*$  for all  $i \in \Lambda$  and  $\sigma(t, x_i^*, x_i^*) = 0$ .

*Definition 2.2.* The equilibrium  $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T \in \mathbb{R}^n$  of system (2.3) is said to be  $p$ th moment exponentially stable if there exist  $\lambda > 0$  and  $M \geq 1$  such that

$$E \|x(t) - x^*\|^p \leq E \|\varphi - x^*\|^p M e^{-\lambda t} \quad \text{for } t \geq 0, \quad (2.12)$$

where  $x(t)$  is an any solution of system (2.3) with initial value  $\varphi \in \text{PCB}_{F_0}^b((-\infty, 0], \mathbb{R}^n)$ .

*Definition 2.3* (Forti and Tesi, 1995 [25]). A map  $H : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a homeomorphism of  $\mathbb{R}^n$  onto itself if  $H$  is continuous and one-to-one, and its inverse map  $H^{-1}$  is also continuous.

### 3. Main Result

For convenience, we denote that

$$\begin{aligned} \phi_i &= pa_i - \sum_{j=1}^n \sum_{l=1}^{p-1} \left( |b_{ij}|^{p\alpha_{l,ij}} L_j^{f p \beta_{l,ij}} + |c_{ij}|^{p\gamma_{l,ij}} L_j^{g p \delta_{l,ij}} + |d_{ij}|^{p\xi_{l,ij}} L_j^{h p \eta_{l,ij}} \int_0^\infty \mathcal{K}(s) ds \right) \\ &\quad - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}|^{p\alpha_{p,ji}} L_i^{f p \beta_{p,ji}}, \quad \psi_i = \sum_{j=1}^n \frac{\mu_j}{\mu_i} |c_{ji}|^{p\gamma_{p,ji}} L_i^{g p \delta_{p,ji}}, \\ \Phi_i &= \phi_i - \frac{p(p-1)}{2} e_i - \frac{(p-1)(p-2)}{2} l_i, \quad \Psi_i = \psi_i + (p-1)l_i, \\ \zeta_{ij} &= \frac{\mu_i}{\mu_j} |d_{ij}|^{p\xi_{p,ij}} L_j^{h p \eta_{p,ij}}, \quad \bar{\mu} = \max_i \{\mu_i\}, \quad \underline{\mu} = \min_i \{\mu_i\}, \end{aligned} \tag{3.1}$$

where  $\mu_i$  are positive constant,  $\alpha_{l,ij}, \beta_{l,ij}, \gamma_{l,ij}, \delta_{l,ij}, \xi_{l,ij}$ , and  $\eta_{l,ij}$  are real numbers and satisfy

$$\sum_{l=1}^p \alpha_{l,ij} = 1, \quad \sum_{l=1}^p \beta_{l,ij} = 1, \quad \sum_{l=1}^p \gamma_{l,ij} = 1, \quad \sum_{l=1}^p \delta_{l,ij} = 1, \quad \sum_{l=1}^p \xi_{l,ij} = 1, \quad \sum_{l=1}^p \eta_{l,ij} = 1. \tag{3.2}$$

**Lemma 3.1.** If  $a_i$  ( $i = 1, 2, \dots, p$ ) denote  $p$  nonnegative real numbers, then

$$a_1 a_2 \cdots a_p \leq \frac{a_1^p + a_2^p + \cdots + a_p^p}{p}, \tag{3.3}$$

where  $p \geq 1$  denotes an integer.

A particular form of (3.3), namely,

$$a_1^{p-1} a_2 \leq \frac{(p-1)a_1^p}{p} + \frac{a_2^p}{p}, \quad \text{for } p = 1, 2, 3, \dots \tag{3.4}$$

**Lemma 3.2.** If  $H : \mathbb{R}^n \rightarrow \mathbb{R}^n$  is a continuous function and satisfies the following conditions.

(1)  $H(x)$  is injective on  $\mathbb{R}^n$ , that is,  $H(x) \neq H(y)$  for all  $x \neq y$ .

(2)  $\|H(x)\| \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ .

Then,  $H(x)$  is homeomorphism of  $\mathbb{R}^n$ .

**Theorem 3.3.** *System (2.3) exists a unique equilibrium  $x^*$  under the assumptions (H1)–(H3) if the following condition is also satisfied:*

$$(H6) \quad \phi_i > \varphi_i + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds.$$

*Proof.* Defining a map  $H(x) = (h_1(x), h_2(x), \dots, h_n(x))^T \in C^0(\mathbb{R}^n, \mathbb{R}^n)$ , where

$$h_i(x) = -a_i(x_i) + \sum_{j=1}^n b_{ij} f_j(x_j) + \sum_{j=1}^n c_{ij} g_j(x_j) + \sum_{j=1}^n d_{ij} \int_{-\infty}^t K_{ij}(t-s) h_j(x_j) ds + I_i, \quad (3.5)$$

the map  $H$  is a homeomorphism on  $\mathbb{R}^n$  if it is injective on  $\mathbb{R}^n$  and satisfies  $\|H(x)\| \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ .

In the following, we will prove that  $H(x)$  is a homeomorphism.

Firstly, we claim that  $H(x)$  is an injective map on  $\mathbb{R}^n$ . Otherwise, there exist  $x^T, y^T \in \mathbb{R}^n$ , and  $x^T \neq y^T$  such that  $H(x) = H(y)$ , then

$$\begin{aligned} a_i(x_i) - a_i(y_i) &= \sum_{j=1}^n b_{ij} [f_j(x_j) - f_j(y_j)] + \sum_{j=1}^n c_{ij} [g_j(x_j) - g_j(y_j)] \\ &\quad + \sum_{j=1}^n d_{ij} \int_{-\infty}^t K_{ij}(t-s) [h_j(x_j) - h_j(y_j)] ds. \end{aligned} \quad (3.6)$$

It follows from (H1)–(H3) that

$$\begin{aligned} a_i |x_i - y_i| &\leq \sum_{j=1}^n |b_{ij}| L_j^f |x_j - y_j| + \sum_{j=1}^n |c_{ij}| L_j^g |x_j - y_j| \\ &\quad + \sum_{j=1}^n |d_{ij}| L_j^h |x_j - y_j| \int_{-\infty}^t \mathcal{K}(t-s) ds. \end{aligned} \quad (3.7)$$

Therefore,

$$\begin{aligned} &\sum_{i=1}^n p \mu_i a_i |x_i - y_i|^p \\ &\leq \sum_{i=1}^n \sum_{j=1}^n p \mu_i |b_{ij}| L_j^f |x_i - y_i|^{p-1} |x_j - y_j| + \sum_{i=1}^n \sum_{j=1}^n p \mu_i |c_{ij}| L_j^g |x_i - y_i|^{p-1} |x_j - y_j| \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n p \mu_i |d_{ij}| L_j^h |x_i - y_i|^{p-1} |x_j - y_j| \int_{-\infty}^t \mathcal{K}(t-s) ds \end{aligned}$$

$$\begin{aligned}
&\leq \sum_{i=1}^n \sum_{j=1}^n \mu_i \left( \sum_{l=1}^{p-1} |b_{ij}|^{p\alpha_{l,ij}} L_j^{f p \beta_{l,ij}} |x_i - y_i|^p + |b_{ij}|^{p\alpha_{p,ij}} L_j^{f p \beta_{p,ij}} |x_j - y_j|^p \right) \\
&\quad + \sum_{i=1}^n \sum_{j=1}^n \mu_i \left( \sum_{l=1}^{p-1} |c_{ij}|^{p\gamma_{l,ij}} L_j^{g p \delta_{l,ij}} |x_i - y_i|^p + |c_{ij}|^{p\gamma_{p,ij}} L_j^{g p \delta_{p,ij}} |x_j - y_j|^p \right) \\
&\quad + \sum_{i=1}^n \sum_{j=1}^n \mu_i \int_{-\infty}^t \mathcal{K}(t-s) ds \sum_{l=1}^{p-1} |d_{ij}|^{p\zeta_{l,ij}} L_j^{h p \eta_{l,ij}} |x_i - y_i|^p \\
&\quad + \sum_{i=1}^n \sum_{j=1}^n \mu_i \int_{-\infty}^t \mathcal{K}(t-s) ds |d_{ij}|^{p\zeta_{p,ij}} L_j^{h p \eta_{p,ij}} |x_j - y_j|^p \\
&= \sum_{i=1}^n \mu_i \left( p a_i - \phi_i + \psi_i + \sum_{j=1}^n \zeta_{ji} \int_0^{\infty} \mathcal{K}(s) ds \right) |x_i - y_i|^p.
\end{aligned} \tag{3.8}$$

From (H6), it leads to a contradiction with our assumption. Therefore,  $H(x)$  is an injective map on  $\mathbb{R}^n$ .

To demonstrate the property  $\|H(x)\| \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ , we have

$$\begin{aligned}
&\sum_{i=1}^n \operatorname{sgn}(x_i) p \mu_i (h_i(x) - h_i(0)) |x_i|^{p-1} \\
&= \sum_{i=1}^n \operatorname{sgn}(x_i) p \mu_i |x_i|^{p-1} \left( -a_i(x_i) + \sum_{j=1}^n b_{ij} f_j(x_j) + \sum_{j=1}^n c_{ij} g_j(x_j) \right) \\
&\quad + \sum_{i=1}^n \operatorname{sgn}(x_i) p \mu_i |x_i|^{p-1} \sum_{j=1}^n d_{ij} \int_{-\infty}^t K_{ij}(t-s) h_j(x_j) ds \\
&\leq -\sum_{i=1}^n p a_i \mu_i |x_i|^p + \sum_{i=1}^n \mu_i \left( p a_i - \phi_i + \psi_i + \sum_{j=1}^n \zeta_{ji} \int_0^{\infty} \mathcal{K}(s) ds \right) |x_i|^p \\
&= -\sum_{i=0}^n \mu_i \left( \phi_i - \psi_i - \sum_{j=1}^n \zeta_{ji} \int_0^{\infty} \mathcal{K}(s) ds \right) |x_i|^p \\
&\leq -\omega \|x\|_p^p,
\end{aligned} \tag{3.9}$$

where  $\omega = \min_{1 \leq i \leq n} \{ \mu_i (\phi_i - \psi_i - \sum_{j=1}^n \zeta_{ji} \int_0^{\infty} \mathcal{K}(s) ds) \}$ . Then, we have

$$\omega \|x\|_p^p \leq p \bar{\mu} \sum_{i=1}^n |h_i(x) - h_i(0)| |x_i|^{p-1}. \tag{3.10}$$

Using the Hölder inequality, we obtain

$$\|x\|_p^p \leq \frac{p\bar{\mu}}{\omega} \left( \sum_{i=1}^n |x_i|^p \right)^{1-1/p} \left( \sum_{i=1}^n |h_i(x) - h_i(0)|^p \right)^{1/p}, \quad (3.11)$$

which leads to

$$\|x\|_p \leq \frac{p\bar{\mu}}{\omega} \left( \|H(x)\|_p + \|H(0)\|_p \right). \quad (3.12)$$

From (3.12), we see that  $\|H(x)\| \rightarrow \infty$  as  $\|x\| \rightarrow \infty$ . Thus, the map  $H(x)$  is a homeomorphism on  $\mathbb{R}^n$  under the sufficient condition (H6), and hence it has a unique fixed point  $x^*$ . This fixed point is the unique solution of the system (2.3). The proof is now complete.  $\square$

To establish some sufficient conditions ensuring the  $p$ th moment exponential stability of the equilibrium point of  $x^*$  of system (2.3), we transform  $x^*$  to the origin by using the transformation  $y_i(t) = x_i(t) - x^*$  for  $i \in \Lambda$ . Then system (2.3) can be rewritten as the following form:

$$\begin{aligned} dy_i(t) = & \left[ -\tilde{a}_i(y_i(t)) + \sum_{j=1}^n b_{ij} \tilde{f}_j(y_j(t)) + \sum_{j=1}^n c_{ij} \tilde{g}_j(y_j(t - \tau_j(t))) \right. \\ & \left. + \sum_{j=1}^n d_{ij} \int_{-\infty}^t K_{ij}(t-s) \tilde{h}_j(y_j(s)) ds \right] dt \\ & + \sum_{l=1}^m \tilde{\sigma}_{il}(t, y_i(t), y_i(t - \tau_i(t))) d\omega_l(t), \quad t \neq t_k, \\ y_i(t_k) = & \tilde{p}_{ik}(y(t_k^-)), \quad k \in Z^+, i \in \Lambda, \end{aligned} \quad (3.13)$$

where

$$\begin{aligned} \tilde{a}_i(y_i(t)) &= a_i(y_i(t) + x_i^*) - a_i(x_i^*), & \tilde{f}_j(y_j(t)) &= f_j(y_j(t) + x_j^*) - f_j(x_j^*), \\ \tilde{g}_j(y_j(t - \tau_j(t))) &= g_j(y_j(t - \tau_j(t)) + x_j^*) - g_j(x_j^*), \\ \tilde{h}_j(y_j(s)) &= h_j(y_j(s) + x_j^*) - h_j(x_j^*), \\ \tilde{\sigma}_{ij}(t, y_j(t), y_j(t - \tau_j(t))) &= \sigma_{ij}(t, y_j(t) + x_j^*, y_j(t - \tau_j(t)) + x_j^*) - \sigma_{ij}(t, x_j^*, x_j^*), \\ \tilde{p}_{ik}(y(t_k)) &= p_{ik}(y(t_k) + x^*) - p_{ik}(x^*). \end{aligned} \quad (3.14)$$

In order to obtain our results, the following assumptions are necessary.



(H7) When  $\eta_i \geq 1$ ,  $\Phi_i > \Psi_i + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds$  for any  $i \in \Lambda$ .

When  $0 < \eta_i \leq 1$ ,  $\Phi_i > (\Psi_i / \eta_i) + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds$  for any  $i \in \Lambda$ .

(H8) There exist constant  $\lambda \in (0, \mu_0)$  and  $\alpha \in [0, \lambda)$ ,  $\rho_k = \max_{i \in \Lambda} \{1, \sum_{j=1}^n (\mu_j / \mu_i) p_{ji}^k\} \leq e^{\alpha(t_k - t_{k-1})}$  such that

$$\begin{aligned} \lambda < \Phi_i - \frac{\Psi_i}{\eta_i} e^{\lambda \tau} - \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) e^{\lambda s} ds \\ \int_0^\infty \mathcal{K}(s) e^{\lambda s} ds \leq \int_0^\infty \mathcal{K}(s) e^{\mu_0 s} ds. \end{aligned} \quad (3.15)$$

**Theorem 3.4.** Assume that (H1)–(H5) and (H7)–(H8) hold, then system (2.3) exists a unique equilibrium  $x^*$ , and the equilibrium point is  $p$ th moment exponentially stable.

*Proof.* From (H7), if  $0 < \eta_i \leq 1$ , then

$$\phi_i > \Phi_i > \frac{\Psi_i}{\eta_i} + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds > \frac{\psi_i}{\eta_i} + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds > \psi_i + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds, \quad (3.16)$$

if  $\eta_i \geq 1$ ,

$$\phi_i > \Phi_i > \Psi_i + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds > \psi_i + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds, \quad (3.17)$$

for any  $i \in \Lambda$ .

Then regardless of cases, (H6) and

$$\Phi_i > \frac{\Psi_i}{\eta_i} + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) ds \quad (3.18)$$

are satisfied. Therefore, system (2.3) exists a unique equilibrium point  $x^*$ .

Now, we define

$$u_i(t, y(t)) = e^{\lambda t} \mu_i |y_i(t)|^p = u_i(t), \quad U(t, y(t)) = \sum_{i=1}^n u_i(t). \quad (3.19)$$

For  $t \neq t_k, k \in \mathbb{Z}^+$ , we obtain

$$\begin{aligned} \mathcal{L}U &= U_t + \frac{1}{2} \text{trace} \left[ \tilde{\sigma}^T(t, y_i(t), y_i(t - \tau_i(t))) U_{y_i y_i}(t, y) \tilde{\sigma}(t, y_i(t), y_i(t - \tau_i(t))) \right] \\ &\quad + \sum_{i=1}^n U_{y_i} \left( -\tilde{a}_i(y_i(t)) + \sum_{j=1}^n b_{ij} \tilde{f}_j(y_j(t)) + \sum_{j=1}^n c_{ij} \tilde{g}_j(y_j(t - \tau_j(t))) \right) \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n U_{y_i} d_{ij} \int_{-\infty}^t K_{ij}(t-s) \tilde{h}_j(y_j(s)) ds \\ &\leq \sum_{i=1}^n \left( \lambda - p a_i + \frac{p(p-1)}{2} e_i \right) u_i(t) \\ &\quad + p e^{\lambda t} \left\{ \sum_{i=1}^n \sum_{j=1}^n \mu_i |b_{ij}| L_j^f |y_i(t)|^{p-1} |y_j(t)| \right. \\ &\quad \quad \left. + \sum_{i=1}^n \sum_{j=1}^n \mu_i \left[ |c_{ij}| L_j^g |y_i(t)|^{p-1} |y_j(t - \tau_j(t))| \right. \right. \\ &\quad \quad \quad \left. \left. + \frac{(p-1)}{2} l_i |y_i(t)|^{p-2} |y_i(t - \tau_i(t))|^2 \right] \right. \\ &\quad \quad \left. + \sum_{i=1}^n \sum_{j=1}^n \mu_i |d_{ij}| L_j^h |y_i(t)|^{p-1} \int_0^\infty \mathcal{K}(s) |y_j(t-s)| ds \right\} \\ &\leq \sum_{i=1}^n \left( \lambda - p a_i + \frac{p(p-1)}{2} e_i \right) u_i(t) + \frac{p(p-1)}{2} e^{\lambda t} \sum_{i=1}^n \mu_i l_i |y_i(t)|^{p-2} |y_i(t - \tau_i(t))|^2 \\ &\quad + e^{\lambda t} \sum_{i=1}^n \sum_{j=1}^n \mu_i \left( \sum_{l=1}^{p-1} |b_{ij}|^{p \alpha_{l,ij}} L_j^{f p \beta_{l,ij}} |y_i(t)|^p + |b_{ij}|^{p \alpha_{p,ij}} L_j^{f p \beta_{p,ij}} |y_j(t)|^p \right) \\ &\quad + e^{\lambda t} \sum_{i=1}^n \sum_{j=1}^n \mu_i \left( \sum_{l=1}^{p-1} |c_{ij}|^{p \gamma_{l,ij}} L_j^{g p \delta_{l,ij}} |y_i(t)|^p + |c_{ij}|^{p \gamma_{p,ij}} L_j^{g p \delta_{p,ij}} |y_j(t)|^p \right) \\ &\quad + e^{\lambda t} \sum_{i=1}^n \sum_{j=1}^n \mu_i \int_0^\infty \mathcal{K}(s) \left( \sum_{l=1}^{p-1} |d_{ij}|^{p \xi_{l,ij}} L_j^{h p \eta_{l,ij}} |y_i(t)|^p + |d_{ij}|^{p \xi_{p,ij}} L_j^{h p \eta_{p,ij}} |y_j(t-s)|^p \right) ds \end{aligned}$$

$$\begin{aligned}
 &\leq \sum_{i=1}^n \left[ \sum_{j=1}^n \sum_{l=1}^{p-1} \left( |b_{ij}|^{p\alpha_{l,ij}} L_j^{f p \beta_{l,ij}} + |c_{ij}|^{p\gamma_{l,ij}} L_j^{g p \delta_{l,ij}} + |d_{ij}|^{p\xi_{l,ij}} L_j^{h p \eta_{l,ij}} \int_0^\infty \mathcal{K}(s) ds \right) \right] u_i(t) \\
 &+ \sum_{i=1}^n \left( \lambda - p a_i + \frac{p(p-1)}{2} e_i + \frac{(p-1)(p-2)}{2} l_i + \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}|^{p\alpha_{p,ji}} L_j^{f p \beta_{p,ji}} \right) u_i(t) \\
 &+ \sum_{i=1}^n \sum_{j=1}^n \frac{\mu_i}{\mu_j} |d_{ij}|^{p\xi_{p,ij}} L_j^{h p \eta_{p,ij}} \int_0^\infty \mathcal{K}(s) e^{\lambda s} u_j(t-s) ds \\
 &+ \sum_{i=1}^n e^{\lambda \tau_i(t)} \left( \sum_{j=1}^n \frac{\mu_j}{\mu_i} |c_{ji}|^{p\gamma_{p,ji}} L_i^{g p \delta_{p,ji}} + (p-1) l_i \right) u_i(t - \tau_i(t)) \\
 &= \sum_{i=1}^n \left( (\lambda - \Phi_i) u_i(t) + \Psi_i e^{\lambda \tau_i(t)} u_i(t - \tau_i(t)) \right) + \sum_{i=1}^n \sum_{j=1}^n \zeta_{ij} \int_0^\infty \mathcal{K}(s) e^{\lambda s} u_j(t-s) ds.
 \end{aligned} \tag{3.20}$$

while Lemma 3.1 is used in the second inequality. Let

$$\begin{aligned}
 V(t, y(t)) &= U(t, y(t)) + \sum_{i=1}^n \Psi_i e^{\lambda \tau_i} \int_{t-\tau_i(t)}^t \frac{u_i(s)}{1 - \tau_i(\varphi_i^{-1}(s))} ds \\
 &+ \sum_{i=1}^n \sum_{j=1}^n \zeta_{ij} \int_0^\infty \mathcal{K}(s) e^{\lambda s} \int_{t-s}^t u_j(r) dr ds,
 \end{aligned} \tag{3.21}$$

for  $t \geq 0$ , where  $\varphi_i(s) = s - \tau_i(s)$ , applying Itô formula to (3.21), we can get

$$\begin{aligned}
 \mathcal{L}V &= \mathcal{L}U + \sum_{i=1}^n \Psi_i e^{\lambda \tau_i} \left( \frac{u_i(t)}{1 - \tau_i(\varphi_i^{-1}(t))} - \frac{u_i(t - \tau_i(t))}{1 - \tau_i(t)} (t - \tau_i(t))' \right) \\
 &+ \sum_{i=1}^n \sum_{j=1}^n \zeta_{ij} \int_0^\infty \mathcal{K}(s) e^{\lambda s} (u_i(t) - u_j(t-s)) ds \\
 &\leq - \sum_{i=1}^n \left\{ \Phi_i - \lambda - \frac{\Psi_i}{\eta_i} e^{\lambda \tau} - \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) e^{\lambda s} ds \right\} u_i(t).
 \end{aligned} \tag{3.22}$$

From (H8), we have

$$\mathcal{L}V < 0. \tag{3.23}$$

On the other hand, we have

$$EU(t_k, \mathbf{y}(t_k)) = Ee^{\lambda t_k} \sum_{i=1}^n \mu_i |y_i(t_k)|^p \leq Ee^{\lambda t_k} \sum_{i=1}^n \mu_i \sum_{j=1}^n p_{ij}^k |y_j(t_k^-)|^p \leq \rho_k EU(t_k^-, \mathbf{y}(t_k^-)), \quad (3.24)$$

$$\begin{aligned} EV(t_k, \mathbf{y}(t_k)) &= EU(t_k, \mathbf{y}(t_k)) + E \left( \sum_{i=1}^n \Psi_i e^{\lambda \tau_i} \int_{t_k - \tau_i(t_k)}^{t_k} \frac{u_i(s)}{1 - \hat{\tau}_i(\psi_i^{-1}(s))} ds \right) \\ &\quad + E \left( \sum_{i=1}^n \sum_{j=1}^n \zeta_{ij} \int_0^\infty \mathcal{K}(s) e^{\lambda s} \int_{t_k - s}^{t_k} u_j(r) dr ds \right) \\ &\leq \rho_k EV(t_k^-, \mathbf{y}(t_k^-)), \end{aligned} \quad (3.25)$$

for  $t \in [t_{k-1}, t_k)$ ,  $k \in \mathbb{Z}^+$ , by (3.23), (3.25), and (H8), we have

$$\begin{aligned} EU(t, \mathbf{y}(t)) &\leq EV(t, \mathbf{y}(t)) \leq EV(t_{k-1}, \mathbf{y}(t_{k-1})) \leq \rho_{k-1} EV(t_{k-1}^-, \mathbf{y}(t_{k-1}^-)) \\ &\leq \rho_0 \rho_1 \cdots \rho_{k-1} EV(t_0, \mathbf{y}(t_0)) \leq e^{\alpha t_1} \cdots e^{\alpha(t_{k-1} - t_{k-2})} EV(t_0, \mathbf{y}(t_0)) \leq e^{\alpha t} EV(0, \mathbf{y}(0)), \end{aligned} \quad (3.26)$$

where  $\rho_0 = 1$ .

On the other hand, we observe that

$$\begin{aligned} V(0, \mathbf{y}(0)) &\leq \sum_{i=1}^n \mu_i |y_i(0)|^p + \sum_{i=1}^n \Psi_i e^{\lambda \tau_i} \int_{-\tau}^0 \frac{u_i(s)}{1 - \hat{\tau}_i(\psi_i^{-1}(s))} ds \\ &\quad + \sum_{i=1}^n \sum_{j=1}^n \zeta_{ij} \int_0^\infty \mathcal{K}(s) e^{\lambda s} \int_{-s}^0 u_j(r) dr ds \\ &\leq \sum_{i=1}^n \mu_i \left( 1 + \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) e^{\lambda s} ds + \frac{\Psi_i \tau}{\eta_i} e^{\lambda \tau_i} \right) \sup_{-\infty < s \leq 0} |y_i(s)|^p. \end{aligned} \quad (3.27)$$

It follows that

$$E \left( \sum_{i=1}^n |y_i(t)|^p \right) \leq M e^{-(\lambda - \alpha)t} \sup_{-\infty < s \leq 0} \sum_{i=1}^n |y_i(s)|^p, \quad (3.28)$$

for  $t \geq 0$ , where

$$1 \leq M = \frac{\bar{\mu}}{\underline{\mu}} \left( 1 + \max_{i \in \Lambda} \left\{ \sum_{j=1}^n \zeta_{ji} \int_0^\infty \mathcal{K}(s) e^{\lambda s} ds + \frac{\Psi_i \tau}{\eta_i} e^{\lambda \tau_i} \right\} \right) < \infty, \quad (3.29)$$

which means that

$$E\|x(t) - x^*\|^p \leq E\|\varphi - x^*\|^p M e^{-(\lambda-\alpha)t} \quad \text{for } t \geq t_0. \quad (3.30)$$

Therefore, the equilibrium point of system (2.3) is  $p$ th moment exponentially stable.  $\square$

**Corollary 3.5.** *If  $p \geq 2$ , under the assumptions (H1)–(H5), system (2.3) exists a unique equilibrium point  $x^*$ , and  $x^*$  is  $p$ th moment exponentially stable if the following two conditions are satisfied:*

(H9)

$$\begin{aligned} & pa_i - (p-1) \sum_{j=1}^n \left( |b_{ij}|L_j^f + |c_{ij}|L_j^g + |d_{ij}|L_j^h \int_0^\infty \mathcal{K}(s)ds \right) - \sum_{j=1}^n |b_{ji}|L_i^f \\ & > \sum_{j=1}^n \left( |c_{ji}|L_i^g + |d_{ji}|L_i^h \int_0^\infty \mathcal{K}(s)ds \right) + \frac{p(p-1)}{2} (l_i + e_i). \end{aligned} \quad (3.31)$$

(H10) *There exist constant  $\lambda \in (0, \mu_0)$  and  $\alpha \in [0, \lambda)$ ,  $\rho_k = \max_{i \in \Lambda} \{1, \sum_{j=1}^n (\mu_j / \mu_i) \rho_{ji}^k\} \leq e^{\alpha(t_k - t_{k-1})}$  such that*

$$\begin{aligned} \lambda < & pa_i - (p-1) \sum_{j=1}^n \left( |b_{ij}|L_j^f + |c_{ij}|L_j^g + |d_{ij}|L_j^h \int_0^\infty \mathcal{K}(s)ds \right) - \sum_{j=1}^n |b_{ji}|L_i^f - \frac{p(p-1)}{2} e_i \\ & - \frac{(p-2)(p-1)}{2} l_i - \left( \sum_{j=1}^n |c_{ji}|L_i^g + (p-1)l_i \right) e^{\lambda\tau} - \sum_{j=1}^n |d_{ji}|L_i^h \int_0^\infty \mathcal{K}(s)e^{\lambda s} ds. \end{aligned} \quad (3.32)$$

*Proof.* In Theorem 3.4, let  $\alpha_{l,ij} = \beta_{l,ij} = \gamma_{l,ij} = \delta_{l,ij} = \xi_{l,ij} = \eta_{l,ij} = 1/p$ ,  $\mu_i = 1$  for all  $i, j, l \in \Lambda$ . The result is obtained directly.  $\square$

*Remark 3.6.* In Corollary 3.5, if  $p = 2$ , the conditions (H9) and (H10) are less weak than the following conditions:

(H9)

$$\begin{aligned} & 2 \min_{i \in \Lambda} a_i - \max_{i \in \Lambda} \left( \sum_{j=1}^n |b_{ij}|L_j^f \right) - \max_{i \in \Lambda} \left( \sum_{j=1}^n |c_{ij}|L_j^g \right) - \max_{i \in \Lambda} \left( \sum_{j=1}^n |d_{ij}|L_j^h \int_0^\infty \mathcal{K}(s)ds \right) \\ & > \sum_{j=1}^n \max_{i \in \Lambda} \left( |b_{ji}|L_i^f \right) + \sum_{j=1}^n \max_{i \in \Lambda} \left( |c_{ji}|L_i^g \right) + \sum_{j=1}^n \max_{i \in \Lambda} \left( |d_{ji}|L_i^h \int_0^\infty \mathcal{K}(s)ds \right) + \max_{i \in \Lambda} l_i \\ & + \sum_{j=1}^n \max_{i \in \Lambda} e_i. \end{aligned} \quad (3.33)$$

(H<sub>10</sub>) There exist constant  $\lambda \in (0, \mu_0)$  and  $\alpha \in [0, \lambda)$ ,  $\rho_k = \max_{i \in \Lambda} \{1, \sum_{j=1}^n (\mu_j / \mu_i) p_{ji}^k\} \leq e^{\alpha(t_k - t_{k-1})}$  such that

$$\begin{aligned} \lambda < 2 \min_{i \in \Lambda} a_i - \max_{i \in \Lambda} \left( \sum_{j=1}^n |b_{ij}| L_j^f \right) - \max_{i \in \Lambda} \left( \sum_{j=1}^n |c_{ij}| L_j^g \right) - \max_{i \in \Lambda} \left( \sum_{j=1}^n |d_{ij}| L_j^h \int_0^\infty \mathcal{K}(s) ds \right) \\ - \sum_{j=1}^n \max_{i \in \Lambda} |b_{ji}| L_i^f - \max_{i \in \Lambda} e_i - \left( \sum_{j=1}^n \max_{i \in \Lambda} |c_{ji}| L_i^g + \max_{i \in \Lambda} l_i \right) e^{\lambda \tau} \\ - \sum_{j=1}^n \max_{i \in \Lambda} |d_{ji}| L_i^h \int_0^\infty \mathcal{K}(s) e^{\lambda s} ds, \end{aligned} \quad (3.34)$$

while (H<sub>9</sub>) and (H<sub>10</sub>) were asked in Theorem 3.2 in [18].

**Corollary 3.7.** *Under the assumptions (H2) and (H8), system (2.1) exists a unique equilibrium point  $x^*$ , and  $x^*$  is globally exponentially stable if  $0 < \eta_i \leq 1$ , and the following condition is also satisfied:*

$$\begin{aligned} p a_i - (p-1) \sum_{j=1}^n \left( L_j^f |b_{ij}|^{p \alpha_{ij}/(p-1)} + L_j^g |c_{ij}|^{p \gamma_{ij}/(p-1)} \right) \\ - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}|^{p(1-\alpha_{ji})} L_i^f > \frac{1}{\eta_i} \sum_{j=1}^n \frac{\mu_j}{\mu_i} |c_{ji}|^{p(1-\gamma_{ji})} L_i^g. \end{aligned} \quad (3.35)$$

*Proof.* In Theorem 3.4, when  $p > 1$ , choosing  $\alpha_{l,ij} = \alpha_{ij}/(p-1)$ ,  $\gamma_{l,ij} = \gamma_{ij}/(p-1)$  for  $l = 1, 2, \dots, p-1$ , and  $d_{ij} = 0$ ,  $\beta_{l,ij} = \delta_{l,ij} = 1/p$  for all  $l \in \Lambda$ , then the result is obtained.  $\square$

*Remark 3.8.* If  $0 < \eta_i \leq 1$ ,  $L_i^f = L_i^g = L_i$ , it follows from (3.35) that

$$p a_i - (p-1) \sum_{j=1}^n \left( L_j |b_{ij}|^{p \alpha_{ij}/(p-1)} + L_j |c_{ij}|^{p \gamma_{ij}/(p-1)} \right) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}|^{p(1-\alpha_{ji})} L_i > \sum_{j=1}^n \frac{\mu_j}{\mu_i} |c_{ji}|^{p(1-\gamma_{ji})} L_i, \quad (3.36)$$

this is less conservative than the following inequality:

$$\begin{aligned} \min_{i \in \Lambda} \left\{ p a_i - (p-1) \sum_{j=1}^n \left( L_j |b_{ij}|^{p \alpha_{ij}/(p-1)} + L_j |c_{ij}|^{p \gamma_{ij}/(p-1)} \right) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}|^{p(1-\alpha_{ji})} L_i \right\} \\ > \max_{i \in \Lambda} \left\{ \sum_{j=1}^n \frac{\mu_j}{\mu_i} |c_{ji}|^{p(1-\gamma_{ji})} L_i \right\}, \end{aligned} \quad (3.37)$$

while (3.37) was required in Theorem 3.1 in [5] and in the only theorem in [8].

When  $p = 2$ ,  $\alpha_{ij} = \gamma_{ij} = 1/2$ ,  $\mu_i = 1$ , (3.37) is equal to

$$\min_{i \in \Lambda} \left\{ 2a_i - \sum_{j=1}^n (L_j |b_{ij}| + L_j |c_{ij}|) - \sum_{j=1}^n |b_{ji}| L_i \right\} > \max_{i \in \Lambda} \left\{ \sum_{j=1}^n |c_{ji}| L_i \right\}, \quad (3.38)$$

while (3.38) was required in Theorem 3.2 in [19].

Similar to Corollaries 3.5 and 3.7, the following results are directly gained from Theorem 3.4.

**Corollary 3.9.** *Under the assumptions (H2), (H4), and (H8), system (2.2) exists a unique equilibrium point  $x^*$ , and  $x^*$  is  $p$ th moment exponentially stable if  $0 < \eta_i \leq 1$ , and the following condition is satisfied:*

$$\begin{aligned} pa_i - (p-1) \sum_{j=1}^n (|b_{ij}| L_j^f + |c_{ij}| L_j^g) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}| L_i^f - \frac{p(p-1)}{2} e_i - \frac{(p-1)(p-2)}{2} l_i \\ > \frac{1}{\eta_i} \left( \frac{\mu_j}{\mu_i} \sum_{j=1}^n |c_{ji}| L_i^g + (p-1) l_i \right). \end{aligned} \quad (3.39)$$

*Epecially, if  $p = 2$ , then the equilibrium  $x^*$  is exponentially stable in mean square if the following condition is also satisfied:*

$$\begin{aligned} 2a_i - \sum_{j=1}^n (|b_{ij}|^{2\alpha_{ij}} L_j^{f2\beta_{ij}} + |c_{ij}|^{2\gamma_{ij}} L_j^{g2\delta_{ij}}) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}|^{2-2\alpha_{ji}} L_i^{f2-2\beta_{ji}} - e_i \\ > \frac{1}{\eta_i} \left( \frac{\mu_j}{\mu_i} \sum_{j=1}^n |c_{ji}|^{2-2\gamma_{ji}} L_i^{g2-2\delta_{ji}} + l_i \right). \end{aligned} \quad (3.40)$$

*Remark 3.10.* If  $0 < \eta_i \leq 1$ , it follows from (3.39) that

$$\begin{aligned} pa_i - (p-1) \sum_{j=1}^n (|b_{ij}| L_j^f + |c_{ij}| L_j^g) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}| L_i^f - \frac{p(p-1)}{2} e_i - \frac{(p-1)(p-2)}{2} l_i \\ > \frac{\mu_j}{\mu_i} \sum_{j=1}^n |c_{ji}| L_i^g + (p-1) l_i. \end{aligned} \quad (3.41)$$

This is less conservative than the following inequality:

$$\min_{1 \leq i \leq n} \left\{ pa_i - (p-1) \sum_{j=1}^n (|b_{ij}| L_j^f + |c_{ij}| L_j^g) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}| L_i^f - \sum_{j=1}^n \frac{(p-1)(p-2)}{2} (e_i + l_i) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} (p-1) e_i \right\} > \max_{1 \leq i \leq n} \left\{ \frac{\mu_j}{\mu_i} \sum_{j=1}^n |c_{ji}| L_i^g + \frac{\mu_j}{\mu_i} (p-1) l_i \right\}, \quad (3.42)$$

while (3.42) was required in Theorem 3.3 in [26].

*Remark 3.11.* If  $0 < \eta_i \leq 1$ , it follows from (3.40) that

$$2a_i - \sum_{j=1}^n (|b_{ij}|^{2\alpha_{ij}} L_j^{f2\beta_{ij}} + |c_{ij}|^{2\gamma_{ij}} L_j^{g2\delta_{ij}}) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}|^{2-2\alpha_{ji}} L_i^{f2-2\beta_{ji}} - e_i > \frac{\mu_j}{\mu_i} \sum_{j=1}^n |c_{ji}|^{2-2\gamma_{ji}} L_i^{g2-2\delta_{ji}} + l_i, \quad (3.43)$$

this is less conservative than the following inequality:

$$\min_{1 \leq i \leq n} \left\{ 2a_i - \sum_{j=1}^n (|b_{ij}|^{2\alpha_{ij}} L_j^{f2\beta_{ij}} + |c_{ij}|^{2\gamma_{ij}} L_j^{g2\delta_{ij}}) - \sum_{j=1}^n \frac{\mu_j}{\mu_i} |b_{ji}|^{2-2\alpha_{ji}} L_i^{f2-2\beta_{ji}} - \frac{\max_{1 \leq i \leq n} \{\mu_i\}}{\mu_i} e_i \right\} > \max_{1 \leq i \leq n} \left\{ \frac{\mu_j}{\mu_i} \sum_{j=1}^n |c_{ji}|^{2-2\gamma_{ji}} L_i^{g2-2\delta_{ji}} + \frac{\max_{1 \leq i \leq n} \{\mu_i\}}{\mu_i} l_i \right\}, \quad (3.44)$$

while (3.44) was required in Theorem 3.3 in [27].

#### 4. Illustrative Example

In this section, we will give an example to show that the conditions given in the previous sections are less weak than those given in some of the earlier literatures, such as [18].

Consider the following stochastic neural networks with mixed time delays

$$dx_i(t) = \left( -a_i x_i(t) + \sum_{j=1}^2 b_{ij} f_j(x_j(t)) + \sum_{j=1}^2 c_{ij} g_j(x_j(t - \tau_j(t))) + \sum_{j=1}^2 d_{ij} \int_{-\infty}^t K_{ij}(t-s) h_j(x_j(s)) ds + I_i \right) dt + \sigma_i(t, x_i(t), x_i(t - \tau_i(t))) d w_i(t), \quad (4.1)$$



where  $t \neq t_k$ ,  $i \in \Lambda = \{1, 2\}$ ,  $t_k = k$ ,  $f_j(s) = g_j(s) = h_j(s) = (1/2)[|x+1| - |x-1|]$ , and  $K_{ij}(s) = e^{-s}$ .

$$\begin{aligned} (a_i)_{2 \times 1} &= \begin{pmatrix} 1.7 \\ 3 \end{pmatrix}, & (b_{ij})_{2 \times 2} &= \begin{pmatrix} 0.1 & 0.2 \\ 0.4 & 0.3 \end{pmatrix}, & (c_{ij})_{2 \times 2} &= \begin{pmatrix} 0.2 & 0.3 \\ 0.1 & 0.4 \end{pmatrix}, \\ (d_{ij})_{2 \times 2} &= \begin{pmatrix} 0.1 & 0.2 \\ 0.2 & 0.3 \end{pmatrix}, & (I_i)_{2 \times 1} &= \begin{pmatrix} 0.94 \\ 2.5 \end{pmatrix}, & (\sigma_i)_{2 \times 1} &= \begin{pmatrix} 0.1(x_1(t - \tau_1(t)) - 1.2) \\ 0.1(x_2(t - \tau_2(t)) - 1.4) \end{pmatrix}. \end{aligned} \quad (4.2)$$

In the following, we introduce the following nonlinear impulsive controllers:

$$\begin{aligned} x_1(t_k) &= 0.05 \sin(x_1(t_k^-) - 1.2) - 0.02x_2(t_k) + 1.48, & k \in \mathbb{Z}^+, \\ x_2(t_k) &= -0.03x_1(t_k) + 0.04 \cos(x_2(t_k^-) - 1.4) + 1.72, & k \in \mathbb{Z}^+. \end{aligned} \quad (4.3)$$

In this case, we have  $L_j^f = L_j^g = L_j^h = 1$ ,  $\mathcal{K}(s) = e^{-s}$ ,  $e_j = 0$ ,  $l_j = 0.01$  for  $j = 1, 2$ , and  $\mu_0 = 0.9$ . For  $p = 2$ , we can compute that

$$\begin{aligned} 2a_1 - \sum_{j=1}^2 \left[ |b_{1j}|L_j^f + |c_{1j}|L_j^g + |d_{1j}|L_j^h \int_0^\infty \mathcal{K}(s)ds \right] - \sum_{j=1}^2 |b_{j1}|L_1^f &= 1.8 \\ &> \sum_{j=1}^2 \left[ |c_{j1}|L_1^g + |d_{j1}|L_1^h \int_0^\infty \mathcal{K}(s)ds \right] + l_1 + e_1 = 0.61, \\ 2a_2 - \sum_{j=1}^2 \left[ |b_{2j}|L_j^f + |c_{2j}|L_j^g + |d_{2j}|L_j^h \int_0^\infty \mathcal{K}(s)ds \right] - \sum_{j=1}^2 |b_{j2}|L_2^f &= 3.8 \\ &> \sum_{j=1}^2 \left[ |c_{j2}|L_1^g + |d_{j2}|L_1^h \int_0^\infty \mathcal{K}(s)ds \right] + l_2 + e_2 = 1.21, \end{aligned} \quad (4.4)$$

$$P_k = \begin{pmatrix} 0.2 & 0.08 \\ 0.12 & 0.16 \end{pmatrix}.$$

Choosing  $\mu_i = 1$  for  $i = 1, 2$ , we have  $\rho_k = \max_{i \in \Lambda} \{1, \sum_{j=1}^2 p_{ji}^k\} = 1$  and  $\lambda = 0.6$  and  $\alpha = 0.5$ , then

$$\begin{aligned}
0.6 = \lambda &< 2a_1 - \sum_{j=1}^2 \left[ |b_{1j}|L_j^f + |c_{1j}|L_j^g + |d_{1j}|L_j^h \int_0^\infty \mathcal{K}(s)ds \right] - \sum_{j=1}^2 |b_{j1}|L_1^f - e_1 \\
&- \left[ \sum_{j=1}^2 |c_{j1}|L_1^g + l_1 \right] e^{\lambda\tau} - \sum_{j=1}^2 |d_{j1}|L_1^h \int_0^\infty \mathcal{K}(s)e^{\lambda s} ds = 1.05 - 0.31e^{0.12} = 0.7, \\
0.6 = \lambda &< 2a_2 - \sum_{j=1}^2 \left[ |b_{2j}|L_j^f + |c_{2j}|L_j^g + |d_{2j}|L_j^h \int_0^\infty \mathcal{K}(s)ds \right] - \sum_{j=1}^2 |b_{j2}|L_2^f - e_2 \\
&- \left[ \sum_{j=1}^2 |c_{j2}|L_2^g + l_2 \right] e^{\lambda\tau} - \sum_{j=1}^2 |d_{j2}|L_2^h \int_0^\infty \mathcal{K}(s)e^{\lambda s} ds = 2.55 - 0.71e^{0.12} = 1.749, \\
&\int_0^\infty \mathcal{K}(s)e^{\lambda s} ds = 6.25 \leq \int_0^\infty \mathcal{K}(s)e^{\mu_0 s} ds = 10.
\end{aligned} \tag{4.5}$$

Thus, all conditions of Theorem 3.4 in this paper are satisfied; the equilibrium solution is exponentially stable in mean square. From above discussion, it is easy to see that

$$\begin{aligned}
&2\min_{i \in \Lambda} a_i - \max_{i \in \Lambda} \left( \sum_{j=1}^2 |b_{ij}|L_j^f \right) - \max_{i \in \Lambda} \left( \sum_{j=1}^2 |c_{ij}|L_j^g \right) - \max_{i \in \Lambda} \left( \sum_{j=1}^2 |d_{ij}|L_j^h \int_0^\infty \mathcal{K}(s)ds \right) \\
&- \sum_{j=1}^n \max_{i \in \Lambda} \left( |b_{ji}|L_i^f \right) = 1.0 \\
&< \sum_{j=1}^2 \max_{i \in \Lambda} \left( |c_{ji}|L_i^g \right) + \sum_{j=1}^2 \max_{i \in \Lambda} \left( |d_{ji}|L_i^h \int_0^\infty \mathcal{K}(s)ds \right) + \max_{i \in \Lambda} l_i + \sum_{j=1}^2 \max_{i \in \Lambda} e_i = 1.11,
\end{aligned} \tag{4.6}$$

which implies that the condition  $(H_9)$  in [18] do not hold for this example. So our results are less weaker than some previous results.

## 5. Conclusion

In this paper, we investigate the  $p$ th moment exponential stability for stochastic neural networks with mixed delays under nonlinear impulsive effects. By means of Lyapunov functionals, stochastic analysis, and differential inequality technique, some sufficient conditions for the  $p$ th moment exponential stability of this system are derived. The results of this paper are new, and they supplement and improve some of the previously known results [5, 8, 18, 19, 26, 27]. Moreover, examples are given to illustrate the effectiveness of our results. Furthermore, the method given in this paper may be extended to study other neural networks, such as the model in [29] and stochastic Cohen-Grossberg neural networks in [30], and we can get improved results too.

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