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*Research Article*

# Model-Free Adaptive Control Algorithm with Data Dropout Compensation

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The convergence of model-free adaptive control (MFAC) algorithm can be guaranteed when the system is subject to measurement data dropout. The system output convergent speed gets slower as dropout rate increases. This paper proposes a MFAC algorithm with data compensation. The missing data is first estimated using the dynamical linearization method, and then the estimated value is introduced to update control input. The convergence analysis of the proposed MFAC algorithm is given, and the effectiveness is also validated by simulations. It is shown that the proposed algorithm can compensate the effect of the data dropout, and the better output performance can be obtained.

## 1. Introduction

Model free adaptive control (MFAC) is an attractive technique which has gained a large amount of interest in the recent years [1]. The key feature of this technique is to design controller only using the I/O data of the controlled system and can realize the adaptive control both in parametric and structural manner [2–4]. Instead of identifying a, more or less, known global nonlinear model of the plant, a series of equivalent dynamical linearized time varying models is built along the dynamic operation points of the controlled plant using a novel concept called pseudopartial derivative (PPD), which is estimated merely using the I/O data of the controlled plant. Since the model is valid only for a small domain around the operation point, the PPD estimation algorithm has to be repeated at each time instant. Based on the equivalent dynamical linearized model, the analysis and design for the MAFC scheme then should be implemented. The dynamic linearization method includes the compacted form dynamic linearization (CFDL), partial form dynamic linearization (PFDL), and full form dynamic linearization (FFDL). Up to now, this technique has been extensively studied with significant progress in both theoretical aspects and applications [5–12].

When MFAC is used in practical systems, robustness is an important aspect that should be considered. In traditional model-based control theory, robustness refers to the ability to deal with unknown uncertainties or unmodeled dynamics of the plants. However, the unmodeled dynamics has no meanings in MFAC because its controller designs without any model information. In [13], the robustness of model-free control algorithms proposed should focus on the influences of unknown disturbances or data dropouts, and then the robustness of MFAC with disturbance or data dropout is discussed in [13–17]. When the MFAC scheme is implemented via an NCS [16, 17], it includes plant, controller, actuators, sensors, and a network that connects all these components. In this case, missing data is usual due to a failing sensor, actuator, or network failure, resulting in what it is called intermittent MFAC. Thus, when it is said intermittent MFAC, two different kinds of data dropouts are considered. The first one occurs when the control input is updated. During the control signal translation through the network, the signal may miss by actuator failure, network failure or data collision. The second data dropout is due to the measurement data loss during the signal transfer from the sensor to the controller, which is caused by sensor or network failure.

It is shown that the MFAC is still convergent as long as not all the output measurement data is lost [16, 17]. Compared with existing intermittent theories with having critical data dropout rates [18–23], the intermittent MFAC has no critical data dropout rate. However, the output convergent speed gets slower as dropout rate increases, and the tracking performance will be destroyed when data dropout is serious. In this paper, we propose a robust MFAC algorithm with data dropout compensation. The algorithm first estimated the missing measurement output, and then the estimated value is introduced to the control algorithm. The convergence of the proposed MFAC algorithm is given, and effectiveness is also supported by simulations. The result shows that the proposed algorithm can compensate the effect of the data dropout, and the better output performance can be obtained.

The paper is organized as follows. In Section 2, the MFAC algorithm with data dropout compensation is given, and then the convergence of the proposed algorithm is analyzed in Section 3. In Section 4, a numerical example is given to validate the effectiveness of the algorithm. Conclusions are given in Section 5.

## 2. Problem Formulation

For the convenience of understanding, the MFAC algorithm is first given. Considering the following discrete-time SISO nonlinear system

$$y(k+1) = f\left(y(k), \dots, y\left(k-n_y\right), u(k), \dots, u\left(k-n_u\right)\right), \tag{2.1}$$

where $n_y, n_u$ are the unknown orders of output $y(k)$ and input $u(k)$, respectively, and $f(\cdot)$ is an unknown nonlinear function.

The following assumptions are made about the controlled plant:

(A1) the partial derivative of $f(\cdot)$ with respect to control input $u(k)$ is continuous;

(A2) the system (2.1) is generalized *Lipschitz* , that is, $|\Delta y(k+1)| \leq b|\Delta u(k)|$ for any $k$ and $\Delta u(k) \neq 0$ with $\Delta y(k+1) = y(k+1) - y(k)$, $\Delta u(k) = u(k) - u(k-1)$, and $b$ is a positive constant.

*Remark 2.1.* These assumptions of the system are reasonable and acceptable from a practical viewpoint. Assumption (A1) is a typical condition of control system design for general nonlinear system. Assumption (A2) poses a limitation on the rate of change of the system output permissible before the control law to be formulated is applicable. From the "energy" point of view, the energy rate increasing inside a system cannot go to infinite if the energy rate of change of input is in a finite altitude. For instance, in a water tank control system, since the change of the pump flow of water tank is bounded, the liquid level change of the tank caused by the pump flow cannot go to infinity. There exists a maximum ratio factor between the liquid level and the pump flow, just as the positive constant $b$ defined in assumption (A2).

The following theorem illustrates that the general discrete time nonlinear system satisfying assumptions (A1) and (A2) can be transformed into an equivalent dynamical linearized model, called CFDL model.

**Theorem 2.2.** *For the nonlinear system* (2.1) *satisfying assumptions (A1) and (A2), then there must exist a $\phi(k)$, called pseudo-partial derivative (PPD), such that if $\Delta u(k) \neq 0$, the system* (2.1) *can be described as the following CFDL model:*

$$\Delta y(k+1) = \phi(k)\Delta u(k), \tag{2.2}$$

*and $|\phi(k)| \leq b$.*

*Proof.* Equation (2.1) gives

$$\begin{aligned}
\Delta y(k+1) &= f\left(y(k), \dots, y\left(k-n_y\right), u(k), \dots, u\left(k-n_u\right)\right) \\
&\qquad - f\left(y(k-1), \dots, y\left(k-n_y-1\right), u(k-1), u(k-2), \dots, u\left(k-n_u-1\right)\right) \\
&= f\left(y(k), \dots, y\left(k-n_y\right), u(k), \dots, u\left(k-n_u\right)\right) \\
&\quad - f\left(y(k-1), \dots, y\left(k-n_y-1\right), u(k-1), u(k-1), \dots, u\left(k-n_u-1\right)\right) \\
&\quad + f\left(y(k-1), \dots, y\left(k-n_y-1\right), u(k-1), u(k-1), \dots, u\left(k-n_u-1\right)\right) \\
&\quad - f\left(y(k-1), \dots, y\left(k-n_y-1\right), u(k-1), u(k-2), \dots, u\left(k-n_u-1\right)\right).
\end{aligned} \tag{2.3}$$

Using assumption (A2) and the mean value theorem, (2.3) gives

$$\Delta y(k+1) = \frac{\partial f^*}{\partial u(k)} \Delta u(k) + \zeta(k), \tag{2.4}$$

where $\partial f^*/\partial u(k)$ denotes the value of gradient vector of $f(\cdot)$ with respect to $u(k)$, and

$$\begin{aligned}\zeta(k) &= f\left(y(k-1),\ldots,y\left(k-n_y-1\right),u(k-1),u(k-1),\ldots,u(k-n_u-1)\right)\\ &\quad - f\left(y(k-1),\ldots,y\left(k-n_y-1\right),u(k-1),\ldots,u(k-n_u-1)\right).\end{aligned} \tag{2.5}$$

Considering the following equation:

$$\zeta(k) = \eta(k)\Delta u(k), \tag{2.6}$$

where $\eta(k)$ is a variable. Since the condition $\Delta u(k) \neq 0$, (2.6) must have a solution $\eta(k)$. Let

$$\phi(k) = \frac{\partial f^*}{\partial u(k)} + \eta(k), \tag{2.7}$$

then (2.3) can be written as $\Delta y(k+1) = \phi(k)\Delta u(k)$. □

*Remark 2.3.* Equation (2.2) is a dynamic linear system with slowly time-varying parameter if $\Delta u(k) \neq 0$ and $\Delta u(k)$ is not too large. Therefore, when it is used for the control system design, the condition $\Delta u(k) \neq 0$ and not too large altitude of $\Delta u(k)$ should be guaranteed. In other words, some free adjustable parameter should be added in the control input criterion function to keep the change rate of control input signal not too large.

*Remark 2.4.* Theorem 2.2 requires that $|\Delta u(k)| \neq 0$ is satisfied for every $k$. As a matter of fact, if the case $\Delta u(k) = 0$ comes forth at certain sampling time, the new dynamic linearization can be applied after shifting $\sigma_k \in Z^+$ time instants till $u(k) \neq u(k-\sigma_k)$ holds. In this case, the system (2.1) can be transformed into the CFDL model as $y(k+1)-y(k-\sigma_k+1) = \phi(k)(u(k)-u(k-\sigma_k))$. The proof of the conclusion is similar to the proof of Theorem 2.2.

Rewritten (2.2) as

$$y(k+1) = y(k) + \phi(k)\Delta u(k). \tag{2.8}$$

For the control law algorithm, a weighted one-step-ahead control input cost function is adopted, and given by

$$J(u(k)) = \left|y^*(k+1) - y(k+1)\right|^2 + \lambda|u(k) - u(k-1)|^2, \tag{2.9}$$

where $y^*(k+1)$ is the expected system output signal, and $k$ is a positive weighted constant.

Substituting (2.8) into (2.9), solving the equation $\partial J(u(k))/\partial u(k) = 0$ gives the control law as follows:

$$u(k) = u(k-1) + \frac{\rho\phi(k)}{\lambda + \left|\phi(k)\right|^2}\left(y^*(k+1) - y(k)\right), \tag{2.10}$$

where $\rho$ is the step factor.

The objective function for parameter estimation is used as

$$J(\phi(k)) = \left|y(k) - y(k-1) - \phi(k)\Delta u(k-1)\right|^2 + \mu\left|\phi(k) - \widehat{\phi}(k-1)\right|^2. \tag{2.11}$$

Using the similar procedure of control law equations, the parameter estimation algorithm can be obtained as follows:

$$\widehat{\phi}(k) = \widehat{\phi}(k-1) + \frac{\eta\Delta u(k-1)}{\mu + \Delta u(k-1)^2}\left(\Delta y(k) - \widehat{\phi}(k-1)\Delta u(k-1)\right). \tag{2.12}$$

Summarizing, the MFAC algorithm based on CFDL model for a SISO system is given as follows:

$$\widehat{\phi}(k) = \widehat{\phi}(k-1) + \frac{\eta\Delta u(k-1)}{\mu + |\Delta u(k-1)|^2}\left[\Delta y(k) - \widehat{\phi}(k-1)\Delta u(k-1)\right], \tag{2.13}$$

$$\widehat{\phi}(k) = \widehat{\phi}(1), \quad \text{if } \left|\widehat{\phi}(k)\right| \le \varepsilon, \text{ or } |\Delta u(k-1)| \le \varepsilon, \tag{2.14}$$

$$u(k) = u(k-1) + \frac{\rho\widehat{\phi}(k)}{\lambda + \left|\widehat{\phi}(k)\right|^2}\left[y^*(k+1) - y(k)\right], \tag{2.15}$$

where $\eta, \rho$ are the step size, and they are usually set as $\eta, \rho \in (0,1)$. $\mu, \lambda$ are weight factors, $\varepsilon$ is a small positive constant, and $\widehat{\phi}(1)$ is the initial value of $\widehat{\phi}(k)$.

*Remark 2.5.* In order to make the condition $\Delta u(k) \neq 0$ in Theorem 2.2 be satisfied, and meanwhile to make the parameter estimation algorithm have stronger ability in tracking time-varying parameter, a reset algorithm has been added into this MFAC scheme as (2.14).

*Remark 2.6.* The control law (2.15) has no relationship with any structural information (mathematical model, order, structure, etc.) of the controlled plant. It is designed only using input and output data of the plant.

From (2.13) and (2.15), we know that the current control input $u(k)$ update depends on the current measurement output $y(k)$. Due to sensor or network failures, the measurement data $y(k)$ loss occurs. Our previous work shows that the MFAC is still convergent as long as not all the output data is lost (see [16, 17]). The system output convergent speed gets slower as dropout rate increases, and the tracking performance will be destroyed when data dropout is serious. In this paper, we develop a compensation scheme to suppress the influence of data dropout.

From Theorem 2.2, we can obtain that

$$y(k) = y(k-1) + \phi(k-1)\Delta u(k-1); \tag{2.16}$$

which can be viewed as one-step-ahead predict of the output $y(k)$. Therefore, if $y(k)$ is lost, we can estimate it using $\widehat{\phi}(k-1)$ and $\Delta u(k-1)$, then the estimated value $\widehat{y}(k)$ can be used in control algorithm update when $y(k)$ is lost.

Then, we can give the following estimation scheme

$$\widehat{y}(k) = y(k-1) + \widehat{\phi}(k-1)\Delta u(k-1), \tag{2.17}$$

thus, the output signal of the controller received at time instant $k$ is

$$\overline{y}(k) = \begin{cases} y(k), & \text{if } \beta(k) = 1, \\ \widehat{y}(k), & \text{if } \beta(k) = 0, \end{cases} \tag{2.18}$$

where $\beta(k) \in \{0,1\}$ is a binary random variable, and it is uncorrelated with $u(k), y(k)$, and $\widehat{\phi}(k)$. If $\beta(k) = 1$, there is no data dropout, the output signal $\overline{y}(k)$ is exactly $y(k)$. Otherwise, if $\beta(k) = 0$, there could be the measurement data dropout; in this case, $\overline{y}(k)$ is estimated by $\widehat{y}(k)$.

Assuming that the probability of $\beta(k)$ satisfies

$$\begin{aligned} P(\beta(k) = 1) &= E\{\beta(k)\} = \overline{\beta}, \\ P(\beta(k) = 0) &= 1 - E\{\beta(k)\} = 1 - \overline{\beta}, \end{aligned} \tag{2.19}$$

where $E\{\cdot\}$ denotes the expectation, and $\overline{\beta}$ is the data successful transfer rate, it is a constant with $0 \leq \overline{\beta} \leq 1$. In this paper, we assume that $\overline{\beta}$ is known.

Therefore, the MFAC with dropout compensation scheme can be described as

$$\widehat{\phi}(k) = \widehat{\phi}(k-1) + \frac{\eta \Delta u(k-1)}{\mu + |\Delta u(k-1)|^2}\left(\Delta\overline{y}(k) - \widehat{\phi}(k-1)\Delta u(k-1)\right), \tag{2.20}$$

$$\widehat{\phi}(k) = \widehat{\phi}(1), \quad \text{if } \left|\widehat{\phi}(k)\right| \leq \varepsilon, \text{ or } |\Delta u(k-1)| \leq \varepsilon, \tag{2.21}$$

$$u(k) = u(k-1) + \frac{\rho\widehat{\phi}(k)}{\lambda + \left|\widehat{\phi}(k)\right|^2}\left[y^*(k+1) - \overline{y}(k)\right], \tag{2.22}$$

where $\Delta\overline{y}(k) = \overline{y}(k) - \overline{y}(k-1)$.

## 3. Stability Analysis

In order to obtain the convergence of the proposed MFAC algorithm, another assumption about the system should be made.

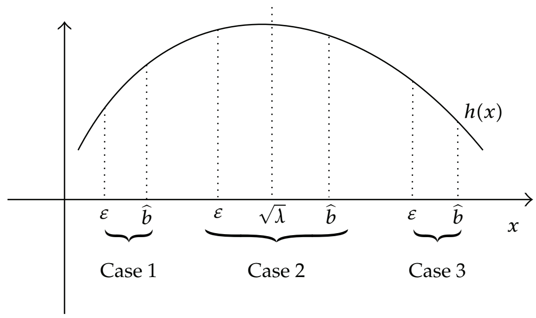



**Figure 1:** The curve of $h(x)$ respects to $x$.

(A3) The PPD satisfies that $\phi(k) \geq b_1 > 0$ (or $\phi(k) \leq -b_1 < 0$), and $b_1$ is a positive constant. Without loss of generality, it is assumed that $\phi(k) \geq b_1 > 0$ in this paper.

*Remark 3.1.* Most of plants in practice satisfy this condition, and its practical meaning is obvious; that is, the plant output should increase (or decrease) when the corresponding control input increase. For example, the water tanks control system, the temperature control system, and so on.

To prove our main result the following lemma is developed first.

**Lemma 3.2.** *Define that* $\vartheta(k) = \rho\hat{\phi}(k)\phi(k)/(\lambda + \hat{\phi}^2(k))$*, if* $\rho, \lambda$ *are chosen as* $\lambda > (\rho b)^2/4$*, then it exists constants* $d_1, d_2$*, such that*

$$0 < d_1 \leq \vartheta(k) \leq d_2 < 1. \tag{3.1}$$

*Proof.* From (2.14), it can be assumed that $0 < \varepsilon \leq \hat{\phi}(k) \leq \hat{b}$, where $\hat{b}$ is a positive constant. □

Denote that $h(x) = x/(\lambda + x^2)$, the change of $h(x)$ respects to $x$ is shown in Figure 1. Obviously, when $x \geq \sqrt{\lambda}$, $h(x)$ is decreasing respect to $x$, and when $x < \sqrt{\lambda}$, it is increasing. Now we discuss the extremum of $h(\hat{\phi}(k)) = \hat{\phi}(k)/(\lambda + \hat{\phi}^2(k))$ when $\hat{\phi}(k) \in [\varepsilon, \hat{b}]$. It has three cases to be discussed as follows:

*Case 1.* when $\hat{b} \leq \sqrt{\lambda}$, $\varepsilon/(\lambda + \varepsilon^2) \leq h(\hat{\phi}(k)) \leq \hat{b}/(\lambda + \hat{b}^2)$,

*Case 2.* when $\varepsilon \leq \sqrt{\lambda} \leq \hat{b}$, $\min(\varepsilon/(\lambda + \varepsilon^2), \hat{b}/(\lambda + \hat{b}^2)) \leq h(\hat{\phi}(k)) \leq 1/2\sqrt{\lambda}$,

*Case 3.* when $\varepsilon \geq \sqrt{\lambda}$, $\hat{b}/(\lambda + \hat{b}^2) \leq h(\hat{\phi}(k)) \leq \varepsilon/(\lambda + \varepsilon^2)$.

Considering $0 < b_1 \leq \phi(k) < b$ gives

*Case 1.* $\rho b_1 \varepsilon^2/(\lambda + \varepsilon^2) \leq \vartheta(k) \leq \rho b\hat{b}/(\lambda + \hat{b}^2)$.

*Case 2.* $\rho b_1 \min(\varepsilon/(\lambda + \varepsilon^2), \hat{b}/(\lambda + \hat{b}^2)) \leq \vartheta(k) \leq \rho b/2\sqrt{\lambda}$.

*Case 3.* $\rho b_1 \hat{b}/(\lambda + \hat{b}^2) \leq \vartheta(k) \leq \rho b\varepsilon/(\lambda + \varepsilon^2)$.

Hence, it exists constants $d_1, d_2$ such that $d_1 \leq \vartheta(k) \leq d_2$, where

*Case 1.* $d_1 = \rho b_1 \varepsilon^2/(\lambda + \varepsilon^2)$, $d_2 = \rho b\widehat{b}/(\lambda + \widehat{b}^2)$,

*Case 2.* $d_1 = \rho b_1 \min(\varepsilon/(\lambda + \varepsilon^2), \widehat{b}/(\lambda + \widehat{b}^2))$, $d_2 = \rho b/2\sqrt{\lambda}$,

*Case 3.* $d_1 = \rho b_1 \widehat{b}/(\lambda + \widehat{b}^2)$, $d_2 = \rho b\varepsilon/(\lambda + \varepsilon^2)$.

Using the fact that $\rho > 0, \lambda > 0$, then $d_1 > 0$ for all the three cases. Now we discuss $d_2$ in detail.

*Case 1.* Since $\lambda > (\rho \mathrm{b})^2/4$, it leads to

$$d_2 = \frac{\rho b\widehat{b}}{\lambda + \widehat{b}^2} = \frac{\rho b}{\lambda/\widehat{b} + \widehat{b}} \leq \frac{\rho b}{2\sqrt{\lambda}} < 1. \tag{3.2}$$

*Case 2.* Since $\lambda > (\rho b)^2/4$, it is obvious that $d_2 = \rho b/2\sqrt{\lambda} < 1$.

*Case 3.* Since $\lambda > (\rho b)^2/4$, it implies that

$$d_2 = \frac{\rho b\varepsilon}{\lambda + \varepsilon^2} = \frac{\rho b}{\lambda/\varepsilon + \varepsilon} \leq \frac{\rho b}{2\sqrt{\lambda}} < 1. \tag{3.3}$$

Hence, we can conclude that if $\rho, \lambda$ satisfies $\lambda > (\rho b)^2/4$, then $0 < d_1 \leq \vartheta(k) \leq d_2 < 1$.

*Remark 3.3.* From the lemma, the parameters should satisfy $\lambda > (\rho b)^2/4$. However, the constant $b$ is difficult to be obtained because MFAC is designed merely using the I/O data and has no model information involved. In this case, $\lambda$ must be chosen large enough to ensure the condition.

With the above lemma, the following result can be given.

**Theorem 3.4.** *For the system* (2.1)*, using the MFAC algorithm* (2.20)–(2.22) *and dropout compensation scheme* (2.17)*, when and $y^* = \mathrm{const}$, if $\overline{\beta} \neq 0$ and $\rho, \lambda$ are chosen as $\lambda > (\rho b)^2/4$, then the expectation of output error is convergent.*

*Proof.* The estimated algorithm (2.17) gives

$$\begin{aligned}\widehat{y}(k) &= y(k-1) + \left(\phi(k-1) + \widetilde{\phi}(k-1)\right)\Delta u(k-1) \\ &= y(k-1) + \phi(k-1)\Delta u(k-1) + \widetilde{\phi}(k-1)\Delta u(k-1) \\ &= y(k) + \widetilde{\phi}(k-1)\Delta u(k-1).\end{aligned} \tag{3.4}$$

From (2.18) and (3.4), we can describe $\overline{y}(k)$ as

$$\overline{y}(k) = y(k) + (1 - \beta(k))\widetilde{\phi}(k-1)\Delta u(k-1). \tag{3.5}$$

Substituting (3.5) into (2.22) gives

$$\begin{aligned}\Delta u(k) &= \frac{\rho\widehat{\phi}(k)}{\lambda + \left|\widehat{\phi}(k)\right|^2}\left[y^*(k+1) - y(k) - (1-\beta(k))\widetilde{\phi}(k-1)\Delta u(k-1)\right] \\ &= \frac{\rho\widehat{\phi}(k)}{\lambda + \left|\widehat{\phi}(k)\right|^2}\left[e(k) - (1-\beta(k))\widetilde{\phi}(k-1)\Delta u(k-1)\right],\end{aligned} \tag{3.6}$$

then

$$e(k+1) = (1-\vartheta(k))e(k) - \vartheta(k)(1-\beta(k))\widetilde{\phi}(k-1)\Delta u(k-1). \tag{3.7}$$

If we choose $\lambda > (\rho b)^2/4$, it is exists constant $d_1, d_2$ satisfying that

$$0 < d_1 \leq \vartheta(k) \leq d_2 < 1. \tag{3.8}$$

Since $\beta(k) \in \{0,1\}$, from (3.7), we can obtain that

$$\begin{aligned}|e(k+1)| &\leq |(1-\vartheta(k))||e(k)| + |\vartheta(k)|\left|(1-\beta(k))\right|\left|\widetilde{\phi}(k-1)\Delta u(k-1)\right| \\ &\leq (1-d_1)|e(k)| + d_2\left|(1-\beta(k))\right|\left|\widetilde{\phi}(k-1)\Delta u(k-1)\right|,\end{aligned} \tag{3.9}$$

which leads to

$$E\{|e(k+1)|\} \leq (1-d_1)E\{|e(k)|\} + d_2\left(1-\overline{\beta}\right)E\left\{\left|\widetilde{\phi}(k-1)\Delta u(k-1)\right|\right\}. \tag{3.10}$$

Since $E\{|\widetilde{\phi}(k-1)|\}$ and $E\{|\Delta u(k-1)|\}$ are bounded, then it exists a positive constant $\varsigma$ satisfying that

$$E\left\{\left|\Delta u(k-1)\widetilde{\phi}(k-1)\right|\right\} \leq \varsigma, \tag{3.11}$$

from (3.10) and (3.11), we can obtain that

$$E\{|e(k+1)|\} \leq (1-d_1)E\{|e(k)|\} + d_2\varsigma\left(1-\overline{\beta}\right), \tag{3.12}$$

where $0 < 1 - d_1 < 1$, hence $E\{|e(k)|\}$ is convergence. □

*Remark 3.5.* From (3.12), it is obvious that $E\{|e(k)|\}$ is convergence, but not convergent to 0, which results from the second argument $d_2(1-\overline{\beta})E\{|\widetilde{\phi}(k-1)\Delta u(k-1)|\}$ in the right of (3.10). $\widetilde{\phi}(k-1)$ is the estimated error of PPD, it is not equal to 0 in general. From [17], we know that data dropout only affects the convergent speed not convergence, and if $\overline{\beta}\neq 0$, it guarantees $\lim_{k\to\infty}\Delta u(k)=0$. Hence, when $k\to\infty$, we can obtain $\lim_{k\to\infty}E\{|e(k)|\}=0$ from (3.10).

## 4. Simulation Results

Considering the SISO nonlinear system

$$y(k+1)=\begin{cases}\dfrac{y(k)}{1+y(k)^2}+u(k)^3, & k\le 500,\\[2ex] \dfrac{y(k)y(k-1)y(k-2)u(k-1)\big(y(k-2)-1\big)+a(k)u(k)}{1+y(k-1)^2+y(k-2)^2}, & k>500,\end{cases} \tag{4.1}$$

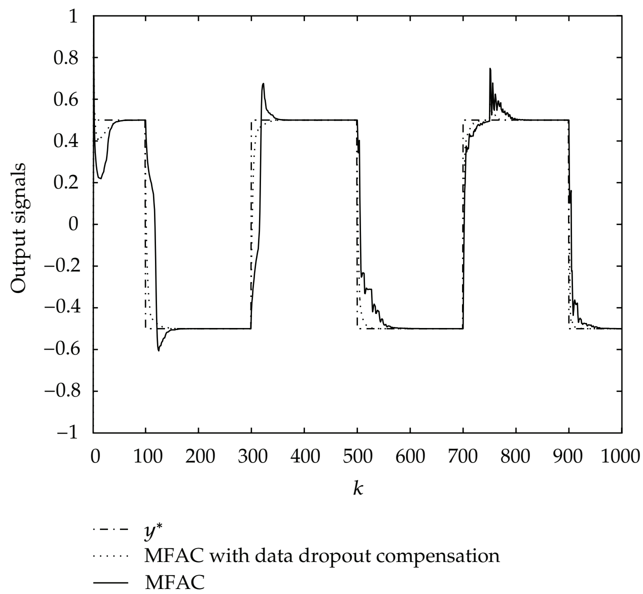
where $a(k)=1+\text{round}(k/500)$, and the desired output is

$$y^*(k+1)=(-1)^{\text{round}(k/200)}. \tag{4.2}$$

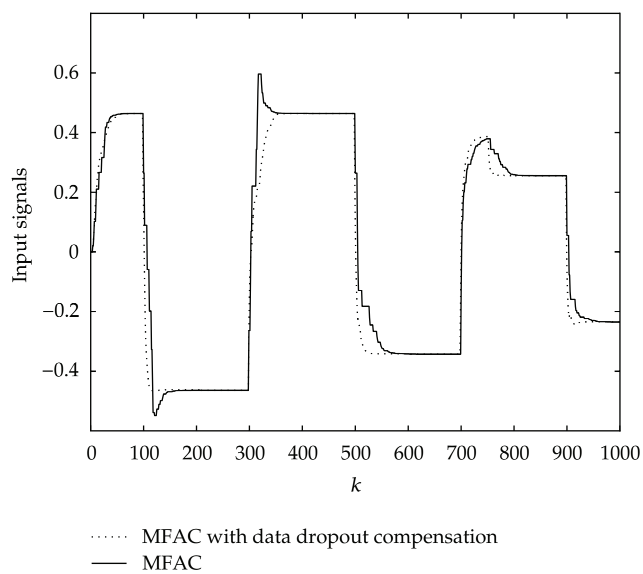
Set the initial conditions as $u(1:2)=0$, $y(1)=-1, y(2)=1$, $y(3)=0.5$, $\widehat{\phi}(1)=2$, $\varepsilon=10^{-5}$, the resetting initial value of PPD is 0.5, and controller parameters are $\rho=0.5$, $\lambda=3$, $\eta=0.5$, $\mu=1$. Considering three different data dropout processes $\overline{\beta}=0.4$ (60% dropout), $\overline{\beta}=0.2$ (80% dropout), and $\overline{\beta}=0.1$ (90% dropout). Using MFAC algorithm and the MFAC with data dropout compensation algorithm, the simulation results are shown in Figures 2, 3, and 4. It is obvious that the MFAC with data dropout compensation algorithm can obtain the better output performance.

## 5. Conclusions

This paper proposes a robust model-free adaptive control algorithm with data dropout compensation. This algorithm first estimated the missing measurement output and then applied the estimated value into the model-free adaptive control algorithm. The convergence of the proposed MFAC algorithm is given, and effectiveness is also supported by simulations. It is shown that the proposed algorithm can compensate the effect of the data dropout, and the better output performance can be obtained.

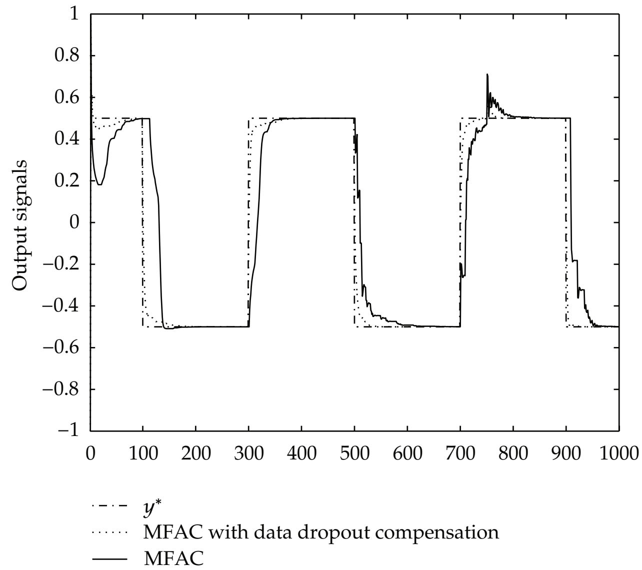



(a)

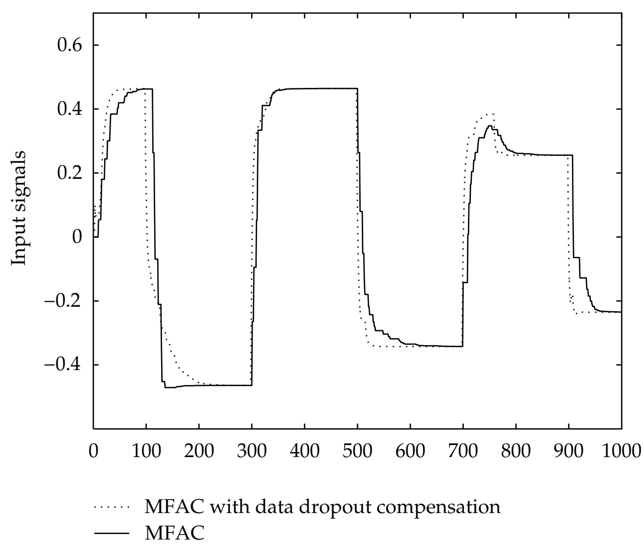



(b)

**Figure 2:** The simulation result for dropout rate is 60%. (a) Output signal, (b) control input signal.

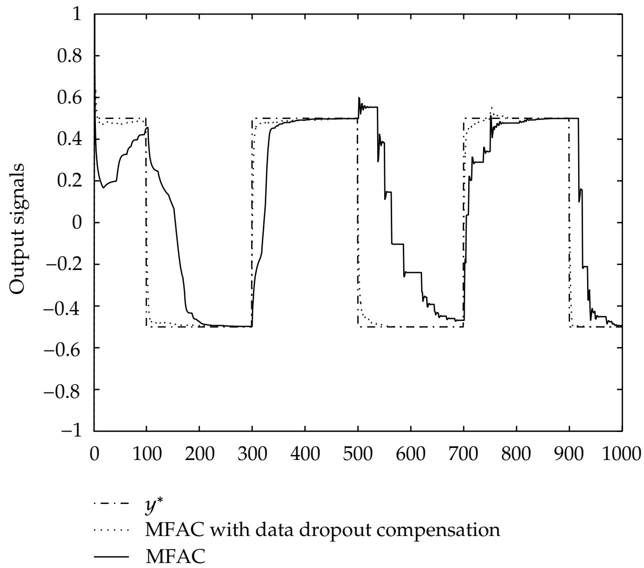



(a)

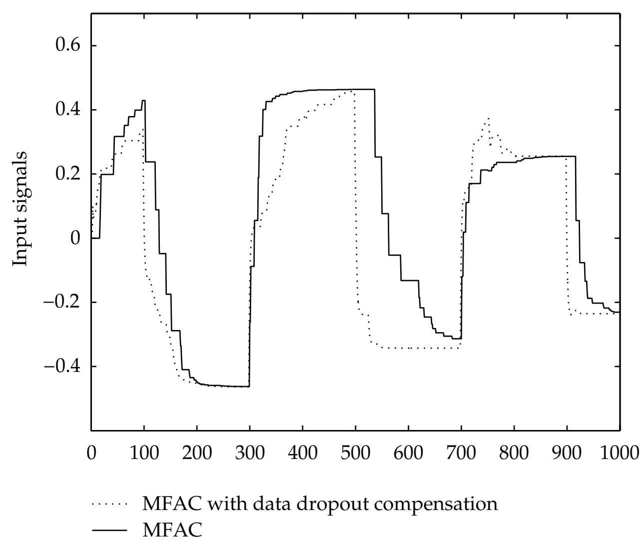



(b)

**Figure 3:** The simulation result for dropout rate is 80%. (a) Output signal, (b) control input signal.




(a)




(b)

**Figure 4:** The simulation result for dropout rate is 90%. (a) Output signal, (b) control input signal.

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