Research Article

Practical Stability in the *p*th Mean for Itô Stochastic Differential Equations

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The *p*th mean practical stability problem is studied for a general class of Itô-type stochastic differential equations over both finite and infinite time horizons. Instead of the comparison principle, a function $\eta(t)$ which is nonnegative, nondecreasing, and differentiable is cooperated with the Lyapunov-like functions to analyze the practical stability. By using this technique, the difficulty in finding an auxiliary deterministic stable system is avoided. Then, some sufficient conditions are established that guarantee the *p*th moment practical stability of the considered equations. Moreover, the practical stability is compared with traditional Lyapunov stability; some differences between them are given. Finally, the results derived in this paper are demonstrated by an illustrative example.

1. Introduction

Lyapunov stability is one of the most important conceptions of stability and has been widely applied to many fields involving nearly all aspects of reality. As we all know, however, the Lyapunov stability is usually employed to study the steady-state property over an infinite horizon and cannot cope with the transient behavior of the trajectory. Therefore, even a stable system in the sense of Lyapunov cannot be applied in the practice since the trajectory exhibits undesirable transient behaviors such as exceeding certain boundary imposed on the trajectory. Moreover, for a Lyapunov stable system, the domain of the desired attractor may be too small to control the initial perturbation in it, which also limits the uses of the Lyapunov stability. On the other hand, for an unstable system in the sense of Lyapunov, it is often the case that its trajectory oscillates sufficiently near by the desired state, which is absolutely acceptable in the practical engineering. As such, we are more interested in the transient behavior over a finite or infinite horizon rather than the steady-state property over

an infinite horizon. For this purpose, a new notion of stability, that is, the practical stability has first been proposed in [1], where it has been shown that the Lyapunov stability may not assure the practical stability and vice versa. Subsequently, the theory on the practical stability has been developed in [2–4].

Up to now, the practical stability problem has been well investigated for deterministic differential equations and many desirable results have been achieved. For example, in [5], a concept of finite time stability, as one special case of practical stability proposed in [6], has been introduced to examine the behavior of systems contained within prespecified bounds during a fixed time interval. The practical stability with respect to a set rather than the particular state $x = 0$ has been extended. In [7, 8], some results on the practical stability have been obtained for discontinuous systems and some differences between the practical stability and the Lyapunov stability have been given. In [9], by using the method of Lyapunov function and Dini derivative, some sufficient conditions have been derived for various types of practical stability. In [10], a new definition of generalized practical stability is introduced. By making use of Lyapunov-like functions, some sufficient conditions are established.

With respect to the stochastic differential systems, we just mention the following representative works. The practical stability in the pth mean has been proposed for discontinuous systems in [11]. In [12], by using the Lyapunov-like functions and the comparison principle, a unified approach is developed to deal with the problems of both the pth mean Lyapunov stability and the pth mean practical stability for the delayed stochastic systems. In [13], some criteria of practical stability in probability have been established in terms of deterministic auxiliary systems with initial conditions. The results obtained in [11, 13] have been further extended to a class of large-scale Itô-type stochastic systems in [14], where the initial conditions of the resulting auxiliary systems are random. In all papers mentioned above, the practical stability of the stochastic systems is determined through testing one corresponding auxiliary deterministic system, whereas, in [15], the sufficient conditions for practical stability in the mean square for a class of stochastic dynamical systems are established by using an integrable function and Lyapunov-like functions instead of the comparison principle.

In this paper, we are concerned with the problem of the practical stability in the pth mean for a general class of Itô-type stochastic differential equations over both finite and infinite time intervals. By using Lyapunov-like functions and a nonnegative, nondecreasing, and differentiable function $\eta(t)$, some criteria are established to ensure the pth mean practical stability for the considered stochastic system. This technique avoids the difficulty in finding an auxiliary deterministic stable system. Moreover, the practical stability is compared with traditional Lyapunov stability and some differences between them are presented. Finally, an illustrative example is provided to demonstrate the results derived in this paper.

Notation. R^n denotes the n-dimensional Euclidean space. T_0 denotes the interval $[t_0, T)$, where $t_0, T \in R_+$ (in this paper, T can be finite or infinite). $M[t_0, T)$ represents the family of nonnegative, nondecreasing, and differentiable functions on $[t_0, T)$. $C^{1,2}(T_0 \times R^n, R_+)$ represents the family of all real-valued functions $V(t, x(t))$ defined on $T_0 \times R^n$ which are continuously twice differentiable in $x(t) \in R^n$ and once differentiable in $t \in R_+$. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space. For a random variable ξ, $E\|\xi\|^p$ means the pth mean of ξ. The followings are the other notions in this paper:

$$S_0(t) = \left\{ x(t) \in R^n : E\|x(t)\|^p < \lambda \right\},$$

$$S(t) = \left\{ x(t) \in R^n : E\|x(t)\|^p \leq A \right\},$$

$$V_M^{S_0}(t) = \sup\{EV(t, x(t)) : x(t) \in S_0(t)\},$$

$$V_m^{\hat{S}}(t) = \inf\{EV(t, x(t)) : x(t) \in \partial S(t)\},$$

$$(1.1)$$

where $\lambda,\ A\ (\lambda < A)$ are given.

2. Preliminaries and Definitions

Consider the stochastic system described by the following n-dimensional stochastic differential equation:

$$dx(t) = f(t, x(t))dt + g(t, x(t))dB(t) \quad \text{on } t \in T_0,$$

$$x(t_0) = x_0,$$

$$(2.1)$$

where $dx(t)$ is the stochastic increment in the sense of Itô and $B(t)$ is an m-dimensional Brownian motion. $f(t, x(t))$ and $g(t, x(t))$ are $n \times 1$ and $n \times m$ matrix functions, respectively. And $x(t_0) = x_0$ is the initial value. Then, we let $x(t) = x(t; t_0, x_0)$ be any solution process of (2.1) with the initial value $x(t_0) = x_0$. Furthermore, we assume that (2.1) satisfies the theorem of the existence and uniqueness of solutions [16] as follows.

(i) (Lipschitz condition) for all $x(t), y(t) \in R^n$, and $t \in T_0$,

$$\|f(t, x(t)) - f(t, y(t))\|^2 \bigvee \|g(t, x(t)) - g(t, y(t))\|^2 \leq K \|x(t) - y(t)\|^2. \qquad (2.2)$$

(ii) (Linear growth condition) for all $x(t), y(t) \in R^n$, and $t \in T_0$,

$$\|f(t, x(t))\|^2 \bigvee \|g(t, x(t))\|^2 \leq K^* \left(1 + \|x(t)\|^2\right), \qquad (2.3)$$

where K and K^* are two positive constants.

Note that $S_0(t)$ and $S(t)$ satisfy the conditions

$$S_0(t) \subset S(t), \qquad \partial S_0(t) \cap \partial S(t) = \emptyset. \qquad (2.4)$$

By using Itô formula, The derivative of the Lyapunov-like function $V(t, x(t)) \in C^{1,2}(T_0 \times R^n, R_+)$ with respect to t along the solution $x(t)$ of (2.1) is given by

$$dV(t, x(t)) = LV(t, x(t))dt + V_x(t, x(t))g(t, x(t))dB(t), \qquad (2.5)$$

where

$$LV(t, x(t)) = V_t(t, x(t)) + V_x(t, x(t))f(t, x(t)) + \frac{1}{2}\text{trace}\left[g^T(t, x(t))V_{xx}(t, x(t))g(t, x(t))\right]. \qquad (2.6)$$

Now, we give the definitions on the practical stability in the pth mean for (2.1).

Definition 2.1. System (2.1) is said to be practically stable in the pth mean (PSM) with respect to (λ, A), $0 < \lambda < A$; if there exist (λ, A), then one has $E\|x_0\|^p < \lambda$; one implies that

$$E\|x(t; t_0, x_0)\|^p < A \quad \forall t \in T_0. \tag{2.7}$$

Remark 2.2. In Definition 2.1, for $T_0 = [t_0, T)$, if the T is finite time, then the system (2.1) is called finite time practically stable, which is one special case of practical stability.

   Noticing the notations of $S_0(t)$ and $S(t)$ above, we can see that $S_0(t_0)$ is a subset of the initial-state set when the initial time is t_0, and $S(t)$ is a subset of the state space at time t. Therefore, it is easy to see that $E\|x_0\|^p < \lambda$; one implies that $x(t_0) \in S_0(t_0)$, $E\|x(t; t_0, x_0)\|^p < A$, and $x(t; t_0, x_0) \in \text{int } S(t)$. Thus, we give the following definition which is equal to Definition 2.1.

Definition 2.3. System (2.1) is said to be PSM with respect to (λ, A), $0 < \lambda < A$, if, for given $S_0(t), S(t)$ with $S_0(t) \subset S(t)$ and $\partial S_0(t) \cap \partial S(t) = \emptyset$, one has $x(t_0) \in S_0(t_0)$ then it is implied that

$$x(t; t_0, x_0) \in \text{int } S(t) \quad \forall t \in T_0. \tag{2.8}$$

Remark 2.4. If the conditions of Definition 2.3 are satisfied, then the system (2.1) is also said to be practically stable in the pth mean with respect to $(S_0(t_0), S(t))$.

   In Section 3, the criteria for practical stability in the pth mean will be established for (2.1).

3. Practical Stability Criteria

In this section, the practical stability in the pth mean will be investigated in detail, and some stability criteria will be derived for (2.1) by using a Lyapunov-like function and a nonnegative, nondecreasing, and differentiable function $\eta(t)$.

Theorem 3.1. *If the following conditions are met:*

   (1) $S_0(t) \subset S(t)$ *and* $\partial S_0(t) \cap \partial S(t) = \emptyset$ *for all* $t \in T_0$,

   (2) *there exists a function* $V(t, x(t)) \in C^{1,2}(T_0 \times R^n, R_+)$, *which is satisfying the following conditions:*

      (a)

$$ELV(t, x(t)) \leq 0 \quad t \in T_0, \; x(t) \in S(t), \tag{3.1}$$

      (b)

$$V_M^{S_0}(t_0) < V_m^{\hat{S}}(t) \quad \forall t \in T_0, \tag{3.2}$$

then (2.1) *is PSM with respect to* (λ, A).

Proof. For all $x_0 \in S_0(t_0)$, let $x(t) = x(t; t_0, x_0)$ be a solution of (2.1) with the initial value x_0. For contradiction, we assume that there exists a first time $t_1 \in T_0$ such that $E\|x(t)\|^p < A$ for $t_0 \leq t < t_1$ and $E\|x(t_1)\|^p = A$.

By the notations of $V_M^{S_0}(t)$, $V_m^{\widehat{S}}(t)$, and (2)-(b), we have

$$V_M^{S_0}(t_0) < V_m^{\widehat{S}}(t_1) \leq EV(t_1, x(t_1)). \tag{3.3}$$

Noticing the $V(t, x(t))$ and (2.5), (2.6), it can be obtained that

$$V(t_1, x(t_1)) - V(t_0, x(t_0)) = \int_{t_0}^{t_1} LV(s, x(s)) ds + \int_{t_0}^{t_1} V_x(s, x(s)) g(s, x(s)) dB(s). \tag{3.4}$$

By the assumption (2)-(a) and taking the expected value on the both sides of (3.4), we have

$$
\begin{aligned}
E[V(t_1, x(t_1)) - V(t_0, x(t_0))] &= E\left[\int_{t_0}^{t_1} LV(s, x(s)) ds\right] \\
&= \int_{t_0}^{t_1} ELV(s, x(s)) ds \\
&\leq 0
\end{aligned}
\tag{3.5}
$$

because

$$E\left[\int_{t_0}^{t_1} V_x(s, x(s)) g(s, x(s)) dB(s)\right] = 0, \tag{3.6}$$

then we have

$$V_M^{S_0}(t_0) < V_m^{\widehat{S}}(t_1) \leq EV(t_1, x(t_1)) \leq EV(t_0, x(t_0)) \leq V_M^{S_0}(t_0). \tag{3.7}$$

This is a contradiction, so the proof is complete. $\qquad\square$

Remark 3.2. In Theorem 3.1, if the T_0 is a finite time interval, then (2.1) is practically stable on finite time. Furthermore, it should be pointed out that the condition $ELV(t, x(t)) \leq 0$ is necessary to guarantee the pth moment stability for (2.1) in the sense of Lyapunov. However, it would be too strict for the pth mean practical stability of (2.1). In the following theorem, this condition is replaced by

$$ELV(t, x(t)) \leq \frac{d\eta(t)}{dt}, \tag{3.8}$$

where $\eta(t) \in M[t_0, T]$.

Theorem 3.3. *If the following conditions are met:*

(1) $S_0(t) \subset S(t)$ *and* $\partial S_0(t) \cap \partial S(t) = \emptyset$ *for all* $t \in T_0$,

(2) *there exists a function* $\eta(t) \in M[t_0, T]$, *which satisfies the following conditions:*

(a)

$$ELV(t, x(t)) \leq \frac{d\eta(t)}{dt} \quad t \in T_0, \ x(t) \in S(t), \tag{3.9}$$

(b)

$$\eta(t_0) = V_M^{S_0}(t_0), \tag{3.10}$$

(c)

$$\eta(t) < V_m^{\hat{S}}(t) \quad \forall t \in T_0, \tag{3.11}$$

then (2.1) *is* PSM *with respect to* (λ, A).

Proof. Let $x(t)$ be a solution of (2.1) with the initial value $x_0 \in S_0(t_0)$. For contradiction, we assume that the result is not true, which means that there exists a first time $t_1 \in T_0$ such that $E\|x(t)\|^p < A$ for $t_0 \leq t < t_1$ and $E\|x(t_1)\|^p = A$.

Noticing the notation of $V_m^{\hat{S}}(t)$, we have

$$V_m^{\hat{S}}(t_1) \leq EV(t_1, x(t_1)). \tag{3.12}$$

By using (2.5), (2.6), it follows that

$$V(t_1, x(t_1)) - V(t_0, x(t_0)) = \int_{t_0}^{t_1} LV(s, x(s))ds + \int_{t_0}^{t_1} V_x(s, x(s))g(s, x(s))dB(s). \tag{3.13}$$

Taking the expectation on the both sides of (3.13), considering

$$E\left[\int_{t_0}^{t_1} V_x(s, x(s))g(s, x(s))dB(s)\right] = 0 \tag{3.14}$$

and the assumption (2)-(a), we obtain

$$EV(t_1, x(t_1)) = EV(t_0, x(t_0)) + \int_{t_0}^{t_1} ELV(s, x(s))ds$$

$$\leq EV(t_0, x(t_0)) + \int_{t_0}^{t_1} d\eta(s)$$

$$\begin{aligned}
&= EV(t_0, x(t_0)) + \eta(t_1) - \eta(t_0) \\
&= \eta(t_1) - \left[V_M^{S_0}(t_0) - EV(t_0, x(t_0)) \right] \\
&\leq \eta(t_1).
\end{aligned}$$

(3.15)

Then, it follows from (3.12) and (3.15) that

$$V_m^{\hat{S}}(t_1) \leq EV(t_1, x(t_1)) \leq \eta(t_1) \tag{3.16}$$

which contradicts with the condition (2)-(c) of Theorem 3.3, and, hence, the proof is complete.
□

Remark 3.4. In Theorem 3.3, we mainly use the function $\eta(t)$ and the Lyapunov-like functions but not the comparison principle in [11–14] to achieve the result, which avoids the difficulty in finding an auxiliary deterministic stable system. Here, we assume that

$$ELV(t, x(t)) \leq \frac{d\eta(t)}{dt} \tag{3.17}$$

holds on $x(t) \in S(t)$. Next, the condition $x(t) \in S(t)$ will be replaced by a weaker one, that is, $x(t) \in S(t)/S_0(t)$.

Theorem 3.5. *If the following conditions are met:*

(1) $S_0(t) \subset S(t)$ and $\partial S_0(t) \cap \partial S(t) = \emptyset$ for all $t \in T_0$,

(2) *there exists a function* $\eta(t) \in M[t_0, T)$, *which satisfies the following conditions:*

(a)

$$ELV(t, x(t)) \leq \frac{d\eta(t)}{dt} \quad t \in T_0, \ x(t) \in \frac{S(t)}{S_0(t)}, \tag{3.18}$$

(b)

$$\eta(t) = V_M^{S_0}(t), \quad x(t) \in S_0(t), \tag{3.19}$$

(c)

$$\eta(t) < V_m^{\hat{S}}(t) \quad \forall t \in T_0, \tag{3.20}$$

then (2.1) *is* PSM *with respect to* (λ, A).

Proof. Let $x(t)$ be a solution of (2.1) with the initial value $x_0 \in S_0(t_0)$. For contradiction, we assume that there exists a first time $t_2 \in T_0$ such that $E\|x(t)\|^p < A$ for $t_0 \leq t < t_2$ and

$E\|x(t_2)\|^p = A$. Due to the continuity of $E\|x(t)\|^p$ and the connectivity of $S(t), S_0(t)$, there exists such a time t_1 and $E\|x(t_1)\|^p = \lambda$ holds for the last time before the time t_2. So, we get that $x(t) \in S(t)/S_0(t)$ when $t \in [t_1, \ t_2]$.

Noticing the (2.5), (2.6), it can be obtained that, when $t \in [t_1, t_2]$,

$$V(t_2, x(t_2)) - V(t_1, x(t_1)) = \int_{t_1}^{t_2} LV(s, x(s))\mathrm{d}s + \int_{t_1}^{t_2} V_x(s, x(s))g(s, x(s))\mathrm{d}B(s). \qquad (3.21)$$

By virtue of

$$E\left[\int_{t_1}^{t_2} V_x(s, x(s))g(s, x(s))\mathrm{d}B(s)\right] = 0, \qquad (3.22)$$

we take the conditional expectation of (3.21) conditioning on the initial value $x(t_0) = x_0$; it can be seen from condition (2)-(a) that

$$\begin{aligned}
E[V(t_2, x(t_2)) - V(t_1, x(t_1)) \mid x(t_0) = x_0] &= E\left[\int_{t_1}^{t_2} LV(s, x(s))\mathrm{d}s \mid x(t_0) = x_0\right] \\
&= \int_{t_1}^{t_2} ELV(s, x(s))\mathrm{d}s \\
&\leq \int_{t_1}^{t_2} \mathrm{d}\eta(s) \\
&= \eta(t_2) - \eta(t_1).
\end{aligned} \qquad (3.23)$$

Taking the expectation on the both sides of (3.23) and using the assumption (2)-(b), we obtain

$$\begin{aligned}
EV(t_2, x(t_2)) &= EV(t_1, x(t_1)) + \eta(t_2) - \eta(t_1) \\
&= \eta(t_2) - \left[\eta(t_1) - EV(t_1, x(t_1))\right] \\
&\leq \eta(t_2) - \left[\eta(t_1) - V_M^{S_0}(t_1)\right] \\
&= \eta(t_2).
\end{aligned} \qquad (3.24)$$

Then,

$$V_m^{\hat{S}}(t_2) \leq EV(t_2, x(t_2)) \leq \eta(t_2). \qquad (3.25)$$

Noticing the assumption (2)-(c), this is a contradiction, Then, the proof is complete. $\qquad \square$

In the theorems above, some sufficient conditions that guarantee the pth mean practical stability are derived for (2.1). It is worth mentioning that the establishment of the practical stability criteria here avoids introducing other auxiliary stable systems, which make

it convenient to determine whether an Itô-type stochastic differential system is the pth mean practically stable. In Section 4, an example will be employed to demonstrate the obtained results.

4. Example

In this section, one numerical example is given to demonstrate the result in Theorem 3.3. The results obtained in Theorems 3.1 and 3.5 can be verified in the same way.

Example 4.1. Consider the one-dimensional stochastic differential equation as follow:

$$dx(t) = x(t)\sin(t)dt + dB(t) \quad \text{on } t \in [t_0, T),$$
$$x(t_0) = x_0, \tag{4.1}$$

where $B(t)$ is a one-dimensional Brownian motion.

Let $K = K^* = 1$; it is obvious that (4.1) satisfies both the Lipschitz condition and the Linear growth condition, so the existence and uniqueness of the solution $x(t)$ of (4.1) is guaranteed.

Now, we investigate the practical stability in the 1st mean for (4.1) with respect to $\lambda = 1$ and $A = 2$. One assumes that the initial value $x(t_0)$ satisfies the conditions $E|x(t_0)| < 1$ and $E|x(t; t_0, x_0)| < 2$ for $t \in T_0$. Then, we approximate the value of t_0 and T.

We define a Lyapunov-like function as

$$V(t, x(t)) = |x(t)|. \tag{4.2}$$

Due to the fact that $V(t, x(t))$ is a positive-definite function, one can easily get $V(t, x(t)) > 0$ when $x(t) \neq 0$.
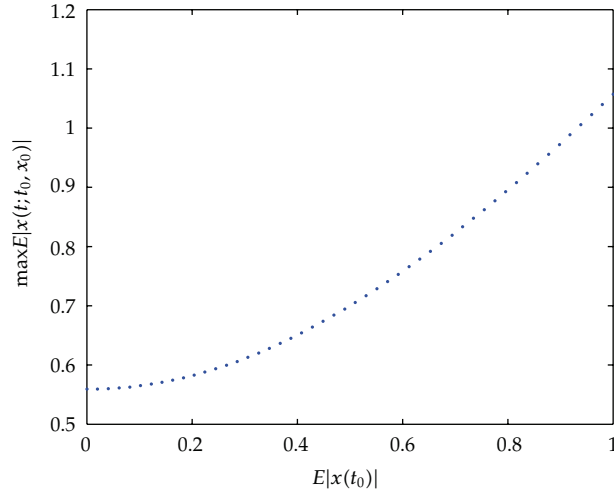
So, when $x(t) \neq 0$, it is obvious that

$$V_t(t, x(t)) = 0, \qquad V_x(t, x(t)) = \begin{cases} 1, & x > 0, \\ -1, & x < 0, \end{cases} \qquad V_{xx}(t, x(t)) = 0. \tag{4.3}$$

By using the Itô formula, we calculate the derivative of the Lyapunov-like function $V(t, x(t))$ along the solution $x(t)$ of (4.1), and noticing the (2.6), we have

$$LV(t, x(t)) = V_t(t, x(t)) + V_x(t, x(t))f(t, x(t)) + \frac{1}{2}\text{trace}\left[g^T(t, x(t))V_{xx}(t, x(t))g(t, x(t))\right]$$
$$= 0 \pm x(t)\sin(t) + 0 \tag{4.4}$$
$$\leq |x(t)|.$$

Taking the expectation on both sides of (4.4), one obtains

$$E[LV(t, x(t))] \leq E|x(t)| < A = 2, \tag{4.5}$$

Figure 1: Illustration of the practical stability in the 1st mean.

so we define

$$\eta(t) = 2t. \tag{4.6}$$

From (4.4)–(4.6), it can be easily verified that the condition (2)-(a) of Theorem 3.3 is satisfied. Then, by the condition (2)-(b) of Theorem 3.3, we have

$$\eta(t_0) = V_M^{S_0}(t_0) = \sup\{E|x(t_0)|; x(t_0) \in S_0(t_0)\} = \lambda = 1 \tag{4.7}$$

and hence, it can be obtained from (4.6) that

$$t_0 = \frac{1}{2}. \tag{4.8}$$

On the other hand, from the condition (2)-(c) of Theorem 3.3, we have

$$\eta(t) < V_m^{\hat{S}}(t) = \inf\{E|x(t)| : x(t) \in \partial S(t)\} = A = 2. \tag{4.9}$$

So, we have

$$t < 1. \tag{4.10}$$

Now, we have the fact that $t_0 = 1/2$ and $T = 1$. According to Theorem 3.3, (4.1) is practically stable in the 1st mean with respect to $\lambda = 1$ and $A = 2$ on $t \in [1/2, 1)$. In the simulation, we take 50 initial values satisfying $E|x(1/2)| < 1$. For every initial value, the 1st mean orbit and the maximum of $E|x(t)|$ for $t \in [1/2, 1)$ are computed numerically. The simulation result is depicted in Figure 1.

5. Conclusion

This paper mainly establishes the sufficient conditions of practical stability in the pth mean for the Itô-type stochastic differential equation over finite or infinite time interval. By using Lyapunov-like functions and a nonnegative, nondecreasing, and differentiable function $\eta(t)$ instead of the comparison principle, the difficulty in finding an auxiliary deterministic stable system is avoided. Moreover, this paper indicates that the practical stability can be examined over finite or infinite time interval and it can be used to depict the transient behavior of the trajectory.

For further studies, we can extend practical stability in the pth mean to uniformly practical stability and strict practical stability in the pth mean by the same methods in this paper. And, we can also consider other techniques to establish the sufficient conditions for the practical stability in probability and the almost sure practical stability instead of the comparison principle. Other future research topics include the investigation on the filtering and control problems for uncertain nonlinear stochastic systems; see, for example, [17–26].

Acknowledgment

References

[1] J. P. LaSalle and S. Lefschetz, *Stability Theory by Liapunov's Direct Method with Applications*, Academic Press, New York, NY, USA, 1961.

[2] S. R. Bernfeld and V. Lakshmikantham, "Practical stability and Lyapunov functions," *The Tôhoku Mathematical Journal*, vol. 32, no. 4, pp. 607–613, 1980.

[3] A. A. Martynyuk, "Methods and problems of practical stability of motion theory," *Nonlinear Vibration Problems*, vol. 22, pp. 19–46, 1984.

[4] A. A. Martynyuk and Z. Q. Sun, *Practical Stability and its Application*, Science Press, Beijing, China, 2003.

[5] L. Weiss and E. F. Infante, "On the stability of systems defined over a finite time interval," *Proceedings of the National Academy of Sciences of the United States of America*, vol. 54, pp. 44–48, 1965.

[6] L. Weiss and E. F. Infante, "Finite time stability under perturbing forces and on product spaces," *IEEE Transactions on Automatic Control*, vol. 12, pp. 54–59, 1967.

[7] J. X. He, "Practical stability of discontinuous systems with external perturbations," *Journal of Mathematics*, vol. 3, no. 4, pp. 385–398, 1983.

[8] J. X. He, "Practical stability of discontinuous systems," *Journal of Mathematical Research and Exposition*, vol. 5, pp. 55–60, 1985.

[9] X. X. Liao, *Stability Theory and Applications*, Huazhong Normal University Press, 1988.

[10] G. Zhai and A. N. Michel, "Generalized practical stability analysis of discontinuous dynamical systems," in *Proceedings of the 42nd IEEE Conference on Decision and Control*, pp. 1663–1668, December 2003.

[11] Z. S. Feng, Y. Q. Liu, and F. W. Guo, "Criteria for practical stability in the pth mean of nonlinear stochastic systems," *Applied Mathematics and Computation*, vol. 49, no. 2-3, pp. 251–260, 1992.

[12] Z. S. Feng, "Lyapunov stability and practical stability of nonlinear delay stochastic systems: a unified approach," in *Proceedings of the 32nd Conference on Decision and Control*, pp. 865–870, December 1993.

[13] A. H. Tsoi and B. Zhang, "Practical stabilities of Itô type nonlinear stochastic differential systems and related control problems," *Dynamic Systems and Applications*, vol. 6, no. 1, pp. 107–124, 1997.

[14] S. Sathananthan and S. Suthaharan, "Practical stability criteria for large-scale nonlinear stochastic systems by decomposition and aggregation," *Dynamics of Continuous, Discrete & Impulsive Systems Series A*, vol. 8, no. 2, pp. 227–248, 2001.

[15] A. N. Michel and L. Hou, "Finite-time and practical stability of a class of stochastic dynamical systems," in *Proceedings of the 47th IEEE Conference on Decision and Control*, pp. 3452–3456, December 2008.

[16] X. Mao, *Stochastic Differential Equations and Applications*, International Publishers in Science and Technology, England, UK, 2nd edition, 2008.

[17] H. Dong, Z. Wang, D. W. C. Ho, and H. Gao, "Robust H_∞ filtering for Markovian jump systems with randomly occurring nonlinearities and sensor saturation: the finite-horizon case," *IEEE Transactions on Signal Processing*, vol. 59, no. 7, pp. 3048–3057, 2011.

[18] Y. Tang, Z. Wang, W. K. Wong, J. Kurths, and J. Fang, "Multiobjective synchroniza- tion of coupled chaotic systems," *Chaos*, vol. 21, no. 2, Article ID 025114, 2011.

[19] Z. Wang, J. Lam, L. Ma, Y. Bo, and Z. Guo, "Variance-constrained dissipative observer-based control for a class of nonlinear stochastic systems with degraded measurements," *Journal of Mathematical Analysis and Applications*, vol. 377, no. 2, pp. 645–658, 2011.

[20] B. Shen, Z. Wang, Y. S. Hung, and G. Chesi, "Distributed H_∞ filtering for polynomial nonlinear stochastic systems in sensor networks," *IEEE Transactions on Industrial Electronics*, vol. 58, no. 5, pp. 1971–1979, 2011.

[21] J. Liang, Z. Wang, and X. Liu, "Distributed state estimation for discrete-time sensor networks with randomly varying nonlinearities and missing measurements," *IEEE Transactions on Neural Networks*, vol. 22, no. 3, pp. 486–496, 2011.

[22] X. He, Z. Wang, Y. Ji, and D. Zhou, "Robust fault detection for networked systems with distributed sensors," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 47, no. 1, pp. 166–177, 2011.

[23] B. Shen, Z. Wang, and X. Liu, "Bounded H_∞ synchronization and state estimation for discrete time-varying stochastic complex networks over a finite-horizon," *IEEE Transactions on Neural Networks*, vol. 22, no. 1, pp. 145–157, 2011.

[24] B. Shen, Z. Wang, H. Shu, and G. Wei, "H_∞ filtering for uncertain time-varying sys- tems with multiple randomly occurred nonlinearities and successive packet dropouts," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 14, pp. 1693–1709, 2011.

[25] Z. Wang, D. W. C. Ho, H. Dong, and H. Gao, "Robust H_∞ finite-horizon control for a class of stochastic nonlinear time-varying systems subject to sensor and actuator saturations," *IEEE Transactions on Automatic Control*, vol. 55, no. 7, pp. 1716–1722, 2010.

[26] Z. Wang, Y. Liu, and X. Liu, "Exponential stabilization of a class of stochastic system with Markovian jump parameters and mode-dependent mixed time-delays," *IEEE Transactions on Automatic Control*, vol. 55, no. 7, pp. 1656–1662, 2010.