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Research Article

Function Projective Synchronization of a Class of Chaotic Systems with Uncertain Parameters

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This paper investigates the function projective synchronization of a class of chaotic systems with uncertain parameters. Based on Lyapunov stability theory, the nonlinear adaptive control law and the parameter update law are derived to make the state of two chaotic systems function projective synchronized. Numerical simulations are presented to demonstrate the effectiveness of the proposed adaptive scheme.

1. Introduction

Synchronization of chaotic systems has attracted considerable attention from scientists and engineers and has been investigated intensively both theoretically and experimentally over the last decade. A variety of methods have been introduced for the synchronization of chaotic systems which include complete synchronization, generalized synchronization, phase synchronization, lag synchronization, adaptive synchronization, time scale synchronization, intermittent lag synchronization, projective synchronization, and function projective synchronization, see [1–6] and references therein. Amongst all these synchronization schemes, it is worthwhile to mention that function projective synchronization has attracted much attention recently due to its potential application in secure communications. Function projective synchronization is the more general definition of projective synchronization. As compared with projective synchronization, function projective synchronization means that the master and slave systems could be synchronized up to a scaling function, but not a constant. This feature could be used to get more secure communication in application to secure communications, because it is obvious that the unpredictability of the scaling function in function projective synchronization can additionally enhance the security of communication. Motivated by the aforementioned reasons, this paper investigates function projective synchronization of recently developed chaotic systems with uncertain parameters using nonlinear adaptive controller. The rest of this paper is organized as follows. In Section 2, the nonlinear adaptive controller is designed to synchronize the chaotic system. In Section 3, numerical simulations are presented to demonstrate the effectiveness of the theoretical analysis.

2. Adaptive Control Scheme

The chaotic system introduced by Liu [7] recently is described by

$$\begin{aligned}
\dot{x}_1 &= a(x_2 - x_1 + x_2x_3),\\
\dot{x}_2 &= bx_2 - cx_1x_3,\\
\dot{x}_3 &= gx_2 - hx_3,
\end{aligned} \tag{2.1}$$

where a, b, c, g, h are the system parameters, when $a = 1, b = 2.5, c = 1, g = 1, h = 4$, the system is chaotic.

We take the system (2.1) as the master system, and the slave system with an adaptive control scheme is given by

$$\begin{aligned}
\dot{y}_1 &= a_1(y_2 - y_1 + y_2y_3) + u_1,\\
\dot{y}_2 &= b_1y_2 - c_1y_1y_3 + u_2,\\
\dot{y}_3 &= g_1y_2 - h_1y_3 + u_3,
\end{aligned} \tag{2.2}$$

where a_1, b_1, c_1, g_1, h_1 are parameters of the slave system which needs to be estimated, and u_1, u_2, u_3 are the nonlinear controllers such that the two chaotic systems are function projective synchronized in the sense that

$$\lim_{t\to\infty} \left\| y_i - \alpha(t)x_i \right\| = 0, \quad i = 1, 2, 3, \tag{2.3}$$

where $\alpha(t)$ is the scaling function.

From system (2.1) and system (2.2), we get the error dynamical system which can be written as

$$\begin{aligned}
\dot{e}_1 &= a_1(y_2 - y_1 + y_2y_3) + u_1 - a\alpha(t)(x_2 - x_1 + x_2x_3) - \dot{\alpha}(t)x_1,\\
\dot{e}_2 &= b_1y_2 - c_1y_1y_3 + u_2 - b\alpha(t)x_2 - c\alpha(t)x_1x_3 - \dot{\alpha}(t)x_2,\\
\dot{e}_3 &= g_1y_2 - h_1y_3 + u_3 - g\alpha(t)x_2 + h\alpha(t)x_3 - \dot{\alpha}(t)x_3,
\end{aligned} \tag{2.4}$$

where $e_i(t) = y_i(t) - \alpha(t)x_i(t)$, $i = 1, 2, 3$.

In order to stabilize the error variables of system (2.3) at the origin, we propose the adaptive control law and the parameter update law for system (2.3) as follows:

$$\begin{aligned}
\dot{u}_1 &= -a_1(y_2 - y_1 + y_2y_3) + \dot{\alpha}(t)x_1 + a_1\alpha(t)(x_2 - x_1 + x_2x_3) - k_1e_1,\\
\dot{u}_2 &= -b_1(y_2 - x_2\alpha(t)) + c_1(y_1y_3 + x_1x_3\alpha(t)) + \dot{\alpha}(t)x_2 - k_2e_2,\\
\dot{u}_3 &= -g_1(y_2 - x_2\alpha(t)) + h_1(y_3 - x_3\alpha(t)) + \dot{\alpha}(t)x_3 - k_3e_3,
\end{aligned} \tag{2.5}$$

and the update rules for the five uncertain parameters a_1, b_1, c_1, g_1, and h_1 are

$$\begin{aligned}
\dot{a}_1 &= (x_1 - x_2 - x_2x_3)\alpha(t)e_1 - k_4e_a, \\
\dot{b}_1 &= -x_2\alpha(t)e_2 - k_5e_b, \\
\dot{c}_1 &= -x_1x_3\alpha(t)e_2 - k_6e_c, \\
\dot{g}_1 &= -x_2\alpha(t)e_3 - k_7e_g, \\
\dot{h}_1 &= x_3\alpha(t)e_3 - k_8e_h,
\end{aligned} \tag{2.6}$$

where $k_i > 0$ $(i = 1, 2, \ldots, 8)$ and $e_a = a_1 - a$, $e_b = b_1 - b$, $e_c = c_1 - c$, $e_g = g_1 - g$, $e_h = h_1 - h$.

Theorem 2.1. *For the given scaling function $\alpha(t)$, the function projective synchronization between the master system* (2.1) *and the slave system* (2.2) *can be achieved if the nonlinear controller* (2.5) *and the update law* (2.6) *are adopted.*

Proof. Construct the following Lyapunov function:

$$V = \frac{1}{2}\sum_{i=1}^{3} e_i^T e_i + \frac{1}{2}\left(e_a^2 + e_b^2 + e_c^2 + e_g^2 + e_h^2\right). \tag{2.7}$$

Calculating the time derivative of V along the trajectory of error system (2.4), we have

$$\begin{aligned}
\dot{V} &= e_1(t)\left[a_1(y_2 - y_1 + y_2y_3) + u_1 - a\alpha(t)(x_2 - x_1 + x_2x_3) - \dot{\alpha}(t)x_1\right] \\
&\quad + e_2(t)\left[b_1y_2 - c_1y_1y_3 + u_2 - b\alpha(t)x_2 - c\alpha(t)x_1x_3 - \dot{\alpha}(t)x_2\right] \\
&\quad + e_3(t)\left[g_1y_2 - h_1y_3 + u_3 - g\alpha(t)x_2 + h\alpha(t)x_3 - \dot{\alpha}(t)x_3\right] \\
&\quad + e_a(t)\left[(x_1 - x_2 - x_2x_3)\alpha(t)e_1 - k_4e_a\right] + e_b(t)\left[-x_2\alpha(t)e_2 - k_5e_b\right] \\
&\quad + e_c(t)\left[-x_1x_3\alpha(t)e_2 - k_6e_c\right] + e_g(t)\left[-x_2\alpha(t)e_3 - k_7e_g\right] \\
&\quad + e_h(t)\left[x_3\alpha(t)e_3 - k_8e_h\right] \\
&= -\mathbf{e}^T\mathbf{K}\mathbf{e},
\end{aligned} \tag{2.8}$$

where $\mathbf{e} = [e_1, e_2, e_3, e_a, e_b, e_c, e_g, e_h]^T$ and $\mathbf{K} = \operatorname{diag}\{k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8\}$.

It is clear that $\dot{V}$ is negative definite and $\dot{V} = 0$ if and only if $\mathbf{e}(t) = \mathbf{0}$. According to the Lyapunov stability theorem, the function projective synchronization is achieved. This completes the proof. □

3. Numerical Simulations

In this section, we will perform some numerical simulations to verify the effectiveness of the proposed adaptive synchronization controllers. The fourth-order Runge-Kutta method is used to solve the master system (2.1) and the slave system (2.2) with time step size 0.001. The initial conditions of the master system are $x_1(0) = 0.2$, $x_2(0) = -1.2$, $x_3(0) = 0.6$, and those of the slave system are $y_1(0) = 0.4$, $y_2(0) = 0.2$, $y_3(0) = -1.8$. Moreover, the true values of the uncertain parameters are $a = 1$, $b = 2.5$, $c = 1$, $g = 1$, $h = 4$, and the initial values of the estimated parameters are chosen as $a_1(0) = 2$, $b_1(0) = 3$, $c_1(0) = 3$, $g_1(0) = 5$, $h_1(0) = 5$, the

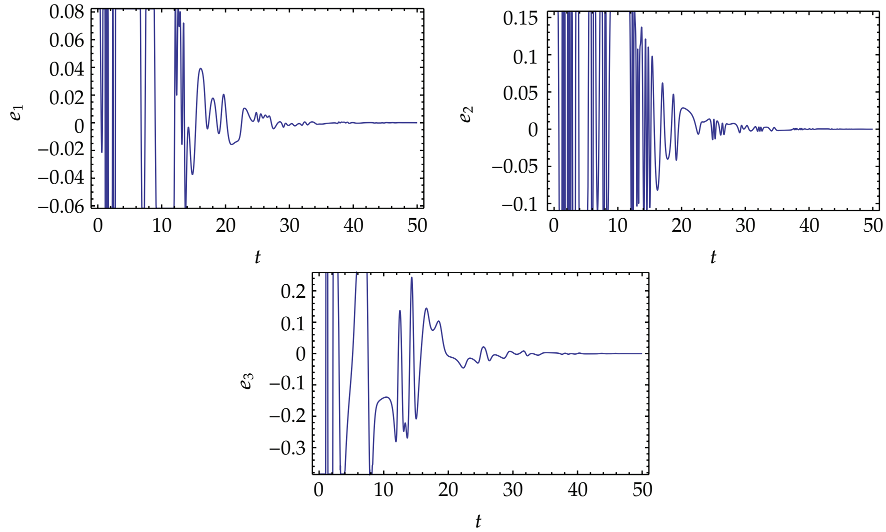

Figure 1: Error dynamics between the master system (2.1) and the slave system (2.2).

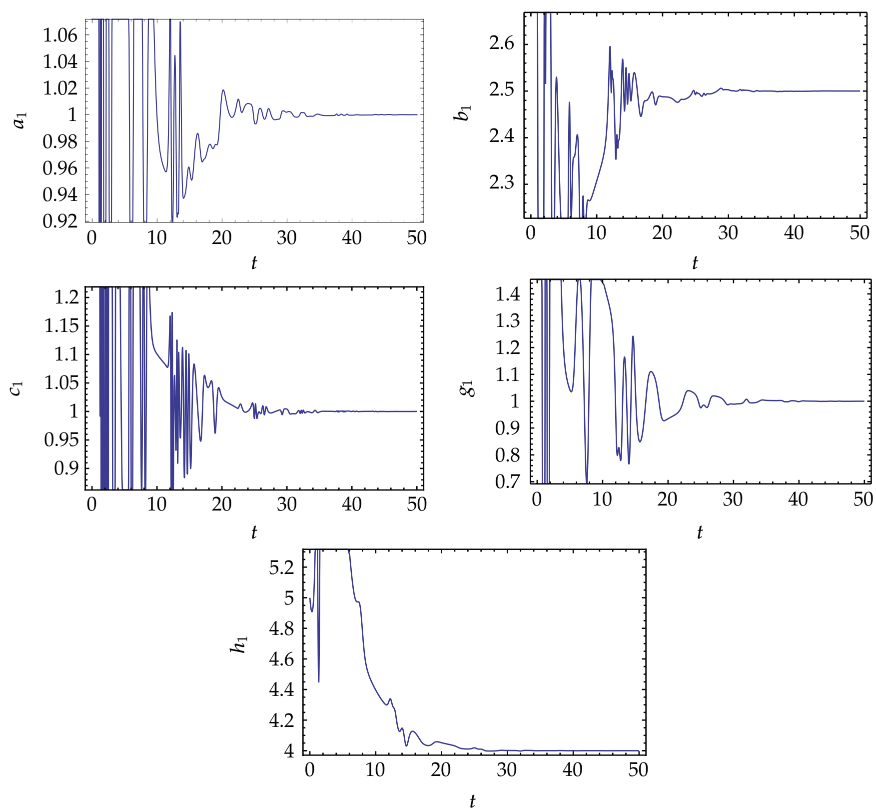

Figure 2: The time evolution of the estimated parameters.

scaling function is chosen as $\alpha(t) = 0.3 + 0.2\sin(t)$. Furthermore, the control gains are chosen as $(k_1, k_2, k_3, k_4, k_5, k_6, k_7, k_8) = (0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2)$. The error variables e_1, e_2, e_3 which tend to zero with $t \to \infty$ are illustrated in Figure 1. Figure 2 shows that the estimated values of the uncertain parameters converge to $a = 1$, $b = 2.5$, $c = 1$, $g = 1$, $h = 4$ as $t \to \infty$.

4. Conclusion

In summary, this work has investigated the function projective synchronization of the new chaotic systems with uncertain parameters. Based on the Lyapunov stability theory, we have designed the adaptive synchronization controllers with corresponding parameter update laws to stabilize the error dynamics between the master and the slave systems. All the theoretical results are verified by numerical simulations to illustrate the effectiveness of the proposed adaptive function projective synchronization scheme.

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