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Research Article

Optimal PMU Placement with Uncertainty Using Pareto Method

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This paper proposes a method for optimal placement of Phasor Measurement Units (PMUs) in state estimation considering uncertainty. State estimation has first been turned into an optimization exercise in which the objective function is selected to be the number of unobservable buses which is determined based on Singular Value Decomposition (SVD). For the normal condition, Differential Evolution (DE) algorithm is used to find the optimal placement of PMUs. By considering uncertainty, a multiobjective optimization exercise is hence formulated. To achieve this, DE algorithm based on Pareto optimum method has been proposed here. The suggested strategy is applied on the IEEE 30-bus test system in several case studies to evaluate the optimal PMUs placement.

1. Introduction

The Phasor Measurement Units (PMUs) are used to measure the positive sequence of voltage and current phasors and are synchronized by a global positioning system (GPS) satellite transmission. Application of PMUs results in making the state estimation equations linear, leading to easier and more accurate solution [1–3]. There is limitation in the number of PMUs being installed due to cost considerations and its effectiveness as far as the appropriate placement are concerned; hence, optimal placement of PMUs is of utmost importance in this matter. In recent years, significant work on the optimum number of PMUs and their placement has been carried out [4–7].

If the entire state vector of bus voltage magnitudes and angles can be estimated from the set of available measurements, the power system with the specified measurement set is said to be observable. Observability is hence one of the most important characteristics needed to be determined in state estimation of power systems [8, 9]; the minimum unobservability can therefore be an accurate criterion. One of the methods used to determine the unobservability is Singular Value Decomposition (SVD) [10–15]. This method can be applied on to the system topology matrix in order to find the unobservable nodes.

PMU placement in the normal condition is demonstrated by several workers [1, 13, 14] who have also considered uncertainty in order to reach full observability. Disrespecting of PMU cost/number employed is the disadvantage of this approach.

In [15], the objective is minimization of the PMUs number and maximization of the redundancy measurement buses number for a power system. A multiobjective function is introduced with two mentioned objectives. Disrespecting of consideration to normal condition and explanation about existent different cases for minimum unobservability which is proposed a method for that here (to find minimum value for single PMU outage suggested objective function as the best) and single line outage in a system as a contingency are the disadvantages of this one.

In this paper, both PMU cost/number and uncertainty are considered in PMU placement using a Pareto multi-objective optimization by Differential Evolution algorithm. Several case studies are considered to verify the proposed method.

2. SVD Application for Observability

For a network with N buses and m measurements of voltage and current phasors, the linear equation relating the measurements and the state vector is [16]

$$z = Hx + e, \tag{2.1}$$

where the vector z is linearly relevant to the n-dimensional state vector x comprising N-bus voltage phasors, that is, $n = 2N - 1$. H is the $(m \times n)$ Jacobian matrix, and e is the $(m \times 1)$ additive measurement error vector. The state vector is therefore

$$x = \begin{bmatrix} \delta_2 & \delta_3 & \cdots & \delta_N & V_1 & V_2 & \cdots & V_N \end{bmatrix}^T. \tag{2.2}$$

The first phase angle, δ_1, is set to an arbitrary value, such as 0 as a reference bus [17].

To evaluate the components of the Jacobian matrix H, the derivatives of measurements proportion to δ_i, δ_j, V_i, and V_j should first be determined. The sparse matrix technique is then used to build this matrix.

Numerical observability is defined as the ability of the system model to be solved for the state estimation. If the matrix H is of full rank, that is, $2N - 1$, then the system is considered to be numerically observable [18]. Moreover, topological observability is defined as the existence of at least one spanning measurement tree of full rank in the network [5]. In a system, the normal equations might be very singular or ill conditioned. Several methods have been suggested to solve ill-conditioned problems [19, 20]. In this paper, SVD method is used to solve ill-conditioned problems and evaluate the numerical observability of a system. Then, rank of H with regards to detection of observable and unobservable areas is discussed.

2.1. Detection Unobservable Buses Using SVD

In solving the observability problem by SVD the matrix $H(m \times n)$ of (2.1) can be replaced by product of three matrices, that is,

$$H = UWV^T, \tag{2.3}$$

where W is a diagonal matrix $(n \times n)$ with positive or zero elements, which are the singular values of H. U is a column orthogonal $(m \times n)$ matrix that is eigenvector matrix of HH^T and V^T is the transpose of an $(n \times n)$ orthogonal matrix that is the eigenvector matrix of $H^T H$ [10, 11].

SVD is capable of solving complex problems in singular measurement systems where despite lack of unique solution, a null space vector is often provided for each singularity. The infinite solutions of such system may be expressed as [12]:

$$[\mathrm{X}] = [\mathrm{X}_P] + \sum_{i=1}^{n-\mathrm{rank}(H)} k_i [\mathrm{X}_{ni}], \tag{2.4}$$

where $[X_P]$ is the particular solution, k_i is a constant, and $[X_{ni}]$ is the null space vector.

The null space vectors can be multiplied by any constant and added to the particular solution to give another valid solution to the set of equations, thereby specifying the infinite number of solutions. Variables corresponding to zeros in all the null space vectors will not be changed by this process and, hence, are completely specified by the particular solution. These variables correspond to estimates of quantities in the observable islands. The variables corresponding to nonzero elements in the null space vectors are in the unobservable regions as they cannot be uniquely determined [12].

3. Optimal PMU Placement

3.1. Placement in Normal Condition

In this study, PMU is assumed to have a sufficient number of channels to measure the voltage and current phasors, respectively, at the PMU bus and through all branches incident to it. Using DE algorithm, the optimal PMU placement in a system is obtained. The fitness function for this DE algorithm is the number of unobservable buses in the system:

$$\text{O.F.} = f_{\text{normal}} = \text{Unobser}_{\text{normal}}. \tag{3.1}$$

This objective function has been evaluated by SVD method as mentioned in Section 2.

3.2. Placement with Uncertainty

The state estimation should be able to accommodate all types of contingencies (such as single line/PMU outage), otherwise the system may become unobservable. Hence, the objective function has to consider both normal and contingency conditions for PMU placement. Two types of contingencies are considered in this study as expressed in the following.

3.2.1. Single PMU Contingency

So far it is assumed that all PMUs are in good working condition. In order to guard against single PMU loss, an additional term is added to the normal objective function. The proposed function then becomes

$$\text{O.F.} = f_{\text{normal}} + f_{\text{S.P.C.}}, \tag{3.2}$$

where

$$f_{\text{S.P.C.}} = \left(\frac{\sum_{K=1}^{N_{\text{PMU}}} \text{Unobser}_{K\text{th PMU}}}{N_{\text{PMU}}} \right) \tag{3.3}$$

and N_{PMU} is the network PMU number and $\text{Unobser}_{K\text{th PMU}}$ is the unobservable bus number for the Kth PMU loss.

3.2.2. Single Line Contingency

This occurs when the link between two adjacent buses is disconnected, leading to change in the network topology and reduction of system observability. This happens frequently and therefore should be considered in the PMU placement. For this, an objective function (O.F.) is proposed below where the mean of unobservable buses at single line outage ($f_{\text{S.L.C.}}$) is added to the unobservable buses in normal condition (f_{normal}):

$$\text{O.F.} = f_{\text{normal}} + f_{\text{S.L.C.}}, \tag{3.4}$$

where

$$f_{\text{S.L.C.}} = \left(\frac{\sum_{K=1}^{N_{\text{Line}}} \text{Unobser}_{K\text{th Line}}}{N_{\text{Line}}} \right) \tag{3.5}$$

and N_{line} is the network line number and $\text{Unobser}_{K\text{th Line}}$ is the unobservable bus number for the Kth line outage.

The O.F. expressed in (3.2) and (3.4) are multiobjective and have to be solved accordingly. In this paper, a combination of Pareto method and DE algorithm is employed.

4. Differential Evolution Algorithm

Differential Evolution (DE) algorithm is a simple but effective intelligent optimization algorithm presented firstly by Rainer Storn and Kenneth Price in 1995. Though it is originated from genetic algorithm, it needs no encoding and decoding operation. And with its fast convergence, well stability, and strong adaptability to all kinds of nonlinear functions, it is proved to be better than those algorithms such as genetic algorithm, particle swarm optimization, evolution strategy, and adaptive simulated annealing [21–24].

4.1. Schemes and Mechanism of DE

DE algorithm employs stochastic search technique and is one of later type amongst the evolutionary algorithms. At every generation G, DE maintains a population $P^{(G)}$ of N_p vectors of candidate solutions to the problem, which evolve throughout the optimization process to find global solutions [25]:

$$P^{(G)} = \left[X_1^{(G)}, \ldots, X_{N_p}^{(G)}\right]. \tag{4.1}$$

The population size, N_p, does not change during the optimization process. The dimension of each vector of candidate solutions correspond, to the number of the decision parameters, D, to be optimized. Therefore,

$$X_i^{(G)} = \left[X_{1,i}^{(G)}, \ldots, X_{D,i}^{(G)}\right]^T, \quad i = 1, \ldots, N_p. \tag{4.2}$$

The optimization process includes three main operations: mutation, crossover, and selection. For any generation, each parameter vector of the current population becomes a target vector. For each target vector, the mutation operation produces a new parameter vector (called mutant vector), by adding the weighted difference between two randomly chosen vectors to a third (also randomly chosen) vector. By mixing the parameters of the mutant vector with those of the target vector, the crossover operation generates a new vector (the trial vector). If the trial vector obtains a better fitness value than the target vector, then the trial vector replaces the target vector in the following generation.

DE has several reproduction schemes. The following one, denoted as DE1 here, is recommended by Corne et al. [26] as the first choice:

$$x'_{i,j}[k] = x_{i,j}[k] + K \cdot \left(x_{r3,j}[k] - x_{i,j}[k]\right) + F \cdot \left(x_{r1,j}[k] - x_{r2,j}[k]\right), \tag{4.3}$$

where $r1 \neq r2 \neq r3 \neq i$ are integers randomly selected from 1 to population size. The parameters K and F are taken within the range of $[0, 1]$. If $x'_{i,j}$ $[k]$ is found outside the range of $[x_{j\min}, x_{j\max}]$, it will then be fixed to the violated limit $x_{j\max}$ or $x_{j\min}$. Equation (4.3) demonstrates that, unlike other Evolutionary Algorithms (EAs) which relies on a predefined probability distribution function [25], the reproduction of DE is driven by the differences between randomly sampled pairs of individuals in the current population. This reproduction scheme, although simple, elegantly endows DE with the features of self-tuning and rotational invariance, which are crucial for an efficient EA scheme and have long been pursued in the EA community [26]. An even simpler DE scheme, denoted as DE2 here, can be derived from (4.3):

$$x'_{i,j}[k] = x_{i,j}[k] + K \cdot \left(x_{\text{best},j}[k] - x_{i,j}[k]\right) + F \cdot \left(x_{r1,j}[k] - x_{r2,j}[k]\right), \tag{4.4}$$

where $x_{\text{best},j}$ is the value of the jth control variable of the historical best individual (x_{best}) obtained from the former generations. DE2 can be viewed as a greedier version of DE1, because it exploits the information of the best individual to guide the search. This can speed up the convergence, because the way the best individual being utilized here is a kind of population acceleration [27]. At kth generation, a search point $x_i[k]$ is first accelerated by the term $K \times (x_{\text{best}}[k] - x_i[k])$, in (4.4), towards the historical best individual, x_{best}, and to some point that is located between $x_i[k]$ and x_{best}. This kind of acceleration forms a kind of contractive pressure and may cause premature convergence. But fortunately, the contractive pressure can be balanced by the diffuse pressure provided by the last term $F \times (x_{r1,j}[k] - x_{r2,j}[k])$ in (4.4), which diverts the search point in a random direction based on the difference between the randomly sampled $x_{r1}[k]$ and $x_{r2}[k]$. Therefore, the global searching ability can hopefully be maintained [21, 27]. So DE2 is used in this study. Used DE algorithm parameters include $N_p = 50$, Maximum Iteration = 100, $K = 0.7$, and $F = 0.5$.

So far, the O.F. is assumed to be single objective; however, for a multi-objective O.F., DE algorithm cannot be used alone. Hence, Pareto optimum method is added to this algorithm here as below.

4.2. Overview of Pareto Optimum Method

Multiobjective optimization supplies information about all possible alternative solutions available for a given set of objectives. The most appropriate solution would then have to be selected amongst the spectrum of solutions considering treatment carried out and aims expressed.

In multi-objective optimization, a vector of decision variables, say, x_j, $j = 1, \ldots, N$ which satisfies constraints and optimizes a vector function $\{$say $f = (f_1(x), \ldots, f_M(x))\}$ whose elements represent M objective functions, forms a mathematical description of a performance criteria expressed as computable functions of the decision variables, that are usually in conflict with each other. Hence, means are optimized to find a solution which would give the values of all objective functions acceptable to the treatment planner [28].

The constraints define the feasible region X and any point x in X defines a feasible solution. The vector function $f(x)$ is a function that maps the set X in the set F that represents all possible values of the objective functions. Normally, we never have a situation in which all the $f_i(x)$ values have an optimum in X at a common point x. We, therefore, have to establish certain criteria to determine an optimal solution. One interpretation of the term optimum in multiobjective optimization is the Pareto optimum, see Figure 1.

A solution x_1 dominates a solution x_2 if and only if the two following conditions are true:

(i) x_1 is no worse than x_2 in all objectives, that is, $f_j(x_1) \le f_j(x_2)$ for $j = 1, \ldots, M$;

(ii) x_1 is strictly better than x_2 in at least one objective, that is, $f_j(x_1) < f_j(x_2)$ for at least one $j \in \{1, \ldots, M\}$.

Let us assume this is a minimization problem. As illustrated in Figure 1 the Pareto front is the boundary between points P_1 and P_2 of the feasible set F.

Solutions 1 and 3 are nondominated Pareto optimal solutions. Solution 2 is not Pareto optimal as solution 1 has simultaneously smaller values for both objectives. Hence, there is no reason why solution 2 should be accepted over solution 1. So x_2 is dominated by x_1. Therefore, the aim of multiobjective optimization becomes to obtain a representative set of nondominated solutions [29].

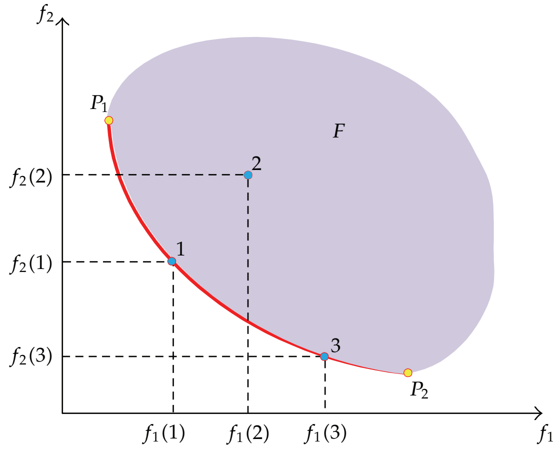

Figure 1: Example of a two objective space (f_1, f_2) for Pareto optimal solutions.

Table 1: PMU placement considering minimum unobservability.

Case	Number of PMU	PMU inbus	Unobservable buses	f_{normal}
1	3	10, 12, 27	**1, 2, 3, 5, 7, 8, 11, 18, 19, 23, 24, 26**	12
2	6	1, 5, 10, 12, 15, 27	**8, 11, 19, 24, 26**	5
		1, 6, 10, 12, 24, 27	**5, 11, 18, 19, 26**	
		2, 6, 10, 12, 19, 27	**3, 11, 23, 24, 26**	
		1, 5, 10, 12, 19, 27	**8, 11, 23, 24, 26**	
		1, 6, 10, 12, 19, 25	**5, 11, 23, 29, 30**	
3	10	1, 2, 6, 9, 10, 12, 15, 19, 25, 27	—	0
		1, 2, 6, 9, 10, 12, 15, 18, 25, 27	—	
		1, 6, 7, 9, 10, 12, 18, 24, 25, 30	—	
		1, 5, 6, 9, 10, 12, 18, 24, 25, 30	—	
		1, 2, 6, 9, 10, 12, 19, 24, 25, 30	—	

4.3. DE Algorithm Based on Pareto Optimum Method

The flow chart for a proposed strategy using DE algorithm based on Pareto method presented in this paper, is shown in Figure 2. Following the first step, that is, initialization of DE algorithm parameters, subfunctions of major objective function are evaluated using the initial population as the first iteration. Then a set of first solutions are selected using Pareto optimum method, in order to obtain the nondominated answers. Next, population is updated with DE mechanism. The process is then repeated for the preset Iter_{max}. Finally, the best solution is identified amongst the Pareto optimum set depending on the user preference and appropriate constraints.

5. Case Study

To verify the proposed method for PMU placement, a number of case studies are carried out on the IEEE 30-bus test system [30].

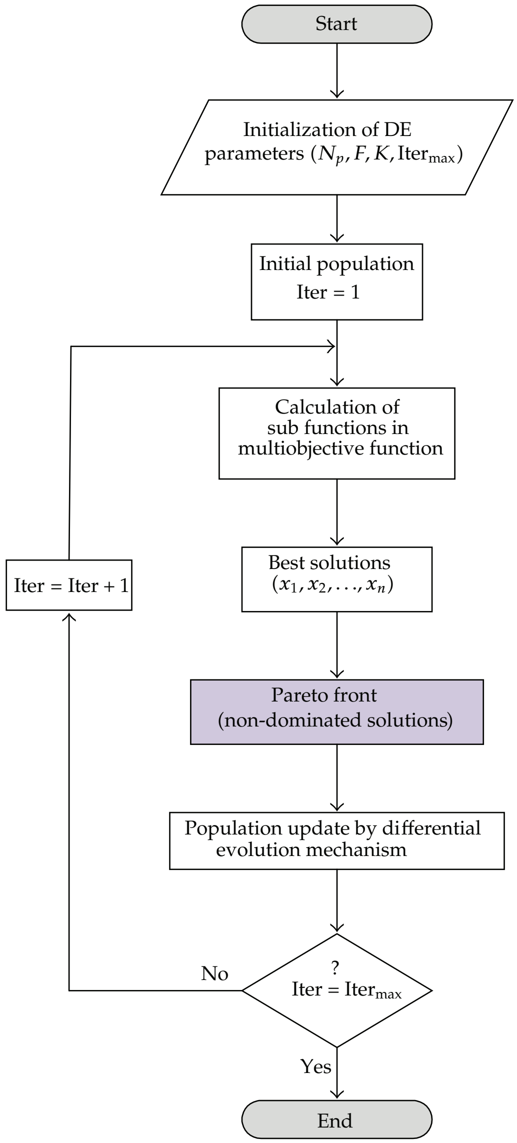

Figure 2: Proposed strategy flowchart based on Pareto optimum method using DE algorithm.

5.1. Optimal PMU Placement Results in Normal Condition

The main purpose of the PMU placement process is the determination of the minimum number of PMUs that can make the minimum unobservability for a system under normal operating conditions.

In order to evaluate the method more comprehensively, different number of PMUs are employed in the 3 case studies attempted here. It should be noted that the number of state variables for IEEE 30-bus test system is 59, and Table 1 summarizes the results obtained. It is interesting to note that the system has become fully observable only after 10 PMU placements. Columns of this show number of case study, number of employed PMU, placement of PMU in

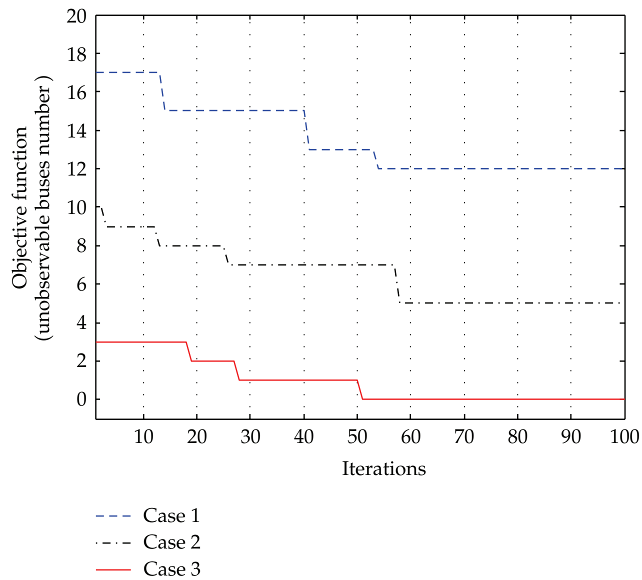

Figure 3: Convergence rate of DE algorithm for PMU placement of IEEE 30-bus test system.

mentioned buses, buses those are unobservable with existing PMUs and f_{normal} values which show number of unobservable buses, respectively. As it is determined in this table, several solutions are found for case studies 2 and 3. Unobservable buses are shown in this table too.

To find the best placement between mentioned solutions is the next step. For this, single PMU outage problem is used. The $f_{\text{S.P.C.}}$ function which is determined in Section 3 is objective function for optimization problem in this step. The best placement has minimum value of $f_{\text{S.P.C.}}$ between possible solutions. Results are shown in Table 2.

In case studies 2 and 3, minimum values of 7.83 and 1.6 for $f_{\text{S.P.C.}}$ are obtained respectively. In case study 1, only one possible state has been found that is optimum. Then Table 3 can show optimal PMU placement for IEEE 30-bus system in normal condition.

As can be seen in Figure 3 convergence rates of DE algorithm for all cases are quick. Approximately at half of iterations, case studies are converged.

5.2. Optimal PMU Placement Results Considering Single Line Contingency

The purpose of placement in this part is finding the minimum number of PMU for full observability considering single line contingency. The proposed method for this work is optimization problem with DE algorithm based on Pareto optimum method. Objective function is expressed in Section 3 too. It is noteworthy that IEEE 30-bus test system has 41 lines.

To verify the performance of proposed method, a comparison between this method and a sequential increasing one is carried out. In sequential increasing method, the aim is obtaining zero value for $f_{\text{S.L.C.}}$ which is expressed in Section 3 while f_{normal} is zero (full observability in normal condition). For this, placement process is started with 10 PMUs those are allocated in normal condition. To obtain the purpose, one PMU is added for the search process until reaching the $f_{\text{S.L.C.}}$ to zero. As it is shown in Table 4 after adding PMUs at 3, 7,

Table 2: $f_{\text{S.P.C.}}$ values for applied PMU Placement in Normal Condition.

Case	Number of PMU	PMU in Bus	$f_{\text{S.P.C.}}$	f_{normal}
1	3	10,12,27	18	12
2	6	1, 5, 10, 12, 15, 27	8.5	5
		1, 6, 10, 12, 24, 27	7.83	5
		2, 6, 10, 12, 19, 27	8	5
		1, 5, 10, 12, 19, 27	8.83	5
		1, 6, 10, 12, 19, 25	8.17	5
3	10	1, 2, 6, 9, 10, 12, 15, 19, 25, 27	1.6	0
		1, 2, 6, 9, 10, 12, 15, 18, 25, 27	1.7	0
		1, 6, 7, 9, 10, 12, 18, 24, 25, 30	1.9	0
		1, 5, 6, 9, 10, 12, 18, 24, 25, 30	1.9	0
		1, 2, 6, 9, 10, 12, 19, 24, 25, 30	1.9	0

Table 3: Optimal PMU placement in normal condition.

Case	Number of PMU	PMU in Bus	$f_{\text{S.P.C.}}$	f_{normal}
1	3	10, 12, 27	18	12
2	6	1, 6, 10, 12, 24, 27	7.83	5
3	10	1, 2, 6, 9, 10, 12, 15, 19, 25, 27	1.6	0

21, 22, 28 and 30 buses, $f_{\text{S.L.C.}}$ will be zero. Then with 16 PMUs, normal condition and single line contingency condition are observable.

Using the proposed method, placement results can be seen in Tables 4, 5, 6, 7, and 8. These show case studies for 10, 12, 14, and 15 PMU numbers. Optimum values for two terms of (3.4) (f_{normal} and $f_{\text{S.L.C.}}$) are shown here.

In this strategy, user can consider different importance degrees and using weighting method, he/she can find the best solution between the optimum sets. Then it is one of the most important benefits of this method in which choosing of the best placement depends on user selection attending to the importance degree of objective function terms. Figures 4 and 5 show pareto optimum values curve of single line contingency for 10 and 12 number of PMU, respectively.

As it is clear from the results, normal and contingency condition terms are zero with 15 PMUs while it was obtained with 16 PMUs using sequential increasing strategy. It is another advantage for suggested method which is very effective for cost reduction.

To compare the effectiveness and speed of proposed method and sequential increasing method, Figure 6 is drawn. This shows a comparison between $f_{\text{S.L.C.}}$ values when f_{normal} is zero in different case studies. As it can be seen, proposed method has an appropriate speed to reach the full observability of single line contingency term.

6. Conclusion

In this paper a method based on SVD is used for identifying the unobservable nodes in a power system due to importance of PMU placement considering observability. The most prominent characteristic of SVD concept is accuracy and speed to diagnosis the unobservable buses and appropriate operation in singularity condition. A methodology with

Table 4: PMU placement using sequential increasing method considering single line contingency.

Number of PMU	f_{normal}	$f_{\text{S.L.C.}}$	PMU in bus
10	0	0.6585	1, 2, 6, 9, 10, 12, 15, 19, 25, 27
11	0	0.5609	1, 2, 3, 6, 9, 10, 12, 15, 19, 25, 27
12	0	0.4634	1, 2, 3, 6, 9, 10, 12, 15, 19, 21, 25, 27
13	0	0.3414	1, 2, 3, 6, 9, 10, 12, 15, 19, 21, 22, 25, 27
14	0	0.2439	1, 2, 3, 6, 7, 9, 10, 12, 15, 19, 21, 22, 25, 27
15	0	0.1219	1, 2, 3, 6, 7, 9, 10, 12, 15, 19, 21, 22, 25, 27, 30
16	0	0	1, 2, 3, 6, 7, 9, 10, 12, 15, 19, 21, 22, 25, 27, 28, 30

Table 5: Optimal PMU placement with proposed method (number of PMU = 10).

Point	f_{normal}	$f_{\text{S.L.C.}}$	PMU in bus
1	0	0.5365	1,6,7,9,10,12,18,24,25,30
2	1	0.4146	3,4,5,17,23,24,25,27,29,30
3	2	0.3171	6,7,8,10,12,15,19,21,24,25
4	3	0.2926	1,4,9,10,14,15,16,19,28,30
5	4	0.1951	2,8,11,12,19,23,24,25,27,28

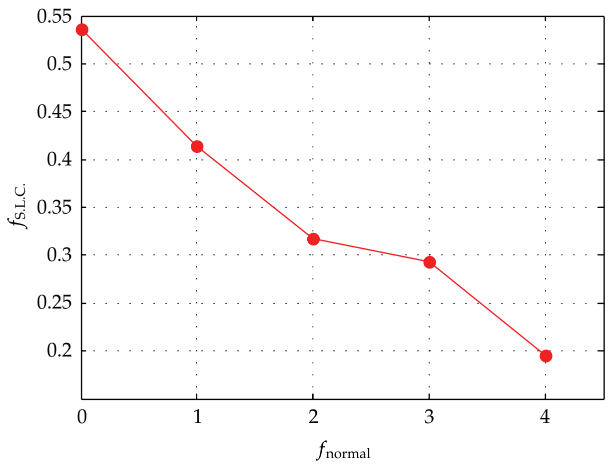

Figure 4: Pareto optimum values curve for single line contingency (number of PMU = 10).

Table 6: Optimal PMU placement with proposed method (number of PMU = 12).

Point	f_{normal}	$f_{\text{S.L.C.}}$	PMU in Bus
1	0	0.3902	6, 10, 11, 12, 15, 17, 19, 21, 25, 26, 27, 28
2	1	0.2682	2, 3, 7, 8, 11, 13, 15, 19, 20, 25, 28, 29
3	2	0.1707	1, 4, 8, 10, 11, 12, 14, 16, 18, 20, 23, 27
4	3	0.0731	3, 5, 7, 9, 10, 13, 17, 18, 21, 25, 28, 30

Table 7: Optimal PMU placement with proposed method (number of PMU = 14).

Point	f_{normal}	$f_{\text{S.L.C.}}$	PMU in Bus
1	0	0.0975	3, 4, 6, 9, 11, 13, 15, 18, 20, 23, 26, 27, 29, 30
2	1	0.0243	5, 7, 10, 12, 13, 14, 15, 17, 18, 20, 22, 24, 25, 28

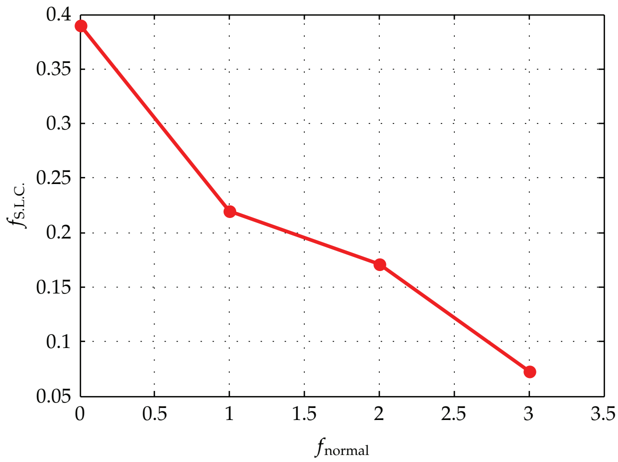

Figure 5: Pareto optimum values curve for single line contingency (number of PMU = 12).

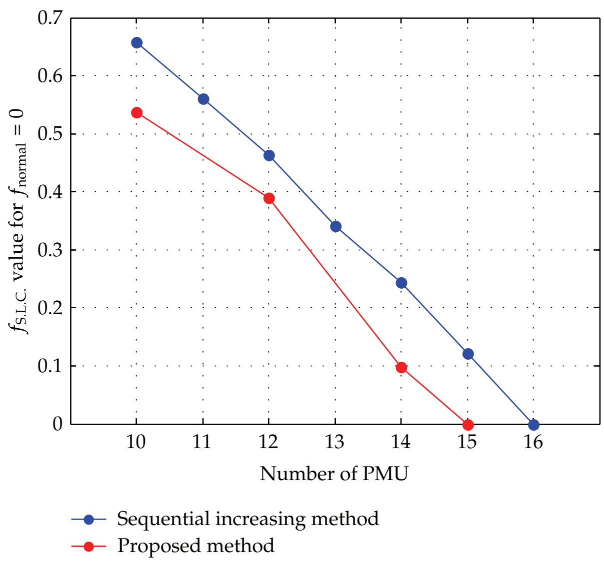

Figure 6: Comparison of $f_{\text{S.L.C.}}$ values of sequential increasing and proposed methods for the full observability in normal condition.

DE algorithm to determine the minimum number of PMUs required to obtain the minimum number of unobservable buses in normal operating condition is presented in this work. The main work of this paper was on PMU placement under uncertainty condition with DE algorithm based on Pareto optimum method. Some uncertainties are considered here. Objective functions were multiobjective in uncertainty conditions, then a proposed strategy using DE algorithm based on Pareto optimum method was introduced here to solve PMU placement problem. The IEEE 30-bus test system was selected to apply the strategies on it to

Table 8: Optimal PMU placement with proposed method (number of PMU = 15).

Point	f_{normal}	$f_{\text{S.L.C.}}$	PMU in Bus
1	0	0	2, 3, 7, 8, 9, 10, 12, 15, 16, 19, 22, 24, 25, 27, 29

prove the performance of proposed method. A sequential increasing method is carried out here for PMU placement and its results were compared with proposed method too.

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