Hindawi Publishing Corporation
Mathematical Problems in Engineering
Volume 2012, Article ID 521675, 15 pages
doi:10.1155/2012/521675

Research Article

Robust H_∞ Filtering of 2D Roesser Discrete Systems: A Polynomial Approach

Chakir El-Kasri,[1] Abdelaziz Hmamed,[1] Teresa Alvarez,[2] and Fernando Tadeo[2]

[1] *Laboratory of Electronics, Signal-Systems and Information Science (LESSI), Department of Physics, Faculty of Sciences Dhar El Mehraz, P.O. Box 1796, Fes-Atlas 30000, Morocco*
[2] *Department of Systems Engineering and Automatic Control, University of Valladolid, 47005 Valladolid, Spain*

Correspondence should be addressed to Fernando Tadeo, fernando@autom.uva.es

Received 5 October 2011; Accepted 23 November 2011

Academic Editor: J. Rodellar

The problem of robust H_∞ filtering is investigated for the class of uncertain two-dimensional (2D) discrete systems described by a Roesser state-space model. The main contribution is a systematic procedure for generating conditions for the existence of a 2D discrete filter such that, for all admissible uncertainties, the error system is asymptotically stable, and the H_∞ norm of the transfer function from the noise signal to the estimation error is below a prespecified level. These conditions are expressed as parameter-dependent linear matrix inequalities. Using homogeneous polynomially parameter-dependent filters of arbitrary degree on the uncertain parameters, the proposed method extends previous results in the quadratic framework and the linearly parameter-dependent framework, thus reducing its conservatism. Performance of the proposed method, in comparison with that of existing methods, is illustrated by two examples.

1. Introduction

Many practical systems can be modeled as two-dimensional (2D) systems, such as many systems in image data processing and transmission, in thermal processes, in gas absorption and water stream heating, and so forth [1, 2]. Therefore, in recent years much attention has been devoted to the analysis and synthesis problems for 2D discrete systems, and many important results are available in the literature. For example, the stability of 2D systems based on Lyapunov approaches was investigated in [3–8]; a 2D dynamic output feedback control, based on solving a set of 2D polynomial equations, was investigated in [9], whereas the model approximation problem for these systems was addressed in [10].

In the filtering literature, the most popular method is probably the celebrated Kalman filtering approach, which provides an optimal estimation of the state variables, in the sense that the covariance of the estimation error is minimized [11]. For the 2D system filtering problem, there are already a significant number of results based on the Kalman filtering approach [12–16], using state-space or polynomial approaches.

This paper concentrates on H_∞ filtering, as it makes it possible to consider uncertainty explicitly and provides guaranteed bounds. For 1D systems, H_∞ filtering has been extensively studied: see, for example, [17–21] and the references therein. However, for 2D systems we can just cite [22, 23], where H_∞ filter designs were proposed, by using an LMI approach, for 2D Roesser models and Fornasini-Marchesini second models, respectively. The major difficulty in developing results in this framework is the lack of the bounded real lemma for 2D systems: most of the 1D system H_∞ filtering results are based on this lemma, which relates the H_∞ performance measure to the solution of certain Riccati equations or inequalities, see [24, 25].

Thus, in this paper we propose a solution to the robust H_∞ filtering problem for uncertain 2D systems by using a structured polynomially parameter-dependent method. The idea exploits the positivity of the uncertain parameters belonging to the unit simplex, constructed in such a way that additional free variables are generated when the degree $\mathfrak{g}$ of the polynomial filter matrices increases. This makes it possible to define a sequence of sufficient LMI conditions of increasing precision and smaller conservatism. For any fixed degree, these LMI conditions are constructed following simple rules derived from the vertices of the polytope. It is also shown that if the LMI conditions are fulfilled for a certain degree, then a feasible solution exists for higher degrees. Moreover, the condition proposed reduces, when $\mathfrak{g} = 0$, to the filter design method in the quadratic framework given in [26] and are equivalent to the sufficient LMI tests based on an affine parameter-dependent Lyapunov function for $\mathfrak{g} = 1$, which shows the reduced conservatism of the proposed approach.

2. Problem Formulation

Consider a 2D discrete system described by the following Roesser's state-space model:

$$\begin{aligned}
\begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} &= A(\alpha)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + B(\alpha)w(i,j), \\
y(i,j) &= C_1(\alpha)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + D_1(\alpha)w(i,j), \\
z(i,j) &= C(\alpha)\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + D(\alpha)w(i,j),
\end{aligned} \tag{2.1}$$

where $x^h(i,j) \in \Re^{n_h}$ and $x^v(i,j) \in \Re^{n_v}$ are the horizontal and vertical states, respectively, with the boundary conditions $x^h(0,k) = x_0^h$, $x^v(k,0) = x_0^v$ for all k, $y(i,j) \in \Re^p$ is the measured output, $z(i,j) \in \Re^r$ is the signal to be estimated and $w(i,j) \in \Re^m$ is the exogenous input.

To describe the uncertainty, all matrices are assumed to be real, belonging to the polytope

$$\wp \triangleq \left\{ \begin{bmatrix} A(\alpha) & B(\alpha) \\ C_1(\alpha) & D_1(\alpha) \\ C(\alpha) & D(\alpha) \end{bmatrix} = \sum_{i=1}^{N} \alpha_i \begin{pmatrix} A_i & B_i \\ C_{1i} & D_{1i} \\ C_i & D_i \end{pmatrix}, \alpha \in \Omega \right\}, \tag{2.2}$$

where Ω is the unit simplex, defined by

$$\Omega = \left\{ (\alpha_1, \alpha_2, \ldots, \alpha_N) : \sum_{i=1}^{N} \alpha_i = 1, \alpha_i \geq 0 \right\}. \tag{2.3}$$

The 2D transfer function from the noise $\omega(t_1, t_2)$ to the estimated output $z(t_1, t_2)$ is then given by

$$T_{zw}(z_1, z_2) = C(\alpha)(I(z_1, z_2) - A(\alpha))^{-1} B(\alpha) + D(\alpha), \tag{2.4}$$

where

$$I(z_1, z_2) = \operatorname{diag}(z_1 I_{n_h}, z_2 I_{n_v}). \tag{2.5}$$

This 2D transfer function will be central in the rest of the paper, so the following definition of H_∞ norm is given explicitly for this transfer function.

Definition 2.1. The H_∞ norm of the transfer function $T_{zw}(\omega_1, \omega_2)$ of the 2D discrete system (2.1) is given by

$$\|T_{zw}(z_1, z_2)\|_\infty = \sup_{\omega_1,\, \omega_2 \in [0\ 2\pi]} \sigma_{\max}\left[T_{zw}\left(e^{j\omega_1}, e^{j\omega_2} \right) \right]. \tag{2.6}$$

In this paper, the basic objective is to find a filter of the form

$$\begin{gathered} \begin{bmatrix} x_f^h(i+1, j) \\ x_f^v(i, j+1) \end{bmatrix} = A_f(\alpha) \begin{bmatrix} x_f^h(i, j) \\ x_f^v(i, j) \end{bmatrix} + B_f(\alpha) y(i, j), \\ z_f(i, j) = C_f(\alpha) \begin{bmatrix} x_f^h(i, j) \\ x_f^v(i, j) \end{bmatrix}, \end{gathered} \tag{2.7}$$

in order to estimate the signal z from the measurements of y, where $x_f^h(i, j) \in \Re^{n_h}$ and $x_f^v(i, j) \in \Re^{n_v}$ are, respectively, the horizontal and vertical states of the filter, $z_f(i, j) \in \Re^p$ is the estimate of $z(i, j)$, and $A_f(\alpha)$, $B_f(\alpha)$, and $C_f(\alpha)$, are the filter parameter matrices, to be determined using the technique developed in this paper.

We now give a proper definition of the H_∞ norm of the filtering error. First, define the augmented state vectors and the filtering error output signal, respectively, by

$$\begin{aligned}\tilde{x}^h(i,j) &= \left[x^h(i,j)^T \;\; x_f^h(i,j)^T\right]^T,\\ \tilde{x}^v(i,j) &= \left[x^v(i,j)^T \;\; x_f^v(i,j)^T\right]^T,\\ \tilde{z}(i,j) &= z(i,j) - z_f(i,j).\end{aligned} \tag{2.8}$$

Then, combining these definitions with (2.1) and (2.7), the error dynamic equations are just

$$\begin{aligned}\begin{bmatrix}\tilde{x}^h(i+1,j)\\ \tilde{x}^v(i,j+1)\end{bmatrix} &= \tilde{A}(\alpha)\begin{bmatrix}\tilde{x}^h(i,j)\\ \tilde{x}^v(i,j)\end{bmatrix} + \tilde{B}(\alpha)w(i,j),\\ \tilde{z}(i,j) &= \tilde{C}(\alpha)\begin{bmatrix}\tilde{x}^h(i,j)\\ \tilde{x}^v(i,j)\end{bmatrix} + \tilde{D}(\alpha)w(i,j),\end{aligned} \tag{2.9}$$

where

$$\tilde{A}(\alpha) = \Phi\overline{A}(\alpha)\Phi^T, \qquad \tilde{C}(\alpha) = \overline{C}(\alpha)\Phi^T, \qquad \tilde{B}(\alpha) = \Phi\overline{B}(\alpha), \qquad \tilde{D}(\alpha) = D(\alpha), \tag{2.10}$$

and the augmented matrices are given by

$$\overline{A}(\alpha) = \begin{bmatrix} A(\alpha) & 0\\ B_f(\alpha)C_1(\alpha) & A_f(\alpha)\end{bmatrix}, \tag{2.11}$$

$$\overline{B}(\alpha) = \begin{bmatrix} B(\alpha)\\ B_f(\alpha)D_1(\alpha)\end{bmatrix}, \tag{2.12}$$

$$\overline{C}(\alpha) = \left[C(\alpha) \;\; -C_f(\alpha)\right], \tag{2.13}$$

$$\Phi = \begin{bmatrix} I_{n_h} & 0 & 0 & 0\\ 0 & 0 & I_{n_h} & 0\\ 0 & I_{n_v} & 0 & 0\\ 0 & 0 & 0 & I_{n_v}\end{bmatrix}. \tag{2.14}$$

The problem addressed in this paper is then defined as follows.

Definition 2.2. The robust H_∞ filtering problem consists of finding a filter (2.7) such that the filtering error dynamics (2.9) is asymptotically stable and the transfer function of the error system, given as

$$T_{\tilde{z}w} = \tilde{C}(\alpha)\left[I(z_1,z_2) - \tilde{A}(\alpha)\right]^{-1}\tilde{B}(\alpha) + \tilde{D}(\alpha), \tag{2.15}$$

satisfies

$$\|T_{\tilde{z}w}\|_\infty < \gamma, \tag{2.16}$$

with $\gamma > 0$ a given real number.

3. Robust H_∞ Filter Design

3.1. Preliminaries: Constant P Matrix

In order to solve the filtering problem presented in the previous section we first introduce a lemma presented in [27] which considers a parameter-independent structure for $P(\alpha)$, that is, $P(\alpha) = P = P^T$. This still corresponds to a quadratic framework, but it will be the base behind the nonquadratic approach presented in the rest of the paper (alternative nonquadratic approaches can be found in [17, 18]).

Lemma 3.1. *Given a scalar $\gamma > 0$, the 2D discrete system* (2.9) *is asymptotically stable and satisfies the H_∞ performance $\|T_{\tilde{z}w}\| < \gamma$ if there exists a matrix $\tilde{P} = \operatorname{diag}(\tilde{P}_h, \tilde{P}_v) > 0$, with $\tilde{P}_h \in \Re^{n_h \times n_h}$ and $\tilde{P}_v \in \Re^{n_v \times n_v}$, such that the following LMI holds:*

$$\begin{pmatrix} -\tilde{P} & \tilde{P}\tilde{A}(\alpha) & \tilde{P}\tilde{B}(\alpha) & 0 \\ * & -\tilde{P} & 0 & \tilde{C}(\alpha)^T \\ * & * & -\gamma^2 I & \tilde{D}(\alpha)^T \\ * & * & * & -I \end{pmatrix} < 0. \tag{3.1}$$

Now, we are in a position to present a preliminary solvability condition for the robust H_∞ filtering problem.

Theorem 3.2. *Given a scalar $\gamma > 0$ and the uncertain 2D discrete system* (2.1)*, then, the robust H_∞ filtering problem is solvable if there exists matrices $Z(\alpha), \Theta(\alpha), \Psi(\alpha), X = \operatorname{diag}(X_h, X_v) > 0$, and $Y = \operatorname{diag}(Y_h, Y_v) > 0$ with X_h, $Y_h \in \mathbb{R}^{n_h \times n_h}$ and X_v, $Y_v \in \mathbb{R}^{n_v \times n_v}$ such that the following LMIs hold:*

$$\begin{bmatrix} -Y & -Y & YA(\alpha) & YA(\alpha) & YB(\alpha) & 0 \\ * & -X & XA(\alpha) + \Psi(\alpha)C_1(\alpha) + Z(\alpha) & XA(\alpha) + \Psi(\alpha)C_1(\alpha) & M_{24} & 0 \\ * & * & -Y & -Y & 0 & C(\alpha)^T - \Theta(\alpha)^T \\ * & * & * & -X & 0 & C(\alpha)^T \\ * & * & * & * & -\gamma^2 I & D(\alpha)^T \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{3.2}$$

$$X - Y > 0, \tag{3.3}$$

where $M_{24} = XB(\alpha) + \Psi(\alpha)D_1(\alpha)$.

In this case, a 2D discrete filter in the form of (2.7) is obtained when the parameters are selected as follows:

$$\begin{aligned} A_f(\alpha) &= X_{12}^{-1} Z(\alpha) Y^{-1} Y_{12}^{-T}, \\ B_f(\alpha) &= X_{12}^{-1} \Psi(\alpha), \\ C_f(\alpha) &= \Theta(\alpha) Y^{-1} Y_{12}^{-T}, \end{aligned} \tag{3.4}$$

where

$$X_{12} = \begin{bmatrix} X_{h12} & 0 \\ 0 & X_{v12} \end{bmatrix}, \qquad Y_{12} = \begin{bmatrix} Y_{h12} & 0 \\ 0 & Y_{v12} \end{bmatrix}, \tag{3.5}$$

in which X_{h12}, X_{v12}, Y_{h12}, and Y_{v12} are nonsingular matrices satisfying

$$X_{12} Y_{12}^T = I - X Y^{-1}. \tag{3.6}$$

Proof. Let $\overline{Y}_h = Y_h^{-1}$, $\overline{Y}_v = Y_v^{-1}$, $\overline{Y} = Y^{-1}$ then the relations (3.3), can be written as

$$\begin{pmatrix} X & I \\ I & \overline{Y} \end{pmatrix} > 0. \tag{3.7}$$

By the Schur complement formula, it follows from (3.7) that

$$\overline{Y} - X^{-1} > 0, \tag{3.8}$$

which implies $I - X\overline{Y}$ is nonsingular. Therefore, by noting the structure of X and Y, there always exist nonsingular matrices X_{h12}, X_{v12}, Y_{h12}, and Y_{v12} such that (3.6) is satisfied, that is,

$$\begin{aligned} X_{h12} Y_{h12}^T &= I - X_h \overline{Y}_h, \\ X_{v12} Y_{v12}^T &= I - X_v \overline{Y}_v. \end{aligned} \tag{3.9}$$

Set

$$\begin{aligned} \Pi_{h1} &= \begin{bmatrix} \overline{Y}_h & I \\ Y_{h12}^T & 0 \end{bmatrix}, \qquad \Pi_{v1} = \begin{bmatrix} \overline{Y}_v & I \\ Y_{v12}^T & 0 \end{bmatrix}, \\ \Pi_{h2} &= \begin{bmatrix} I & X_h \\ 0 & X_{h12}^T \end{bmatrix}, \qquad \Pi_{v2} = \begin{bmatrix} I & X_v \\ 0 & X_{v12}^T \end{bmatrix}, \\ \Pi_1 &= \begin{bmatrix} \Pi_{h1} & 0 \\ 0 & \Pi_{v1} \end{bmatrix}, \qquad \Pi_2 = \begin{bmatrix} \Pi_{h2} & 0 \\ 0 & \Pi_{v2} \end{bmatrix}. \end{aligned} \tag{3.10}$$

Then, by some calculations, it can be verified that

$$\widetilde{P} := \Pi_2 \Pi_1^{-1} = \begin{bmatrix} \widetilde{P}_h & 0 \\ 0 & \widetilde{P}_v \end{bmatrix}, \tag{3.11}$$

where

$$\begin{aligned} \widetilde{P}_h &= \begin{bmatrix} X_h & X_{h12} \\ X_{h12}^T & X_{h12}^T (X_h - Y_h)^{-1} X_{h12} \end{bmatrix}, \\ \widetilde{P}_v &= \begin{bmatrix} X_v & X_{v12} \\ X_{v12}^T & X_{v12}^T (X_v - Y_v)^{-1} X_{v12} \end{bmatrix}. \end{aligned} \tag{3.12}$$

Observe that

$$X_h - X_{h12}\left[X_{h12}^T(X_h - Y_h)^{-1}X_{h12}\right]^{-1}X_{h12}^T = Y_h > 0,$$
$$X_v - X_{v12}\left[X_{v12}^T(X_v - Y_v)^{-1}X_{v12}\right]^{-1}X_{v12}^T = Y_v > 0. \tag{3.13}$$

Therefore, it is easy to see that $\tilde{P}_h > 0$ and $\tilde{P}_v > 0$. Now, before and after multiplying (3.2) by $\mathrm{diag}\{\overline{Y}, I, \overline{Y}, I, I, I\}$, we obtain

$$\begin{bmatrix} -\overline{Y} & -I_n & A(\alpha)\overline{Y} & A(\alpha) & B(\alpha) & 0 \\ * & -X & M_{23} & XA(\alpha) + X_{12}B_f(\alpha)C_1(\alpha) & XB(\alpha) + X_{12}B_f(\alpha)D_1(\alpha) & 0 \\ * & * & -\overline{Y} & -I_n & 0 & \overline{Y}C(\alpha)^T - Y_{12}(\alpha)^T \\ * & * & * & -X & 0 & C(\alpha)^T \\ * & * & * & * & -\gamma^2 I & D(\alpha)^T \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{3.14}$$

where $M_{23} = XA(\alpha)\overline{Y} + X_{12}B_f(\alpha)C_1(\alpha)\overline{Y} + X_{12}A_f(\alpha)Y_{12}^T$.

Let $A_f(\alpha)$, $B_f(\alpha)$, and $C_f(\alpha)$ are given in (3.4), Φ is given in (2.14). By (3.11), the inequality (3.14) can be rewritten as

$$\begin{bmatrix} -\Phi^T\Pi_1^T\tilde{P}\Pi_1\Phi & \Phi^T\Pi_1^T\tilde{P}\Phi\overline{A}(\alpha)\Phi^T\Pi_1\Phi & \Phi^T\Pi_1^T\tilde{P}\Phi\overline{B}(\alpha) & 0 \\ \Phi^T\Pi_1^T\Phi\overline{A}(\alpha)^T\Phi^T\tilde{P}\Pi_1\Phi & -\Phi^T\Pi_1^T\tilde{P}\Pi_1\Phi & 0 & \Phi^T\Pi_1^T\Phi\overline{C}(\alpha)^T \\ \overline{B}(\alpha)^T\Phi^T\tilde{P}\Pi_1\Phi & 0 & -\gamma^2 I & \tilde{D}(\alpha)^T \\ 0 & \overline{C}(\alpha)\Phi^T\Pi_1\Phi & \tilde{D}(\alpha) & -I \end{bmatrix} < 0. \tag{3.15}$$

Pre- and postmultiplying (3.15) by $\mathrm{diag}(\Pi_1^{-T}\Phi^{-T}, \Pi_1^{-T}\Phi^{-T}, I, I)$ and $\mathrm{diag}(\Phi^{-1}\Pi_1^{-1}, \Phi^{-1}\Pi_1^{-1}, I, I)$ result in

$$\begin{pmatrix} -\tilde{P} & \tilde{P}\tilde{A}(\alpha) & \tilde{P}\tilde{B}(\alpha) & 0 \\ * & -\tilde{P} & 0 & \tilde{C}(\alpha)^T \\ * & * & -\gamma^2 I & \tilde{D}(\alpha)^T \\ * & * & * & -I \end{pmatrix} < 0. \tag{3.16}$$

Finally, by Lemma 3.1, it follows that the error system (2.9) is asymptotically stable, and the transfer function of the error system satisfies (2.16). This completes the proof. □

Remark 3.3. Theorem 3.2 provides a method for designing H_∞ for fixed α, which casts the nonlinear matrix inequality in Lemma 3.1 into a linear matrix inequality. It is noted that the condition in Theorem 3.2 is dependent on the parameter α, and therefore the decision variables X and Y cannot be used due to the infinite-dimensional nature of the parameter α. In what follows, based on Theorem 3.2, we propose a new method for designing robust H_∞ filters via a structured polynomially parameter-dependent approach.

3.2. Homogenous Polynomially Parameter-Dependent (HPPD) Matrices

Before presenting the main result, some definitions and preliminaries are needed to represent and to handle products and sums of homogeneous polynomials. First, define the HPPD matrices of arbitrary degree $\mathfrak{g}$ as follows:

$$\Psi_g(\alpha) = \sum_{j=1}^{J(\mathfrak{g})} \alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N}\Psi_{\mathfrak{K}_j(\mathfrak{g})},$$
$$\Theta_g(\alpha) = \sum_{j=1}^{J(\mathfrak{g})} \alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N}\Theta_{\mathfrak{K}_j(\mathfrak{g})}, \tag{3.17}$$
$$Z_g(\alpha) = \sum_{j=1}^{J(\mathfrak{g})} \alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N}Z_{\mathfrak{K}_j(\mathfrak{g})},$$

with

$$k_1k_2\cdots k_N = \mathfrak{K}_j(\mathfrak{g}). \tag{3.18}$$

The above notations are explained as follows: $\alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N}$, $\alpha \in \Omega$, $k_i \in \mathbb{N}$, $i = 1,\ldots,N$ are the monomials; $\Psi_{\mathfrak{K}_j(\mathfrak{g})}$, $\Theta_{\mathfrak{K}_j(\mathfrak{g})}$, and $Z_{\mathfrak{K}_j(\mathfrak{g})}$, are matrices with the corresponding coefficients; $\mathfrak{K}_j(\mathfrak{g})$ is the jth N-tuple of $\mathfrak{K}(\mathfrak{g})$, lexically ordered, with $j = 1,\ldots,\mathfrak{J}(\mathfrak{g})$; finally, $\mathfrak{K}(\mathfrak{g})$ is the set of N-tuples obtained as all possible combinations of $k_1k_2\cdots k_N$, with $k_i \in \mathbb{N}$, $i = 1,\ldots,N$, that fulfill $k_1 + k_2 + \cdots + k_N = \mathfrak{g}$. Since the number of vertices in the polytope $\not{D}$ is equal to N, the number of elements in $\mathfrak{K}(\mathfrak{g})$ is given by $\mathfrak{J}(\mathfrak{g}) = (N + \mathfrak{g} - 1)!/(\mathfrak{g}!(N-1)!)$.

Next, for each $i = 1,\ldots,N$ define the N-tuples $\mathfrak{K}_j^i(\mathfrak{g})$, that are equal to $\mathfrak{K}_j(\mathfrak{g})$, but with $k_i > 0$ replaced by $k_{\mathrm{i}} - 1$. Note that these $\mathfrak{K}_j^i(\mathfrak{g})$ are defined only when the corresponding k_i is positive. Note also that, when applied to the elements of $\mathfrak{K}(\mathfrak{g}+1)$, the N-tuples $\mathfrak{K}_j^i(\mathfrak{g}+1)$ define subscripts $k_1k_2\cdots k_N$ of matrices $\Psi_{k_1k_2\cdots k_N}$, $\Theta_{k_1k_2\cdots k_N}$, and $Z_{k_1k_2\cdots k_N}$ associated to homogeneous polynomial parameter-dependent matrices of degree $\mathfrak{g}$.

Finally, define the scalar constant coefficients $\beta_j^i(j+1) = \mathfrak{g}!/(k_1!k_2!\cdots k_N!)$, with $k_1k_2\cdots k_N \in \mathfrak{K}_j^i(\mathfrak{g}+1)$.

To clarify this notation, consider as an example a possible polytope with $N = 3$ vertices. The polynomials of degree $\mathfrak{g} = 2$ are obtained as follows: First, $J(2) = 6$, $\mathfrak{K}(2) = \{002, 011, 020, 101, 110, 200\}$, so the polynomials of degree 2 are

$$\Psi_2(\alpha) = \alpha_3^2\Psi_{002} + \alpha_2\alpha_3\Psi_{011} + \alpha_2^2\Psi_{020} + \alpha_1\alpha_3\Psi_{101} + \alpha_1\alpha_2\Psi_{110} + \alpha_1^2\Psi_{200},$$
$$\Theta_2(\alpha) = \alpha_3^2\Theta_{002} + \alpha_2\alpha_3\Theta_{011} + \alpha_2^2\Theta_{020} + \alpha_1\alpha_3\Theta_{101} + \alpha_1\alpha_2\Theta_{110} + \alpha_1^2\Theta_{200}, \tag{3.19}$$
$$Z_2(\alpha) = \alpha_3^2Z_{002} + \alpha_2\alpha_3Z_{011} + \alpha_2^2Z_{020} + \alpha_1\alpha_3Z_{101} + \alpha_1\alpha_2Z_{110} + \alpha_1^2Z_{200}.$$

Moreover, the 3 tuples are $\mathfrak{K}_1^3(2) = 001$, $\mathfrak{K}_2^2(2) = 001$, $\mathfrak{K}_2^3(2) = 010$, $\mathfrak{K}_3^2(2) = 010$, $\mathfrak{K}_4^1(2) = 001$, $\mathfrak{K}_4^3(2) = 100$, $\mathfrak{K}_5^1(2) = 010$, $\mathfrak{K}_5^2(2) = 100$, and $\mathfrak{K}_6^1(2) = 100$: these are the only possible triples (3 tuples) $\mathfrak{K}_j^i(2)$, $j = 1,\ldots,\mathfrak{J}(2)$ associated to $\mathfrak{K}(2)$.

3.3. Main Result

Using Lemma 3.1 and the homogeneous polynomials just presented, we can derive our main result.

Theorem 3.4. *Given a scalar $\gamma > 0$ and the uncertain 2D discrete system* (2.1)*, then, the robust H_∞ filtering problem given in Definition 2.2 is solvable if there exist matrices $Z_{\mathfrak{K}_j(\mathfrak{g})}$, $\Theta_{\mathfrak{K}_j(\mathfrak{g})}$, $\Psi_{\mathfrak{K}_j(\mathfrak{g})}$, $\mathfrak{K}_j(\mathfrak{g}) \in \mathfrak{K}(\mathfrak{g})$, $j = 1, \ldots, \mathfrak{J}(\mathfrak{g})$, $X = \mathrm{diag}(X_h, X_v) > 0$, and $Y = \mathrm{diag}(Y_h, Y_v) > 0$ with X_h, $Y_h \in \mathbb{R}^{n_h}$ and X_v, $Y_v \in \mathbb{R}^{n_v}$, such that for all $\mathfrak{K}_l(\mathfrak{g}+1) \in \mathfrak{K}(\mathfrak{g}+1)$, $l = 1, \ldots, \mathfrak{J}(\mathfrak{g}+1)$ such that the following LMIs hold:*

$$\sum_{i \in \mathfrak{N}_l(g+1)} \begin{bmatrix} -\beta_l^i(\mathfrak{g}+1)Y & -\beta_l^i(\mathfrak{g}+1)Y & \beta_l^i(\mathfrak{g}+1)YA_i & \beta_l^i(\mathfrak{g}+1)YA_i & \beta_l^i(\mathfrak{g}+1)YB_i & 0 \\ * & -\beta_l^i(\mathfrak{g}+1)X & J_{23} & J_{24} & J_{25} & 0 \\ * & * & -\beta_l^i(\mathfrak{g}+1)Y & -\beta_l^i(\mathfrak{g}+1)Y & 0 & J_{36} \\ * & * & * & -\beta_l^i(\mathfrak{g}+1)X & 0 & J_{46} \\ * & * & * & * & -\gamma^2 I & J_{56} \\ * & * & * & * & * & -I \end{bmatrix} < 0, \tag{3.20}$$

$$X - Y > 0, \tag{3.21}$$

where

$$\begin{aligned} J_{13} &= \beta_l^i(\mathfrak{g}+1)YA_i, \\ J_{15} &= \beta_l^i(\mathfrak{g}+1)YB_i, \\ J_{23} &= \beta_l^i(\mathfrak{g}+1)XA_i + \Psi_{\mathfrak{K}_l^i(\mathfrak{g}+1)}C_{1i} + Z_{\mathfrak{K}_l^i(\mathfrak{g}+1)}, \\ J_{24} &= XA(\alpha) + \Psi_{\mathfrak{K}_l^i(\mathfrak{g}+1)}C_{1i}, \\ J_{25} &= \beta_l^i(\mathfrak{g}+1)XB_i + \Psi_{\mathfrak{K}_l^i(\mathfrak{g}+1)}D_{1i}, \\ J_{36} &= \beta_l^i(\mathfrak{g}+1)C_i^T - \Theta^T_{\mathfrak{K}_l^i(\mathfrak{g}+1)}, \\ J_{46} &= \beta_l^i(\mathfrak{g}+1)C_i^T, \\ J_{56} &= \beta_l^i(\mathfrak{g}+1)D_i^T, \end{aligned} \tag{3.22}$$

then the homogeneous polynomially parameter-dependent matrices given by (3.17) *ensure* (3.2) *for all $\alpha \in \Omega$. Moreover, if the LMIs of* (3.20)-(3.21) *are fulfilled for a given degree $\mathfrak{g}$, then the LMIs corresponding to any degree $\mathfrak{g} > \widehat{\mathfrak{g}}$ are also satisfied.*

In this case, the matrices of the 2D discrete-time HPPD filter are given by

$$
\begin{aligned}
A_{f\mathfrak{g}}(\alpha) &= \sum_{j=1}^{\mathfrak{J}(\mathfrak{g})} \alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N} A_{f\mathfrak{K}_j(g)},\\
B_{f\mathfrak{g}}(\alpha) &= \sum_{j=1}^{\mathfrak{J}(\mathfrak{g})} \alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N} B_{f\mathfrak{K}_j(g)},\\
C_{f\mathfrak{g}}(\alpha) &= \sum_{j=1}^{\mathfrak{J}(\mathfrak{g})} \alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N} C_{f\mathfrak{K}_j(g)},
\end{aligned}
\tag{3.23}
$$

with

$$
\begin{aligned}
k_1k_2\cdots k_N &= \mathfrak{K}_j(g),\\
A_{f\mathfrak{K}_j(g)} &= X_{12}^{-1} Z_{\mathfrak{K}_j(g)} Y^{-1} Y_{12}^{-T},\\
B_{f\mathfrak{K}_j(g)} &= X_{12}^{-1} \Psi_{\mathfrak{K}_j(g)},\\
C_{f\mathfrak{K}_j(g)} &= \Theta_{\mathfrak{K}_j(g)} Y^{-1} Y_{12}^{-T}.
\end{aligned}
\tag{3.24}
$$

Proof. Note that (3.2) for $(A(\alpha), B(\alpha), C_1(\alpha), D_1(\alpha), C(\alpha), D(\alpha)) \in \mathcal{D}$, and $\Psi(\alpha), \Theta(\alpha), Z(\alpha)$ given by (3.17) are homogeneous polynomial matrices equations of degree $\mathfrak{g}+1$ that can be written as

$$
\begin{gathered}
\sum_{l=1}^{J(g+1)} \alpha^{\mathrm{k}} \left\{ \sum_{i\in\mathfrak{N}_l(g+1)} \begin{bmatrix}
-\beta_l^i(\mathfrak{g}+1)Y & -\beta_l^i(\mathfrak{g}+1)Y & J_{13} & J_{13} & J_{15} & 0\\
* & -\beta_l^i(\mathfrak{g}+1)X & J_{23} & J_{24} & J_{25} & 0\\
* & * & -\beta_l^i(\mathfrak{g}+1)Y & -\beta_l^i(\mathfrak{g}+1)Y & 0 & J_{36}\\
* & * & * & -\beta_l^i(\mathfrak{g}+1)X & 0 & J_{46}\\
* & * & * & * & -\gamma^2 I & J_{56}\\
* & * & * & * & * & -I
\end{bmatrix} \right\} < 0,\\
k_1k_2\cdots k_N = \mathfrak{K}_l(\mathfrak{g}+1),\\
\alpha^k = \alpha_1^{k_1}\alpha_2^{k_2}\cdots\alpha_N^{k_N}.
\end{gathered}
\tag{3.25}
$$

Condition (3.20), when imposed for all $l = 1,\ldots,\mathfrak{J}(\mathfrak{g}+1)$, ensure condition (3.2) for all $\alpha \in \Omega$, and thus the first part is proved.

Suppose that the LMIs of (3.20)-(3.21) are fulfilled for a certain degree $\widehat{\mathfrak{g}}$, that is, there exist $\mathfrak{J}(\widehat{\mathfrak{g}})$ matrices $\Psi_{\mathfrak{K}_j(\widehat{g})}$, $\Theta_{\mathfrak{K}_j(\widehat{g})}$, and $Z_{\mathfrak{K}_j(\widehat{g})}$, $j = 1,\ldots,\mathfrak{J}(\widehat{\mathfrak{g}})$, such that $\Psi_{\widehat{\mathfrak{g}}}(\alpha)$, $\Theta_{\widehat{\mathfrak{g}}}(\alpha)$, and $Z_{\widehat{\mathfrak{g}}}(\alpha)$ are homogeneous polynomially parameter-dependent matrices assuring condition in (3.2)-(3.3). Then, the terms of the polynomial matrices $\Psi_{\widehat{\mathfrak{g}}+1}(\alpha) = (\alpha_1 + \cdots + \alpha_N)\Psi_{\widehat{\mathfrak{g}}}(\alpha)$, $\Theta_{\widehat{\mathfrak{g}}+1}(\alpha) = (\alpha_1 + \cdots + \alpha_N)\Theta_{\widehat{\mathfrak{g}}}(\alpha)$ and $Z_{\widehat{\mathfrak{g}}+1}(\alpha) = (\alpha_1 + \cdots + \alpha_N)Z_{\widehat{\mathfrak{g}}}(\alpha)$ satisfy the LMIs of Theorem 3.4 corresponding to the degree $\widehat{\mathfrak{g}}+1$, which can be obtained in this case by a linear combination of the LMIs of Theorem 3.4 for $\widehat{\mathfrak{g}}$. □

Remark 3.5. Note that when $\mathfrak{g} = 0$, we have $\Psi_{\mathfrak{g}}(\alpha) = \Psi$, $\Theta_{\mathfrak{g}}(\alpha) = \Theta$, and $Z_{\mathfrak{g}}(\alpha) = Z$, then $A_f(\alpha) = A_f$, $B_f(\alpha) = B_f$, and $C_f(\alpha) = C_f$, which will lead to the standard filtering result in the quadratic framework. In addition, when $\mathfrak{g} = 1$, they are linearly dependent on the parameter α. This is why we say the polynomial α-dependence encompasses the linear α-dependence as a special case. It is also worth noting that since all coefficients $\alpha_i, i = 1, \ldots, N$, are such that $\alpha \in \Omega$. As the degree $\mathfrak{g}$ of the polynomial increases, the conditions become less conservative since new variables are added to LMIs. Although the number of LMIs is also increasing, each LMI becomes easier to be fulfilled due to the extra degrees of freedom provided by the new variables.

4. Illustrative Examples

In this section, some numerical examples are presented to illustrate the proposed technique for robust H_∞ HPPD filters.

Example 4.1. A stationary random field can be modeled as the following 2D system [13]:

$$\eta(i+1, j+1) = a_1\eta(i, j+1) + a_2\eta(i+1, j) - a_1a_2\eta(i, j) + w_1(i, j), \tag{4.1}$$

where $\eta(i, j)$ is the state of the random field of spacial coordinate (i, j), $w_1(i, j)$ is a noise input, $a_1^2 < 1$, and $a_2^2 < 1$ as a_1 and a_2 represent, respectively, the vertical and horizontal correlations of the random field. The output is then

$$y(i, j) = a_1\eta(i, j+1) + (1 - a_1a_2)\eta(i, j) + w_2(i, j), \tag{4.2}$$

where $w_2(i, j)$ is the measurement noise. The signal to be estimated is

$$z(i, j) = C\eta(i, j) + Dw(i, j). \tag{4.3}$$

As in [28], define $x^h(i, j) = \eta(i, j+1) - a_2\eta(i, j)$ and $x^v(i, j) = \eta(i, j)$. It is easy to see that (4.1)–(4.3) can be converted into a 2D Roesser model of the form (2.1) with $w(i, j) = [w_1(i, j) \quad w_2(i, j)]^T$ and the following system matrices:

$$\begin{gathered} A = \begin{bmatrix} a_1 & 0 \\ 1 & a_2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} a_1 & 0 \end{bmatrix}, \\ D_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \qquad D = \begin{bmatrix} 0 & 0 \end{bmatrix}. \end{gathered} \tag{4.4}$$

Suppose that the uncertain parameters a_1 and a_2 are bounded by $0.15 \leq a_1 \leq 0.45$ and $0.35 \leq a_2 \leq 0.85$, so the above system can be represented by a four-vertex polytopic system. The H_∞ filtering design approach presented in this paper was applied to this system. The results of a comparison with the techniques proposed in [29, 30] are shown in Table 1, which shows the smaller conservativeness of the approach proposed in this paper. For the filter designed when $\mathfrak{g} = 2$, the actual H_∞ norms calculated at the four vertices are shown in Table 2; the corresponding frequency responses of the error system are given in Figures 1, 2, 3, and 4 for each of the vertices, all of which are clearly below the guaranteed bound 1.5713.

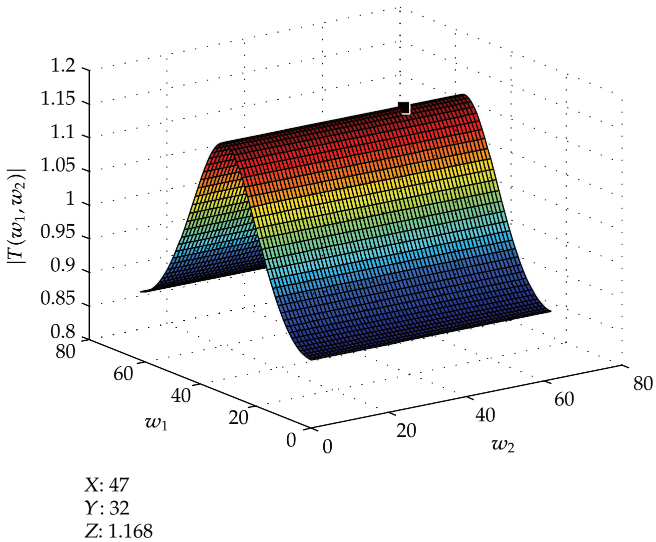


Figure 1: Frequency response of error system for vertex 1.

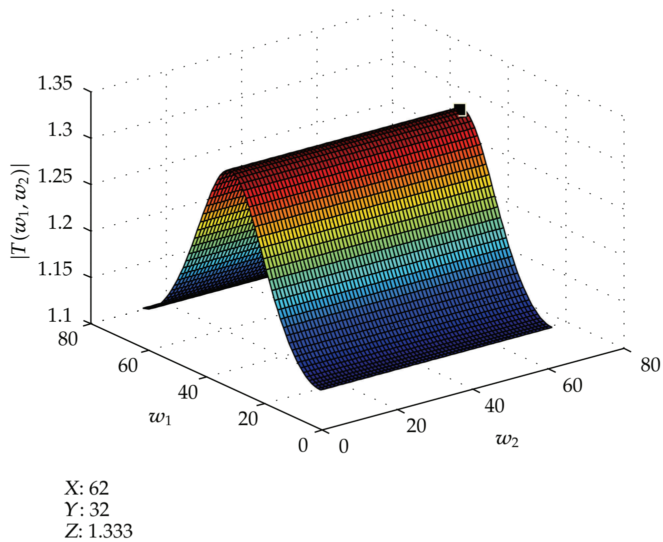


Figure 2: Frequency response of error system for vertex 2.

Example 4.2. In this example we show that less conservative designs are achieved as the degree of the polynomial grows, when applying the HPPD approach. For this, consider the same system of Example 4.1 but this time with $C_1 = [-1 \quad 1.6 - a_1], 0.15 \le a_1 \le 0.65$. The results are given in Table 3, which also provides a comparison with previous results in the literature.

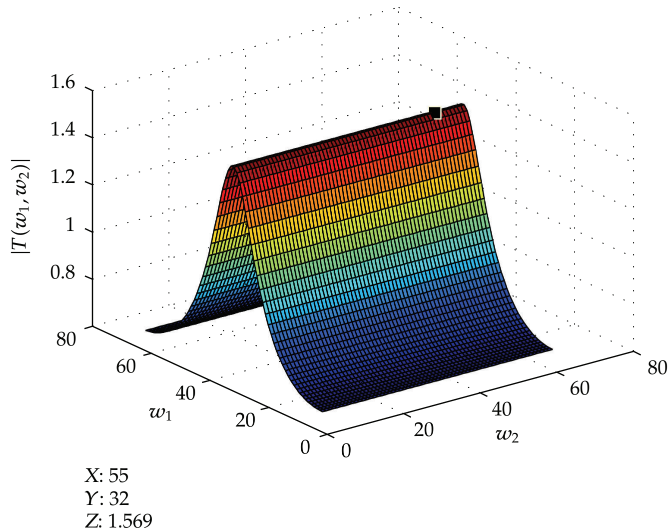


Figure 3: Frequency response of error system for vertex 3.

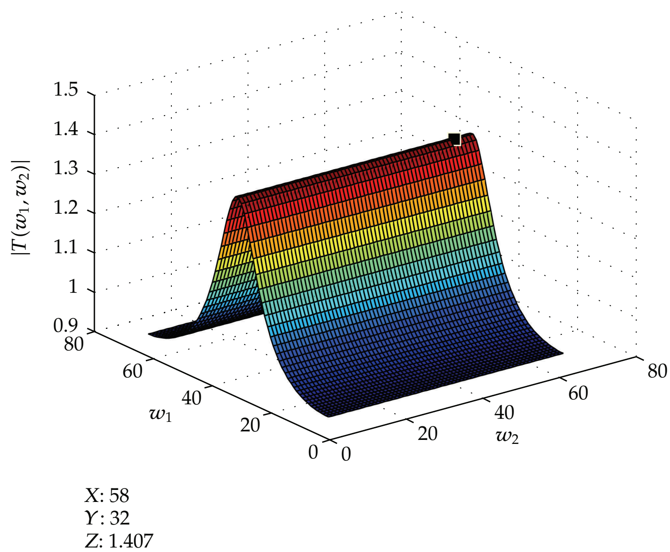


Figure 4: Frequency response of error system for vertex 4.

Table 1: Example 4.1: comparison with previous published results.

g	γ in this paper	γ in [29]	γ in [30]
0	2.4342	2.4373	3.8709
1	1.5713	1.8627	2.5450
2	1.5713	1.8290	2.5028

Table 2: H_∞ norms at the vertices.

a_1	0.15	0.15	0.45	0.45
a_2	0.35	0.85	0.35	0.85
$\|T_{\tilde{z}w}\|_\infty$	1.1682	1.3326	1.5694	1.4070

Table 3: Example 4.2: comparison with previous published results.

g	γ in this paper	γ in [29]	γ in [30]
0	7.5981	7.5981	11.4157
1	5.5216	5.8886	8.2321
2	5.4884	5.7694	8.1109
3	5.4870	5.7636	8.1050
4	5.4870	5.7636	8.1050

5. Conclusions

This paper has studied the robust H_∞ filtering problem for 2D discrete systems described by Roesser state-space model. The proposed method, based on using homogeneous polynomially parameter-dependent matrices of arbitrary degree, is less conservative than previous ones in the literature. Moreover, by increasing the degree of the polynomials involved, the obtained filter gets less conservative, which has been demonstrated by two illustrative examples, which provides comparisons with previous results.

Acknowlegments

This work is funded by AECI (A/030426/10) and MiCInn (DPI2010-21589-c05).

References

[1] T. Kaczorek, *Two-Dimensional Linear Systems*, vol. 68, Springer, Berlin, Germany, 1985.
[2] W. S. Lu and A. Antonio, *Two-Dimensional Digital Filters*, Marcel Deckker, New York, NY, USA, 1992.
[3] T. Hinamoto, “2-D Lyapunov equation and filter design based on Fornasini-Marchesini second model,” *IEEE Transactions on Circuits and Systems I*, vol. 40, no. 2, pp. 102–110, 1993.
[4] T. Hinamoto, “Stability of 2-D discrete systems described by the Fornasini-Marchesini second model,” *IEEE Transactions on Circuits and Systems I*, vol. 44, no. 3, pp. 254–257, 1997.
[5] A. Hmamed, M. Alfidi, A. Benzaouia, and F. Tadeo, “LMI Conditions for robust stability of 2-D linear discrete-time systems,” *Mathematical Problems in Engineering*, vol. 2008, Article ID 356124, 11 pages, 2008.
[6] A. Hmamed, M. Alfidi, A. Benzaouia, and F. Tadeo, “Robust stabilization under linear fractional parametric uncertainties of two-dimensional systems with Roesser models,” *International Journal of Sciences and Techniques of Automatic Control and Computer Engineering*, vol. 1, no. 3, pp. 336–348, 2007.
[7] A. Hmamed, F. Mesquine, F. Tadeo, M. Benhayoun, and A. Benzaouia, “Stabilization of 2D saturated systems by state feedback control,” *Multidimensional Systems and Signal Processing*, vol. 21, no. 3, pp. 277–292, 2010.
[8] N. E. Mastorakis and M. Swamy, “A new method for computing the stability margin of two-dimensional continuous systems,” *IEEE Transactions on Circuits and Systems I*, vol. 49, no. 6, pp. 869–872, 2002.
[9] L. Xu, O. Saito, and K. Abe, “Output feedback stabilizability and stabilization algorithms for 2-D systems,” *Multidimensional Systems and Signal Processing*, vol. 5, no. 1, pp. 41–60, 1994.

[10] C. Du, L. Xie, and Y. C. Soh, "H_∞ Reduced-order approximation of 2-D digital filters," *IEEE Transactions on Circuits and Systems I*, vol. 48, no. 6, pp. 688–698, 2001.

[11] B. D. O. Anderson and J. B. Moore, *Optimal Filtering*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1979.

[12] A. Habibi, "Two-dimensional bayesian estimate of image," *Proceedings of the IEEE*, vol. 60, pp. 878–883, 1972.

[13] T. Katayama and M. Kosaka, "Recursive filtering algorithm for a 2-D dystem," *IEEE Transactions on Automatic Control*, vol. 24, no. 1, pp. 130–132, 1979.

[14] W. A. Porter and J. L. Aravena, "State estimation in discrete m-D systems," *IEEE Transactions on Automatic Control*, vol. 31, no. 3, pp. 280–283, 1986.

[15] J. W. Woods and C. H. Radewan, "Kalman filtering in two dimensions," *IEEE Transactions on Information Theory*, vol. 23, no. 4, pp. 473–482, 1977.

[16] J. W. Woods and V. K. Ingle, "Kalman filtering in two dimensions: further results," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 29, no. 2, pp. 188–197, 1981.

[17] X. Li, Z. Li, and H. Gao, "Further results on H_∞ filtering for discrete-time systems with state delay," *International Journal of Robust and Nonlinear Control*, vol. 21, no. 3, pp. 248–270, 2011.

[18] X. Li and H. Gao, "A delay-dependent approach to robust generalized H-2 filtering for uncertain continuous-time systems with interval delay," *Signal Processing*, vol. 91, no. 10, pp. 2371–2378, 2011.

[19] U. Shaked and Y. Theodor, "H_∞-optimal estimation: a tutorial," in *Proceedings of the 31st IEEE Conference on Decision and Control*, pp. 2278–2286, Tucson, Tex, USA, December 1992.

[20] L. Xie, C. E. de Souza, and M. Fu, "H_∞ estimation for linear discrete-time uncertain systems," *International Journal of Robust and Nonlinear Control*, vol. 2, pp. 111–123, 1992.

[21] I. Yaesh and U. Shaked, "H_∞-optimal estimation: the discrete time case," in *Proceedings of the International Symposium of Recent Advances in Mathematical Theory of Networks and Systems (MTNS '91)*, pp. 261–268, Mita Press, 1991.

[22] C. Du, L. Xie, and Y. C. Soh, "H_∞ filtering for 2-D discrete systems," *IEEE Transactions on Signal Processing*, vol. 48, no. 6, pp. 1760–1768, 2000.

[23] C. Du, L. Xie, and C. Zhang, "Solutions for H_∞ filtering of two-dimensional systems," *Multidimensional Systems and Signal Processing*, vol. 11, no. 4, pp. 301–320, 2000.

[24] C. E. de Souza and L. Xie, "On the discrete-time bounded real lemma with application in the characterization of static state feedback H_∞ controllers," *Systems & Control Letters*, vol. 18, no. 1, pp. 61–71, 1992.

[25] C. Xiao, D. J. Hill, and P. Agathoklis, "Stability and the Lyapunov equation for n-dimensional digital systems," *IEEE Transactions on Circuits and Systems I*, vol. 44, no. 7, pp. 614–621, 1997.

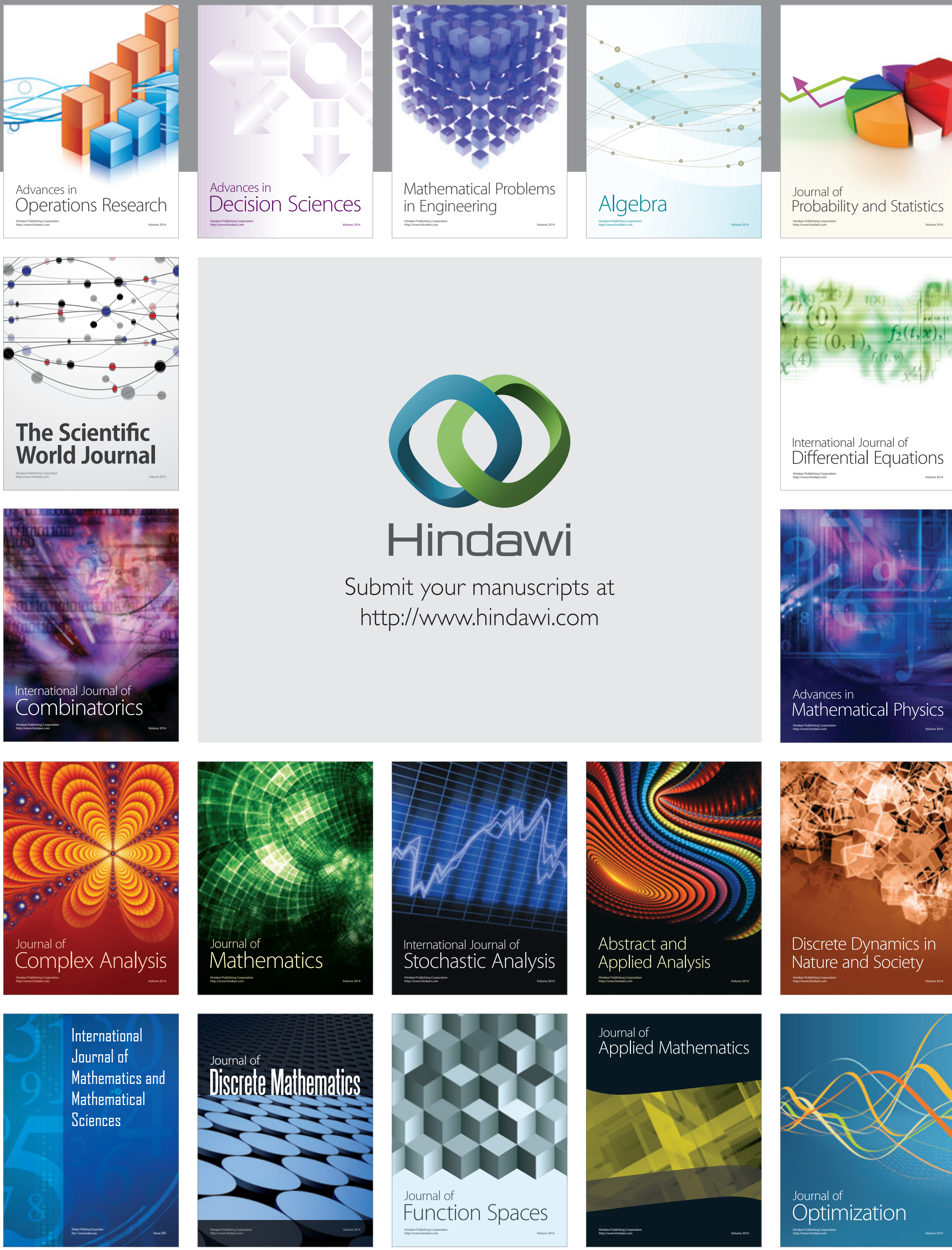
[26] S. Xu, J. Lam, Y. Zou, Z. Lin, and W. Paszke, "Robust H_∞ filtering for uncertain 2-D continuous systems," *IEEE Transactions on Signal Processing*, vol. 53, pp. 1731–1738, 2005.

[27] C. Du, L. Xie, and C. Zhang, "H_∞ control and robust stabilization of two-dimensional systems in Roesser models," *Automatica*, vol. 37, no. 2, pp. 205–211, 2001.

[28] C. Du and L. Xie, *H_∞ Control and Filtering of Two-Dimensional Systems*, vol. 278, Springer, Berlin, Germany, 2002.

[29] C. Y. Gao, G. R. Duan, and X. Y. Meng, "Robust H_∞ filter design for 2-D discrete systems in Roesser model," *International Journal of Automation and Computing*, vol. 5, no. 2, pp. 413–418, 2008.

[30] H. Gao, X. Meng, and T. Chen, "New design of robust H_∞ filters for 2-D systems," *IEEE Signal Processing Letters*, vol. 15, pp. 217–220, 2008.



Advances in
Operations Research
Advances in
Decision Sciences
Mathematical Problems
in Engineering
Algebra
Journal of
Probability and Statistics
The Scientific
World Journal
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of
Differential Equations
International Journal of
Combinatorics
Advances in
Mathematical Physics
Journal of
Complex Analysis
Journal of
Mathematics
International Journal of
Stochastic Analysis
Abstract and
Applied Analysis
Discrete Dynamics in
Nature and Society
International
Journal of
Mathematics and
Mathematical
Sciences
Journal of
Discrete Mathematics
Journal of
Function Spaces
Journal of
Applied Mathematics
Journal of
Optimization